• Find the sequence, $f(k)$, with z-Transform $F(z)$,

$$F(z) = \frac{z-10}{(z+2)(z+5)}$$

Solution:

Decomposition of $\bar{F}(z) = \frac{F(z)}{z}$.

hp: $F(z)$ has n distinct poles, p_i, $i=1,\dots n$, with multiplicity $\mu_i=1$,

- if $F(z)$ has a zero in $z=0$ (origin)

  the decomposition $\bar{F}(z) = \frac{R_1}{z-p_1} + \frac{R_2}{z-p_2} + \dots \frac{R_n}{z-p_n} = \sum_{i=1}^{n} \frac{R_i}{z-p_i}$

- if $F(z)$ has not a zero in $z=0$,

  the decomposition $\bar{F}(z) = \frac{R_0}{z} + \frac{R_1}{z-p_1} + \dots \frac{R_n}{z-p_n} = \sum_{i=0}^{n} \frac{R_i}{z-p_i}$

Then, in this case,

$$\bar{F}(z) = \frac{F(z)}{z} = \frac{(z-10)}{z(z+2)(z+5)} = \frac{R_0}{z} + \frac{R_1}{z+2} + \frac{R_2}{z+5};$$

Compute R_i

$$R_0 = \lim_{z\to 0} z\,\bar{F}(z) = \left.\frac{(z-10)}{(z+2)(z+5)}\right|_{z=0} = -1$$

$$R_1 = \lim_{z\to -2} (z+2)\,\bar{F}(z) = \left.\frac{(z-10)}{z(z+5)}\right|_{z=-2} = 2$$

$$R_2 = \lim_{z\to -5} (z+5)\,\bar{F}(z) = \left.\frac{(z-10)}{z(z+2)}\right|_{z=-5} = -1$$

$$\Rightarrow F(z) = z\,\bar{F}(z) = R_0 + R_1 \frac{z}{z+2} + R_2 \frac{z}{z+5}$$

$$= -1 + 2\cdot\frac{z}{z+2} - \frac{z}{z+5}$$

$$f(k) = Z^{-1}\left(F(z)\right) = -\delta(k) + \left[2\cdot(-2)^k - (-5)^k\right] 1(k)$$