



Course of "Industrial Automation"

z-Transform

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z-Transform definition

- ✧ Given a sequence of discrete values, $f(0), \dots, f(k), \dots$, the z-transform is defined by

$$\mathcal{Z}[f(k)] = F(z) = \sum_{k=0}^{+\infty} z^{-k} f(k)$$

where k is an integer variable and z a complex variable

- ✧ the series needs to converge for some z , i.e. $r < |z| < R_0$, i.e. r and R_0 bounds on the magnitude of z , for which the series converges
- ✧ The zeta-Transform is a bilateral only if the function $f(k)$ is null for $k \leq \underline{0}$



z-Transform of common signals

✧ Unit pulse(Kronecker delta)

$$f(k) = \delta(k) = \begin{cases} 1 & \text{se } k = 0 \\ 0 & \text{se } k \neq 0 \end{cases}$$

✧ z-Transform of the unit pulse

$$\mathcal{Z}[\delta(k)] = 1$$

✧ The unit step

$$f(k) = 1(k) = \begin{cases} 1 & \text{se } k \geq 0 \\ 0 & \text{se } k < 0 \end{cases}$$

✧ z-Transform of the unit step

$$\mathcal{Z}[1(k)] = \frac{z}{z-1}$$



z-Transform main properties (1/3)

✧ *Linearity*

$$\mathcal{Z}[\alpha f(k) + \beta g(k)] = \alpha F(z) + \beta G(z)$$

✧ *Time advance (of one unit)*

$$\mathcal{Z}[f(k+1)1(k)] = zF(z) - zf(0)$$

✧ *Time delay (of one unit)*

$$\mathcal{Z}[f(k-1)1(k-1)] = \frac{1}{z}F(z)$$



z-Transform main properties (2/3)

✧ *Convolution of time sequences*

$$\mathcal{Z}[f(k) * g(k)] = F(z)G(z)$$

✧ *Differentiation in the z -domain*

$$\mathcal{Z}[kf(k)] = -z \frac{d}{dz} F(z)$$



z-Transform main properties (3/3)

✧ *Initial value theorem*

$$f(0) = \lim_{z \rightarrow \infty} F(z)$$

✧ *Final value theorem*

if $\lim_{k \rightarrow \infty} f(k)$ is finite, then

$$\lim_{k \rightarrow \infty} f(k) = \lim_{z \rightarrow 1} \left(\frac{z-1}{z} F(z) \right)$$



Selected set of signals

✦ *Ramp signal $k1(k)$*

✦ *Polynomial inputs*

✦ *Power functions*

✦ *Sinusoidal inputs*



Selected z-Transforms

✧ *Ramp*

$$\mathcal{Z}[k1(k)] = \frac{z}{(z-1)^2}$$

✧ *Power*

$$\mathcal{Z}[a^k 1(k)] = \frac{z}{z-a}$$

$$\mathcal{Z}[ka^k 1(k)] = \frac{az}{(z-a)^2}$$



Selected z-Transforms

✧ *Sinusoidal inputs*

✧ *sine*

$$\mathcal{Z}[\sin(k\theta)1(k)] = \frac{z \sin \theta}{z^2 - 2z \cos \theta + 1}$$

✧ *cosine*

$$\mathcal{Z}[\cos(k\theta)1(k)] = \frac{z(z - \cos \theta)}{z^2 - 2z \cos \theta + 1}$$