Course of "Industrial Automation"

Discrete-time LTI systems – part B

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Forced response

- The forced evolution of a discrete-time LTI system, ($k_0 = 0$)

$$y(k) = C\sum_{h=0}^{k-1} A^{k-h-1}Bu(h) + D\,u(k), \quad k \geq 0 \tag{1}$$

- Hard problem to solve:
  - Calculus of A^k
  - Compute a solution in a closed form due to the discrete convolution .
- Easy calculus for standard canonical signals, as unit pulse and step signal
- ***Otherwise we use z-Transform***

Response to unit pulse

- In the case of pulse signal, $\boldsymbol{u(k) = \delta(k)}$

- It is possible to compute the impulse response, $w_y(k)$, as

$$w_y(k) = C\sum_{h=0}^{k-1} A^{k-h-1}B\delta(h) + D\,\delta(k) = CA^{k-1}B + D\delta(k) \qquad (2)$$

- By exploiting (2), it is possible to rewrite (1), that is

$$y(k) = C\sum_{h=0}^{k-1} A^{k-h-1}Bu(h) + D\,u(k), \quad k \geq 0$$

- as

$$y(k) = \sum_{h=0}^{k} w_y(k-h)u(h) \qquad (3)$$

Impulse response fully characterizes the system

- Therefore, by (3), we can get the response to a general signal
- ***The impulse response is unique***.
- Let us assume to have two equivalent state space representations of a given system. The relationship between these two representation is given by

$$\bar{A} = T^{-1}AT, \qquad \bar{B} = T^{-1}B, \qquad \bar{C} = CT, \qquad \bar{D} = D$$

- For both cases, the impulse response is given by

$$w_y(k) = CA^{k-1}B + D\delta(k)$$

$$\bar{w}_y(k) = \bar{C}\bar{A}^{k-1}\bar{B} + \bar{D}\delta(k)$$

$$= CT(T^{-1}AT)^{k-1}T^{-1}B + D\delta(k)$$

$$= CTT^{-1}A^{k-1}TT^{-1}B + D\delta(k) = w_y(k)$$

Step response

- Let us assume an input step signal $\boldsymbol{u(k) = \bar{u} \cdot 1(k)}$

- In this case, it is possible to compute the response as it follows

$$y(k) = C\sum_{h=0}^{k-1} A^{k-h-1}B\,\bar{u} + D\bar{u} = C(A^{k-1} + A^{k-2} + \cdots I)B\bar{u} + D\bar{u}$$

$$= C\left(A^k - I\right)(A - I)^{-1}B\bar{u} + D\bar{u} = CA^k(A - I)^{-1}B\bar{u} + [C(I - A)^{-1}B + D]\bar{u}$$

If all the modes are convergent, then

$$\bar{y} = \lim_{k\to\infty} y(k) = [C(I - A)^{-1}B + D]\,\bar{u}$$

- The term

$$C(I - A)^{-1}B + D$$

  is the ***static gain of the system.***