Per un sistema del II ordine con due poli e uno zero reali la cui fdt risulta

$$G(s) = \frac{b\,(s-z)}{(s-p_1)(s-p_2)}, \qquad p_1 \text{ e } p_2 \in \mathbb{R}^-,$$

calcolare la risposta al segnale gradino unitario $u(t) = 1(t)$.

$$Y(s) = G(s)U(s) = \frac{b(s-z)}{(s-p_1)(s-p_2)} \cdot \frac{1}{s} =$$

$$= \frac{A}{s-p_1} + \frac{B}{s-p_2} + \frac{C}{s}$$

$$A = \lim_{s\to p_1} (s-p_1)Y(s) = \lim_{s\to p_1} \frac{\cancel{(s-p_1)}\, b(s-z)}{\cancel{(s-p_1)}(s-p_2)} \cdot \frac{1}{s} = \frac{p_1 - z}{p_1 - p_2} \cdot \frac{b}{p_1}$$

$$B = \lim_{s\to p_2} (s-p_2)Y(s) = \lim_{s\to p_2} \frac{\cancel{(s-p_2)}\, b(s-z)}{\cancel{(s-p_2)}(s-p_1)} \cdot \frac{1}{s} = \frac{p_2 - z}{p_2 - p_1} \cdot \frac{b}{p_2}$$

$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} \cancel{s}\frac{G(s)}{\cancel{s}} = G(0) = G_0 = -\frac{bz}{p_1p_2}$$

Riscrivendo in termini di guadagno statico, G_0, e di costanti di tempo, τ_1, τ_2 e τ:

$$G_0 = -\frac{bz}{p_1p_2} \qquad \tau_1 = -\frac{1}{p_1}, \quad \tau_2 = -\frac{1}{p_2} \quad e\ \tau = -\frac{1}{z}$$

$$\Rightarrow A = \frac{p_1 - z}{p_1 - p_2} \cdot \underbrace{\frac{b}{p_1}}_{= -\frac{G_0 p_2}{z}} = \frac{-\frac{1}{\tau_1} + \frac{1}{\tau}}{-\frac{1}{\tau_1} + \frac{1}{\tau_2}} \cdot \left(-\frac{G_0 p_2}{z}\right) = \frac{\frac{-\tau + \tau_1}{\cancel{\tau_1}\tau}}{\frac{-\tau_2 + \tau_1}{\cancel{\tau_1}\cancel{\tau_2}}} \cdot \left(\frac{G_0 \cancel{\tau}}{\cancel{\tau_2}}\right)$$

$$= -G_0 \cdot \frac{\tau_1 - \tau}{\tau_1 - \tau_2}$$

$$B = \frac{p_2 - z}{p_2 - p_1} \cdot \frac{b}{p_2} = \frac{-\frac{1}{\tau_2} + \frac{1}{\tau}}{-\frac{1}{\tau_2} + \frac{1}{\tau_1}} \cdot \left(-\frac{G_0 p_1}{z}\right) = \frac{\frac{-\tau + \tau_2}{\cancel{\tau_2}\cancel{\tau}}}{\frac{-\tau_1 + \tau_2}{\cancel{\tau_2}\cancel{\tau_1}}} \cdot \left(-\frac{G_0 \cancel{\tau}}{\cancel{\tau_1}}\right) =$$

$$= -G_0 \cdot \frac{-\tau + \tau_2}{-\tau_1 + \tau_2} = G_0 \cdot \frac{\tau_2 - \tau}{\tau_1 - \tau_2}$$

$$Y(s) = \frac{A}{s-p_1} + \frac{B}{s-p_2} + \frac{C}{s} = \frac{-G_0 \frac{\tau_1 - \tau}{\tau_1 - \tau_2}}{s - p_1} + \frac{G_0 \frac{\tau_2 - \tau}{\tau_1 - \tau_2}}{s - p_2} + \frac{G_0}{s}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$y(t) = G_0 \left(-\frac{\tau_1 - \tau}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{\tau_2 - \tau}{\tau_1 - \tau_2} e^{-t/\tau_2} + 1\right) 1(t)$$

Se $u(t) = U_0\, 1(t)$,

$$y(t) = G_0 U_0 \left(-\frac{\tau_1 - \tau}{\tau_1 - \tau_2} e^{-t/\tau_2} + \frac{\tau_2 - \tau}{\tau_1 - \tau_2} e^{-t/\tau_2} + 1\right) 1(t)$$