Per il sistema del II ordine con fdt $G(s)$,

$$G(s) = \frac{b}{s^2 + a_1 s + a_0},$$

nelle hp di poli reali negativi,

$$G(s) = \frac{b}{s^2 + a_1 s + a_0} = \frac{b}{(s-p_1)(s-p_2)},$$

dove $p_1, p_2 \in \mathbb{R}^-$,

calcolare la risposta al gradino unitario $u(t) = 1(t)$.

- $$Y(s) = G(s)\,U(s) = \frac{b}{(s-p_1)(s-p_2)} \cdot \frac{1}{s} = \frac{K_1}{(s-p_1)} + \frac{K_2}{(s-p_2)} + \frac{K_3}{s}.$$

$$K_1 = \lim_{s \to p_1} (s-p_1)\,Y(s) = \lim_{s \to p_1} \cancel{(s-p_1)} \frac{b}{\cancel{(s-p_1)}(s-p_2)} \cdot \frac{1}{s} = \frac{b}{p_1-p_2} \cdot \frac{1}{p_1}$$

Nota che $G_0 = \frac{b}{a_0} = \frac{b}{p_1 p_2}$, quindi $\frac{b}{p_1} = G_0 \cdot p_2$

$$\text{e } K_1 = \frac{b}{p_1-p_2} \cdot \frac{1}{p_1} = G_0 \cdot \frac{p_2}{p_1-p_2}$$

$$K_2 = \lim_{s \to p_2} (s-p_2)\,Y(s) = \lim_{s \to p_2} \cancel{(s-p_2)} \frac{b}{(s-p_1)\cancel{(s-p_2)}\, s} = \frac{b}{p_2-p_1} \cdot \frac{1}{p_2}$$

$$= \frac{G_0\, p_1}{p_2-p_1} = -\frac{G_0\, p_1}{p_1-p_2}$$

$$K_3 = \lim_{s \to 0} s\,Y(s) = \lim_{s \to 0} \cancel{s} \frac{G(s)}{\cancel{s}} = G(0) = G_0 = \frac{b}{a_0} = \frac{b}{p_1 p_2}$$

Quindi

$$Y(s) = \frac{G_0 \frac{p_2}{p_1-p_2}}{s-p_1} - \frac{G_0 \frac{p_1}{p_1-p_2}}{s-p_2} + \frac{G_0}{s}$$

$\mathcal{L}^{-1}$

$$y(t) = G_0 \left[\frac{p_2}{p_1-p_2} e^{p_1 t} - \frac{p_1}{p_1-p_2} e^{p_2 t} + 1 \right] 1(t)$$

- Nel caso $u(t) = U_0 \cdot 1(t)$, allora

$$y(t) = G_0 U_0 \left[\frac{p_2}{p_1-p_2} e^{p_1 t} - \frac{p_1}{p_1-p_2} e^{p_2 t} + 1 \right] 1(t)$$

- Nel caso si vuole riscrivere l'uscita in termini di costanti di tempo ($T_1 = -1/p_1$, $T_2 = -1/p_2$)

$$G(s) = \frac{b}{(s-p_1)(s-p_2)} = \frac{b}{(-p_1)\left(\frac{s}{-p_1}+1\right)(-p_2)\left(\frac{s}{-p_2}+1\right)} =$$

$$= \frac{b}{p_1 p_2\,(sT_1+1)(sT_2+1)} =$$

$$= \frac{G_0}{(sT_1+1)(sT_2+1)}$$

$$K_1 = G_0 \frac{p_2}{p_1-p_2} = G_0 \frac{-\frac{1}{T_2}}{-\frac{1}{T_1}+\frac{1}{T_2}} = G_0 \frac{-\frac{1}{\cancel{T_2}}}{\frac{-T_2+T_1}{T_1 \cancel{T_2}}} = -\frac{G_0 T_1}{T_1-T_2}$$

$$K_2 = G_0 \frac{p_1}{p_2-p_1} = -G_0 \cdot \frac{p_1}{p_1-p_2} = -G_0 \frac{-1/T_1}{-1/T_1+\frac{1}{T_2}} = -G_0 \frac{-1/\cancel{T_1}}{\frac{-T_2+T_1}{\cancel{T_1} T_2}} = +\frac{G_0 T_2}{T_1-T_2}$$

Quindi,

$$y(t) = G_0 U_0 \left(-\frac{T_1}{T_1-T_2} e^{-t/T_1} + \frac{T_2}{T_1-T_2} e^{-t/T_2} + 1 \right) 1(t)$$