UNIVERSITÀ DEGLI STUDI DI NAPOLI
PARTHENOPE

Artificial Intelligence

# Adversarial Search

LESSON 7

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# Adversarial Search

- The algorithms discussed so far need to find an answer to a question
- In adversarial search, the algorithm faces an opponent that tries to achieve the opposite goal
- Often, adversarial search is encountered in games

# Types of Games

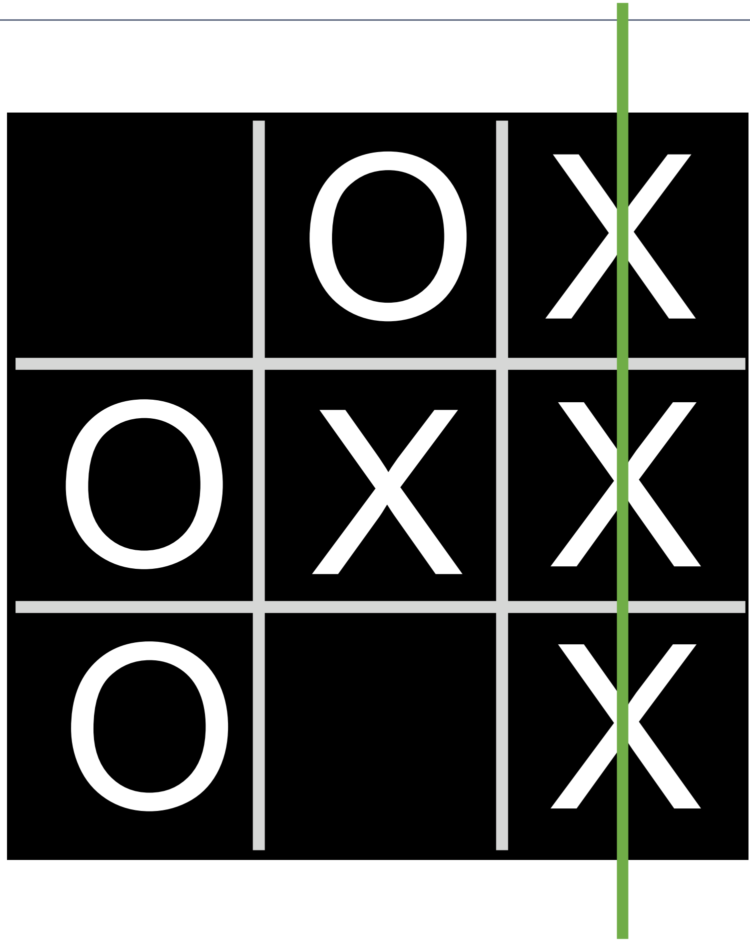
| | deterministic | chance |
|---|---|---|
| perfect information | chess, checkers, go, othello | Backgammon, monopoly |
| imperfect information | battleships, blind tictactoe | bridge, poker, scrabble |

# Perfect Information Zero-Sum Games

- The games most studied within AI (such as chess and Go) are
  - deterministic, two-player turn-taking, perfect information, zero-sum games
- Perfect Perfect information
  - Synonym for fully observable
- Zero-sum
  - means that what is good for one player is just as bad for the other
    - there is no "win-win" outcome
- Terminology
  - Move -> action
  - Position -> state

# Tic-Tac-Toe

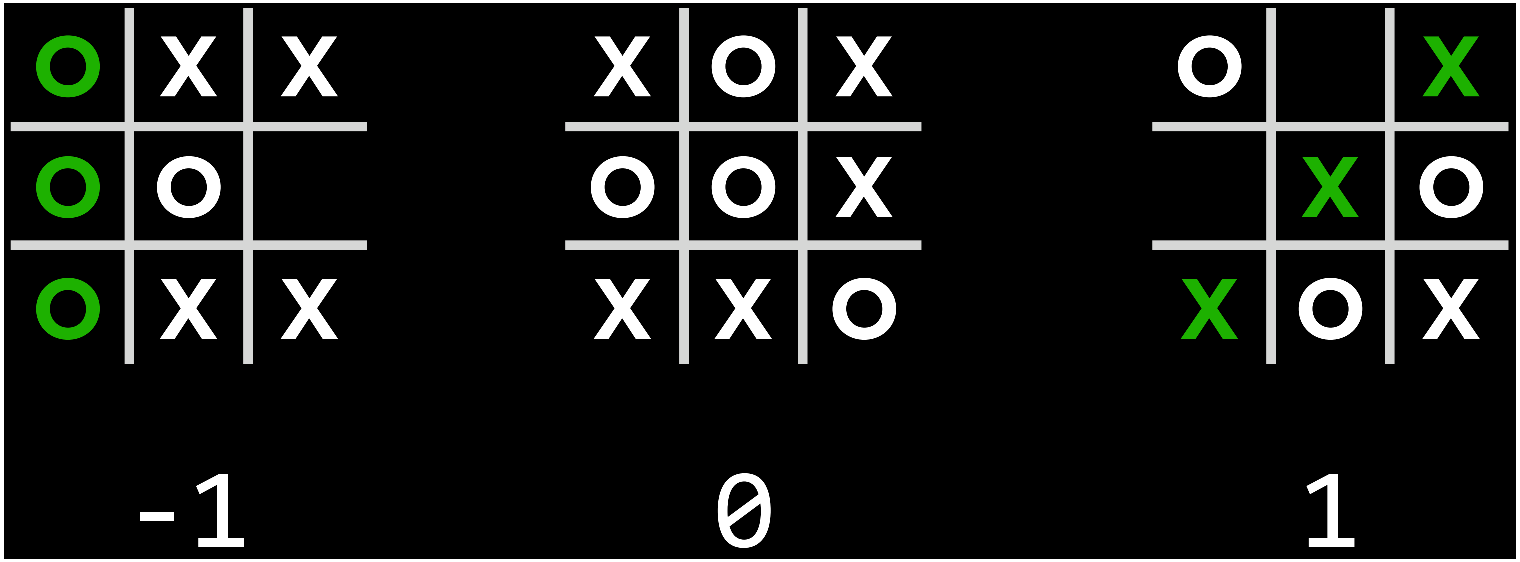
- Two players
  - O
  - X



# Minimax

- A type of algorithm in adversarial search
- Minimax represents winning conditions as (-1) for one side and (+1) for the other side
- Further actions will be driven by these conditions
  - The minimizing side tries to get the lowest score
  - The maximizing side tries to get the highest score

# Minimax for Tic-Tac-Toe

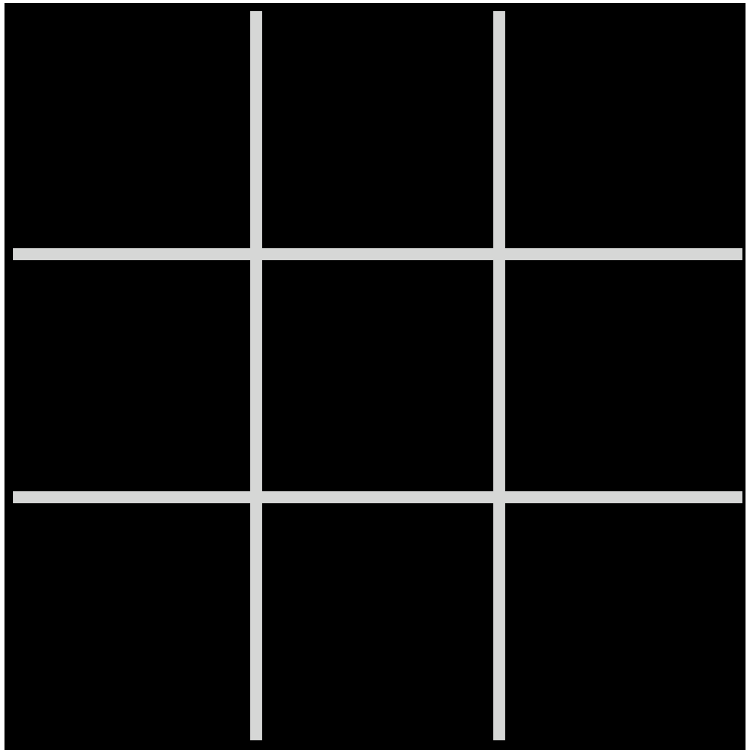



- Max(X) aims to maximize the score
- Min(O) aims to minimize the score

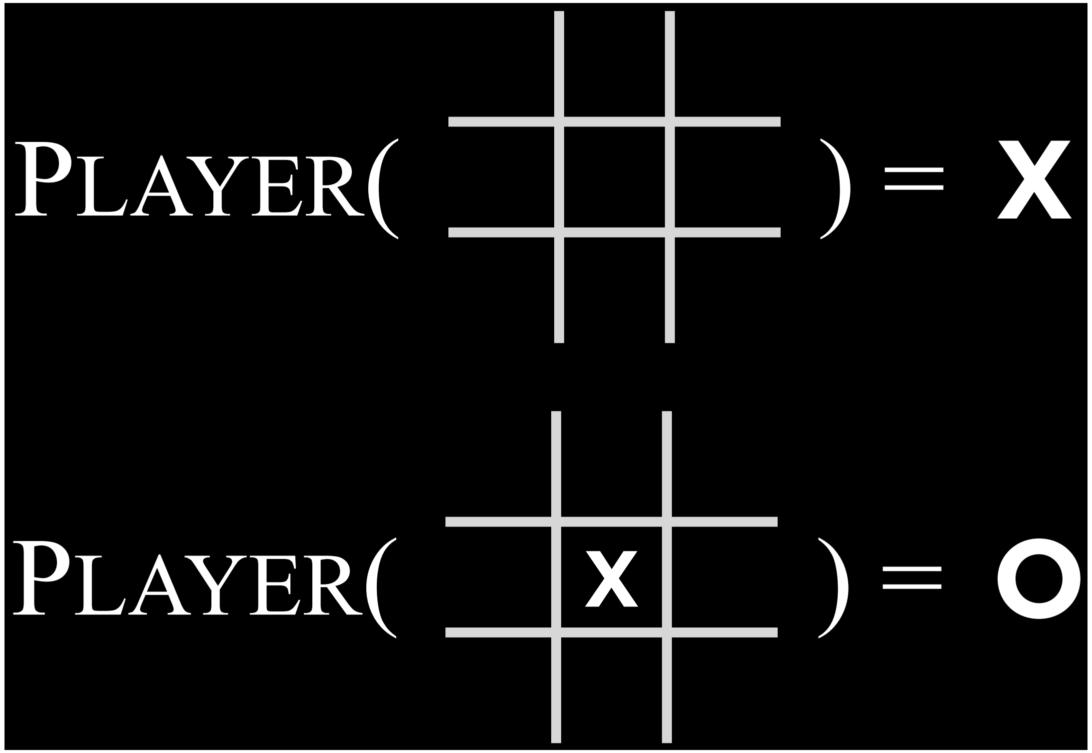
# The Game

- $S_0$: initial state
- PLAYER(s): returns which player (X or O) to move in state s
- ACTIONS(s): returns legal moves in state s
  - What spots are free on the board
- RESULT(s,a): returns state after action a taken in state s
  - The board that resulted from performing the action a on the state s
- TERMINAL(s): checks if state s is a terminal state
  - If someone won or there is a tie
    - Returns True if the game has ended, False otherwise
- UTILITY(s): final numerical value for terminal state s
  - That is, -1, 0 or 1

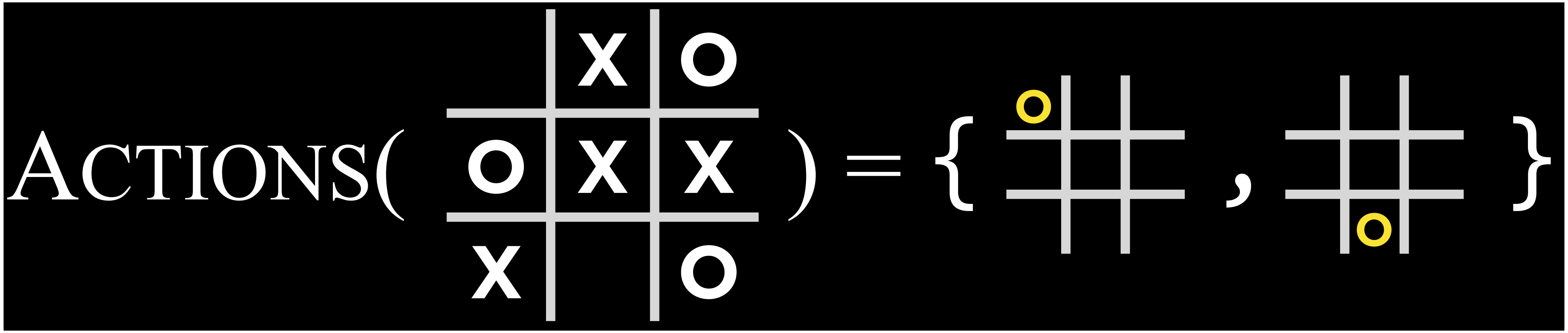
# Initial State



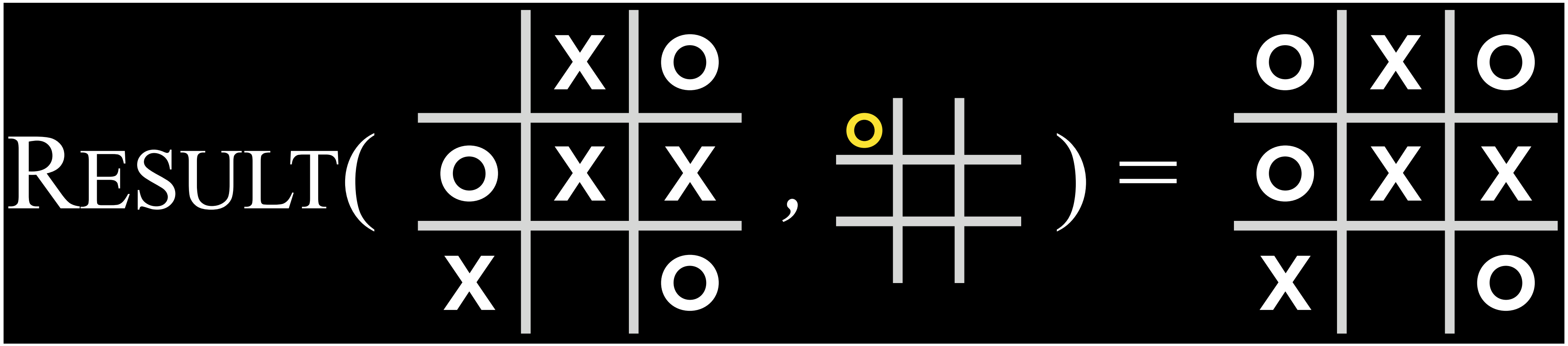
# PLAYER(s)



# ACTION(s)



# RESULTS(s,a)



# TERMINAL(s)

TERMINAL(

| | | |
|---|---|---|
| O | | |
| O | X | |
| X | O | X |

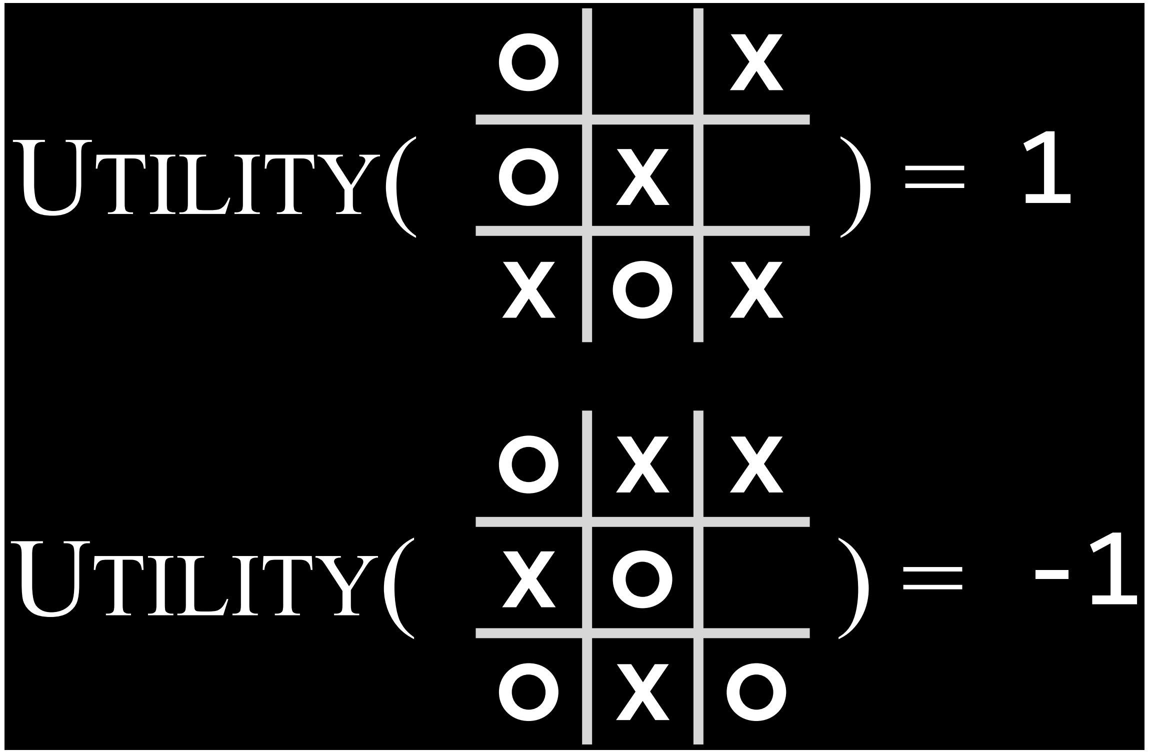
) = false

TERMINAL(

| | | |
|---|---|---|
| O | | X |
| O | X | |
| X | O | X |

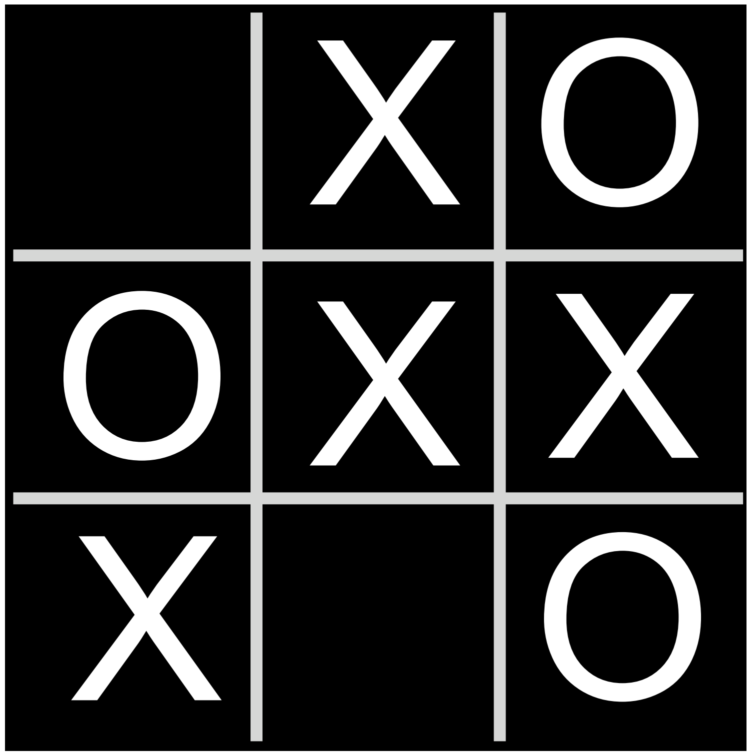
) = true

# UTILITY(s)

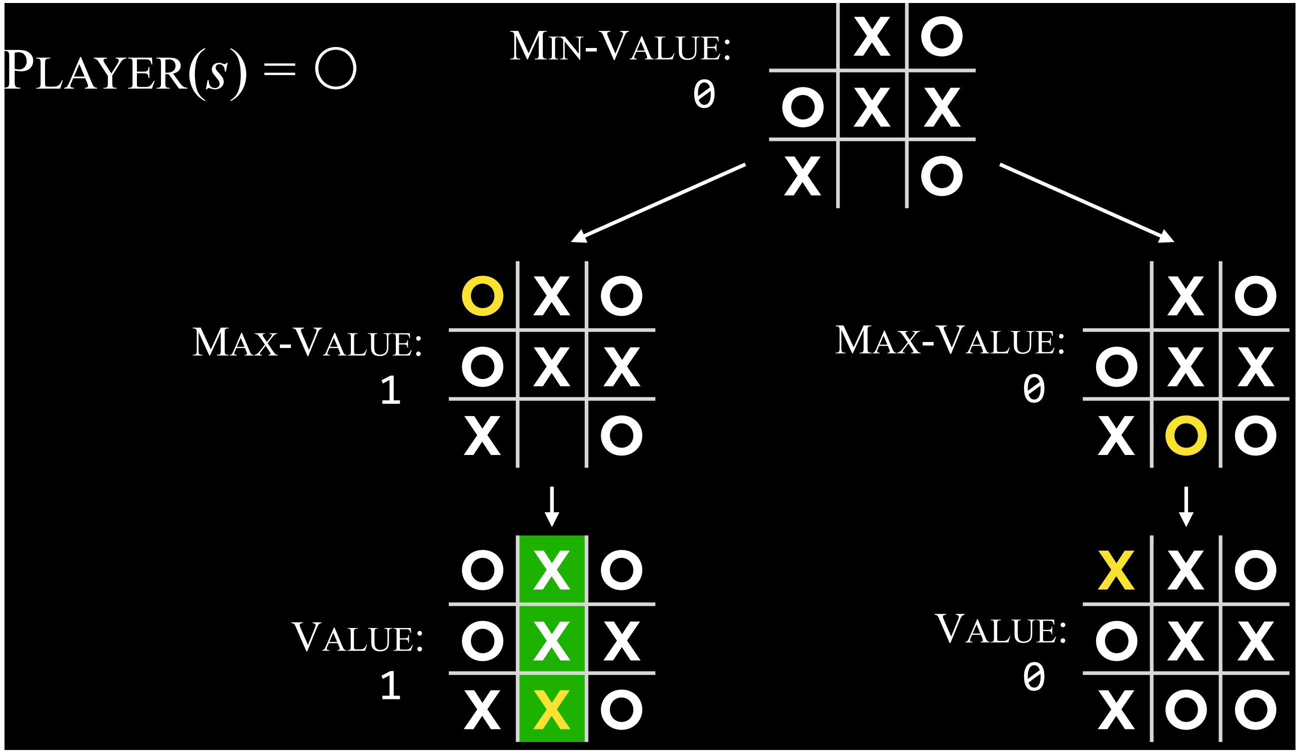


# What Action should O take?

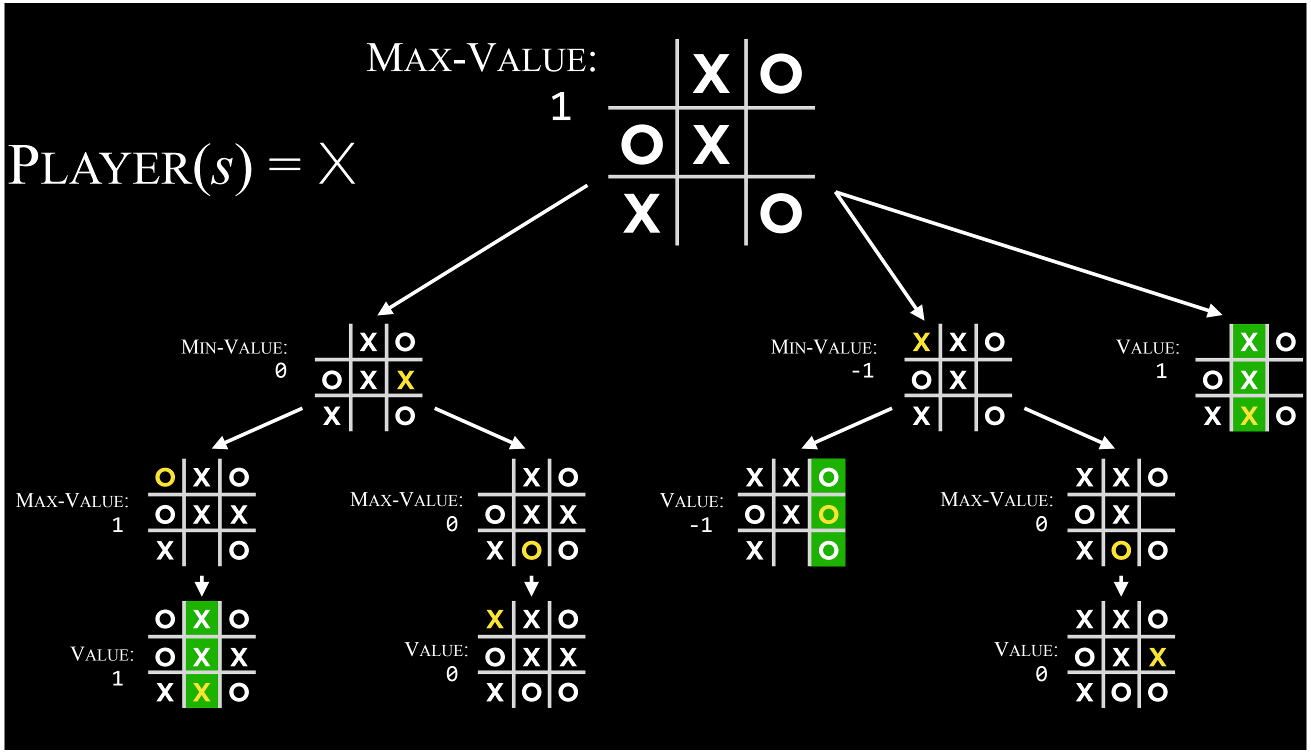
- Player(s) = O



# PLAYER(s) = O

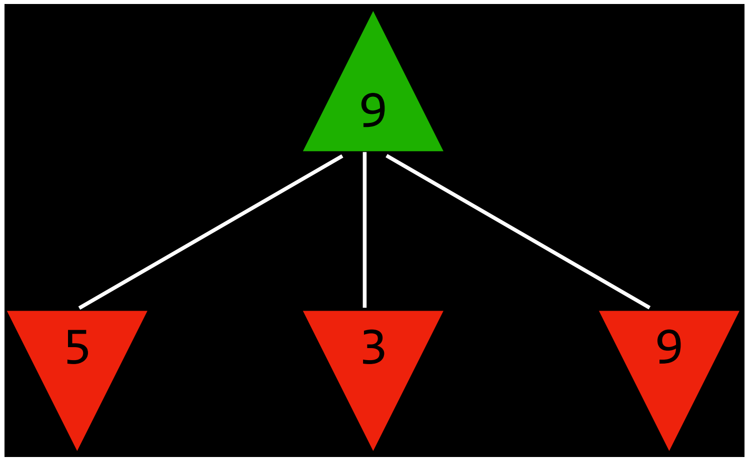


# Minimax Search Tree



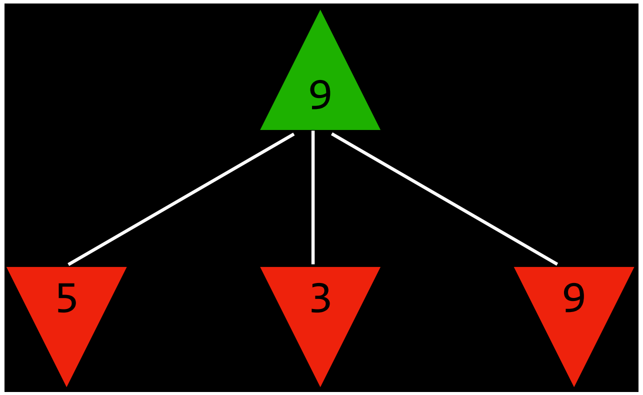
# Generalizing the Game Tree

- We can simplify the diagram into a more abstract Minimax tree
  - each state is just representing some generic game that might be tic-tac-toe or some other game
  - Any of the green arrows that are pointing up, represents a maximizing state
    - the score should be as big as possible
  - Any of the red arrows pointing down are minimizing states, where the player is the min player
    - trying to make the score as small as possible



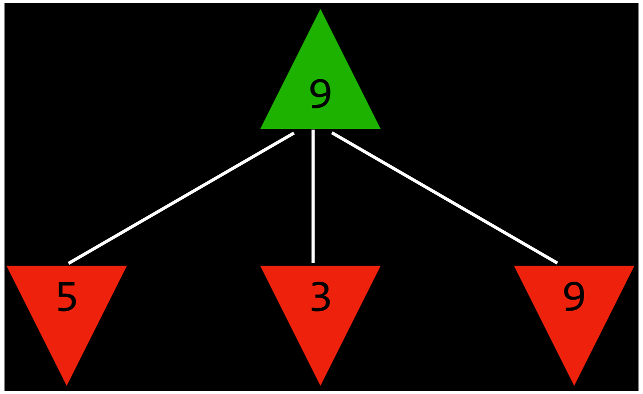
# Generalizing the Game Tree

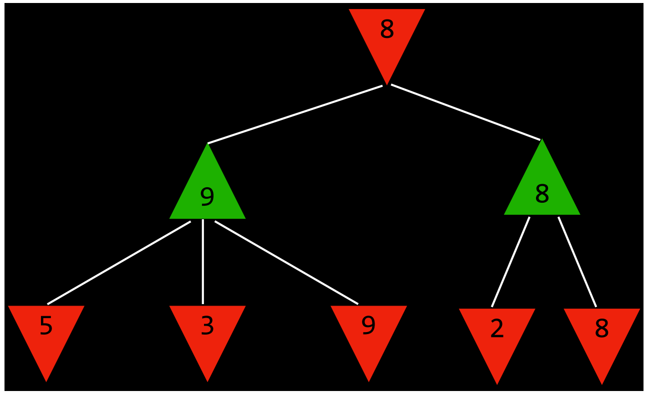
- Let’s consider the maximizing player
  - He has three choices
    - one choice gives a score of 5
    - one choice gives a score of 3
    - one choice gives a score of 9
- Between those three choices, his best option is to choose 9
  - the score that maximizes his options out of all three options



# Generalizing the Game Tree

- Now, one could also ask a reasonable question
  - What might my opponent do two moves away from the end of the game?
  - The opponent is the minimizing player
    - He is trying to make the score as small as possible
    - Imagine what would have happened if they had to pick which choice to make



# How the Algorithm Works

- Recursively, the algorithm simulates all possible games that can take place beginning at the current state and until a terminal state is reached
- Each terminal state is valued as either -1, 0, or +1

# Minimax in Tic-Tac-Toe

- Knowing the state whose turn it is, the algorithm can know whether the current player, if playing optimally, will choose the action that leads to a state with a lower or higher value
- In this way, the algorithm alternates between minimizing and maximizing, generating values for the state that would result from each possible action
- This is a recursive process
  - Eventually, through this recursive reasoning process, the maximizing player generates values for each state that could result from all possible actions at the current state
  - After having these values, the maximizing player chooses the highest one
- The maximizer considers the possible values of future states

# Minimax

- Given a state s:
    - MAX picks action a in ACTIONS(s) that produces highest value of MIN-VALUE(RESULT(s,a))
    - MIN picks action a in ACTIONS(s) that produces smallest value of MAX-VALUE(RESULT(s,a))
- Everyone makes their decision based on trying to estimate what the other person would do

# Minimax Pseudocode

```
function MAX-VALUE(state):
    if TERMINAL(state):
        return UTILITY(state)
    v=-inf
    for action in ACTIONS(state):
        v=MAX(v,MIN-VALUE(RESULT(state,action)))
    return v
```

# Minimax Pseudocode

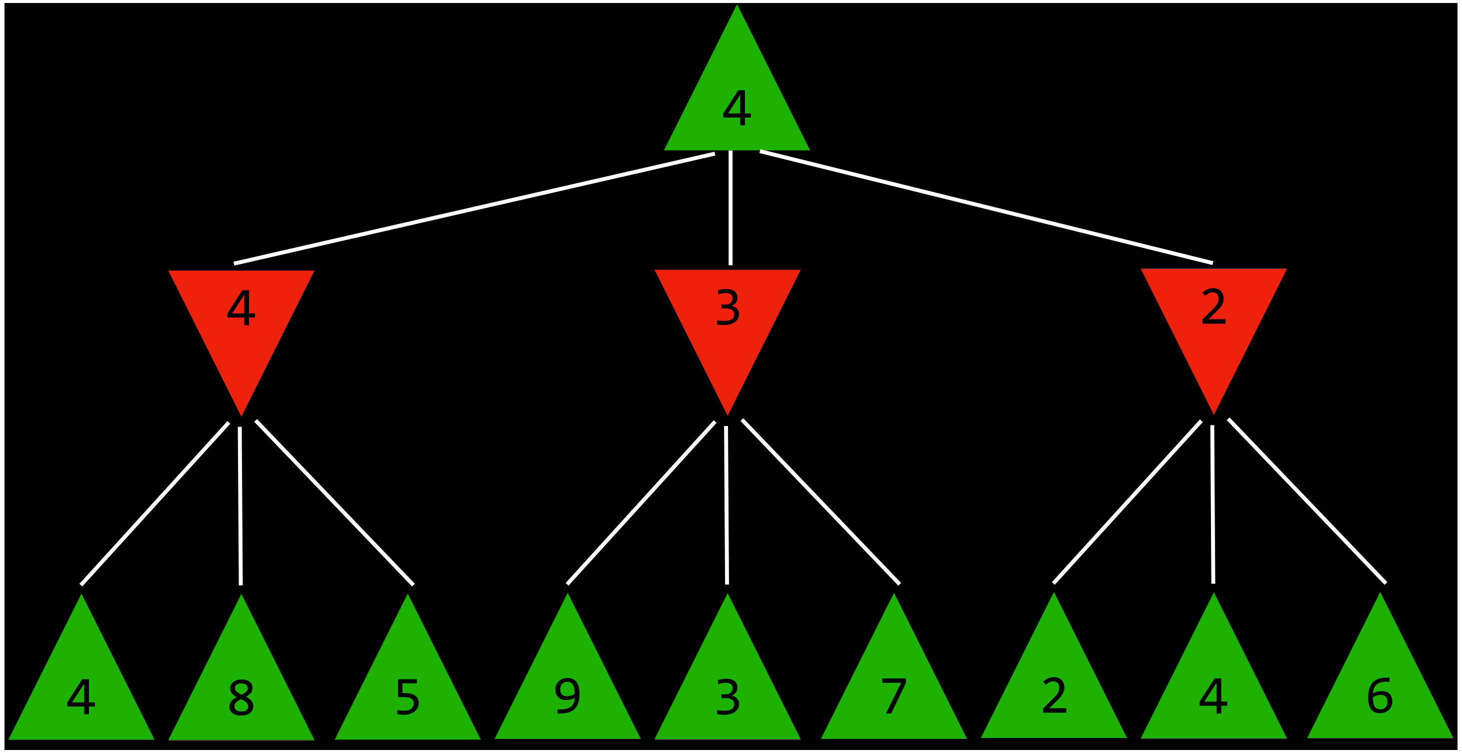
```
function MIN-VALUE(state):
    if TERMINAL(state):
        return UTILITY(state)
    v=+inf
    for action in ACTIONS(state):
        v=MIN(v,MAX-VALUE(RESULT(state,action)))
    return v
```

# Optimizations?

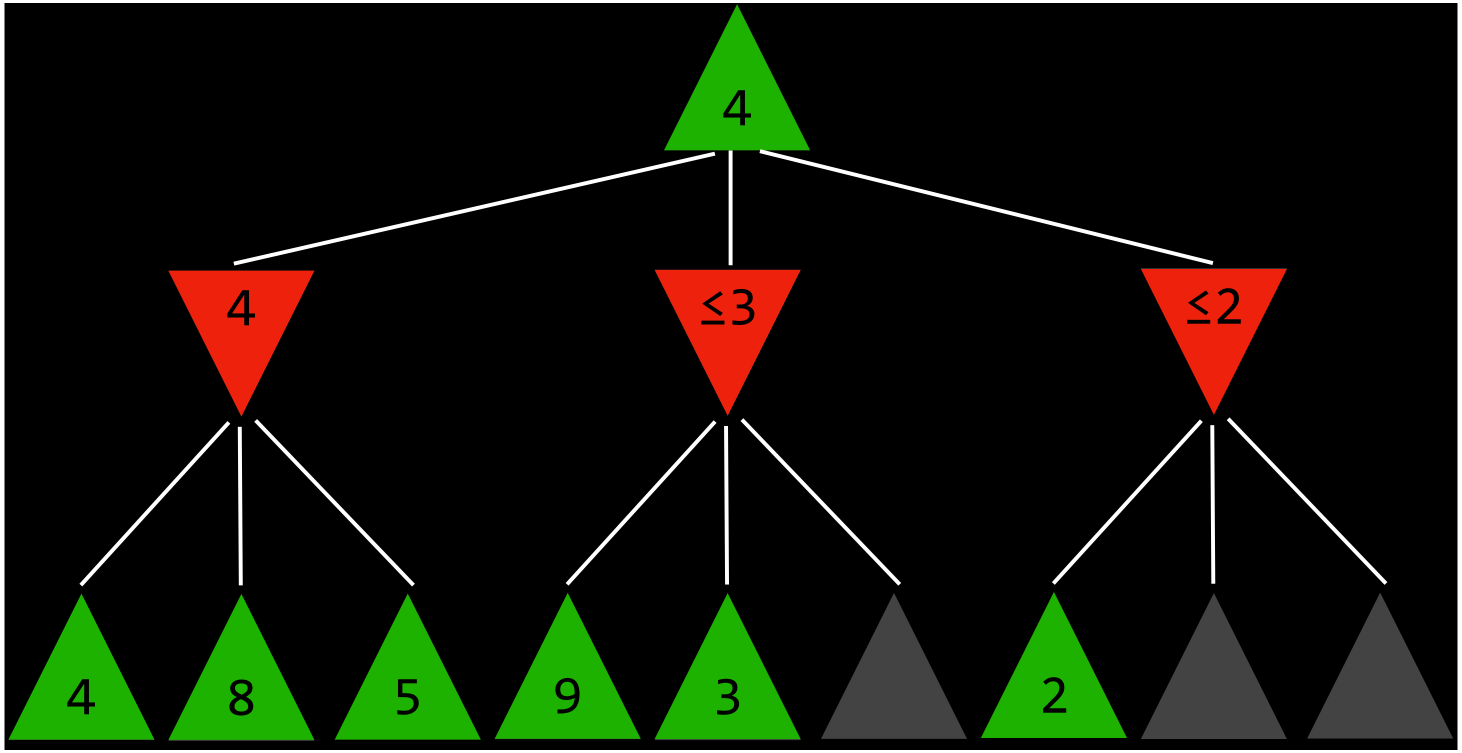
- The entire process could be a long process, especially as the game starts to get more complex, as we start to add more moves and more possible options

- What sort of optimizations can we make here?
  - How can we do better in order to
    - use less space
    - take less time

# What Minimax Does so far



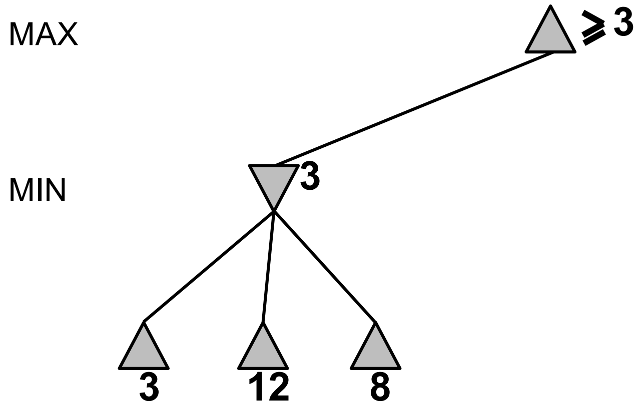
# Pruning Useless Sub-Trees



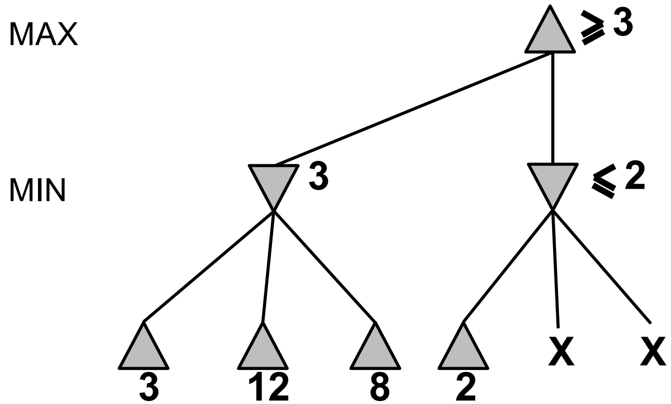
# Alpha-Beta Pruning

- A way to optimize *Minimax*, **Alpha-Beta Pruning** skips some of the recursive computations that are decidedly unfavorable
- If, after establishing the value of an action, there are initial indications that the following action may cause the opponent to achieve a better result than the action already established, there is no need to investigate this action further
  - because it will be decidedly less favorable than the action previously established

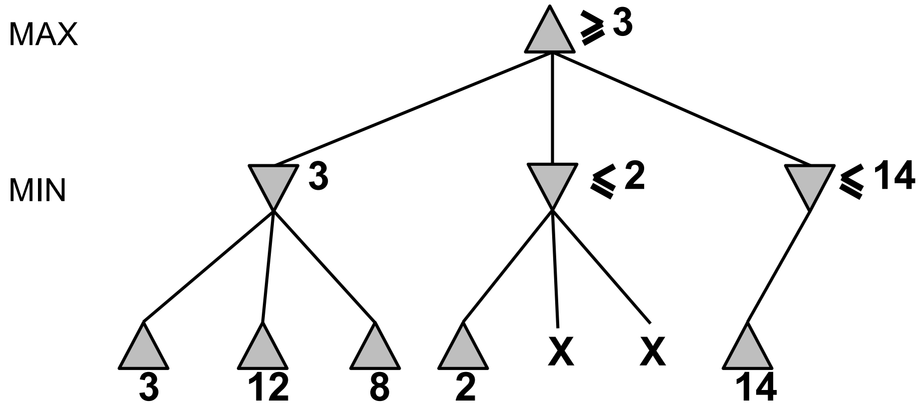
# $\alpha - \beta$ Pruning Example



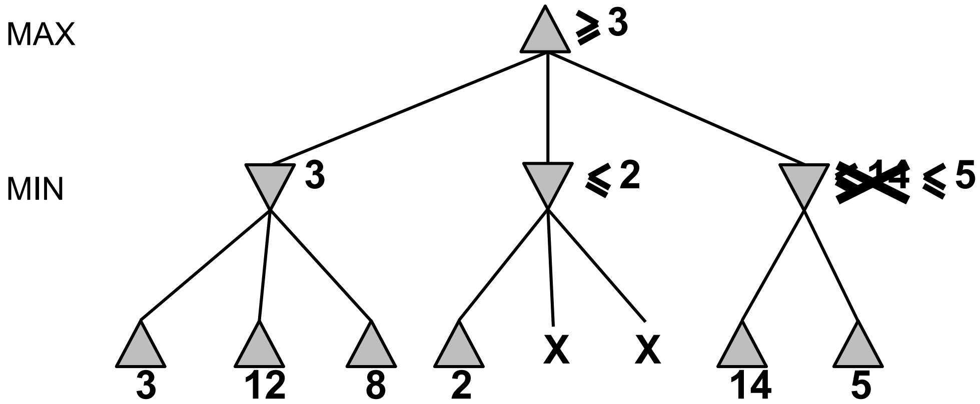
# $\alpha - \beta$ Pruning Example



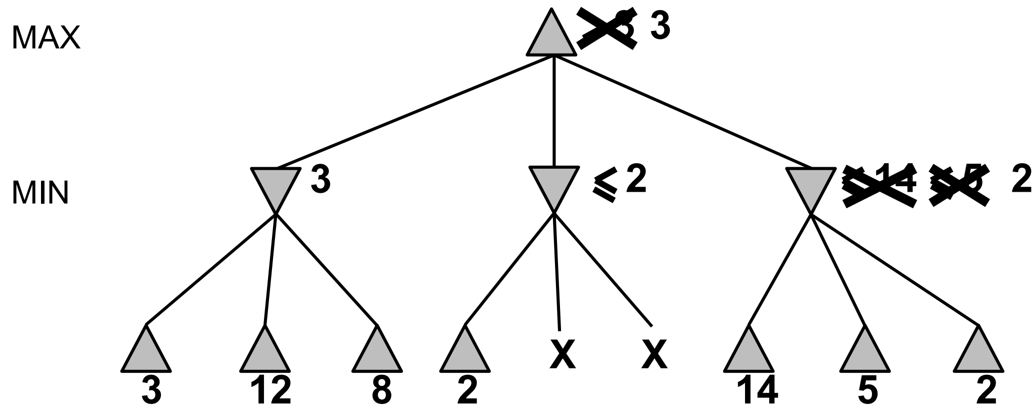
# $\alpha - \beta$ Pruning Example



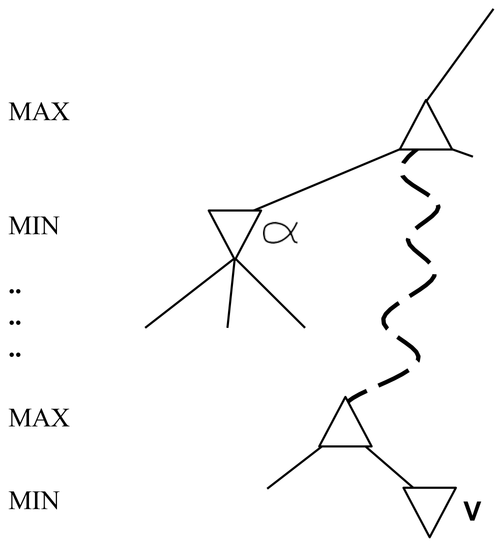
# $\alpha - \beta$ Pruning Example



# $\alpha - \beta$ Pruning Example



# Why is it Called $\alpha - \beta$ ?




- $\alpha$ is the best value (to max) found so far off the current path
- If V is worse than $\alpha$, max will avoid it $\Rightarrow$ prune that branch
- Define $\beta$ similarly for min

# Properties of $\alpha - \beta$

- Pruning does not affect the final result
- Good move ordering improves the effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  - $\Rightarrow$ doubles solvable depth
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)
- For chess ($35^{100}$), unfortunately, $35^{50}$ is still impossible!

# Total Possible Games

- 255.168 total possible Tic-Tac-Toe games
- More complex game
  - 288.000.000.000 total possible chess games
    - after four moves each
  - $10^{29000}$ total possible chess games (lower bound)
- Big problem for Minimax
- So what?
  - Do not  look through all the states
    - Depth-limited Minimax

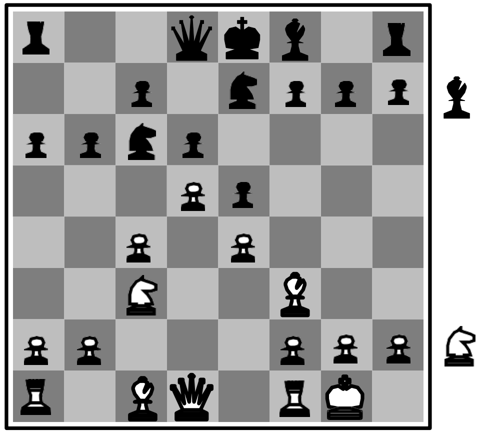
# Depth-Limited Minimax

- **Depth-limited Minimax** considers only a pre-defined number of moves before it stops, without ever getting to a terminal state
  - However, this doesn't allow for getting a precise value for each action, since the end of the hypothetical games has not been reached
- To deal with this problem, *Depth-limited Minimax* relies on an **evaluation function** that estimates the expected utility of the game from a given state, or, in other words, assigns values to states

# Evaluation function

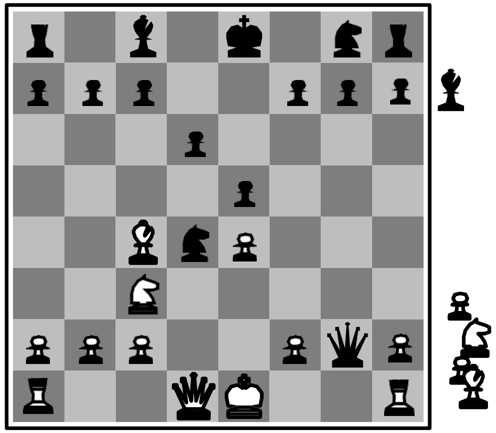
- Evaluation function
  - Function that estimates the expected utility of the game from a given state
- Example
  - In a game like chess, if you imagine that a game value of 1 means white wins, -1 means black wins, 0 means it's a draw
    - A score of 0.8 means white is very likely to win though certainly not guaranteed
    - Depending on how good that evaluation function is, ultimately constrains how good the AI is

# Evaluation Functions



**Black to move**

**White slightly better**

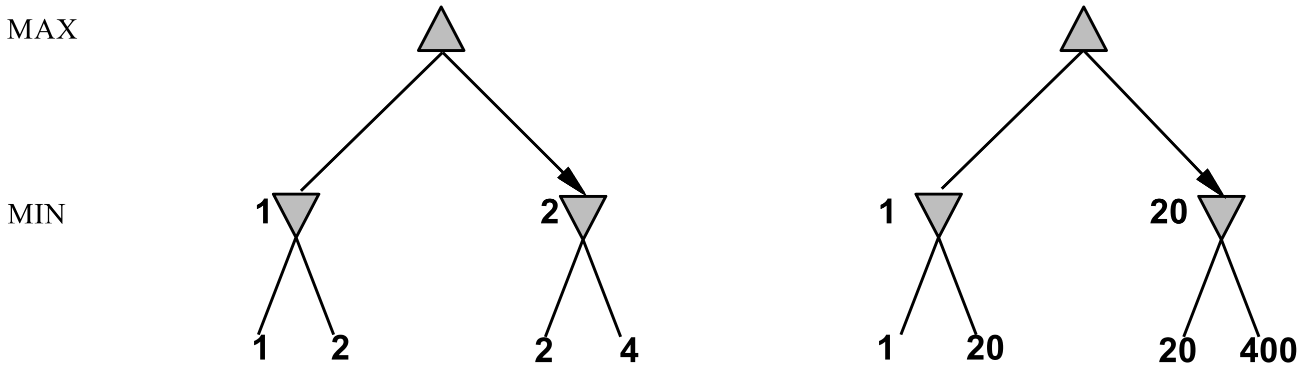


**White to move**

**Black winning**

- For chess, typically linear weighted sum of features
  - $Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
- For instance, $w_1 = 9$ with
  - $f_1(s)$ = (number of white queens) – (number of black queens), etc.

# Exact Values Don't Matter




- Behavior is preserved under any monotonic transformation of Eval
- Only the order matters:
    - payoff in deterministic games acts as an ordinal utility function

# Assignment 1

- Using Minimax, implement an AI to play Tic-Tac-Toe optimally