Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Università degli Studi di Napoli “Parthenope”

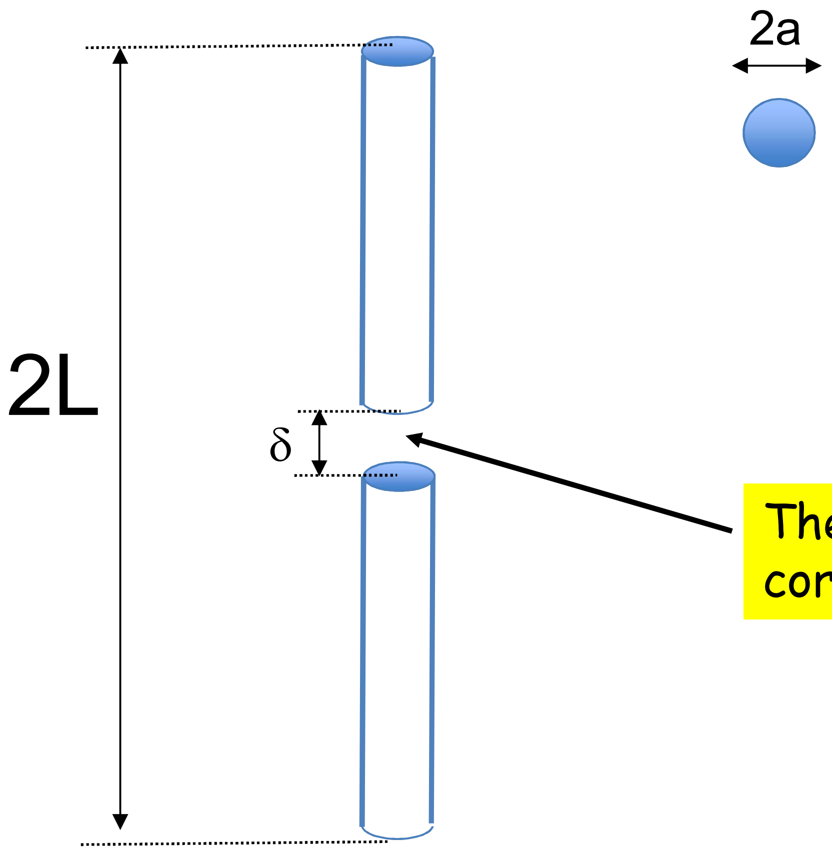
a.a. 2023–2024 – Laurea “Triennale” – Secondo semestre – Terzo anno

Ing. Stefano Perna

Wire antennas

Wire antennas

Wire antennas



$2a \ll \lambda$

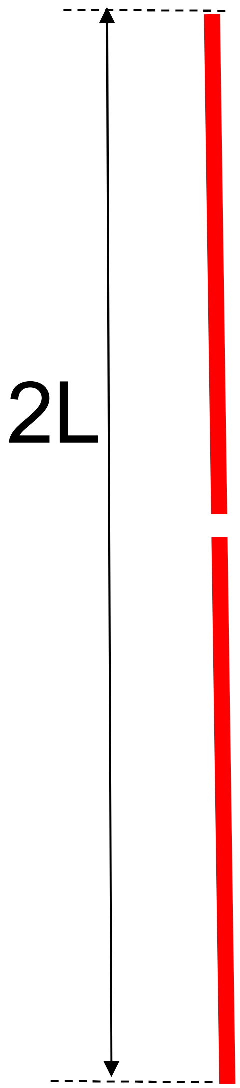
$2a \ll L$

$\delta \ll \lambda$

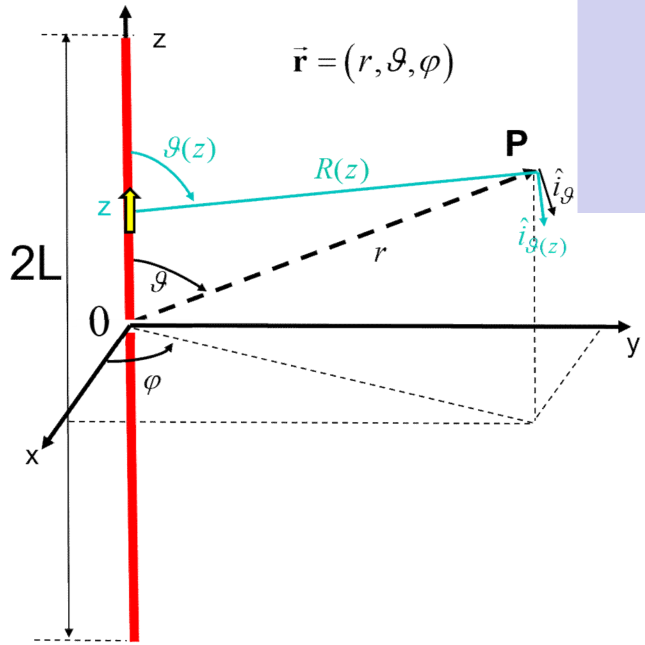
$\delta \ll L$

The terminals are located in correspondence of this gap

Wire antennas



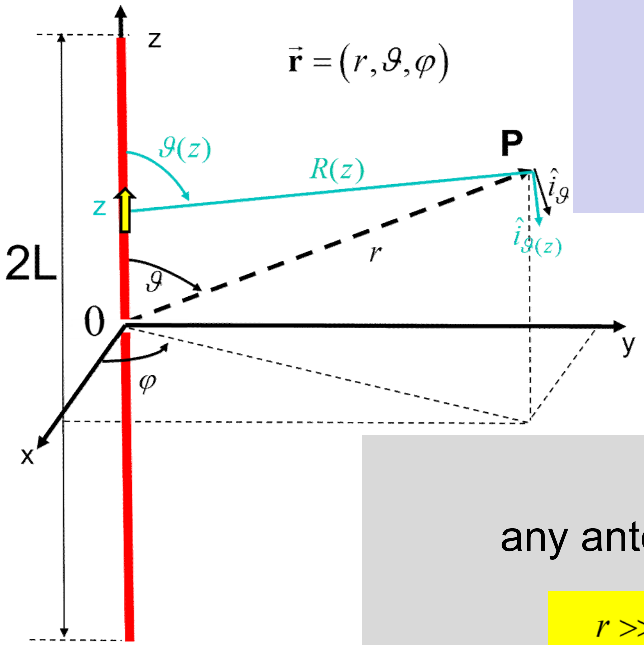
Wire antennas



In the Fraunhofer Region the expression of the radiated field simplifies as

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta \, \hat{i}_\vartheta \int_{-l}^{l} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta)$$

Wire antennas



In the Fraunhofer Region the expression of the radiated field simplifies as

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta \left[\int_{-L}^{L} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta)\right] \hat{i}_\vartheta$$

Effective length of the wire antenna

…. Memo

any antenna, in the Fraunhofer region, behaves as follows

$r \gg D$

$r > \frac{2D^2}{\lambda}$

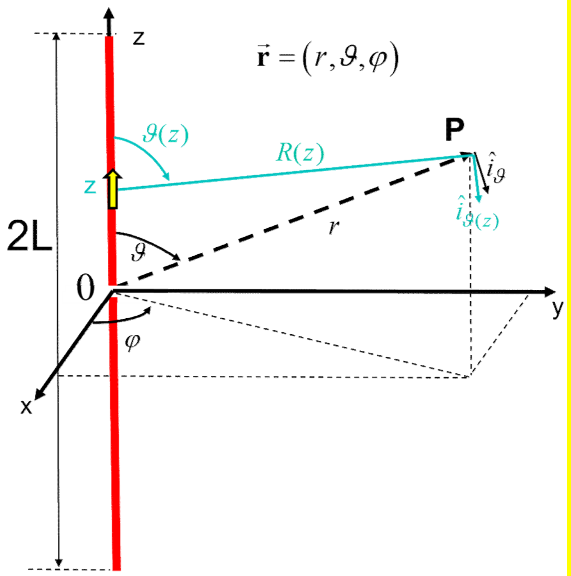
$r \gg \lambda$

$$\begin{cases} \mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi) \\ \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E} \end{cases}$$

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$ **Effective length**

Wire antennas: effective length

$$\vec{\mathbf{l}}(\vartheta) = l_\vartheta(\vartheta)\hat{i}_\vartheta = \sin\vartheta\left[\int_{-L}^{L} dz \frac{I(z)}{I_0}\exp(j\beta z\cos\vartheta)\right]\hat{i}_\vartheta$$



- The effective length is independent of φ

  … absolutely not surprising

- The effective length depends by the current distribution *I*(z)

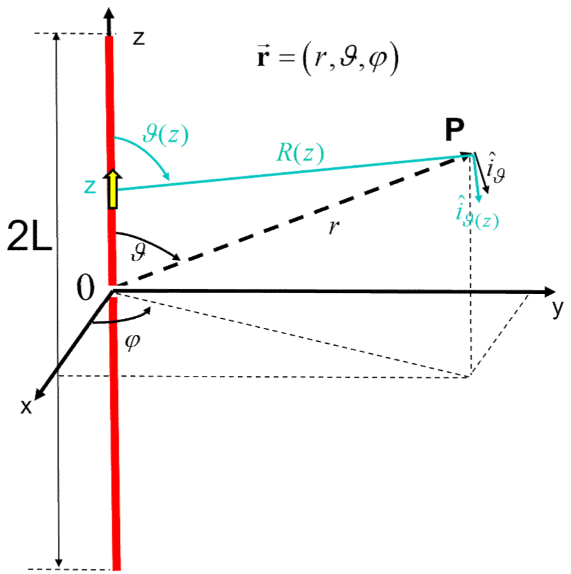
  … absolutely not surprising

- The effective length depends on L

  … absolutely not surprising

Wire antennas: effective length

$$\vec{\mathbf{l}}(\vartheta) = l_\vartheta(\vartheta)\hat{i}_\vartheta = \sin\vartheta\left[\int_{-L}^{L} dz \frac{I(z)}{I_0}\exp\left(j\beta z\cos\vartheta\right)\right]\hat{i}_\vartheta$$



$$u = -\beta\cos\vartheta \qquad \tilde{I}(z) = \frac{I(z)}{I_0}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

For the wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rule

Wire antennas: effective length

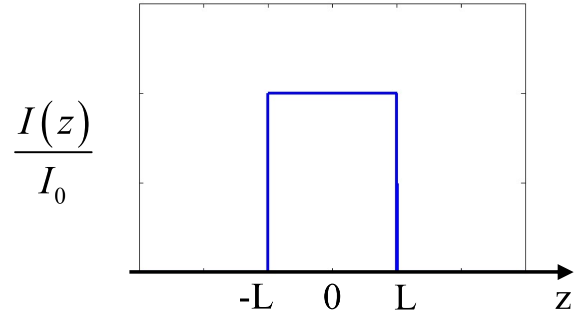
$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

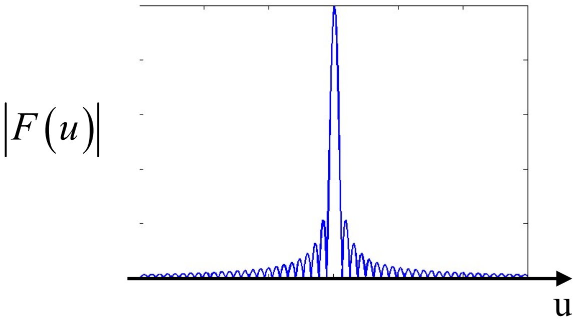
For the wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rule

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$





The properties of the Fourier Transformation suggest some interesting considerations

- **Antenna's size and beamwidth**
- **Scanning of the pattern**
- **Synthesis of the pattern**

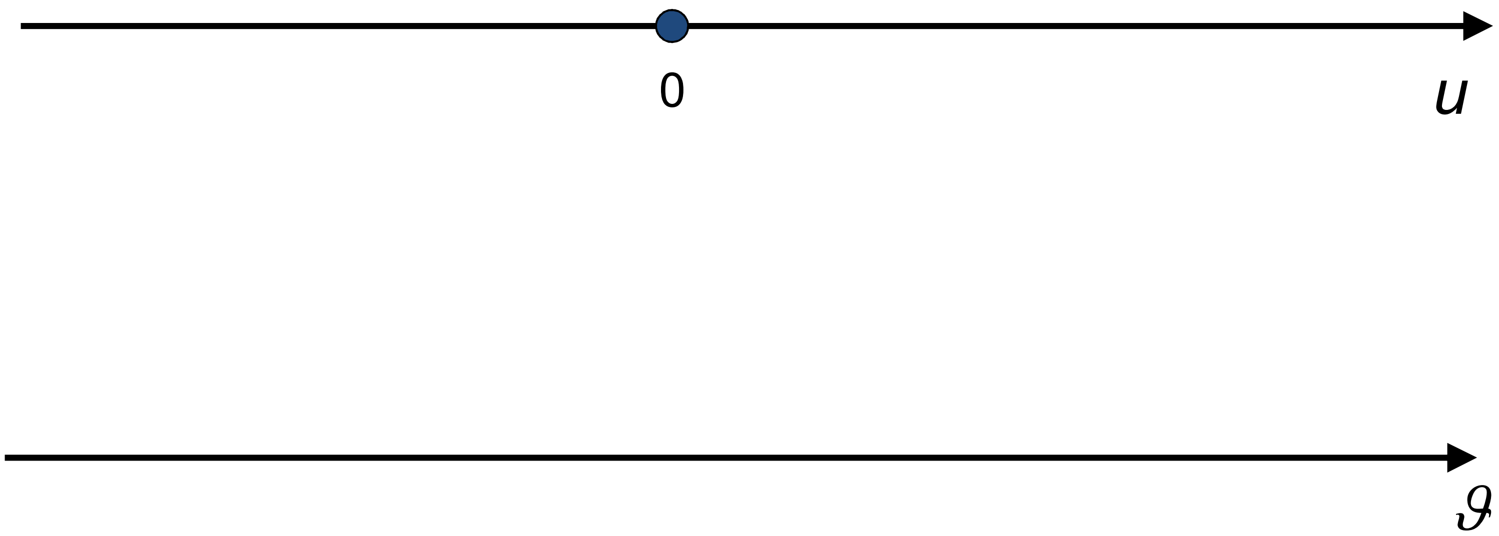
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



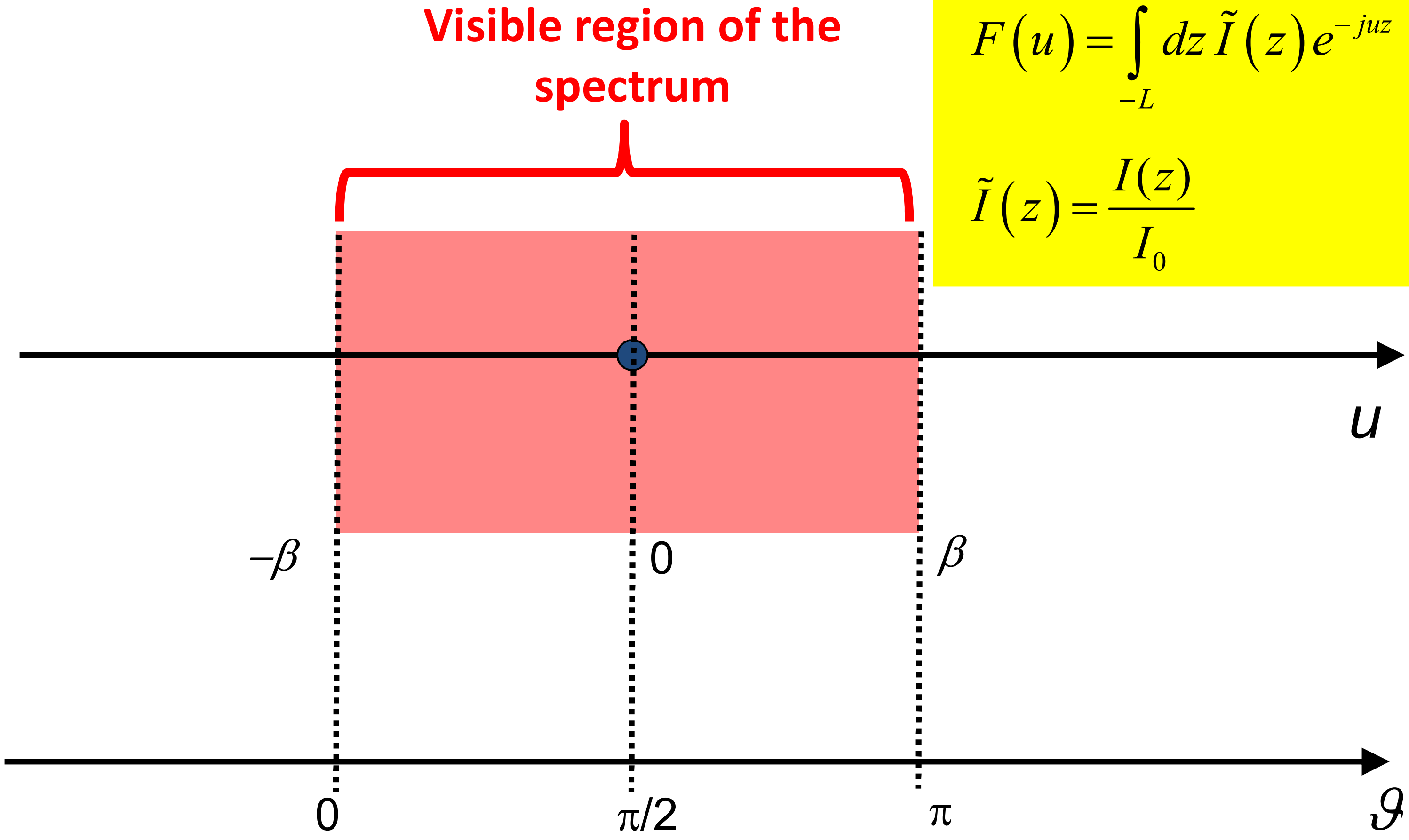
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}I_0\frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta)=F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u)=\int_{-L}^{L}dz\,\tilde{I}(z)e^{-juz}$$

$$\tilde{I}(z)=\frac{I(z)}{I_0}$$



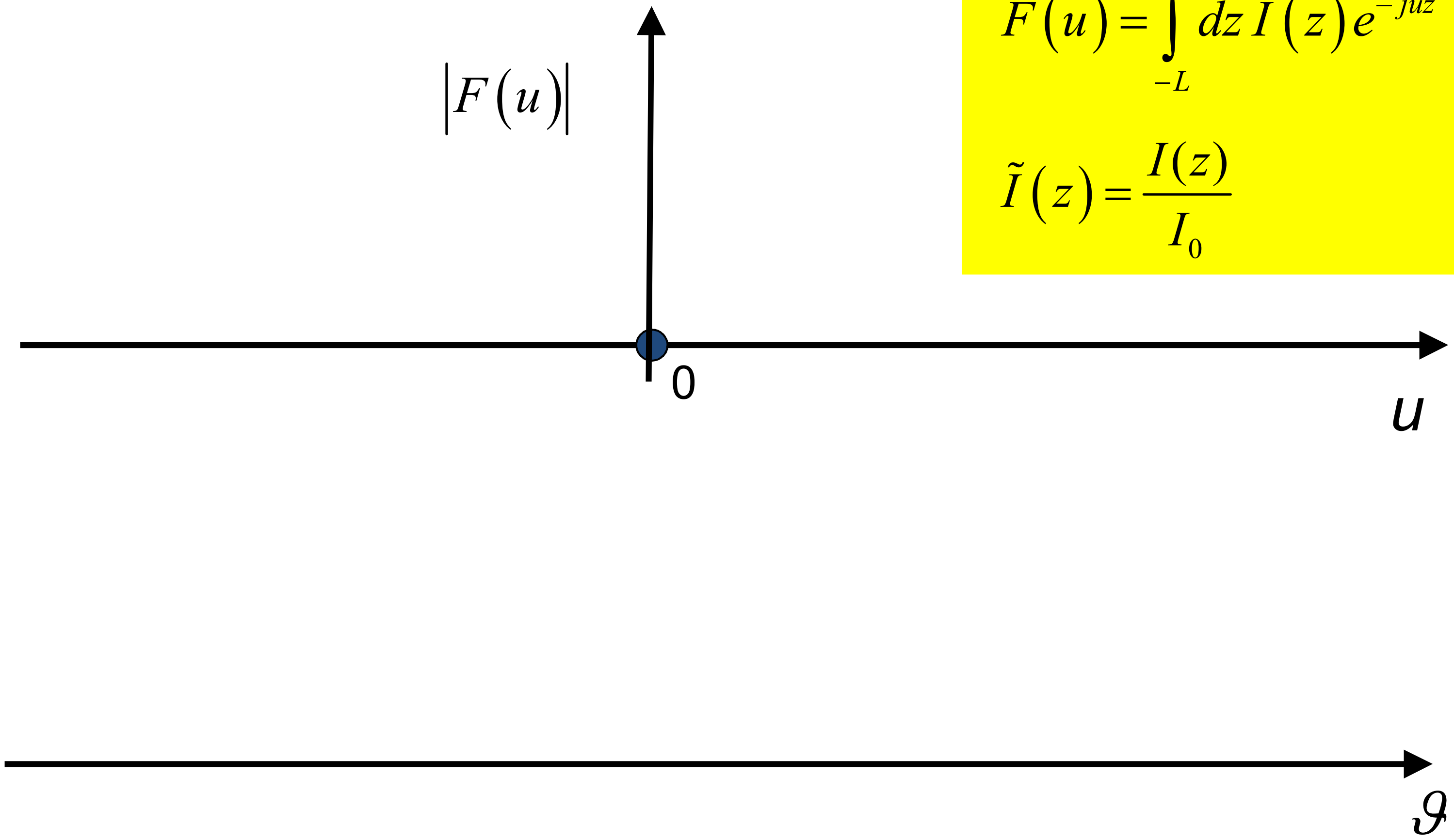
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



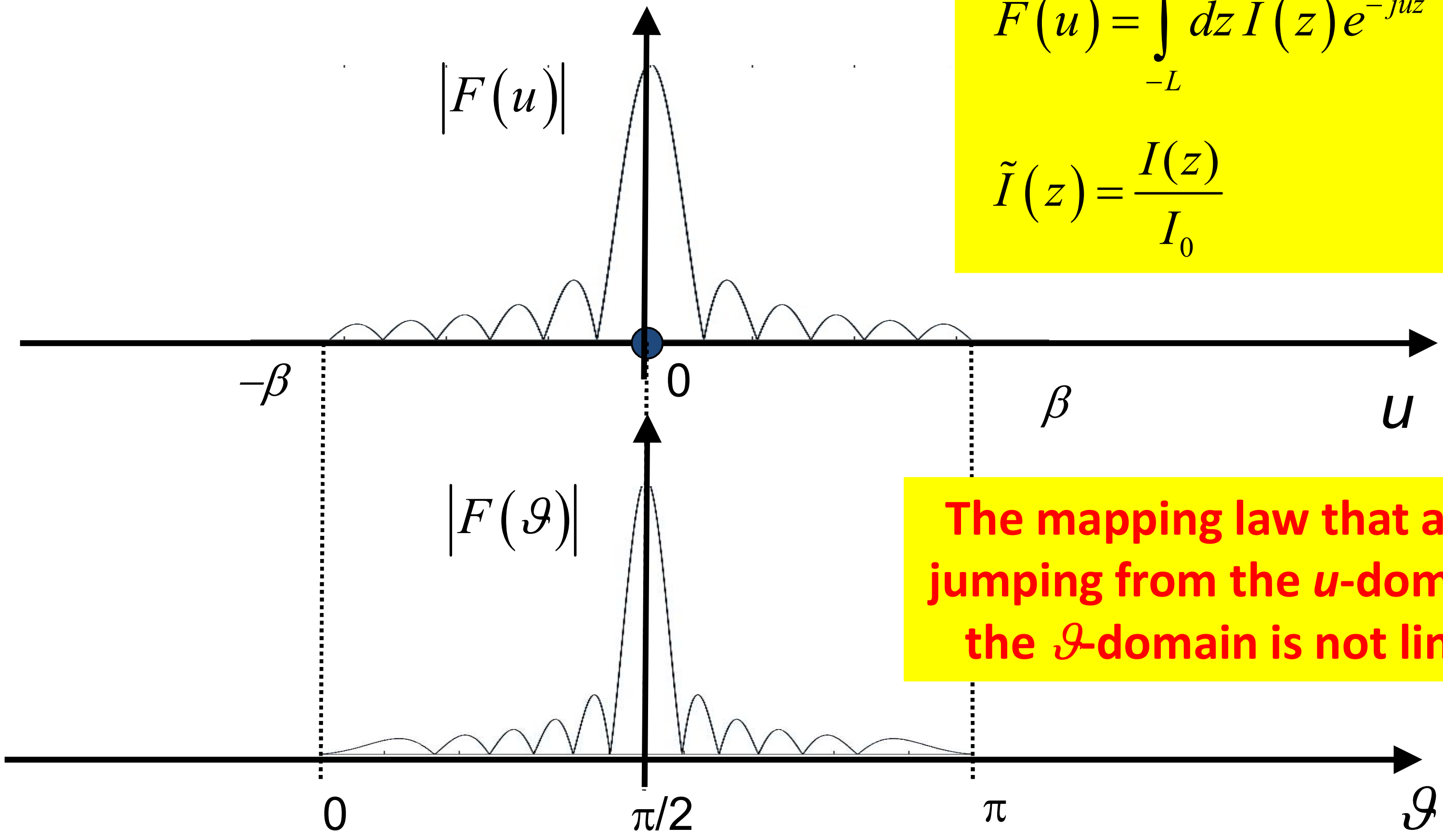
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\,\tilde{I}(z)e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



The mapping law that allows jumping from the *u*-domain to the ϑ-domain is not linear!

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

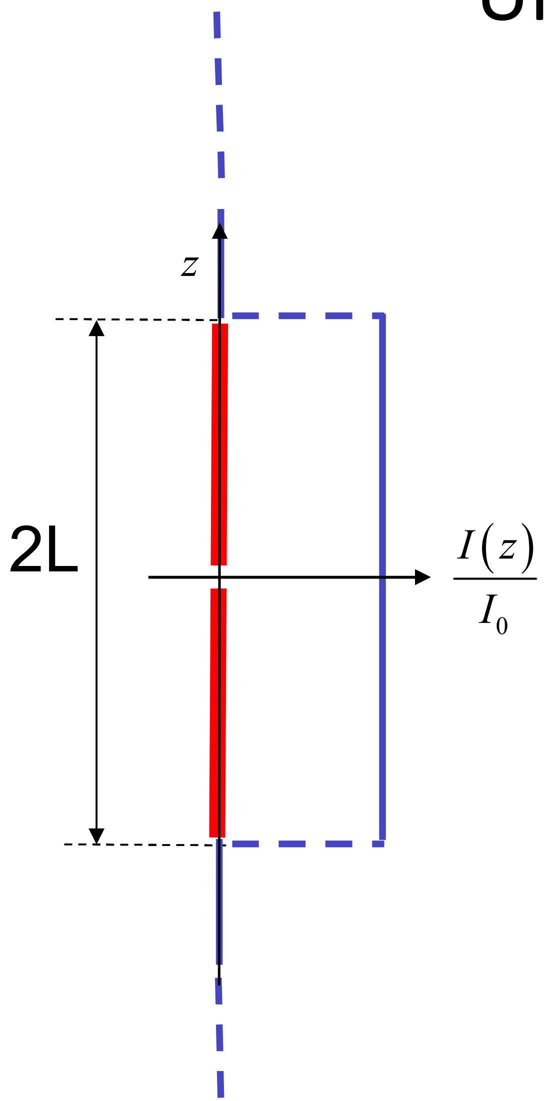
Memo

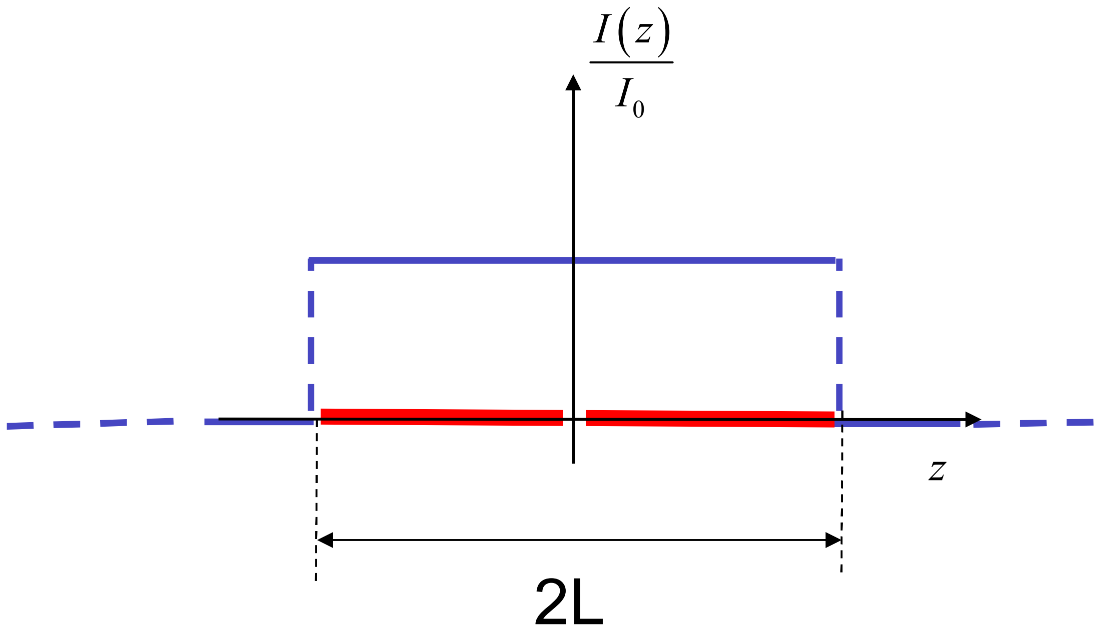
Mathematical tools to be exploited

Mathematics

Wire antennas: an ideal case

Uniform current distribution





$$\frac{I(z)}{I_0} = \tilde{I}(z) = \text{rect}\left[\frac{z}{2L}\right]$$

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right]$$

$$F(u) = \int_{-L}^{L} dz\tilde{I}(z)e^{-juz} = \int_{-L}^{L} dze^{-juz} = \frac{1}{-ju}\int_{-L}^{L} dz(-ju)e^{-juz} = \frac{1}{-ju}\left[e^{-juz}\right]_{-L}^{L} =$$

$$= \frac{1}{-ju}\left[e^{-juL} - e^{juL}\right] = \frac{1}{ju}\left[e^{juL} - e^{-juL}\right]\frac{2L}{2L} = \frac{2L}{uL}\frac{\left[e^{juL} - e^{-juL}\right]}{2j} =$$

$$= \frac{2L}{uL}\sin(uL)$$

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

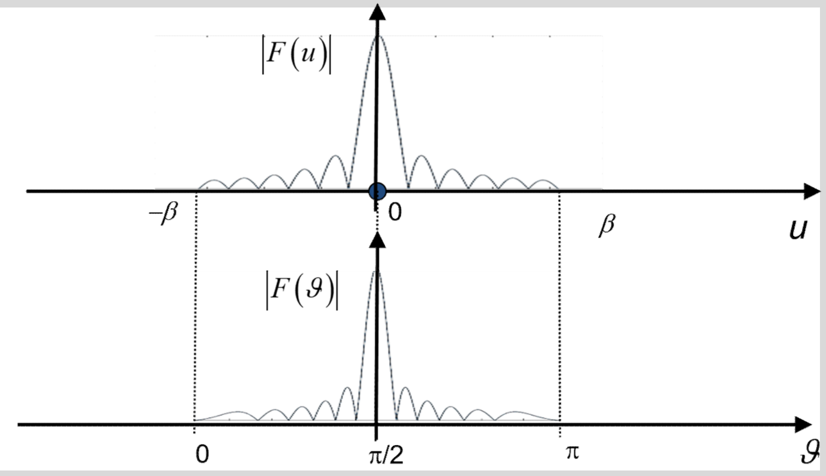
$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

.... Memo

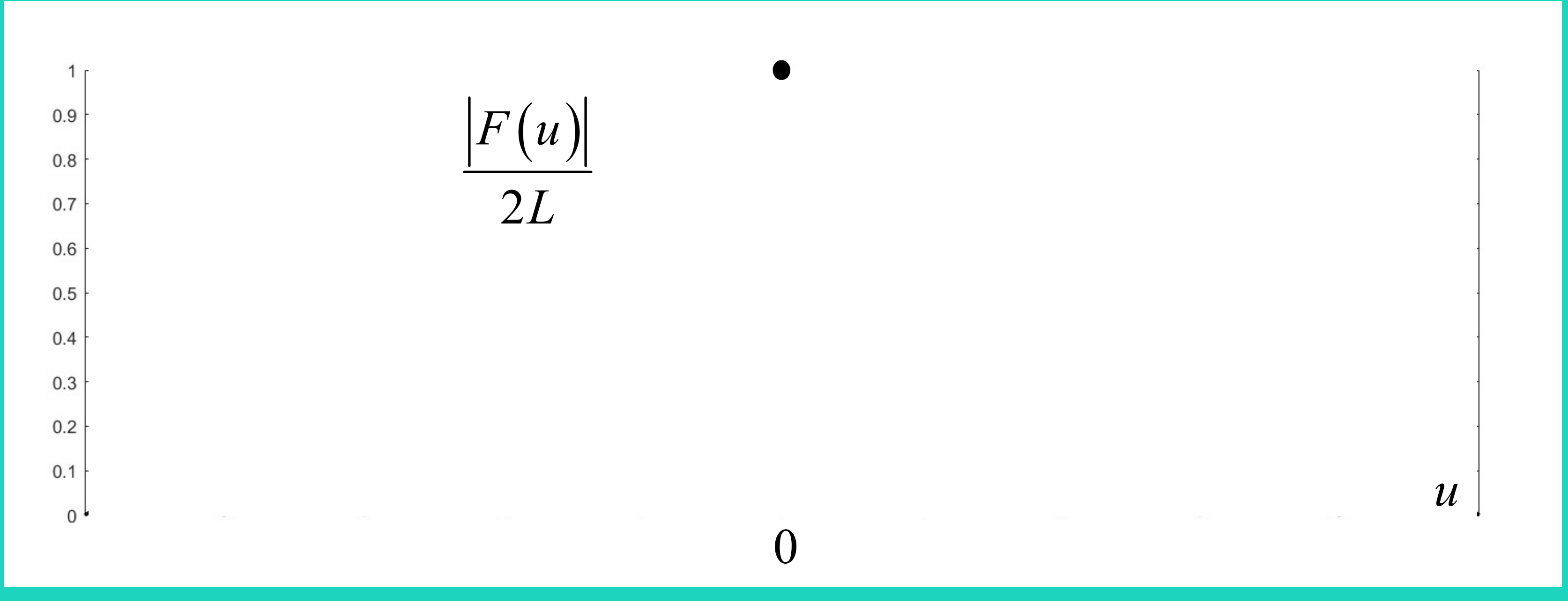


1. **Let's depict F(*u*)**
2. **Let's jump from *u* to ϑ and calculate:**
   - **The direction of the Main Lobe**
   - **The NNBW / HPBW**
   - **The SLL**
   - **The Directivity**

Wire antennas: an ideal case

$$F(u) = 2L\frac{\sin(uL)}{uL}$$

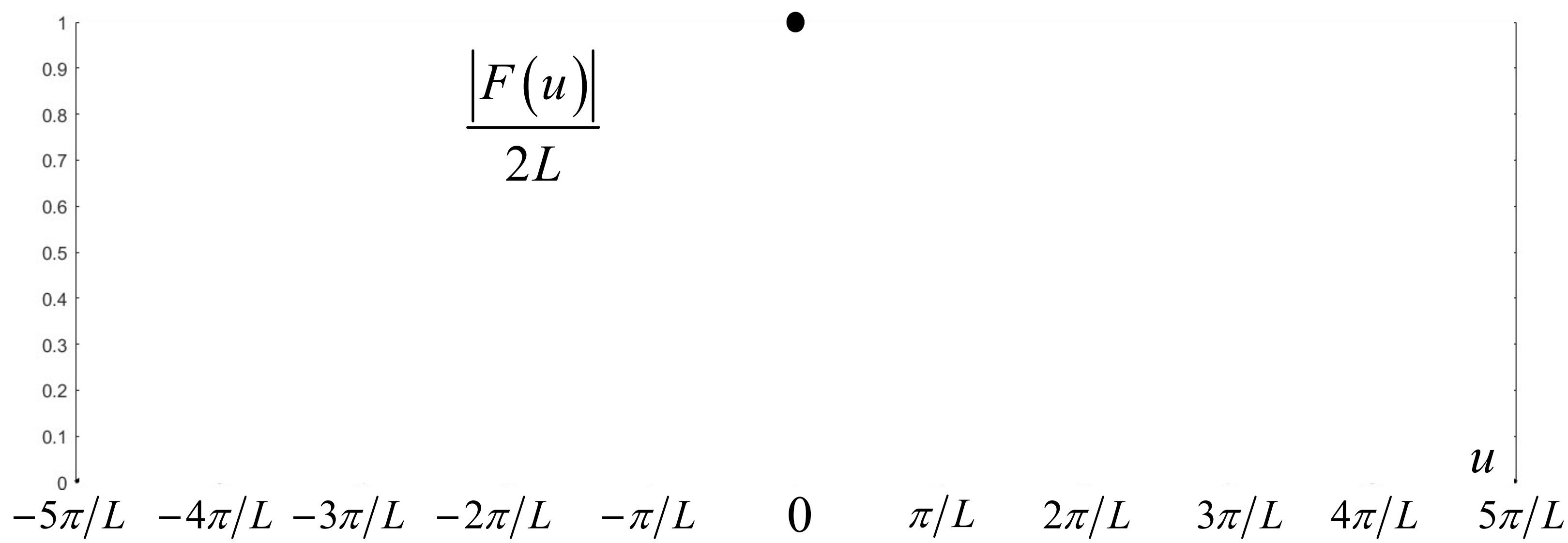
$u = 0 \Rightarrow F(0) = \frac{0}{0}$ ... application of the de l'Hopital rule leads to $F(0) = 2L$



Wire antennas: an ideal case

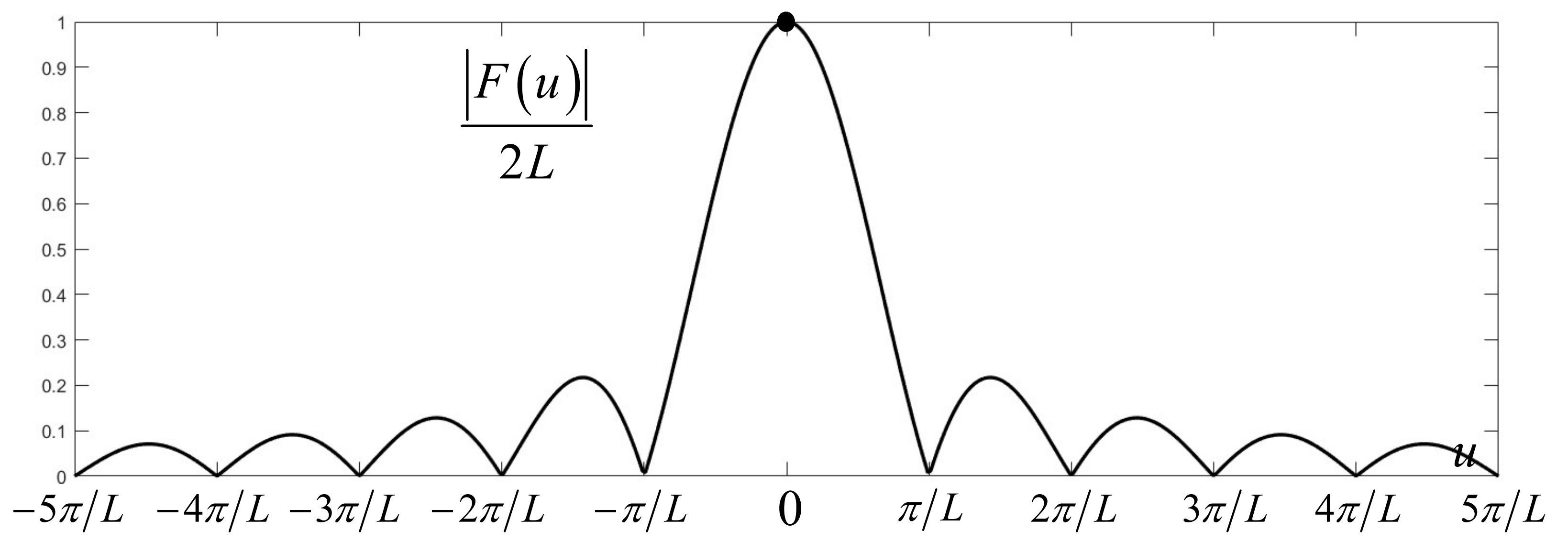
$$F(u) = 2L\frac{\sin(uL)}{uL}$$

Zeroes: $\sin(uL) = 0 \quad \Rightarrow uL = k\pi \quad (k \neq 0) \quad \Rightarrow u = \frac{k\pi}{L} \quad (k \neq 0)$



Wire antennas: an ideal case

$$F(u) = 2L\frac{\sin(uL)}{uL}$$



Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\,\tilde{I}(z)e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

1. Let's depict F(*u*) ✓

2. Let's jump from *u* to ϑ and calculate:

The direction of the Main Lobe

The NNBW / HPBW

The SLL

The Directivity

Wire antennas: an ideal case

u-domain

$$F(u) = 2L\frac{\sin(uL)}{uL}$$

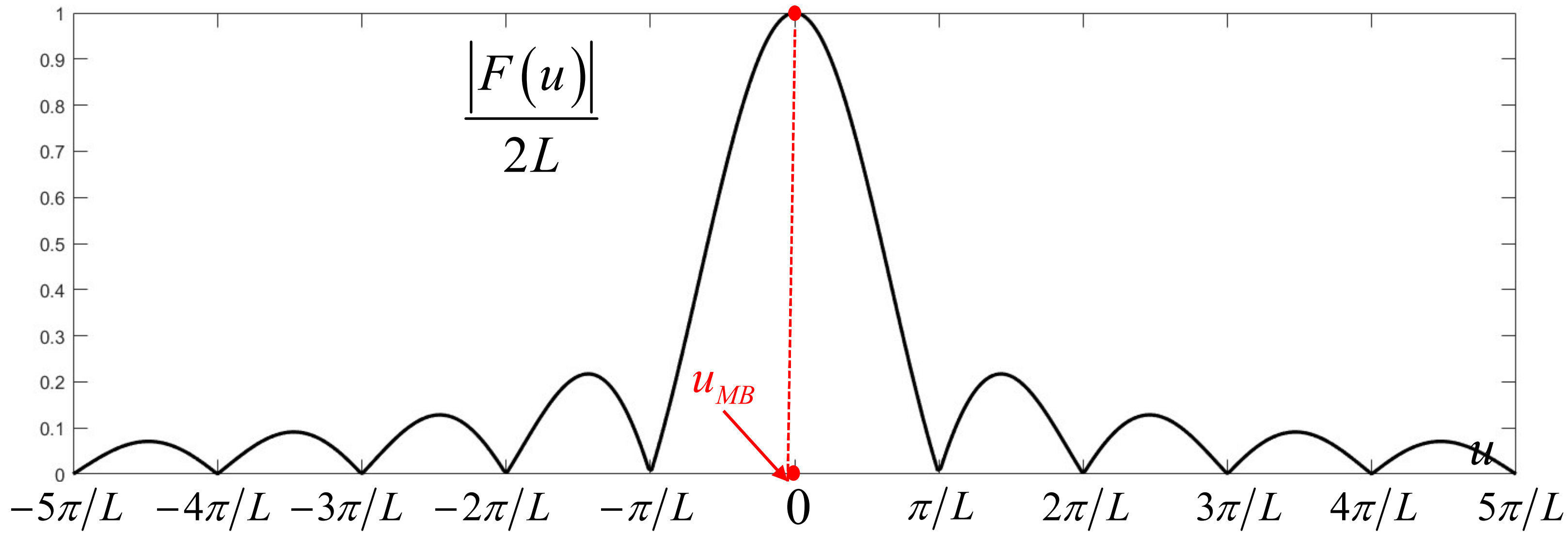
$$u_{MB} = 0$$

ϑ -domain

$$u = -\beta\cos\vartheta$$

$$u_{MB} = -\beta\cos\vartheta_{MB} \Rightarrow 0 = -\beta\cos\vartheta_{MB}$$

$$\Rightarrow \vartheta_{MB} = \frac{\pi}{2}$$



Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

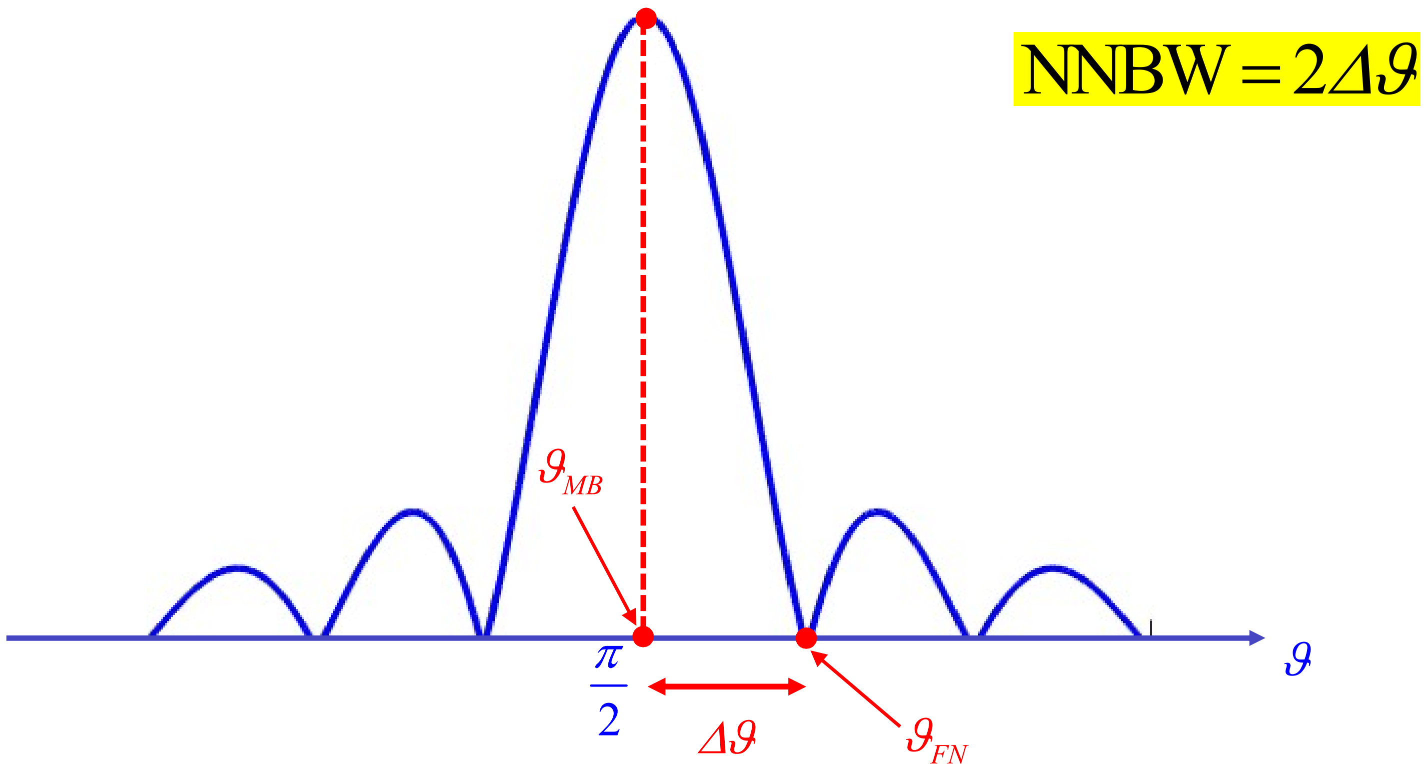
$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

1. **Let's depict F(*u*)** ✓
2. **Let's jump from *u* to ϑ and calculate:**
   - **The direction of the Main Lobe** ✓ $\vartheta_{MB} = \frac{\pi}{2}$
   - **The NNBW / HPBW**
   - **The SLL**
   - **The Directivity**

Main lobe and Beamwidth (NNBW)



Wire antennas: an ideal case

u-domain

$$F(u) = 2L\frac{\sin(uL)}{uL}$$

$$u_{FN} = \frac{\pi}{L}$$

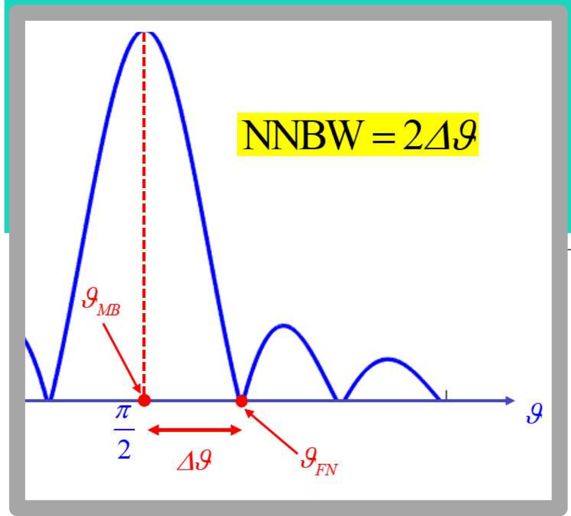
ϑ -domain

$$u = -\beta\cos\vartheta$$

$$u_{FN} = -\beta\cos\vartheta_{FN} = -\beta\cos(\vartheta_{MB} + \Delta\vartheta) \Rightarrow$$

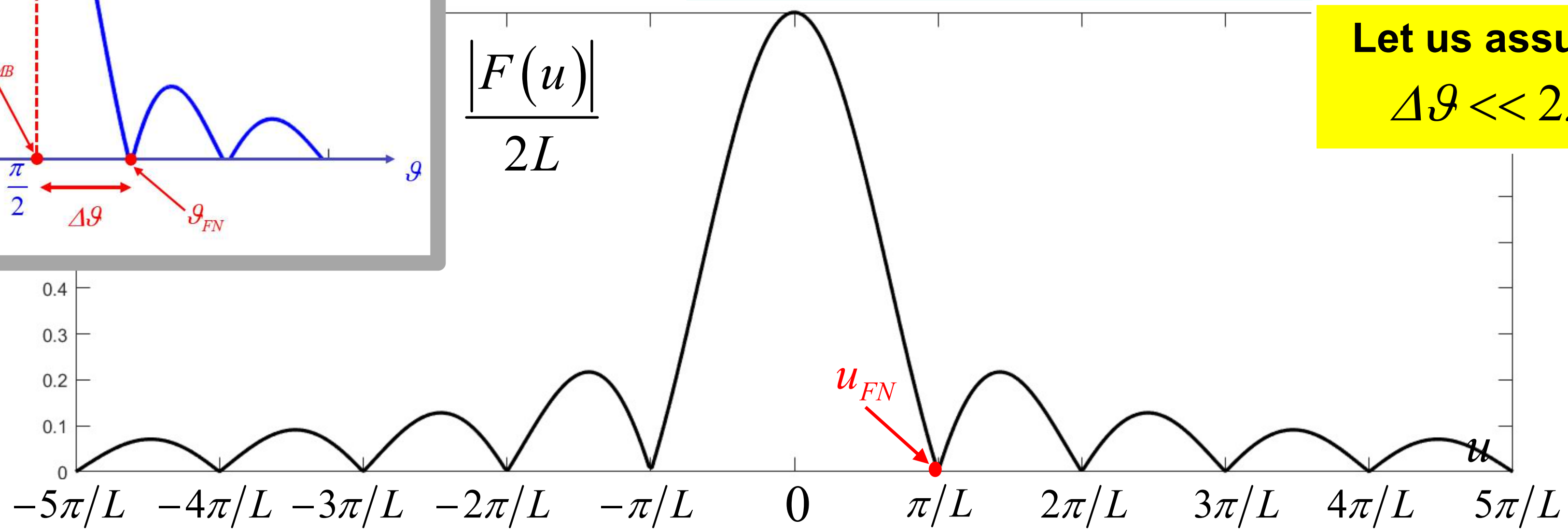
$$\frac{\pi}{L} = -\frac{2\pi}{\lambda}\cos\left(\frac{\pi}{2} + \Delta\vartheta\right) = \frac{2\pi}{\lambda}\sin(\Delta\vartheta) \approx \frac{2\pi}{\lambda}\Delta\vartheta \Rightarrow$$

$$\frac{\pi}{L} \approx \frac{2\pi}{\lambda}\Delta\vartheta \Rightarrow \Delta\vartheta \approx \frac{\lambda}{2L} \Rightarrow \mathrm{NNBW} = 2\Delta\vartheta \approx \frac{\lambda}{L}$$



Let us assume

$$\Delta\vartheta << 2\pi$$



Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \text{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

1. Let's depict F(*u*) ✓

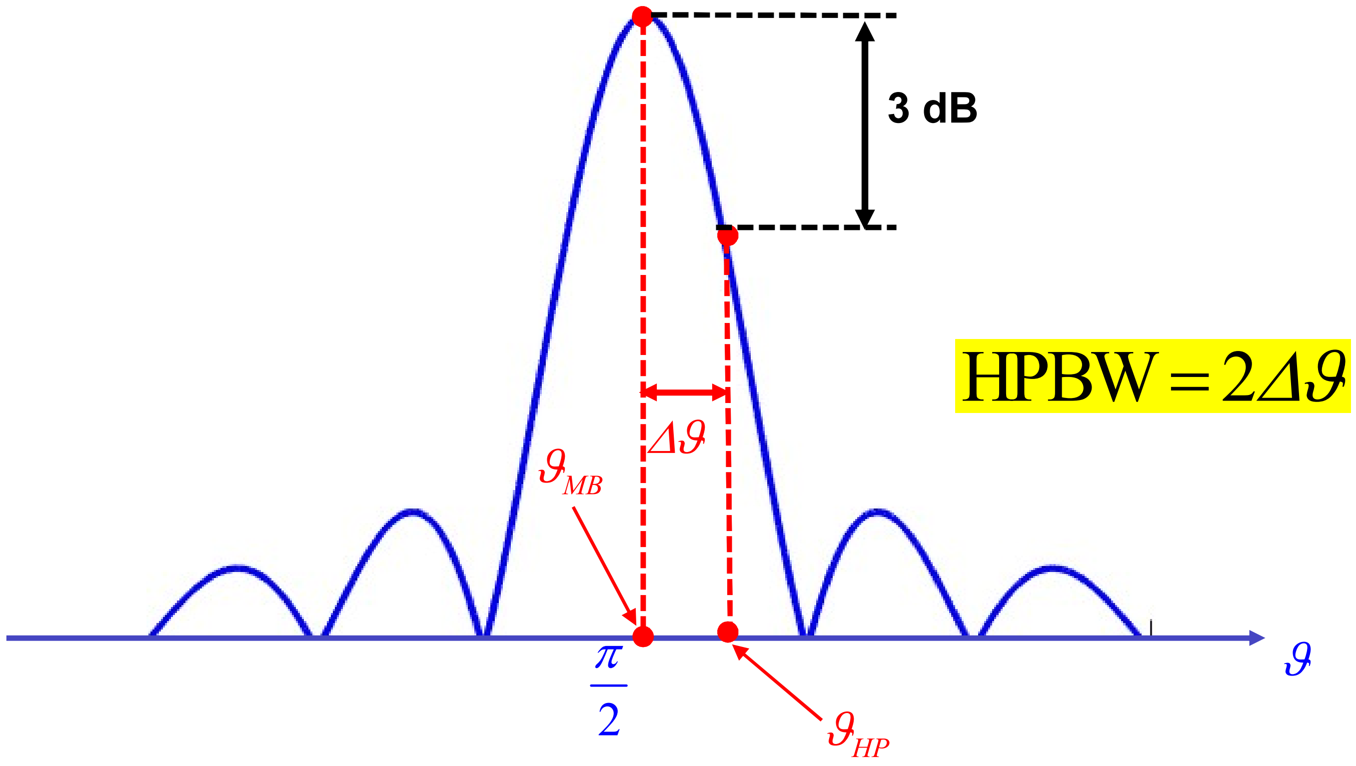
2. Let's jump from *u* to ϑ and calculate:

- **The direction of the Main Lobe** ✓
- **The NNBW / HPBW**
- **The SLL**
- **The Directivity**

$$\vartheta_{MB} = \frac{\pi}{2}$$

$$\text{NNBW} \approx \frac{\lambda}{L}$$

Main lobe and Beamwidth (HPBW)



Wire antennas: an ideal case

u-domain

$$F(u) = 2L\frac{\sin(uL)}{uL}$$

$$u_{HP} = 0.88\frac{\pi}{2L}$$

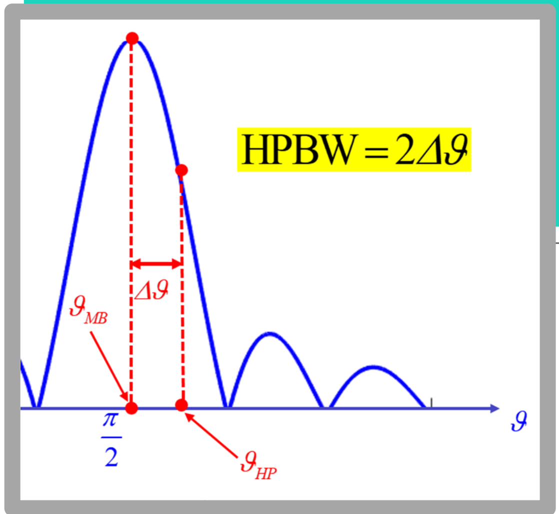
ϑ -domain

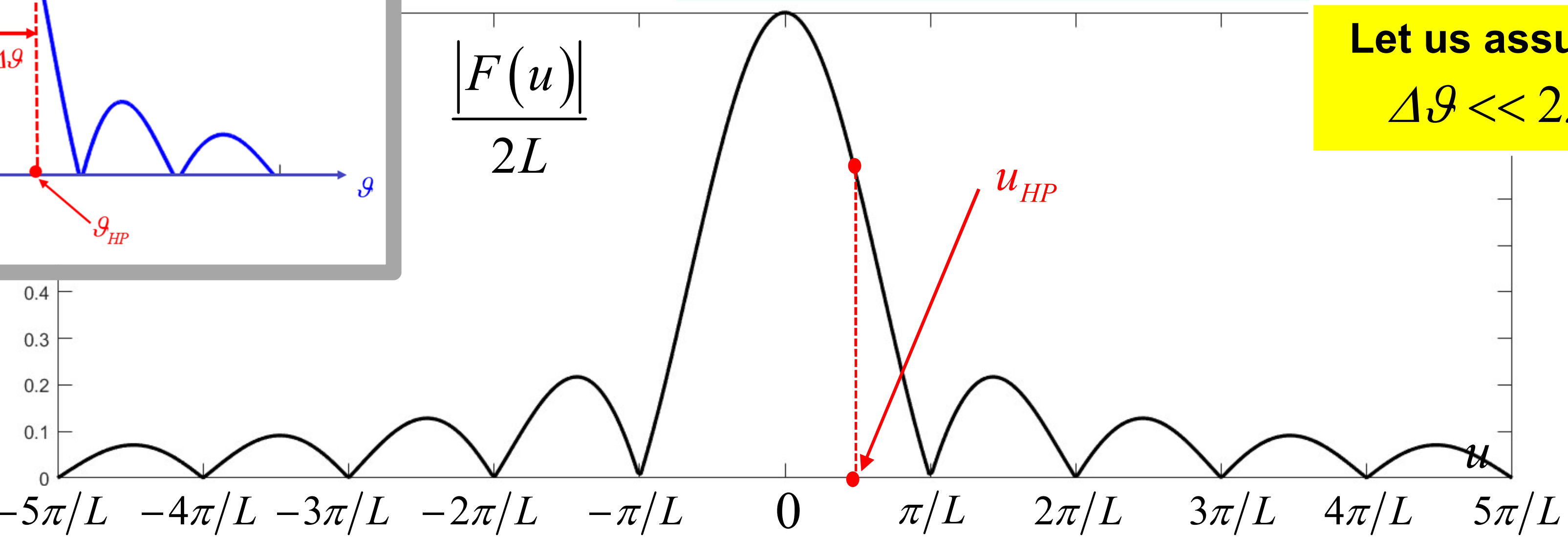
$$u = -\beta\cos\vartheta$$

$$u_{HP} = -\beta\cos\vartheta_{HP} = -\beta\cos(\vartheta_{MB} + \Delta\vartheta) \Rightarrow$$

$$0.88\frac{\pi}{2L} = -\frac{2\pi}{\lambda}\cos\left(\frac{\pi}{2} + \Delta\vartheta\right) = \frac{2\pi}{\lambda}\sin(\Delta\vartheta) \approx \frac{2\pi}{\lambda}\Delta\vartheta$$

$$\Rightarrow \Delta\vartheta \approx 0.88\frac{\lambda}{4L} \Rightarrow \mathrm{HPBW} = 2\Delta\vartheta \approx 0.88\frac{\lambda}{2L}$$





Let us assume

$$\Delta\vartheta << 2\pi$$

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \text{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

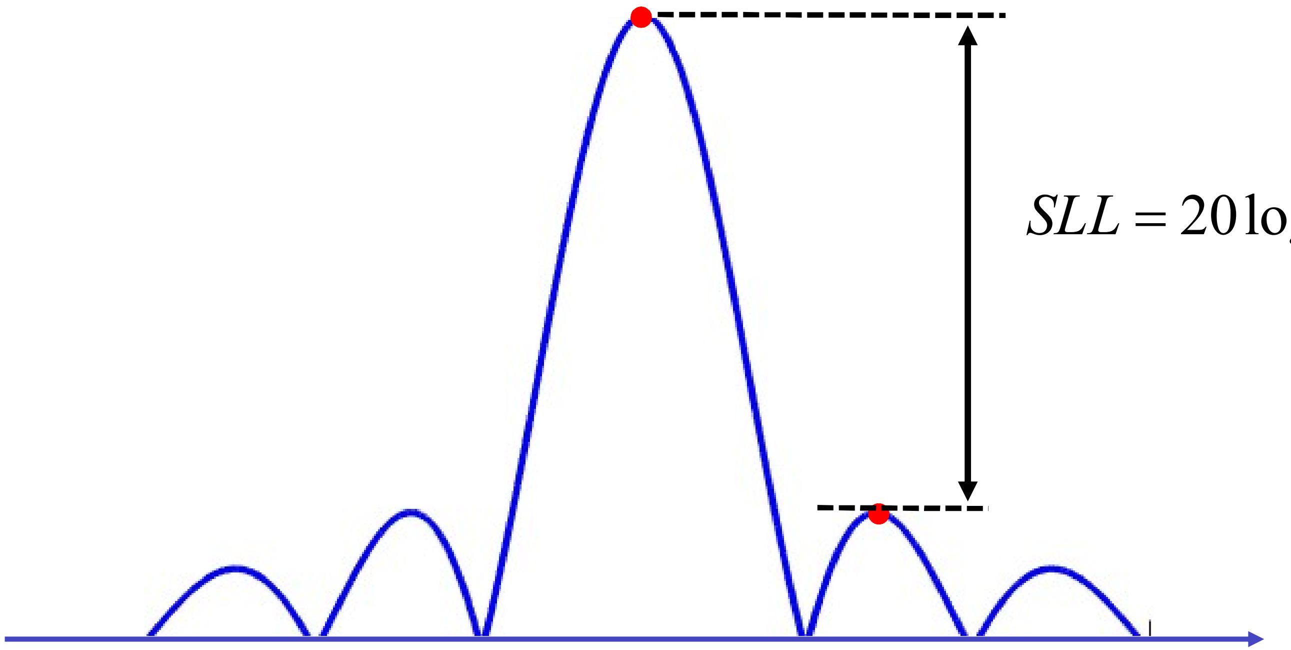
1. **Let's depict F(*u*)** ✓

2. **Let's jump from *u* to ϑ and calculate:**
   - **The direction of the Main Lobe** ✓
   - **The NNBW / HPBW** ✓
   - **The SLL**
   - **The Directivity**

$$\vartheta_{MB} = \frac{\pi}{2}$$

$$\text{NNBW} \approx \frac{\lambda}{L} \qquad \text{HPBW} \approx 0.88\frac{\lambda}{2L}$$

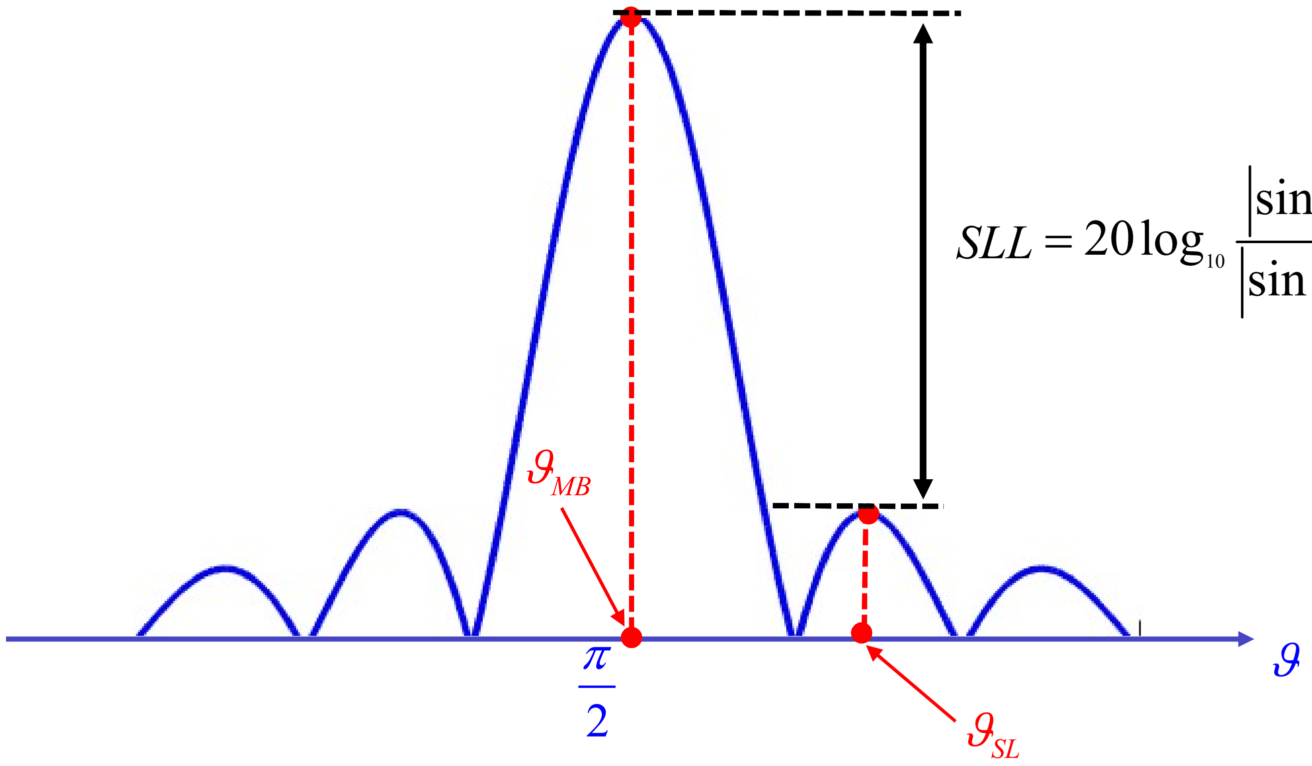
Side Lobe Level (SLL)



$$SLL = 20\log_{10}\frac{\left|\vec{\mathbf{E}}_{SL}\right|}{\left|\vec{\mathbf{E}}_{ML}\right|}$$

Side Lobe Level (SLL)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$



$$SLL = 20\log_{10}\frac{\left|\sin(\vartheta_{SL})F(\vartheta_{SL})\right|}{\left|\sin(\vartheta_{MB})F(\vartheta_{MB})\right|} \approx 20\log_{10}\frac{\left|F(\vartheta_{SL})\right|}{\left|F(\vartheta_{MB})\right|}$$

Let us assume

$$\left|\vartheta_{SL} - \vartheta_{MB}\right| << 2\pi$$

Wire antennas: an ideal case

u-domain

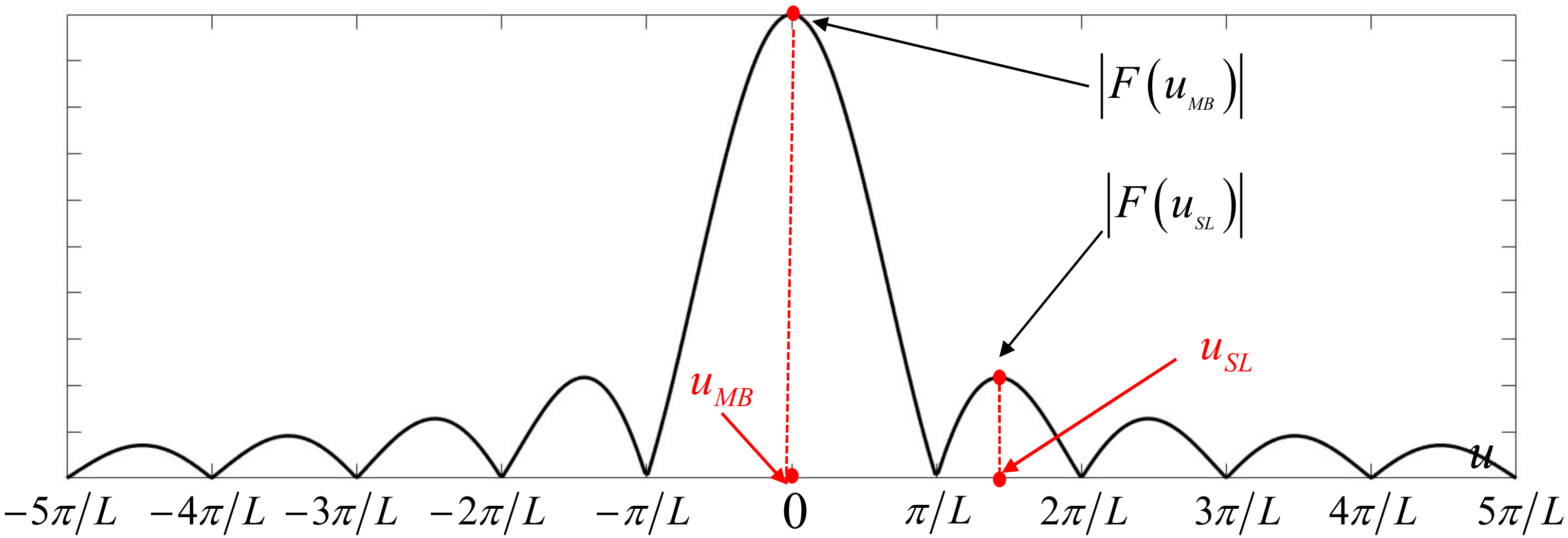
$$F(u) = 2L\frac{\sin(uL)}{uL}$$

$$SLL \approx 20\log_{10}\frac{|F(u_{SL})|}{|F(u_{MB})|}$$

ϑ -domain

$$u = -\beta\cos\vartheta$$

$$SLL \approx 20\log_{10}\frac{|F(\vartheta_{SL})|}{|F(\vartheta_{MB})|}$$



Wire antennas: an ideal case

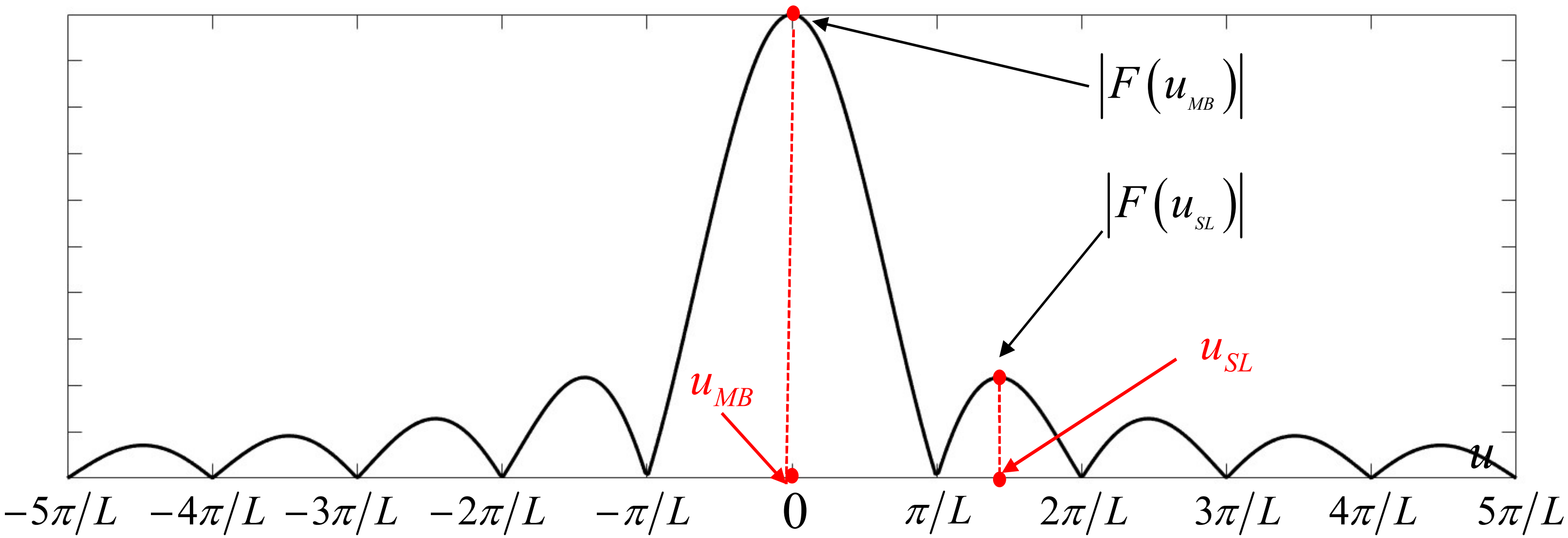
u-domain

$$F(u) = 2L\frac{\sin(uL)}{uL}$$

$$SLL \approx 20\log_{10}\frac{|F(u_{SL})|}{|F(u_{MB})|}$$

$$u_{SL} \approx \frac{3\pi}{2L} \Rightarrow F(u_{SL}) = 2L\frac{\sin\left(\frac{3\pi}{2L}L\right)}{\frac{3\pi}{2L}L} = -2L\frac{2}{3\pi} \Rightarrow SLL \approx 20\log_{10}\frac{|2|}{|3\pi|} = -13.46\,dB$$

$$u_{MB} = 0 \Rightarrow F(u_{MB}) = 2L$$



Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

1. **Let's depict F(*u*)** ✓
2. **Let's jump from *u* to ϑ and calculate:**
   - **The direction of the Main Lobe** ✓
   - **The NNBW / HPBW** ✓
   - **The SLL** ✓
   - **The Directivity**

$$\vartheta_{MB} = \frac{\pi}{2}$$

$$\mathrm{NNBW} \approx \frac{\lambda}{L} \quad \mathrm{HPBW} \approx 0.88\frac{\lambda}{2L}$$

$$\mathrm{SLL} = -13.46\, dB$$

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

1. **Let's depict F(*u*)** ✓

2. **Let's jump from *u* to ϑ and calculate:**
   - **The direction of the Main Lobe** ✓ $\quad \vartheta_{MB} = \frac{\pi}{2}$
   - **The NNBW / HPBW** ✓ $\quad \mathrm{NNBW} \approx \frac{\lambda}{L} \quad \mathrm{HPBW} \approx 0.88\frac{\lambda}{2L}$
   - **The SLL** ✓ $\quad \mathrm{SLL} = -13.46\, dB$
   - **The Directivity**

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics