Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Università degli Studi di Napoli “Parthenope”

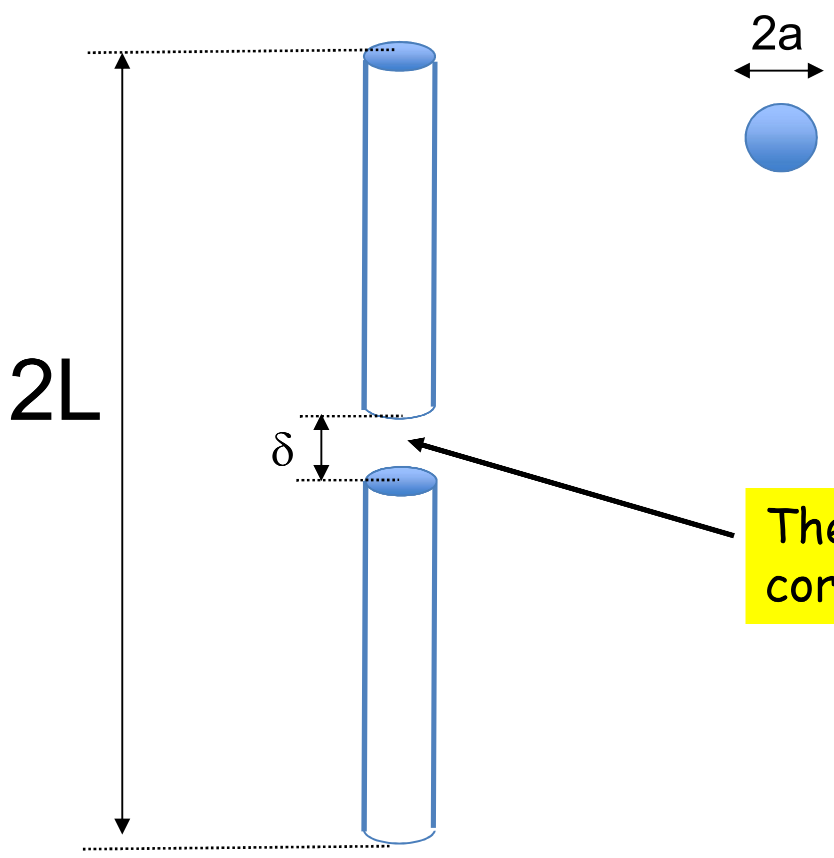
a.a. 2023–2024 – Laurea “Triennale” – Secondo semestre – Terzo anno

Ing. Stefano Perna

Wire antennas

Wire antennas

Wire antennas



$2a << \lambda$

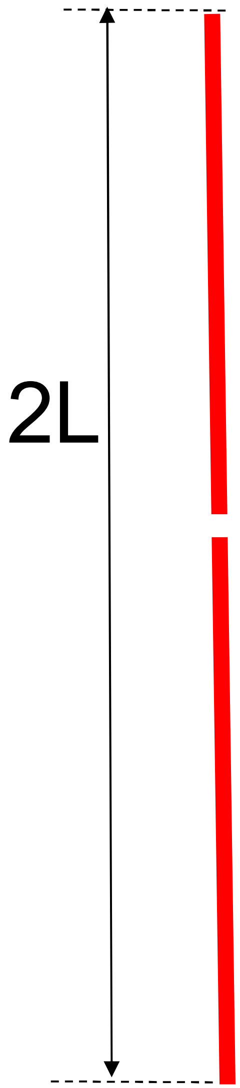
$2a << L$

$\delta << \lambda$

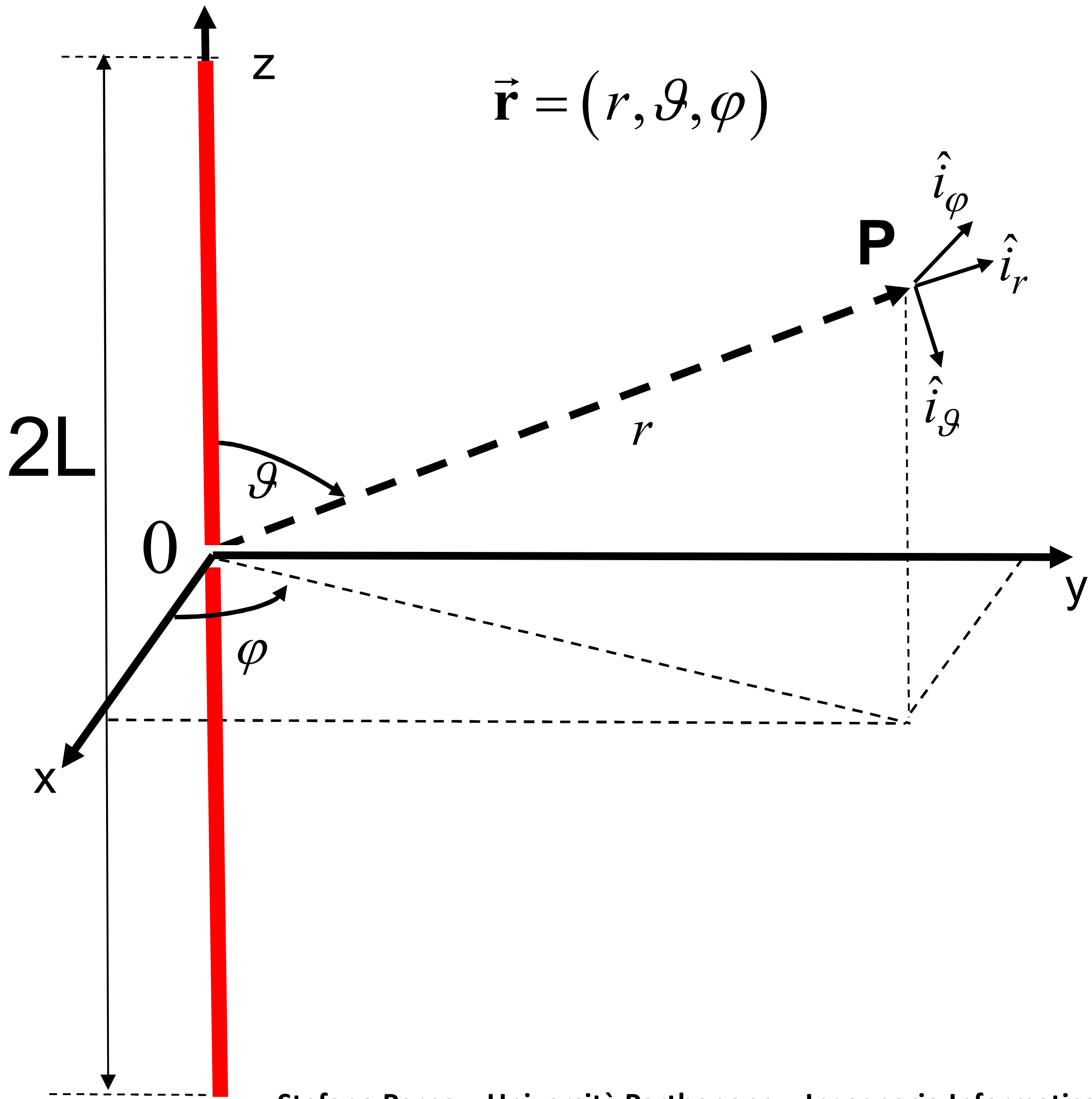
$\delta << L$

The terminals are located in correspondence of this gap

Wire antennas



Wire antennas



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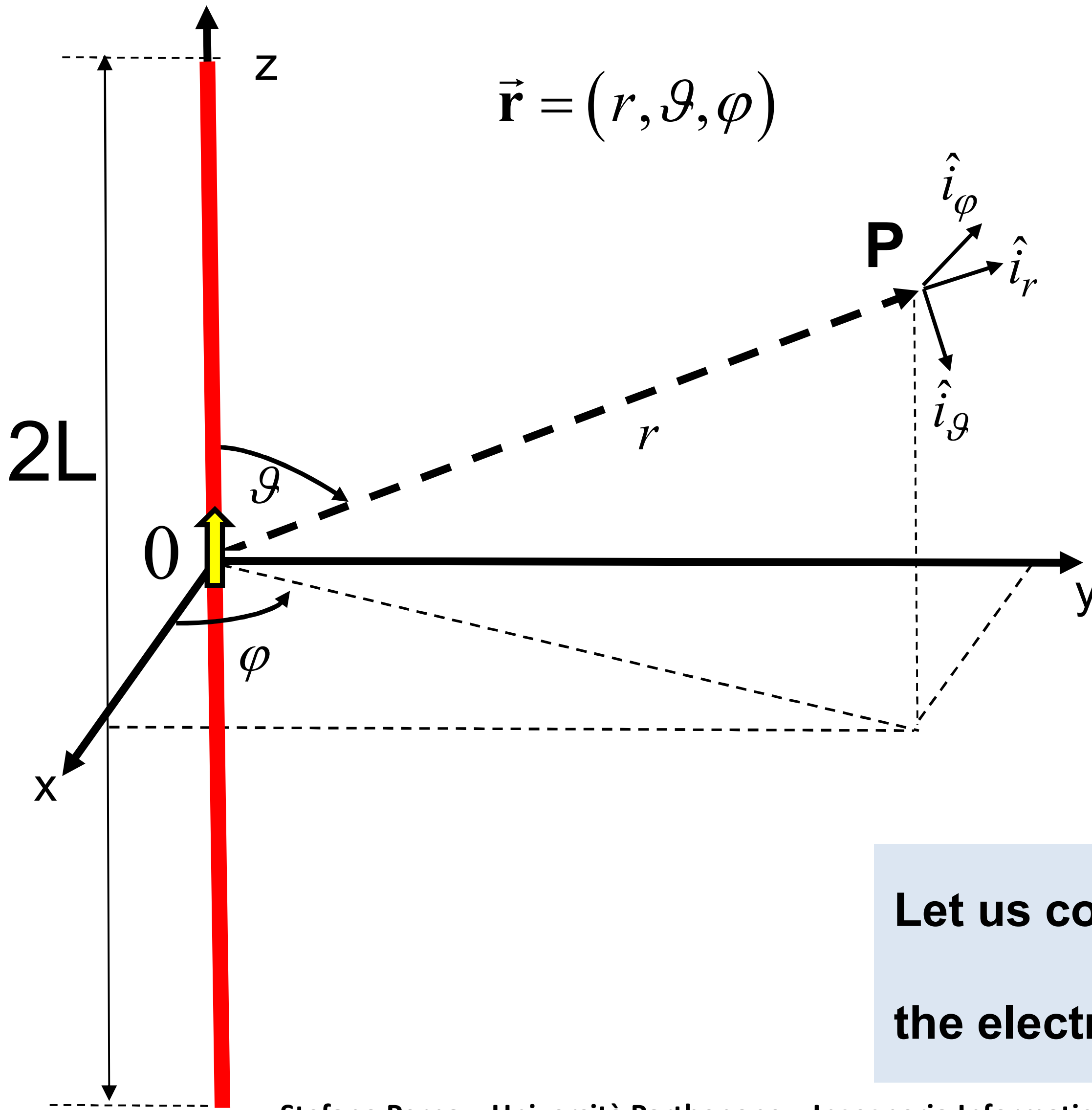
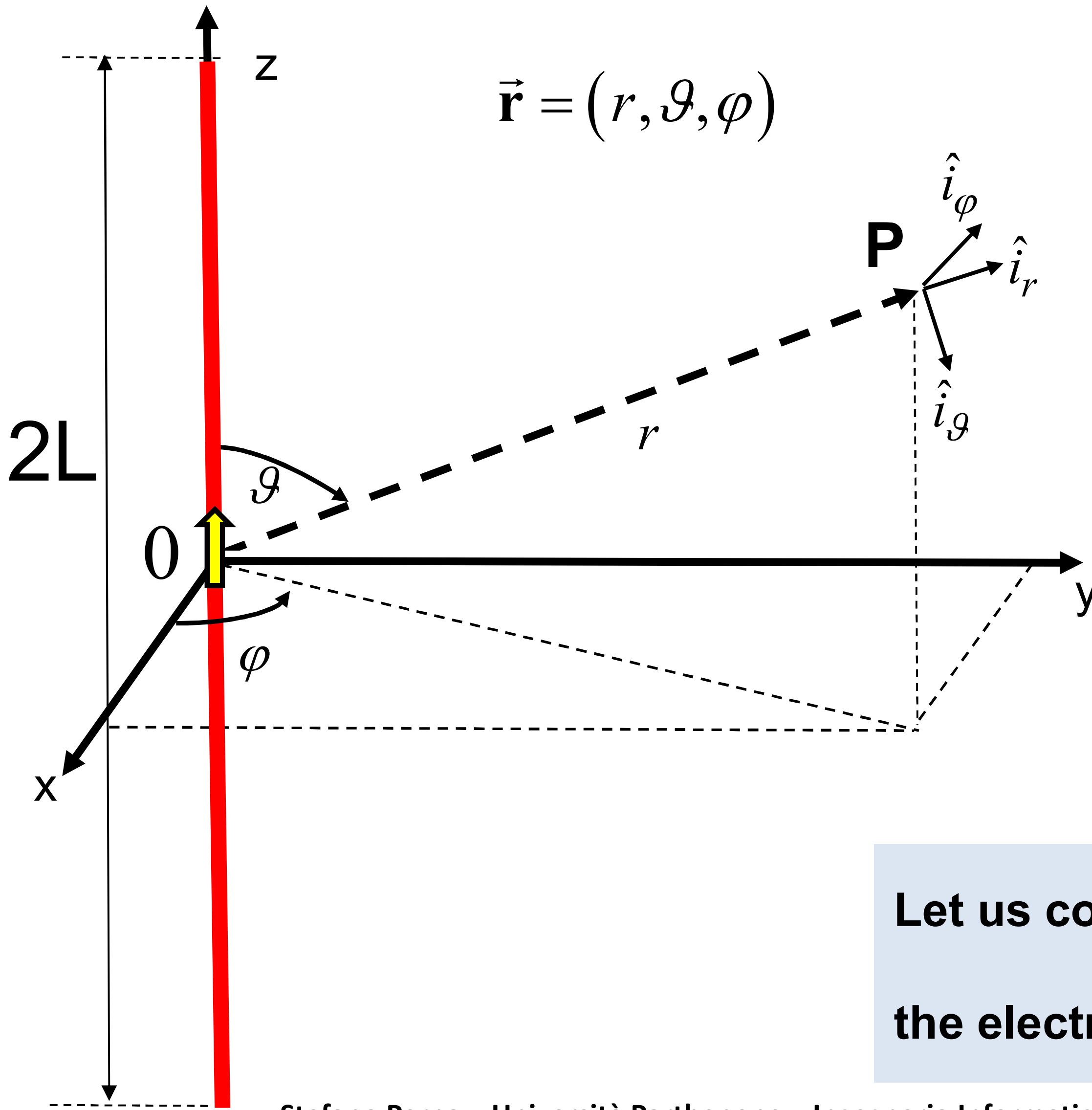


Let us consider the elementary source:

the electric dipole ⇧ located in the origin

Wire antennas



$$\vec{\mathbf{E}}_0 = j\frac{\zeta I_0}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\Delta z \sin\vartheta_0\, \hat{i}_{\vartheta_0}$$

$$\vartheta_0 = \vartheta; \quad R_0 = r; \quad \hat{i}_\vartheta = \hat{i}_{\vartheta_0}$$

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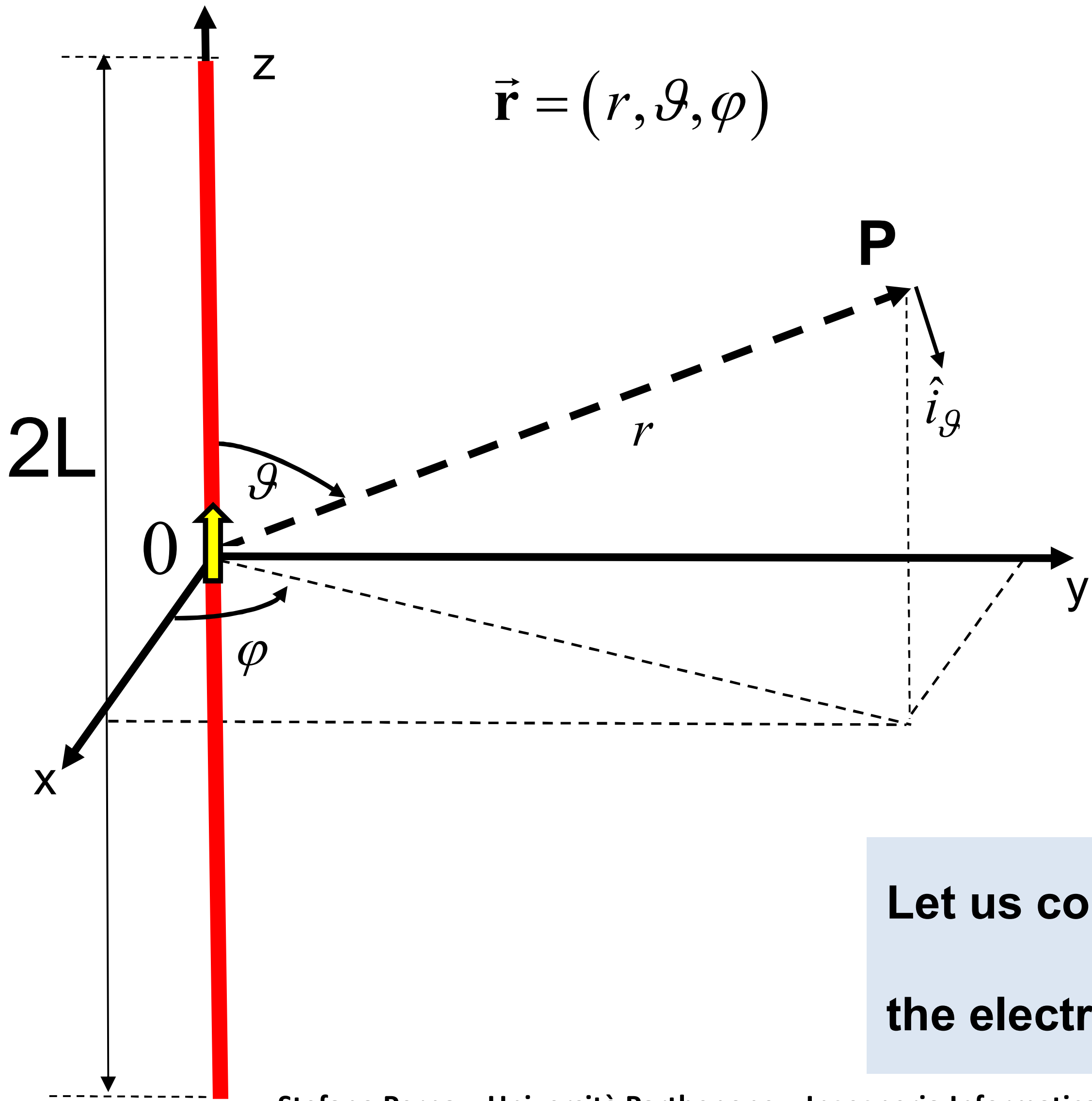
Wire antennas

$$\vec{\mathbf{r}} = \left(r, \vartheta, \varphi\right)$$



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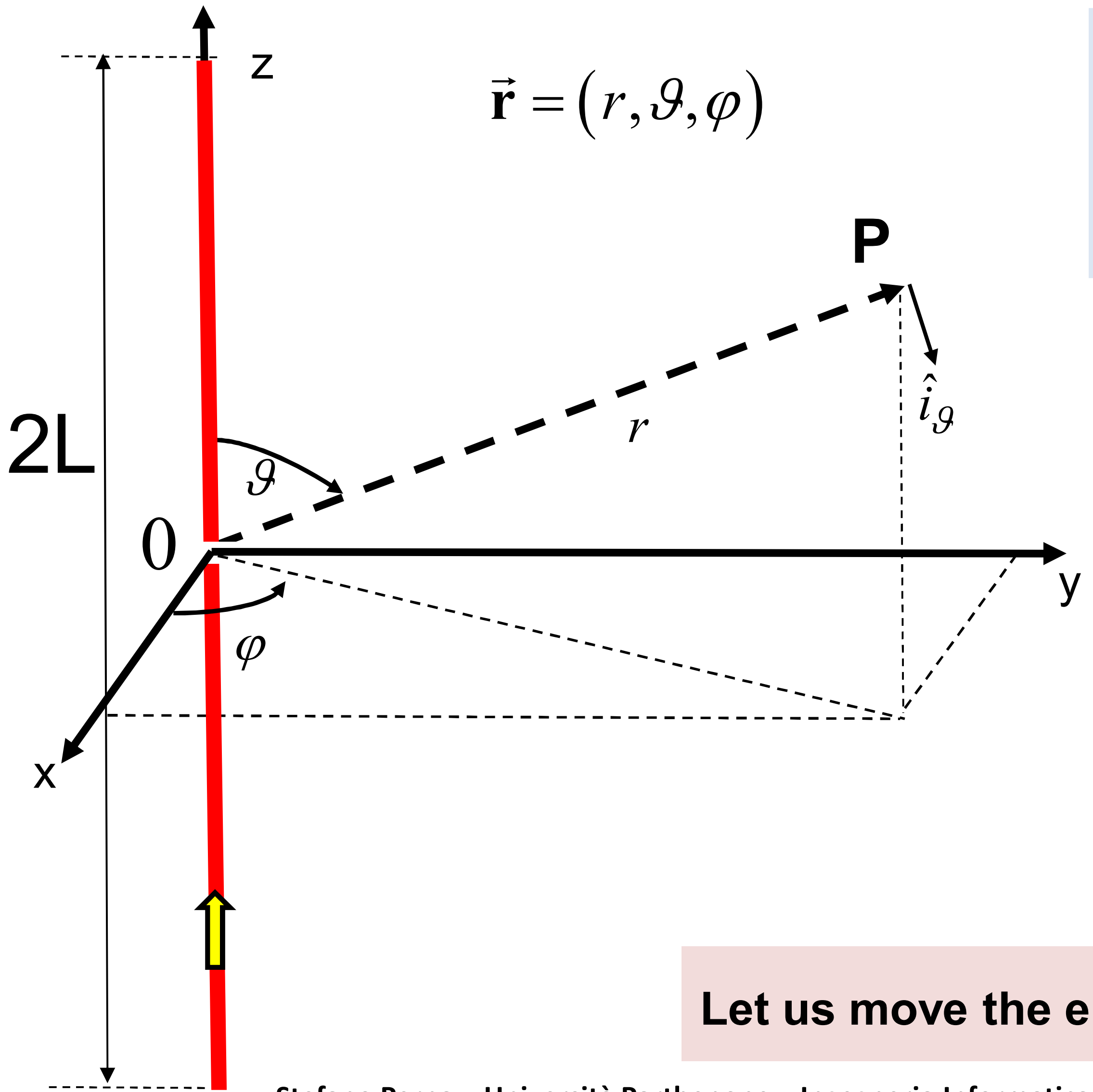

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Let us move the electric dipole ⇧ along the z-axis

Wire antennas

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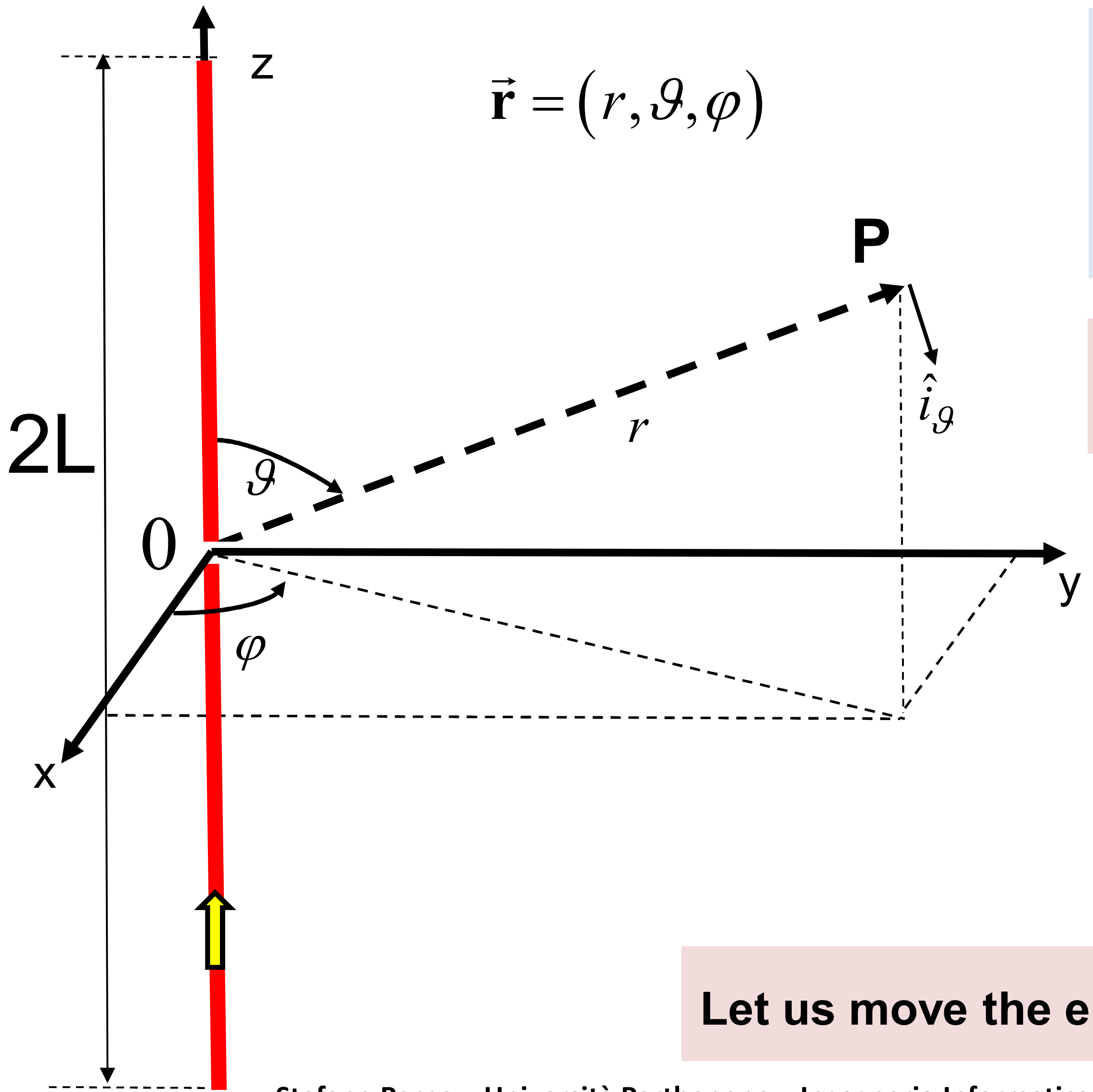

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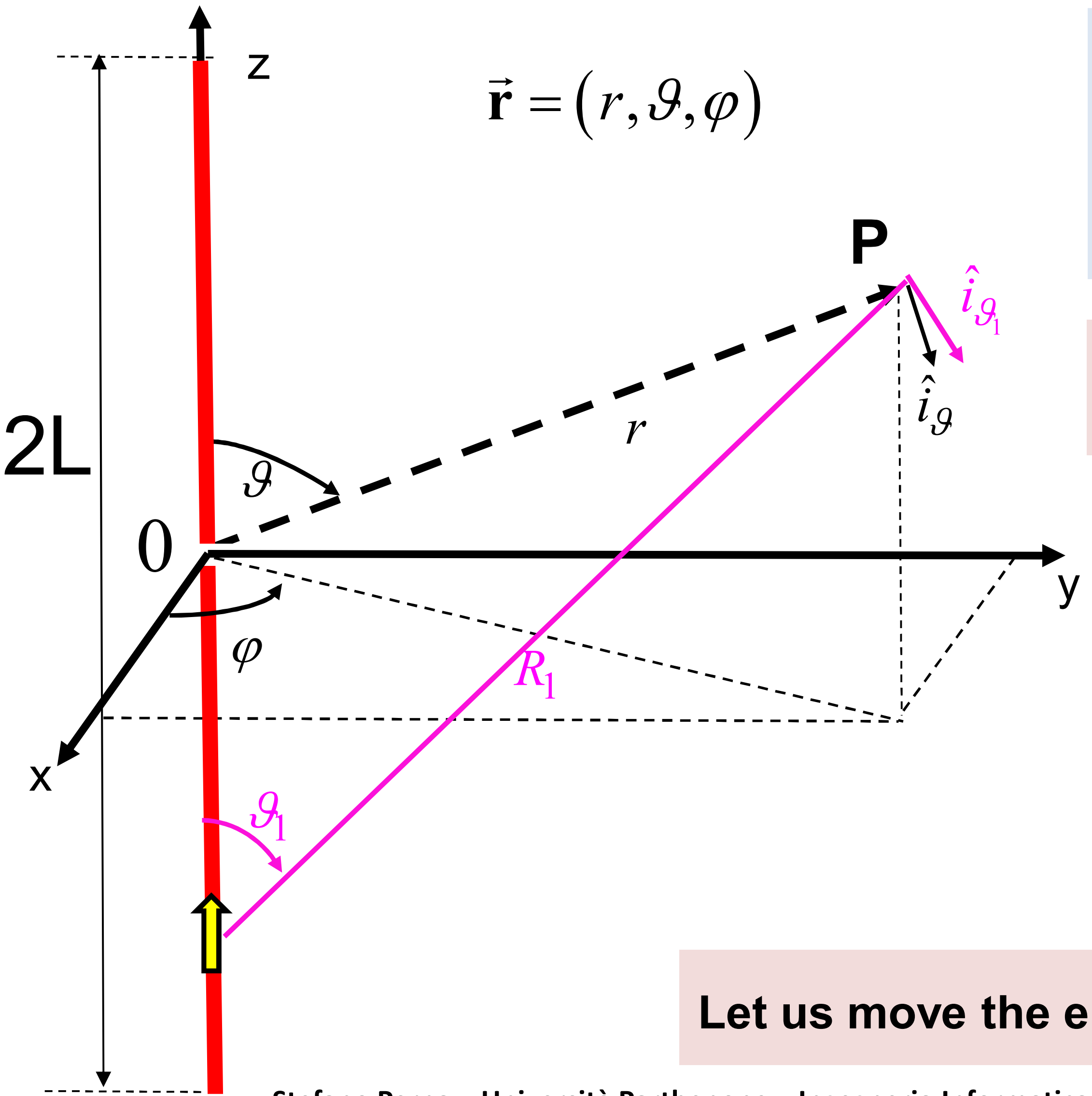
Let us move the electric dipole ⇑ along the z-axis

Wire antennas



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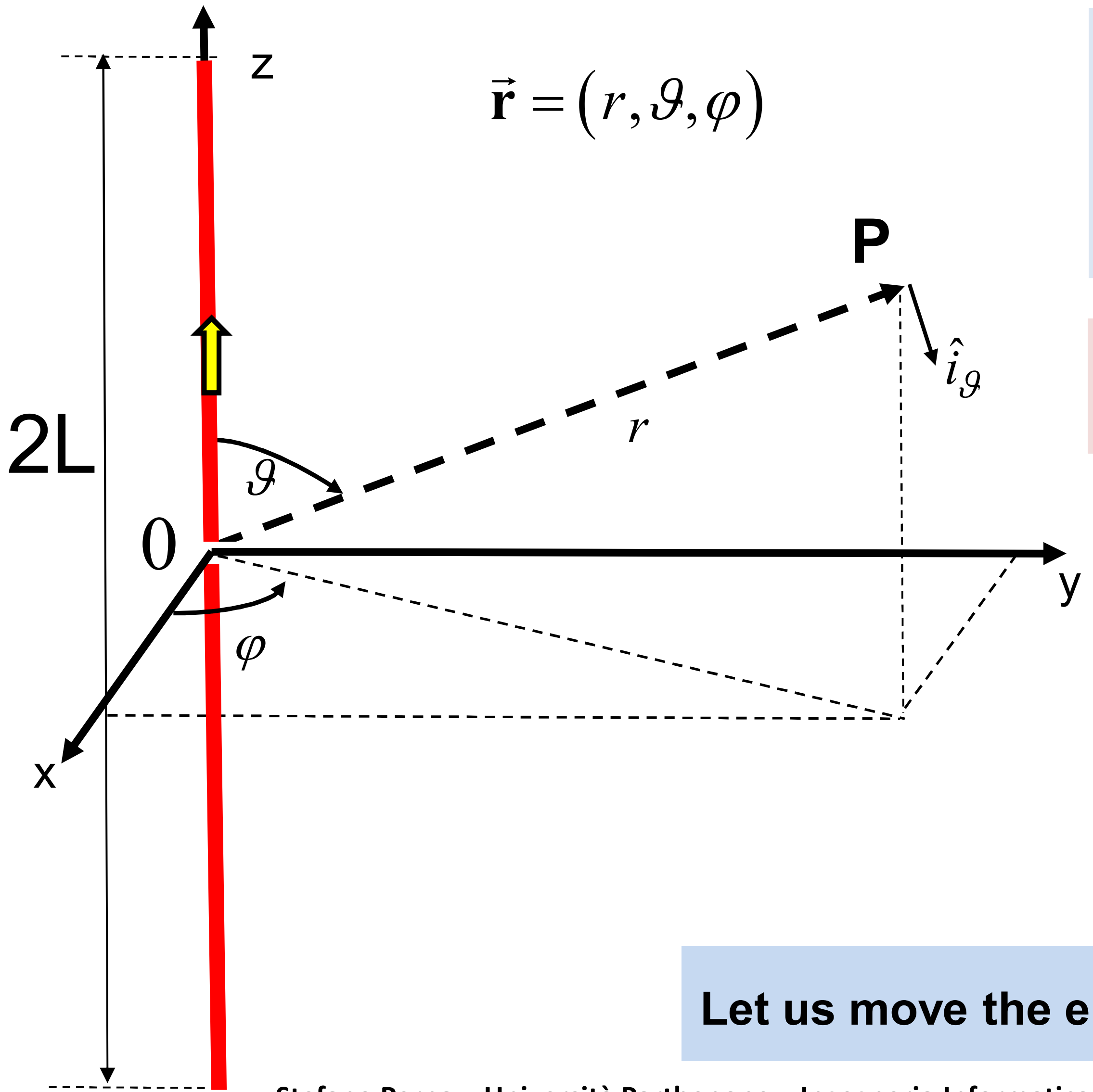

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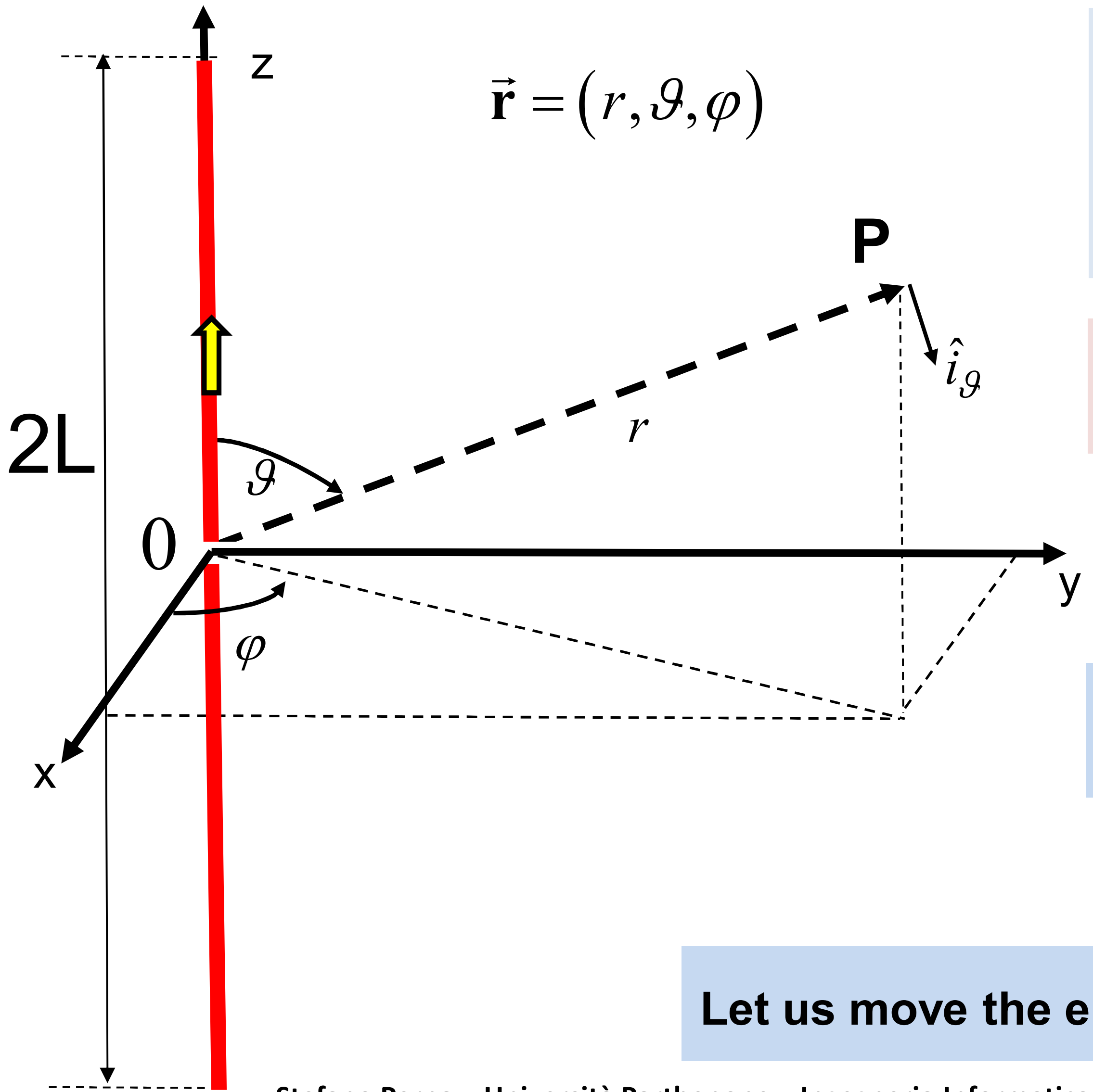
Wire antennas



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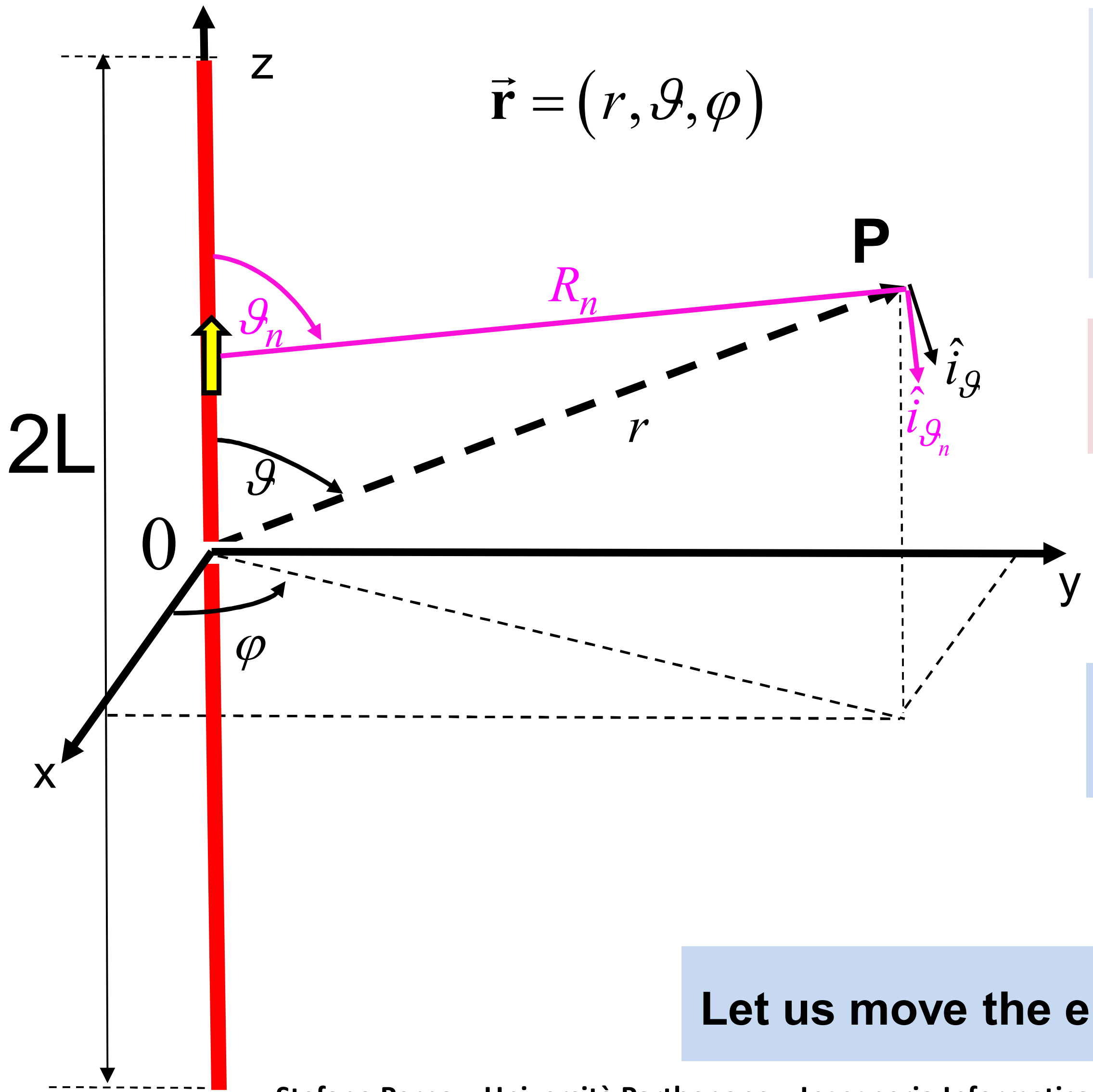
Wire antennas



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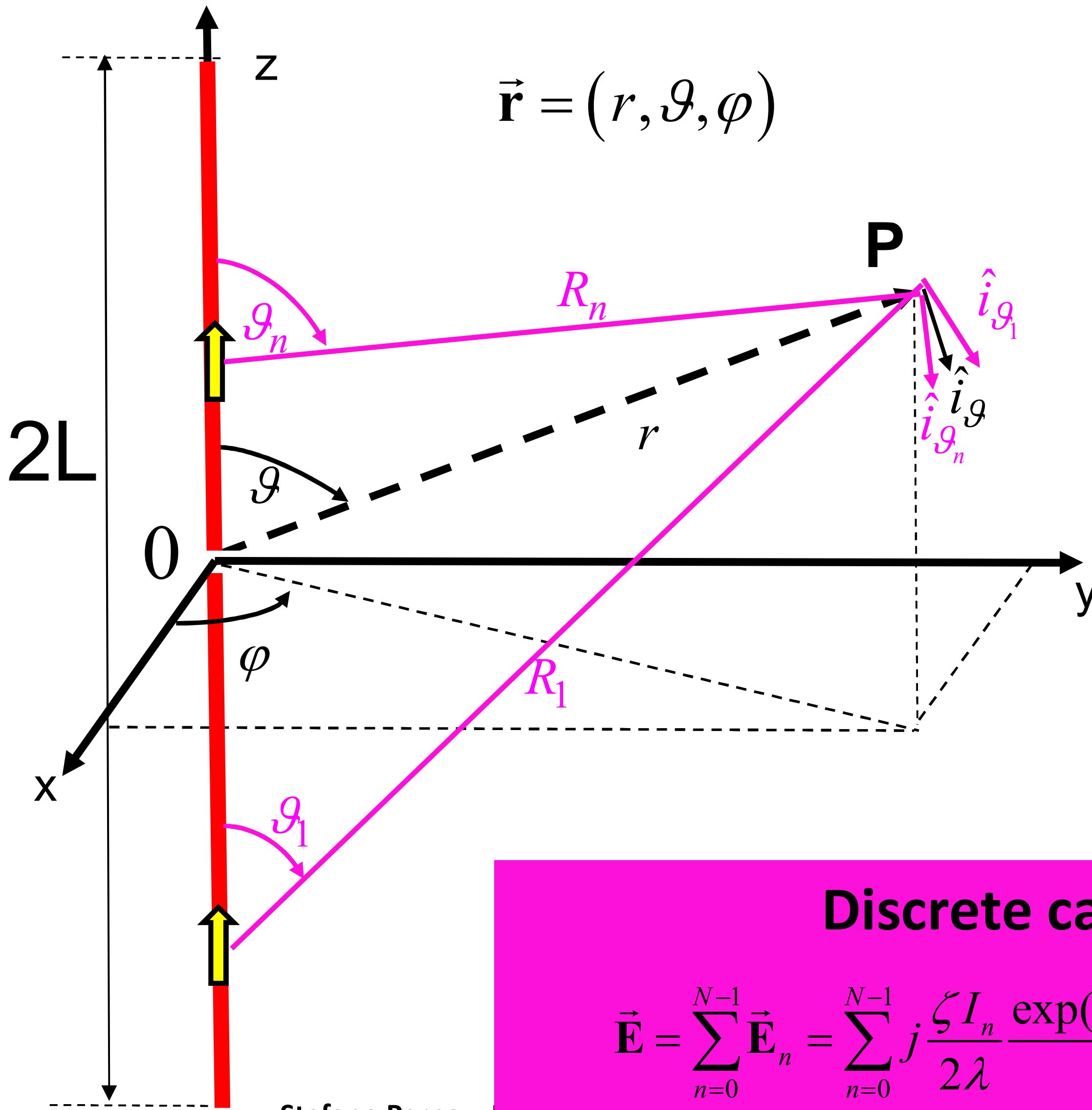

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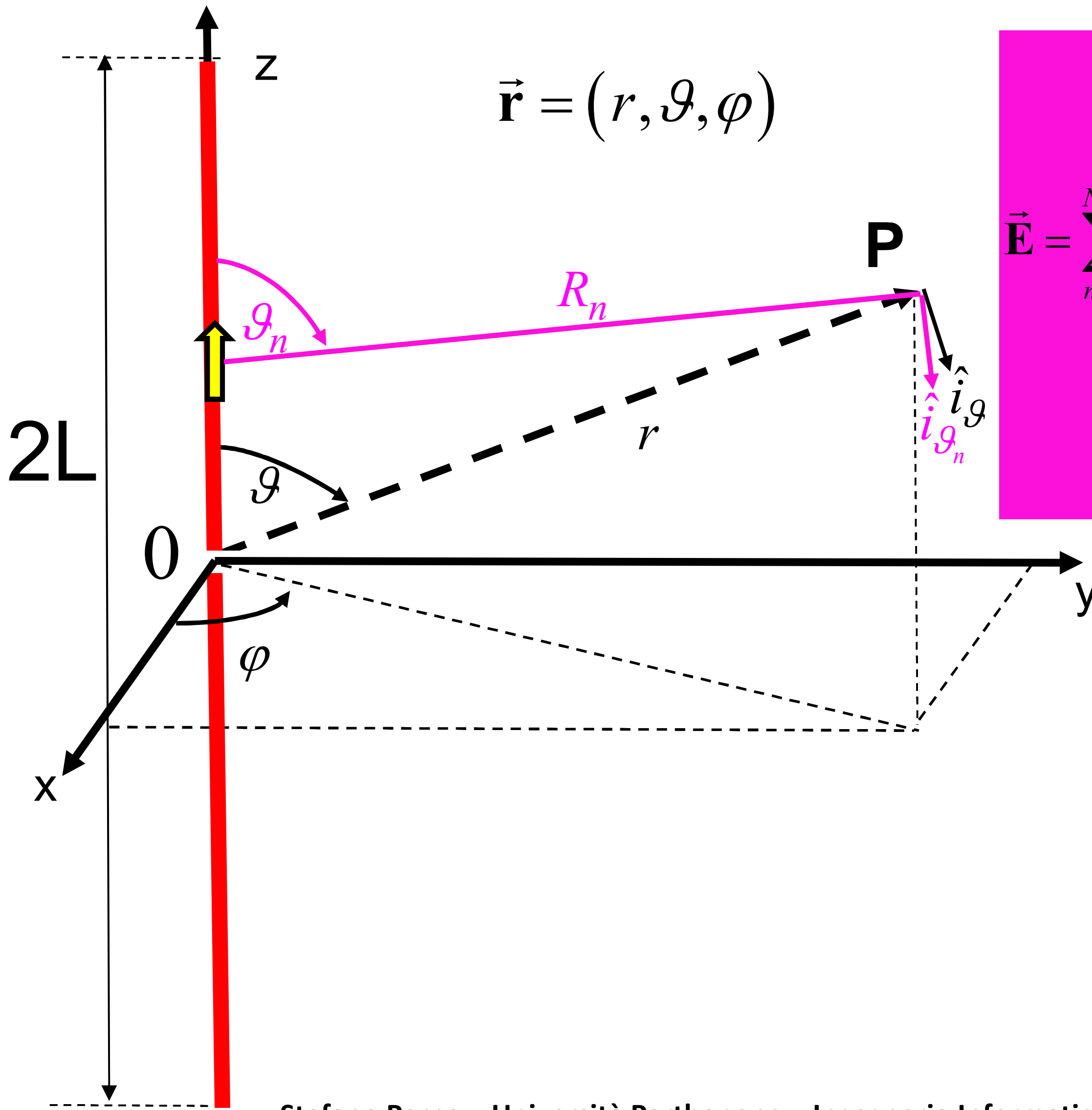
Discrete case

$$\vec{\mathbf{E}} = \sum_{n=0}^{N-1}\vec{\mathbf{E}}_n = \sum_{n=0}^{N-1} j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\Delta z \sin\vartheta_n\, \hat{i}_{\vartheta_n}$$

Wire antennas



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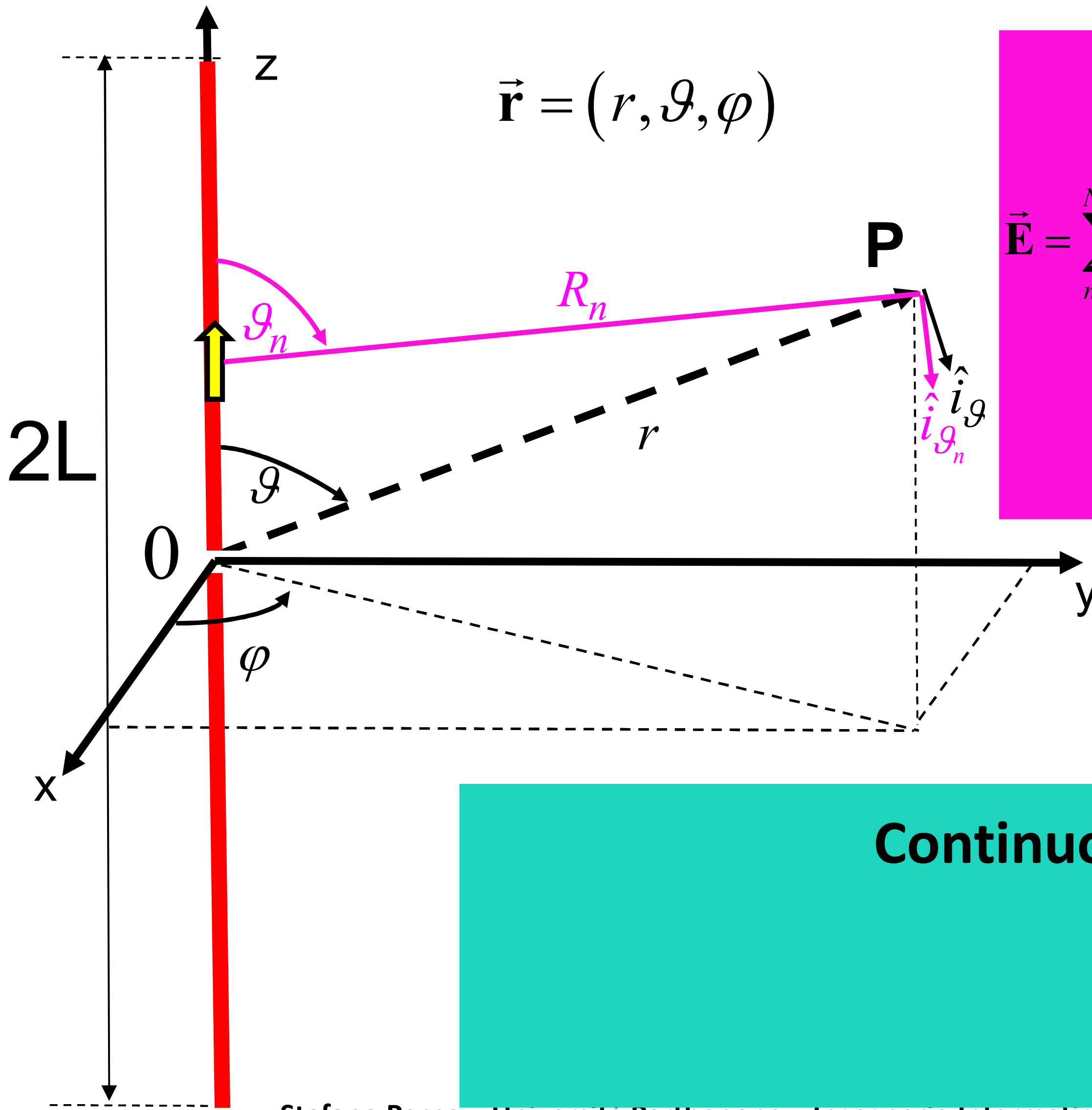
Wire antennas



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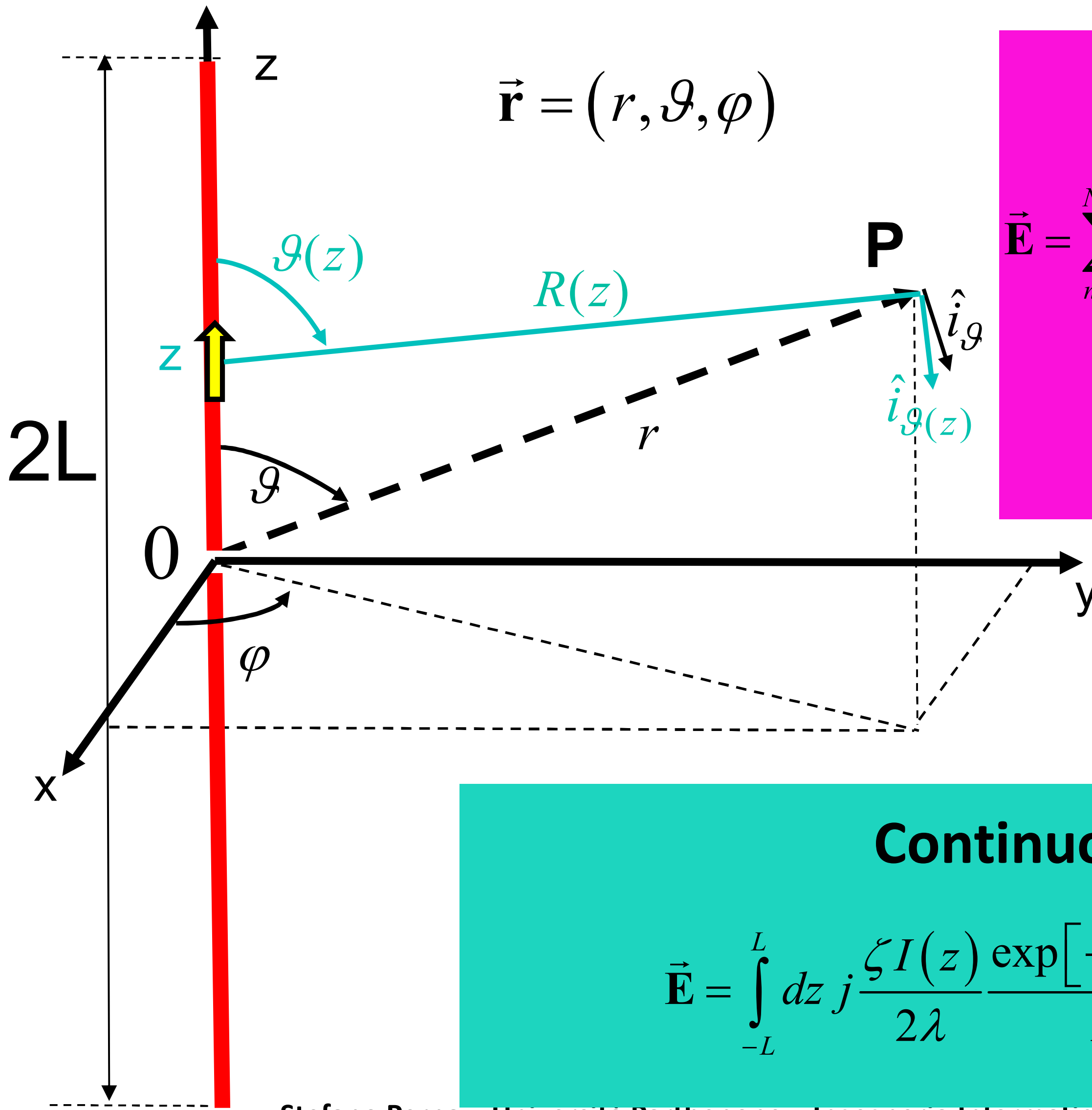
Continuous case

Wire antennas



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$$\vec{\mathbf{E}}_n = j \frac{\zeta I_n}{2\lambda} \frac{\exp(-j\beta R_n)}{R_n} \Delta z \sin\vartheta_n \, \hat{i}_{\vartheta_n}$$

Continuous case

$$\vec{\mathbf{E}} = \int_{-L}^{L} dz \, j \frac{\zeta I(z)}{2\lambda} \frac{\exp\left[-j\beta R(z)\right]}{R(z)} \sin\vartheta(z) \hat{i}_{\vartheta(z)}$$

Wire antennas

$$\vec{\mathbf{E}} = \int_{-L}^{L} dz\, j \frac{\zeta I(z)}{2\lambda} \frac{\exp\left[-j\beta R(z)\right]}{R(z)} \sin\vartheta(z)\, \hat{i}_{\vartheta(z)} = j\frac{\zeta}{2\lambda} \int_{-L}^{L} dz\, I(z) \frac{\exp\left[-j\beta R(z)\right]}{R(z)} \sin\vartheta(z)\, \hat{i}_{\vartheta(z)}$$

Wire antennas

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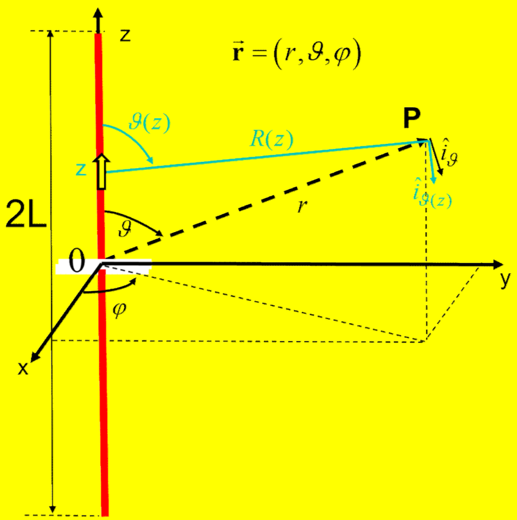
$$= j\frac{\zeta}{2\lambda}\int_{-L}^{L} dz\, I(z)\frac{\exp[-j\beta r]\exp(j\beta z\cos\vartheta)}{r}\sin\vartheta\,\hat{i}_{\vartheta}$$

Let us suppose that P is located in the Fraunhofer Region relevant to the considered wire antenna



- $R(z) \approx r - \vec{\mathbf{r}}' \cdot \hat{i}_r = r - z\hat{i}_z \cdot \hat{i}_r = r - z\cos\vartheta$

  $$\Rightarrow \frac{\exp[-j\beta R(z)]}{R(z)} \approx \frac{\exp[-j\beta r]\exp[j\beta z\cos\vartheta]}{r}$$

- $\vartheta(z) \approx \vartheta$

- $\hat{i}_{\vartheta(z)} \approx \hat{i}_{\vartheta}$

Wire antennas

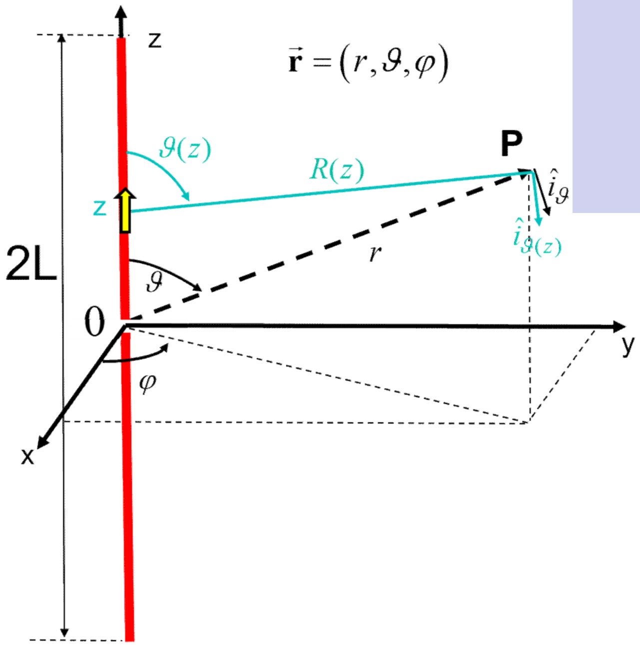
$$\vec{\mathbf{E}} = \int_{-L}^{L} dz\, j \frac{\zeta I(z)}{2\lambda} \frac{\exp\left[-j\beta R(z)\right]}{R(z)} \sin\vartheta(z)\, \hat{i}_{\vartheta(z)} = j\frac{\zeta}{2\lambda}\int_{-L}^{L} dz\, I(z) \frac{\exp\left[-j\beta R(z)\right]}{R(z)} \sin\vartheta(z)\, \hat{i}_{\vartheta(z)}$$

$$= j\frac{\zeta}{2\lambda}\int_{-L}^{L} dz\, I(z) \frac{\exp[-j\beta r]\exp(j\beta z\cos\vartheta)}{r} \sin\vartheta\, \hat{i}_{\vartheta}$$

$$= j\frac{\zeta}{2\lambda} \frac{\exp[-j\beta r]}{r} \sin\vartheta\, \hat{i}_{\vartheta} \left[\int_{-L}^{L} dz\, I(z) \exp(j\beta z\cos\vartheta)\right]$$

$$= j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta\, \hat{i}_{\vartheta} \left[\int_{-L}^{L} dz\, \frac{I(z)}{I_0} \exp(j\beta z\cos\vartheta)\right]$$

Wire antennas



In the Fraunhofer Region the expression of the radiated field simplifies as

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta\, \hat{i}_\vartheta \int_{-l}^{l} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta)$$

Color legend

New formulas, important considerations,
important formulas, important concepts

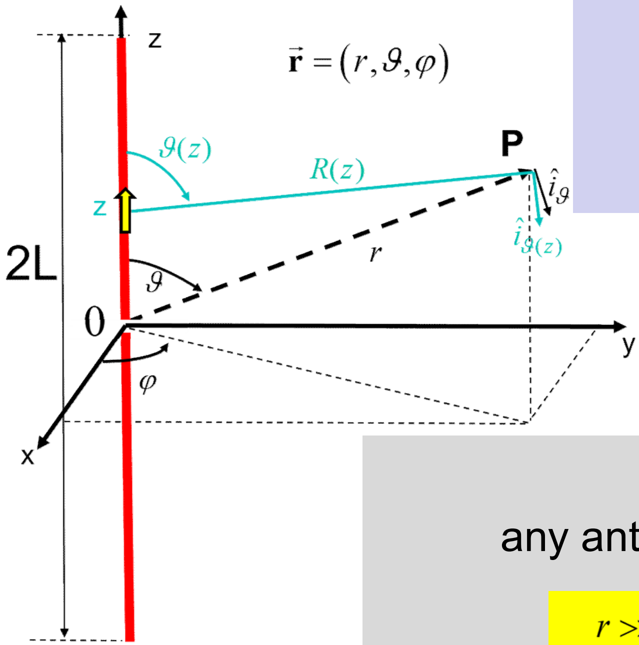
Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Wire antennas



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$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta \left[\int_{-L}^{L} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta) \right] \hat{i}_\vartheta$$

Effective length of the wire antenna

.... Memo

any antenna, in the Fraunhofer region, behaves as follows

$r >> D$

$r > \frac{2D^2}{\lambda}$

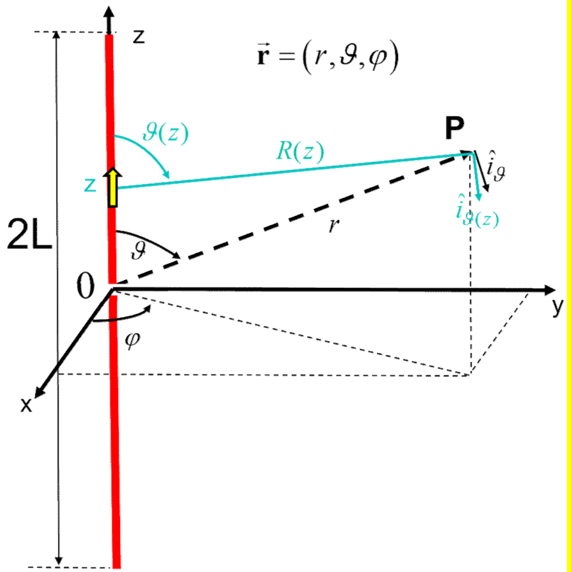
$r >> \lambda$

$$\begin{cases} \mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi) \\ \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E} \end{cases}$$

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$ **Effective length**

Wire antennas: effective length

$$\vec{\mathbf{l}} = \sin\vartheta \left[\int_{-L}^{L} dz \frac{I(z)}{I_0} \exp\left(j\beta z \cos\vartheta \right) \right] \hat{i}_\vartheta$$

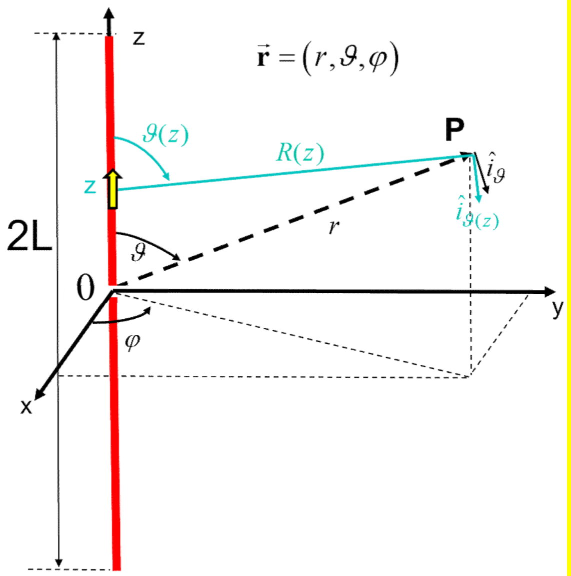


- **The effective length is independent of φ**

  … absolutely not surprising

Wire antennas: effective length

$$\vec{\mathbf{l}}(\vartheta) = l_\vartheta(\vartheta)\hat{i}_\vartheta = \sin\vartheta \left[\int_{-L}^{L} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta) \right] \hat{i}_\vartheta$$



- **The effective length is independent of φ**

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- **The effective length depends by the current distribution *I*(z)**

  … absolutely not surprising

- **The effective length depends on L**

  … absolutely not surprising

Wire antennas: effective length

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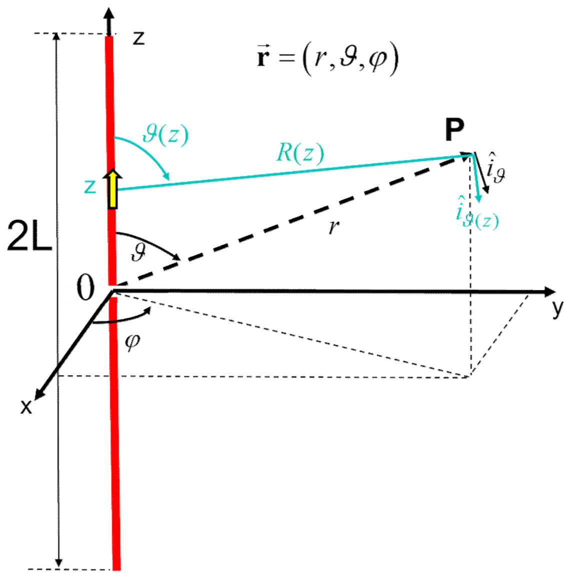

$$u = -\beta\cos\vartheta \qquad \tilde{I}(z) = \frac{I(z)}{I_0}$$

$$F(u) = \int_{-L}^{L} dz\,\tilde{I}(z)\,e^{-juz}$$

For the wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rule

Wire antennas: effective length

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

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$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

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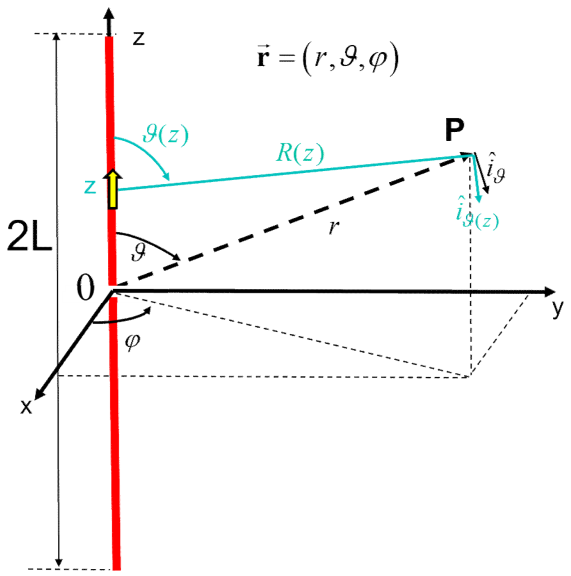

The properties of the Fourier Transformation suggest some interesting considerations

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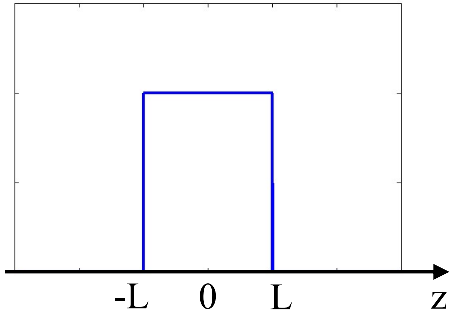
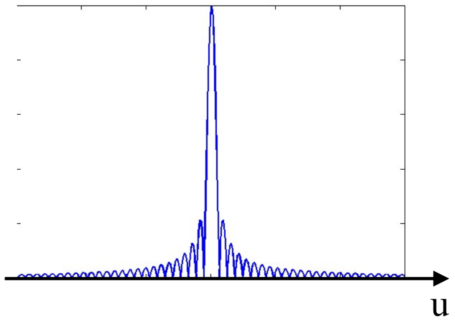
$\frac{I(z)}{I_0}$



$|F(u)|$



The properties of the Fourier Transformation suggest some interesting considerations

- **Antenna's size and beamwidth**

Wire antennas: effective length

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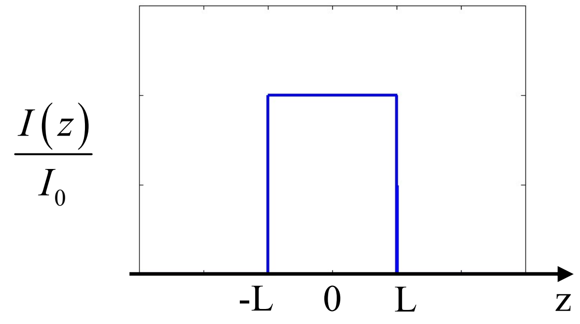

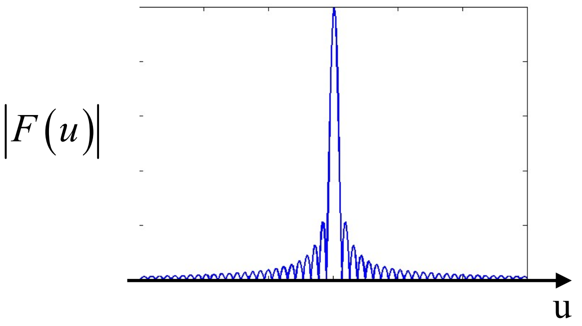

The properties of the Fourier Transformation suggest some interesting considerations

- **Antenna's size and beamwidth**
- **Scanning of the pattern**
- **Synthesis of the pattern**

Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

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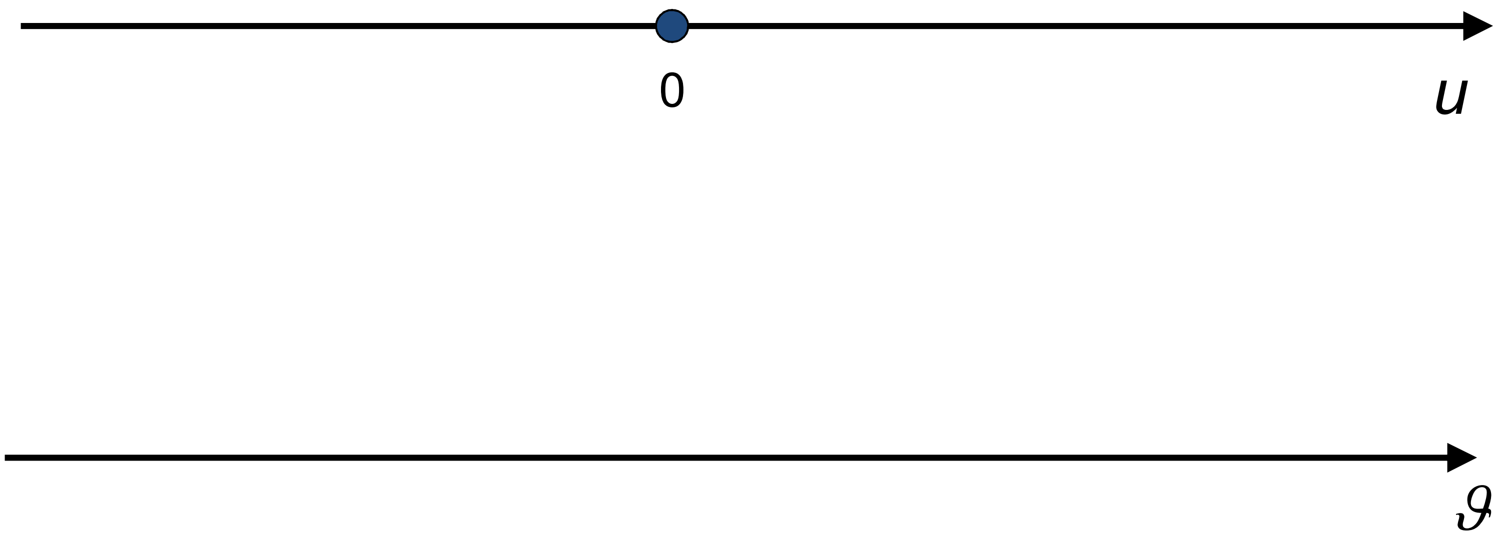

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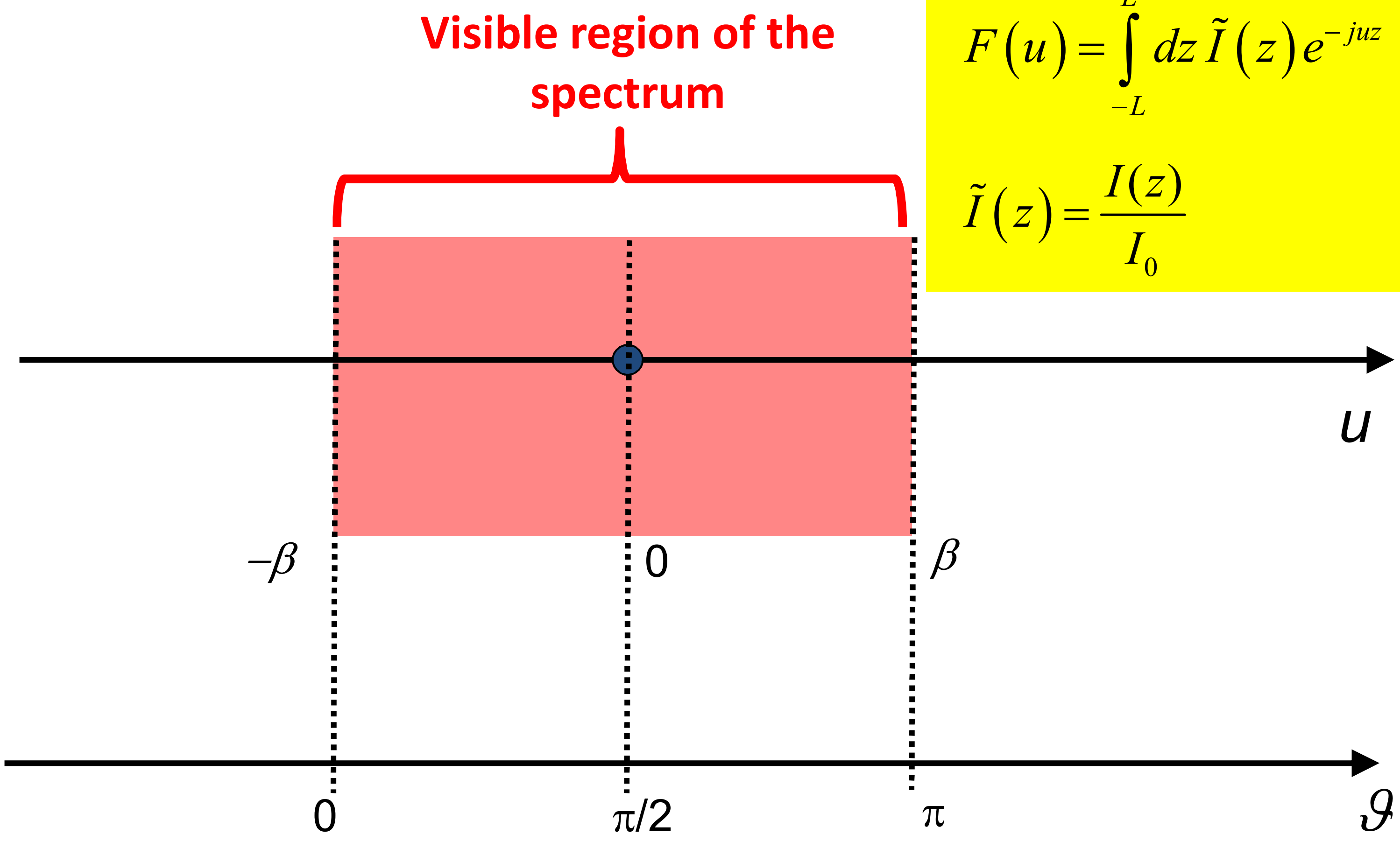

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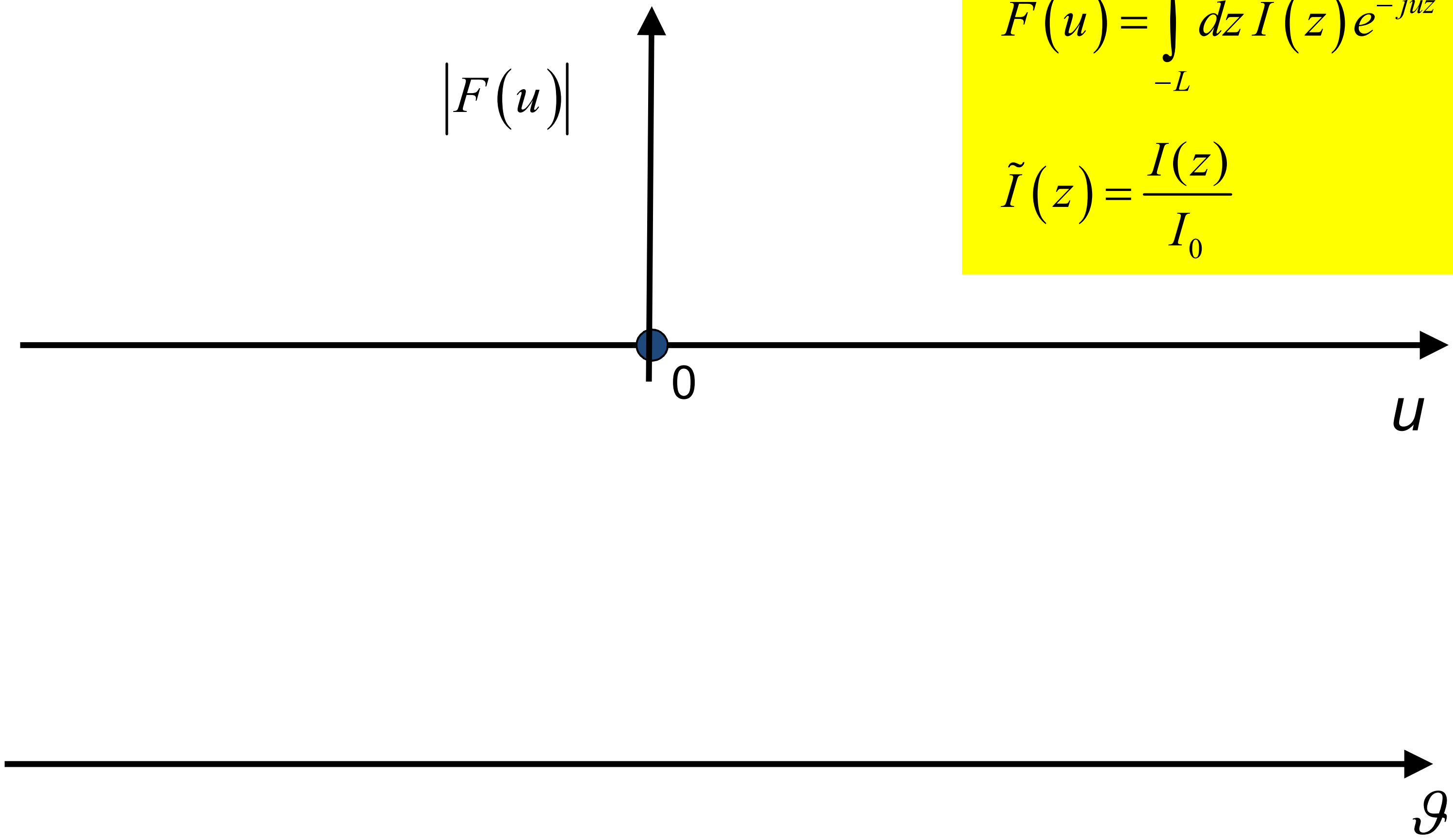
$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



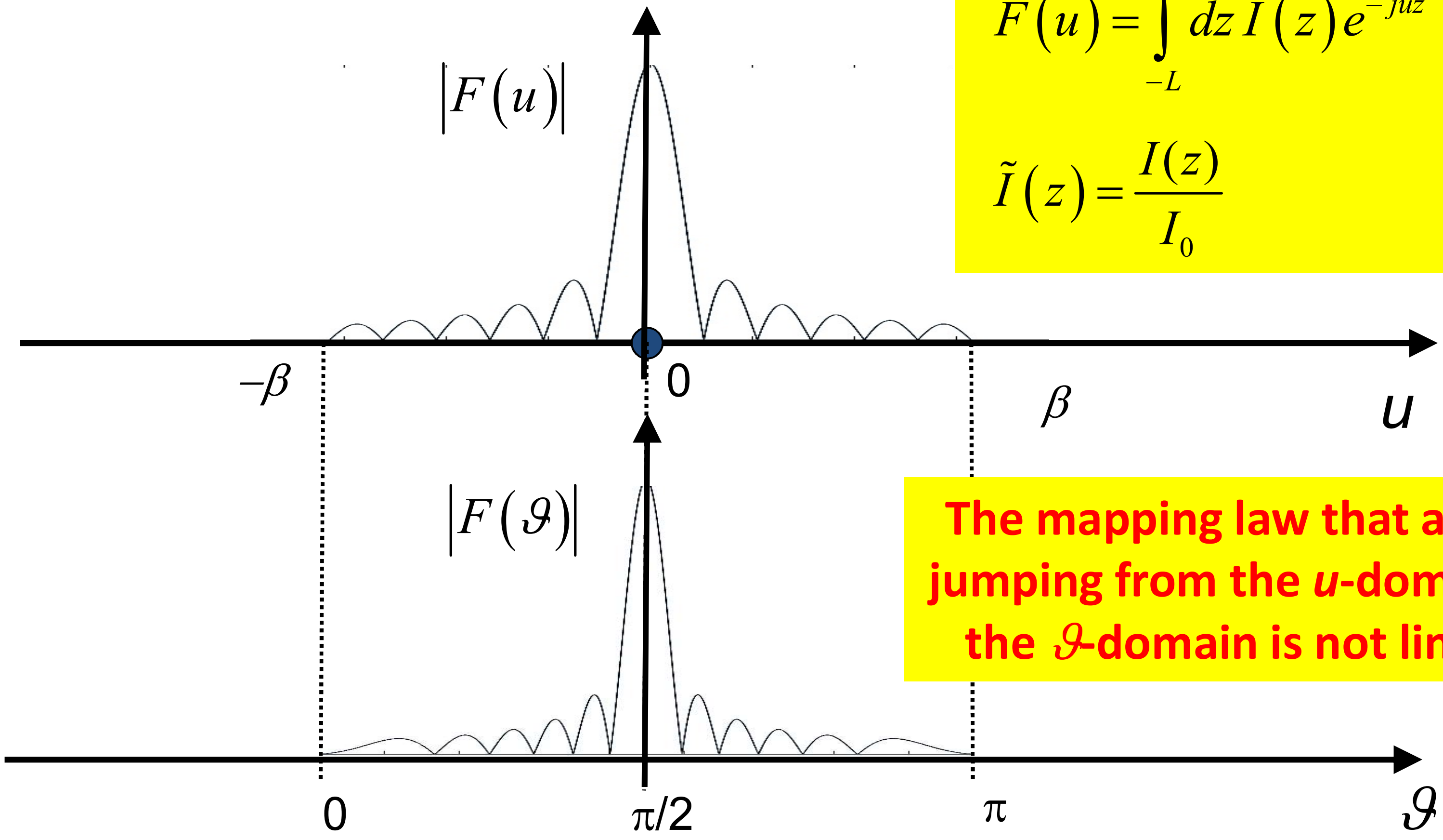
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



The mapping law that allows jumping from the *u*-domain to the ϑ-domain is not linear!

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

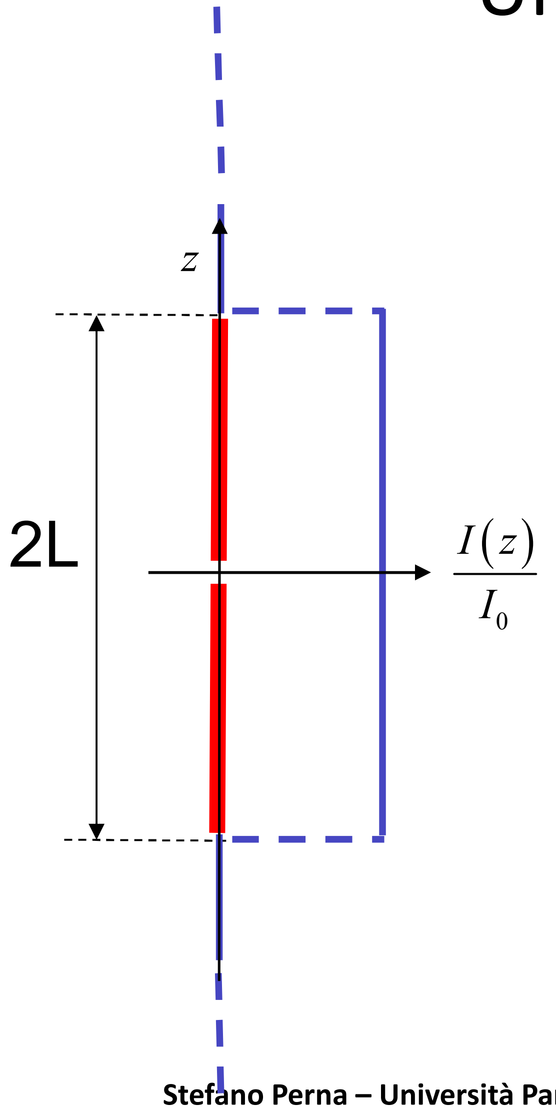
Memo

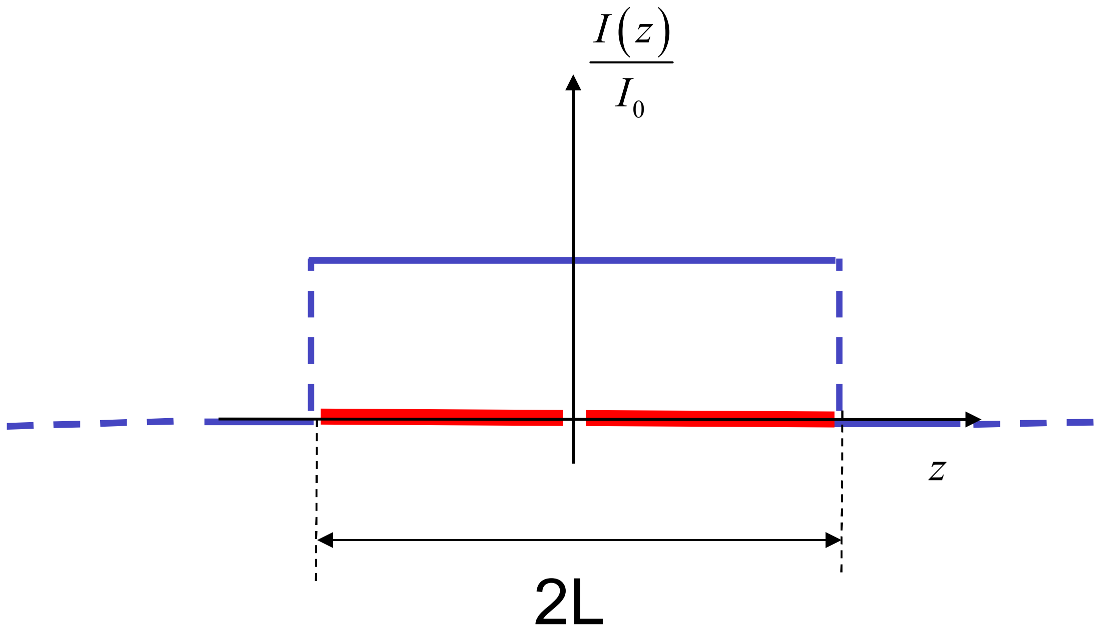
Mathematical tools to be exploited

Mathematics

Wire antennas: an ideal case

Uniform current distribution





$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right]$$