Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

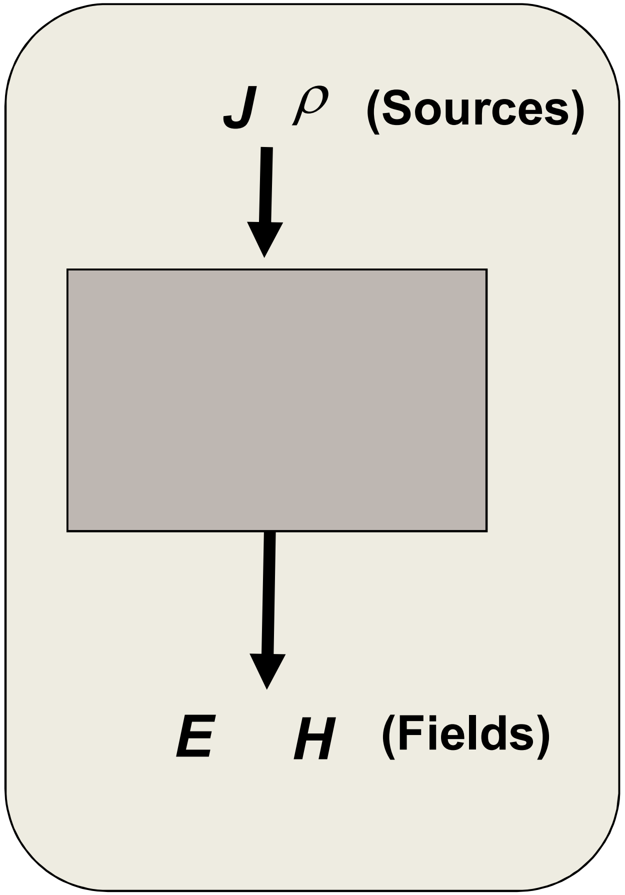
Università degli Studi di Napoli “Parthenope”

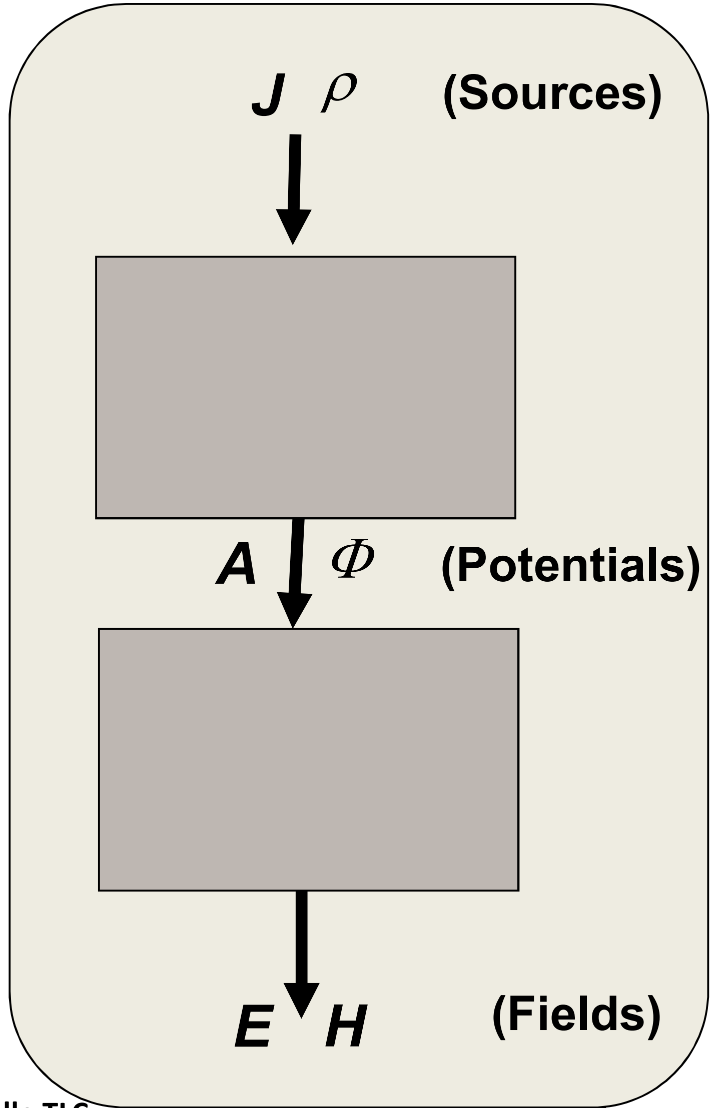
a.a. 2023-2024 - Laurea “Triennale” - Secondo semestre - Terzo anno

Prof. Stefano Perna

Radiation problem

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\mu \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = j\omega\varepsilon \boldsymbol{E} + \boldsymbol{J} \\ \nabla \cdot \varepsilon \boldsymbol{E} = \rho \\ \nabla \cdot \mu \boldsymbol{H} = 0 \end{cases}$$





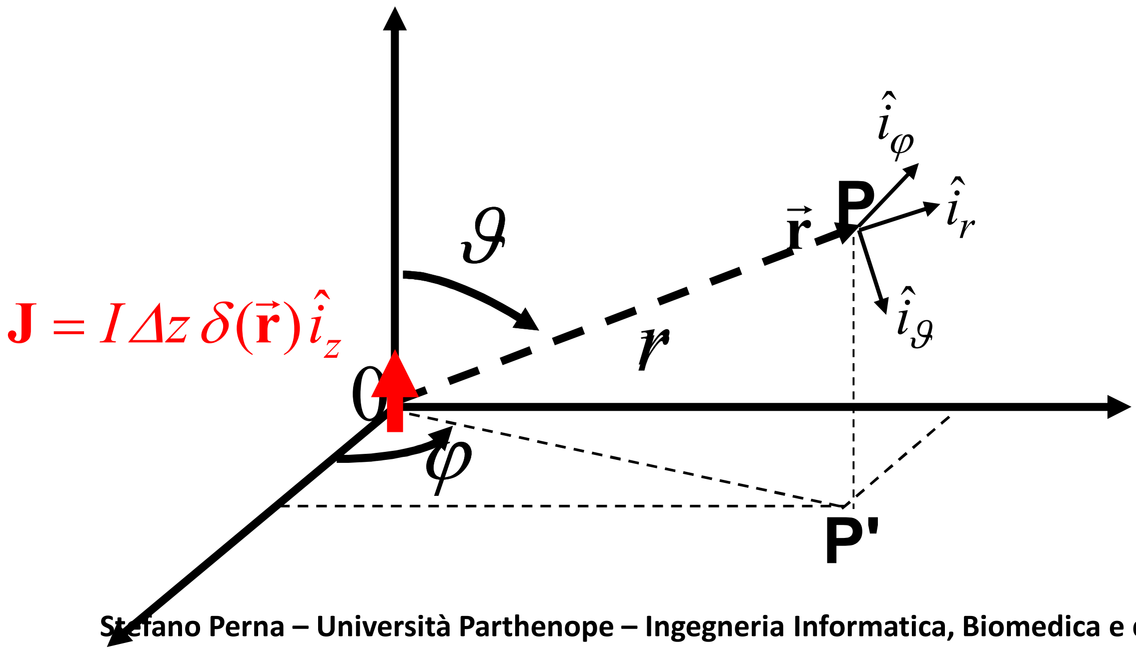
Elementary electrical dipole

for $r >> \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction

Elementary electrical dipole

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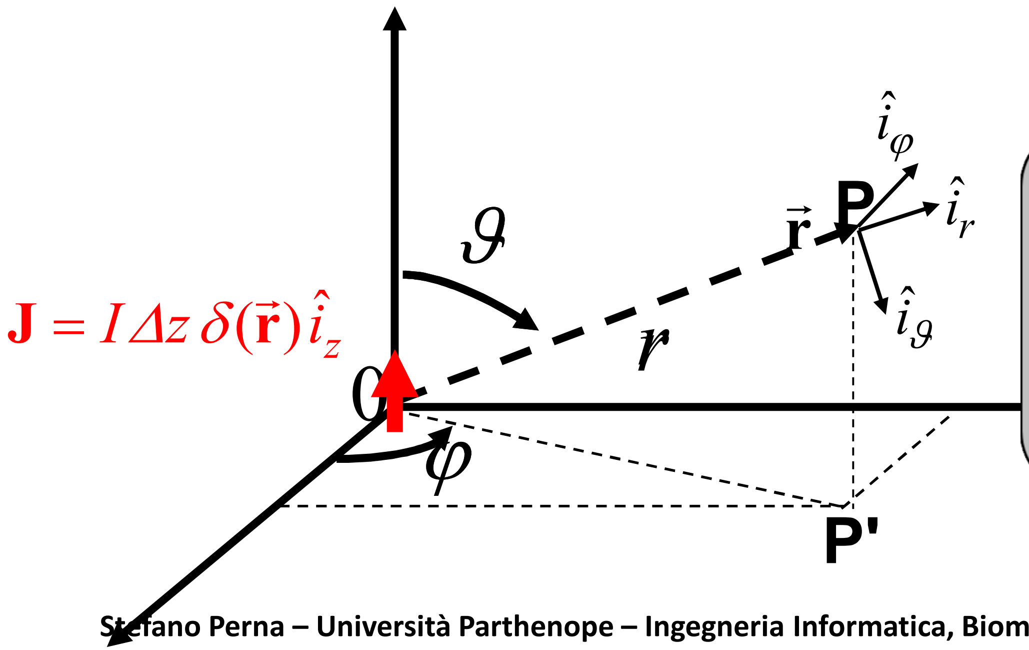
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta \qquad \vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

Elementary electrical dipole

for $r >> \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_{\varphi}(r,\vartheta)\hat{i}_{\varphi} = \frac{E_{\vartheta}}{\zeta}\hat{i}_{\varphi}$$

$\Rightarrow$

$$\zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

and the Poynting vector:

$$\mathbf{S} = \frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}\hat{i}_r$$

Extended antennas

$\downarrow$ $\boldsymbol{J}(\boldsymbol{r})$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

$\downarrow$ $\mathbf{A}(\mathbf{r})$

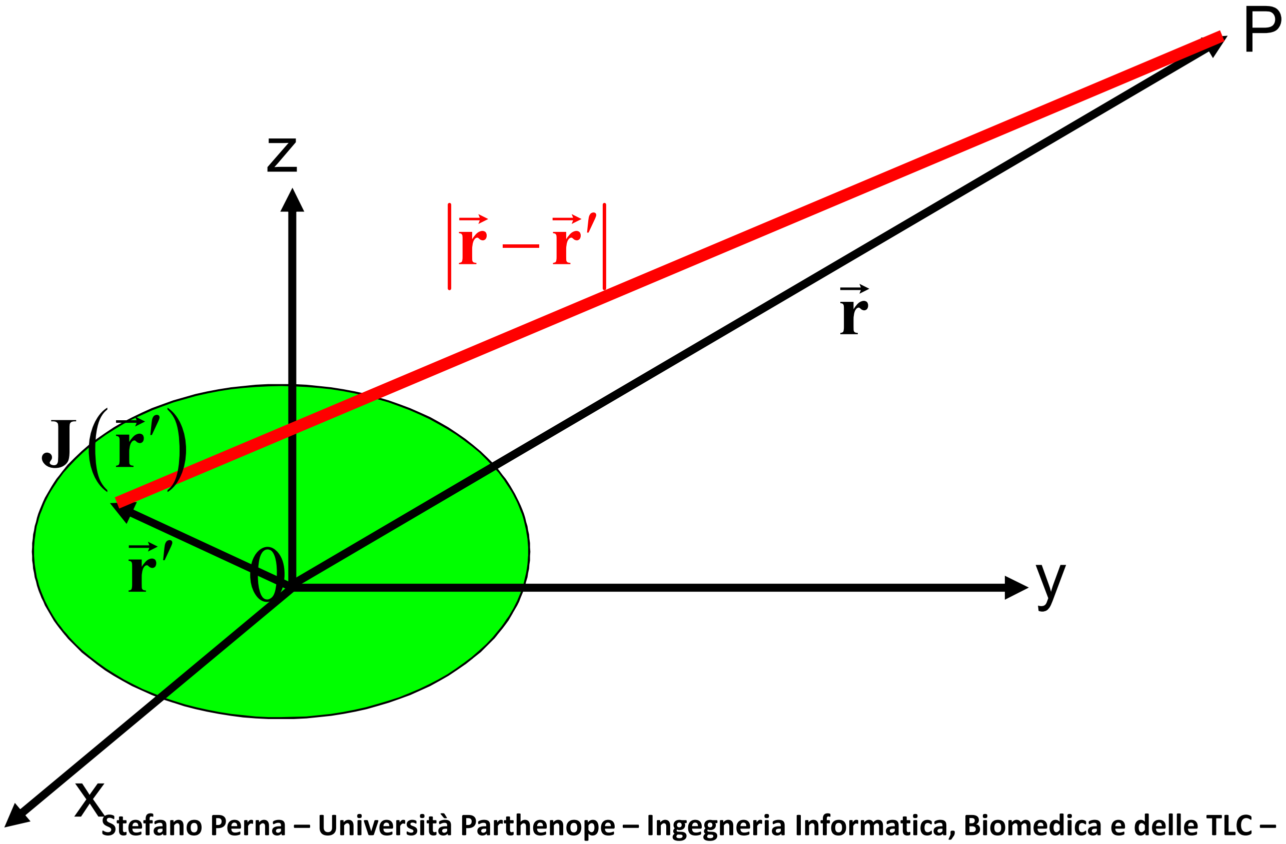
$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\downarrow$ $\boldsymbol{E}(\boldsymbol{r})$

$\boldsymbol{H}(\boldsymbol{r})$

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$



Extended antennas

$$\downarrow \; \boldsymbol{J}(\boldsymbol{r})$$

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

$$\downarrow \; \mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

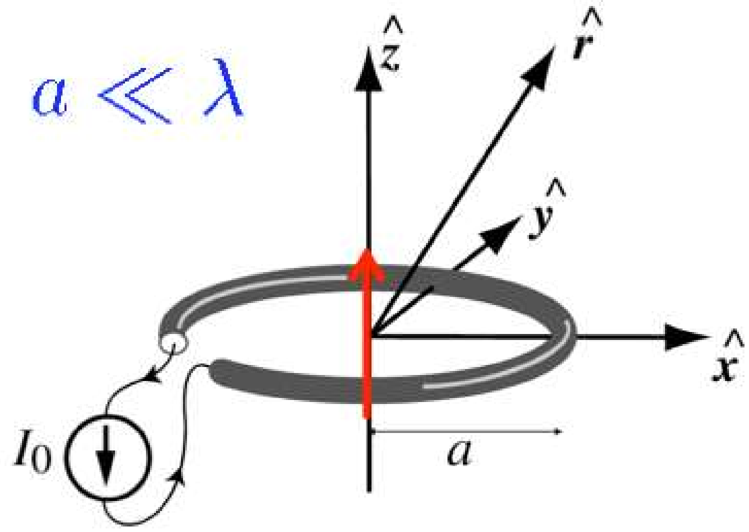
$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$$\downarrow \; \boldsymbol{E}(\boldsymbol{r})$$

$$\boldsymbol{H}(\boldsymbol{r})$$

Small loop antenna

- A simple and inexpensive antenna type is the loop antenna.



$$\Delta S = \pi a^2$$

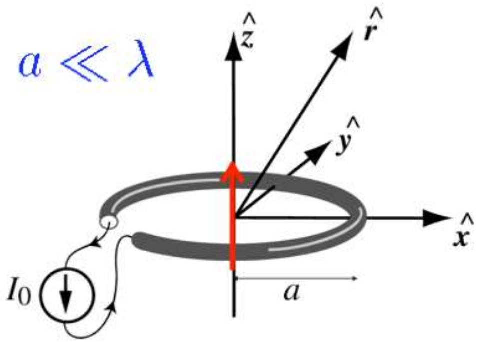
Small loop antenna

for $r >> \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\begin{cases} E_\varphi = \dfrac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r) \\ H_\vartheta = -\dfrac{E_\varphi}{\zeta} \end{cases}$$



$$\Delta S = \pi a^2$$

- the e.m. field propagates along $\hat{i}_r$
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Small loop antenna

for $r >> \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_\varphi \hat{i}_\varphi$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_\vartheta \hat{i}_\vartheta = \frac{-E_\varphi}{\zeta} \hat{i}_\vartheta$$

$$\Rightarrow \quad \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E}$$

and the Poynting vector:

$$\mathbf{S} = \frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta} \hat{i}_r$$