

A large satellite dish antenna is mounted on a mountain peak. The dish is dark and metallic, with a complex support structure. The background shows a sunset or sunrise with a warm, orange and yellow glow on the horizon, transitioning into a darker blue sky. The overall scene is somewhat dimly lit, emphasizing the silhouette of the antenna against the bright sky.

# Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle  
Telecomunicazioni

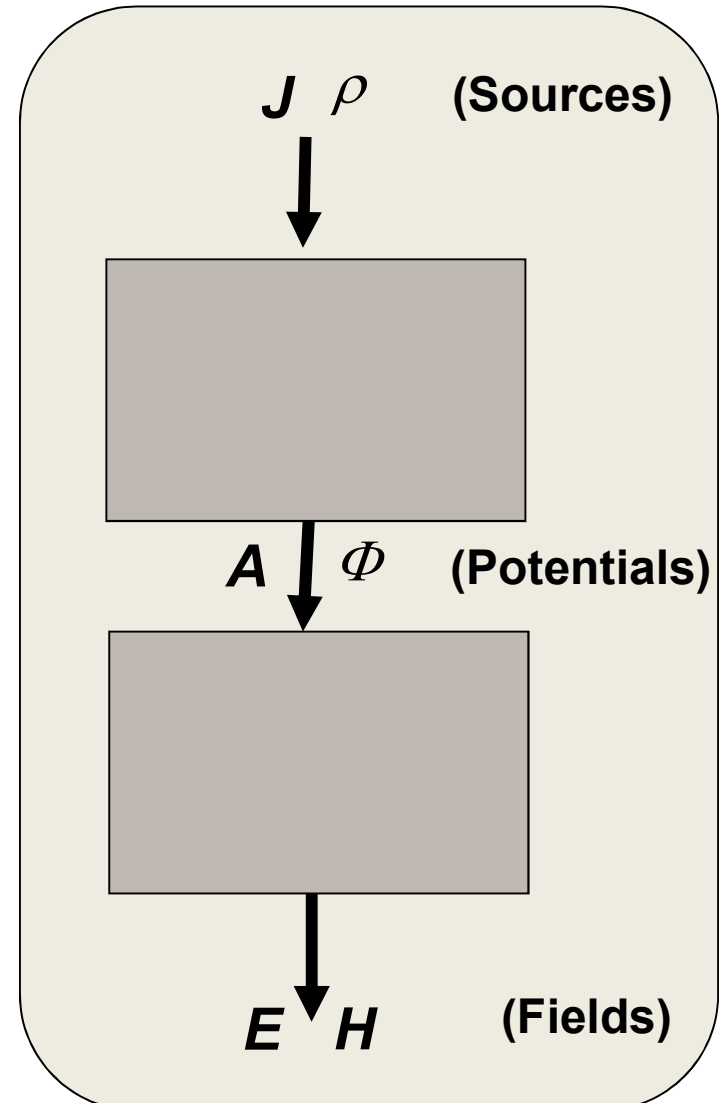
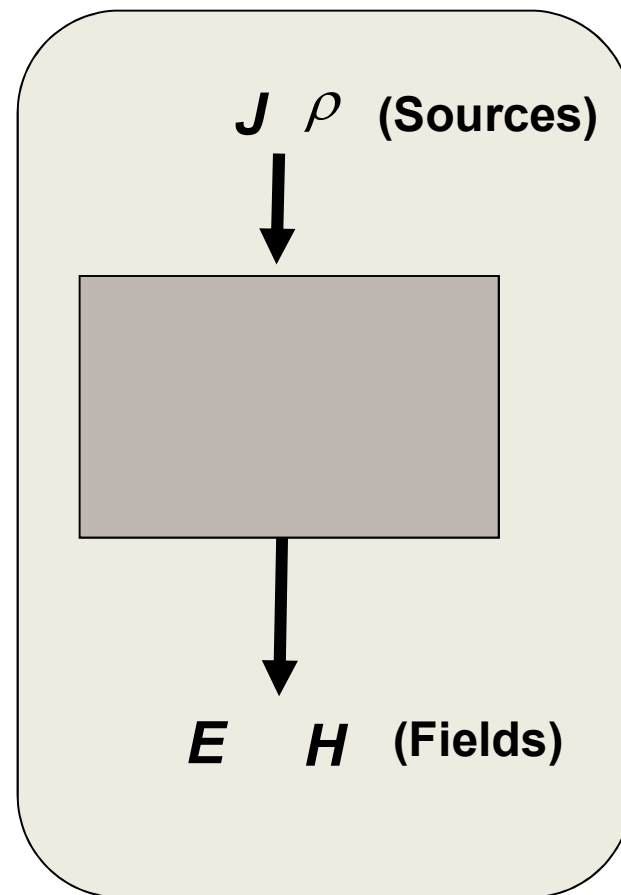
**Università degli Studi di Napoli “Parthenope”**

a.a. 2023–2024 – Laurea “Triennale” – Secondo semestre – Terzo anno

**Prof. Stefano Perna**

# Radiation problem

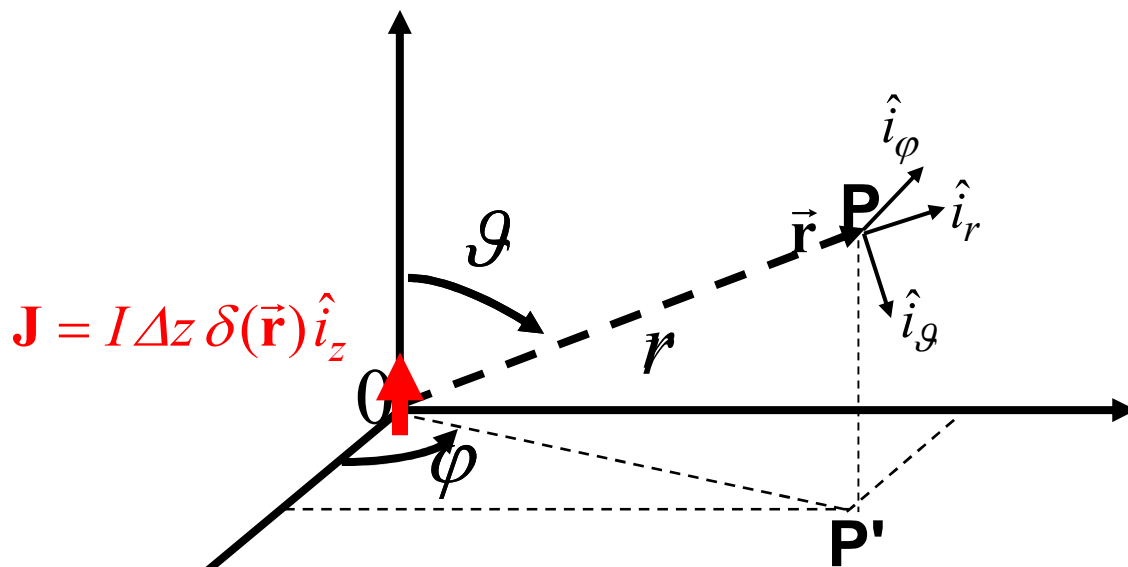
$$\left\{ \begin{array}{l} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \varepsilon\mathbf{E} = \rho \\ \nabla \cdot \mu\mathbf{H} = 0 \end{array} \right.$$



# Elementary electrical dipole

for  $r \gg \lambda$

$$\begin{aligned} \vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) &= E_{\vartheta}(r, \vartheta) \hat{i}_{\vartheta} \\ \vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) &= H_{\varphi}(r, \vartheta) \hat{i}_{\varphi} \end{aligned} \quad \left\{ \begin{aligned} E_{\vartheta} &= j\zeta \frac{I\Delta z}{2\lambda r} \sin \vartheta \exp(-j\beta r) \\ H_{\varphi} &= j \frac{I\Delta z}{2\lambda r} \sin \vartheta \exp(-j\beta r) = \frac{E_{\vartheta}}{\zeta} \end{aligned} \right.$$

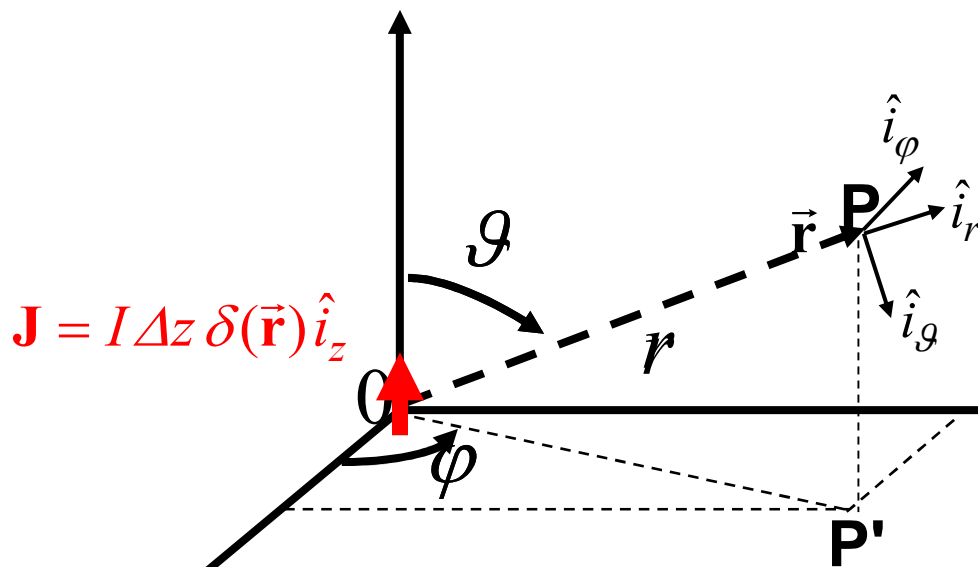


- the e.m. field propagates along  $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction

# Elementary electrical dipole

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$$\begin{aligned} \vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) &= E_{\vartheta}(r, \vartheta) \hat{i}_{\vartheta} \\ \vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) &= H_{\varphi}(r, \vartheta) \hat{i}_{\varphi} \end{aligned} \quad \left\{ \begin{aligned} E_{\vartheta} &= j\zeta \frac{I\Delta z}{2\lambda r} \sin \vartheta \exp(-j\beta r) \\ H_{\varphi} &= j \frac{I\Delta z}{2\lambda r} \sin \vartheta \exp(-j\beta r) = \frac{E_{\vartheta}}{\zeta} \end{aligned} \right.$$



- the e.m. field propagates along  $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- $|E|$  and  $|H|$  exhibit the decaying factor  $1/r$
- $|E|$  and  $|H|$  are proportional through  $\zeta$

# Elementary electrical dipole

for  $r \gg \lambda$

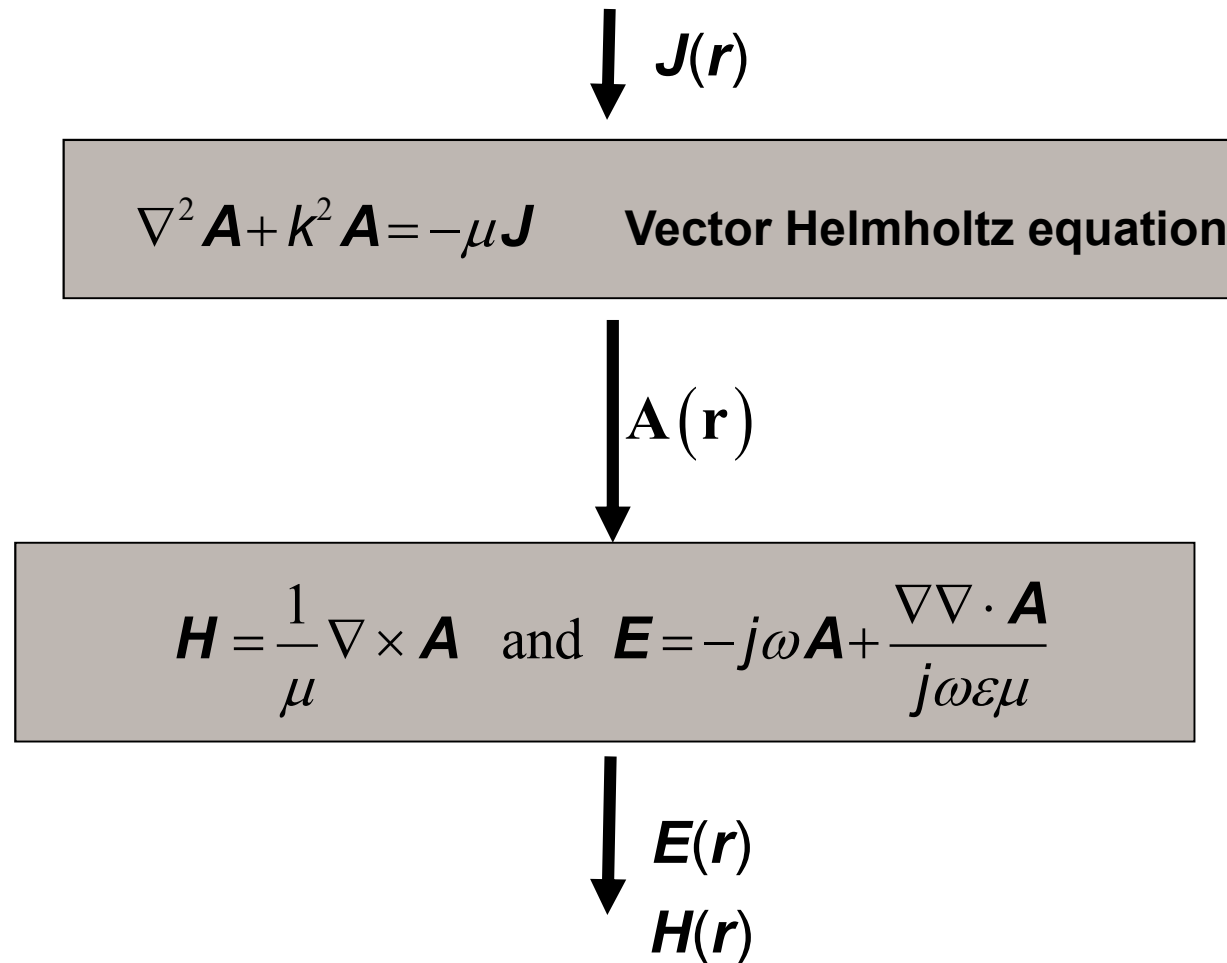
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_{\vartheta}(r, \vartheta) \hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_{\varphi}(r, \vartheta) \hat{i}_{\varphi} = \frac{E_{\vartheta}}{\zeta} \hat{i}_{\varphi} \quad \longrightarrow \quad \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E}$$

and the Poynting vector:

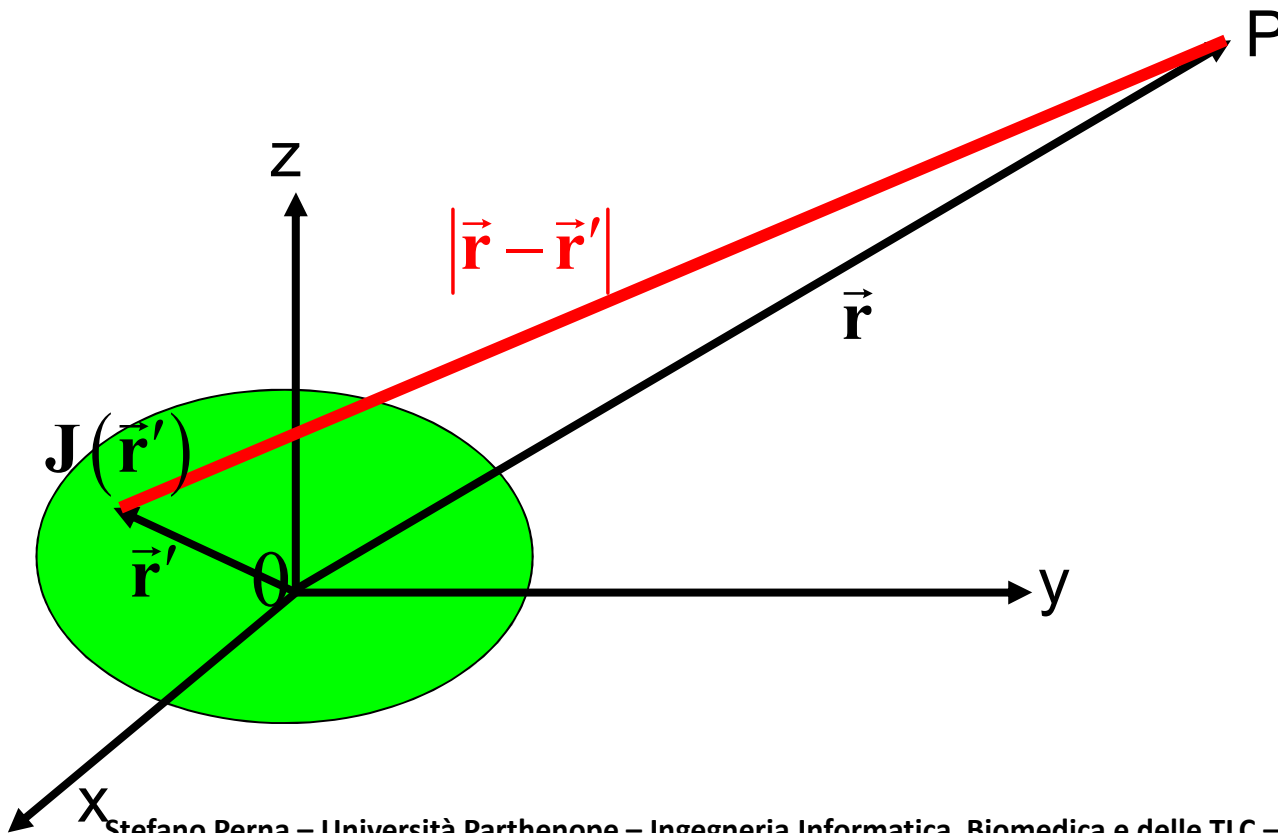
$$\mathbf{S} = \frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta} \hat{i}_r$$

# Extended antennas



# Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}'$$



# Extended antennas

↓  $\mathbf{J}(\mathbf{r})$

$$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J} \quad \text{Vector Helmholtz equation}$$

↓  $\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$

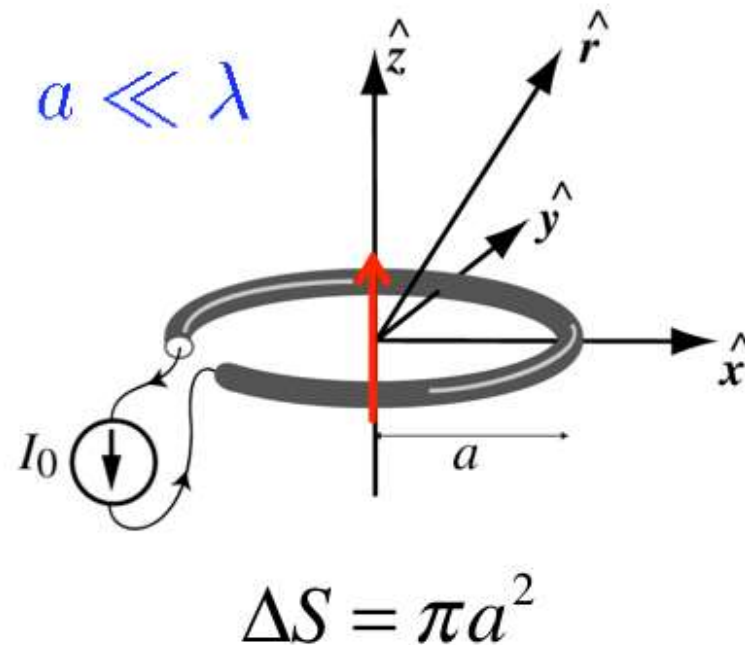
$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \epsilon \mu}$$

↓  $\mathbf{E}(\mathbf{r})$   
 $\mathbf{H}(\mathbf{r})$



# Small loop antenna

- A simple and inexpensive antenna type is the loop antenna.



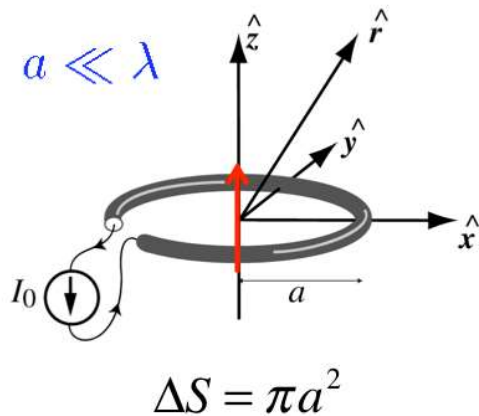
# Small loop antenna

for  $r \gg \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_{\varphi}(r, \vartheta) \hat{\mathbf{i}}_{\varphi}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_{\vartheta}(r, \vartheta) \hat{\mathbf{i}}_{\vartheta}$$

$$\begin{cases} E_{\varphi} = \frac{\zeta \beta \Delta s I}{2 \lambda r} \sin \vartheta \exp(-j \beta r) \\ H_{\vartheta} = -\frac{E_{\varphi}}{\zeta} \end{cases}$$



- the e.m. field propagates along  $\hat{\mathbf{i}}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- $|E|$  and  $|H|$  exhibit the decaying factor  $1/r$
- $|E|$  and  $|H|$  are proportional through  $\zeta$

# Small loop antenna

for  $r \gg \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_{\varphi} \hat{i}_{\varphi}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_{\vartheta} \hat{i}_{\vartheta} = \frac{-E_{\varphi}}{\zeta} \hat{i}_{\vartheta}$$



$$\zeta \mathbf{H} = \hat{i}_r \times \mathbf{E}$$

and the Poynting vector:

$$\mathbf{S} = \frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta} \hat{i}_r$$