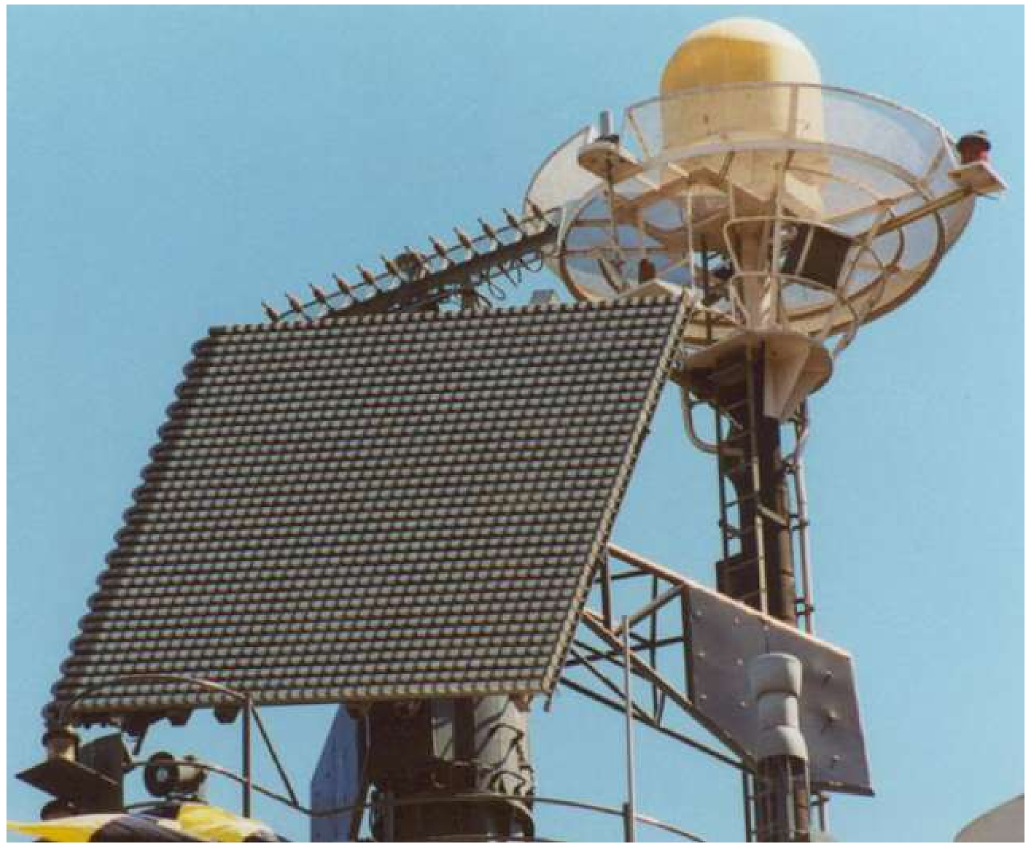
Arrays

Arrays



Color legend

New formulas, important considerations,
important formulas, important concepts

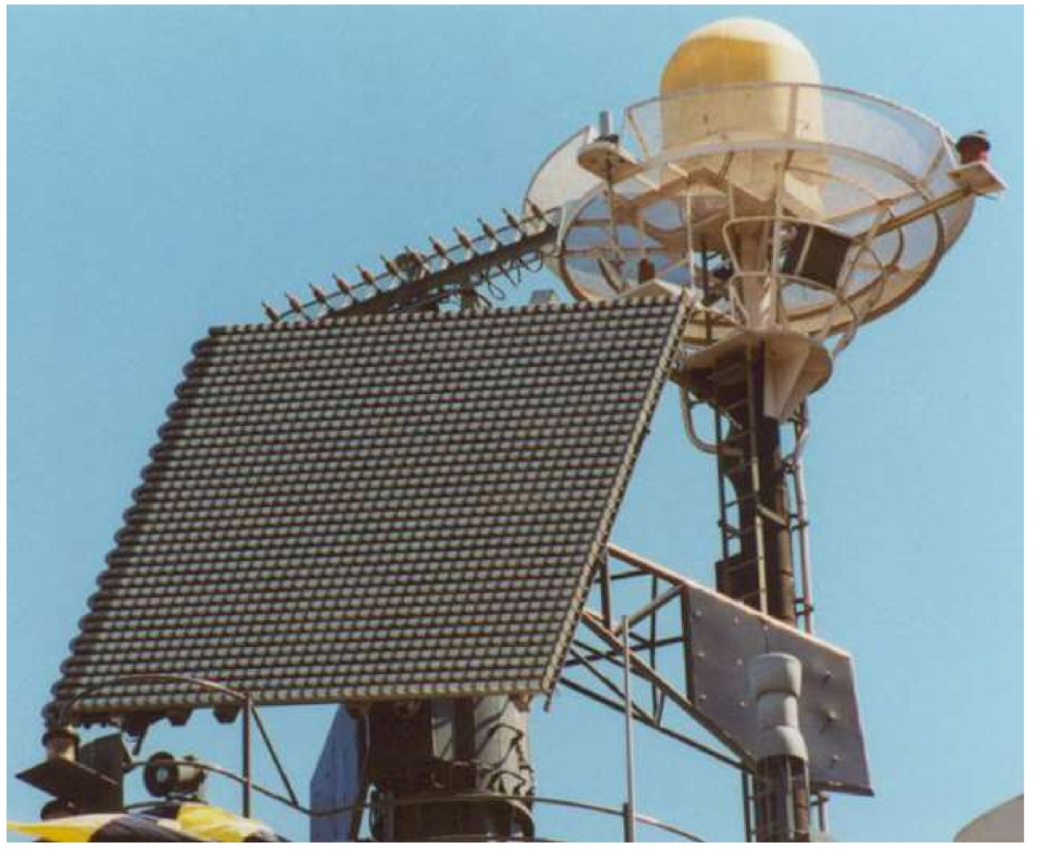
Very important for the discussion

Memo

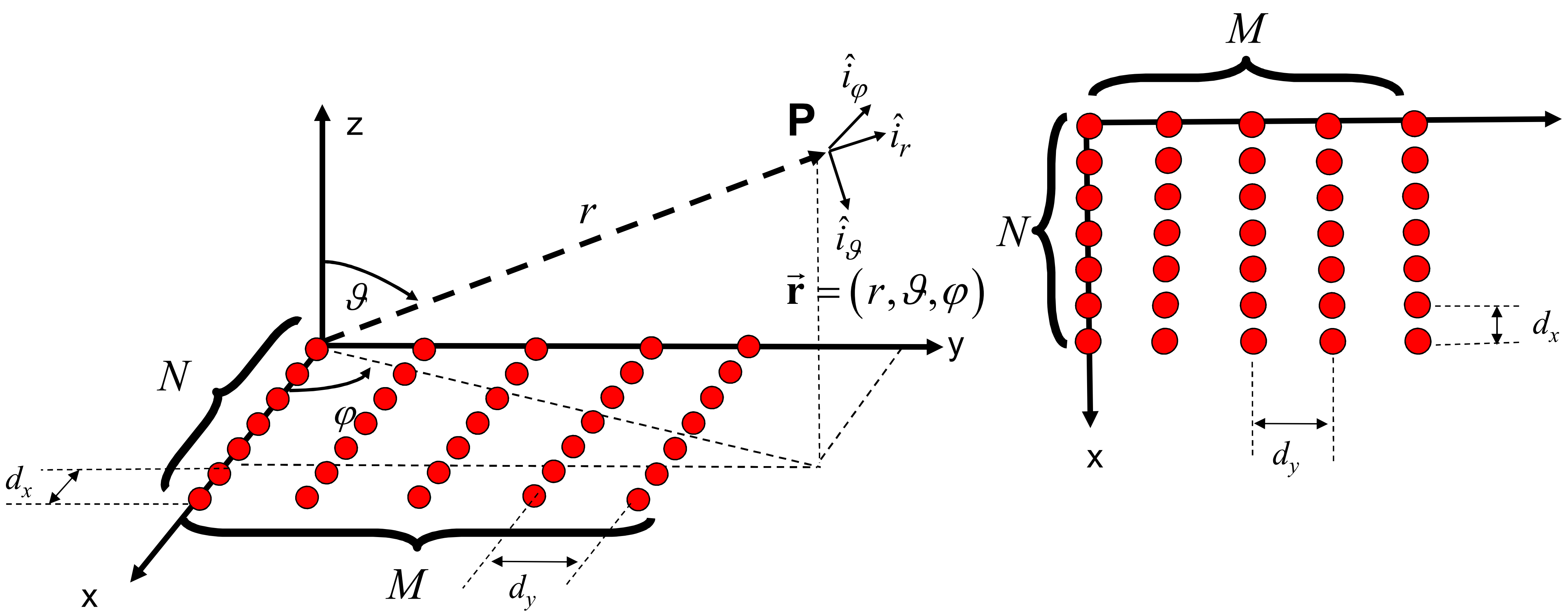
Mathematical tools to be exploited

Mathematics

Planar Arrays



Planar Arrays



Planar Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\ F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{k=0}^{(N\times M)-1} I_k \exp\left(j\beta\vec{\mathbf{r}}'_k \cdot \hat{i}_r\right)$$

*N*x*M* antennas

Periodic Planar Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

The antennas are deployed on the xy plane (planar array)

The antennas are equispaced along both the x and y directions (periodic array)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{I}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v)\Big|_{\substack{u=-\beta d_x \sin\vartheta\cos\varphi \\ v=-\beta d_y \sin\vartheta\sin\varphi}}$$

$$F(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)\exp(-jmv)$$

For the periodic planar arrays the input excitations of the antennas of the array are related to the array factor through the Two Dimensional (2D) Fourier Transformation rule

Periodic Planar Arrays

Grating lobes

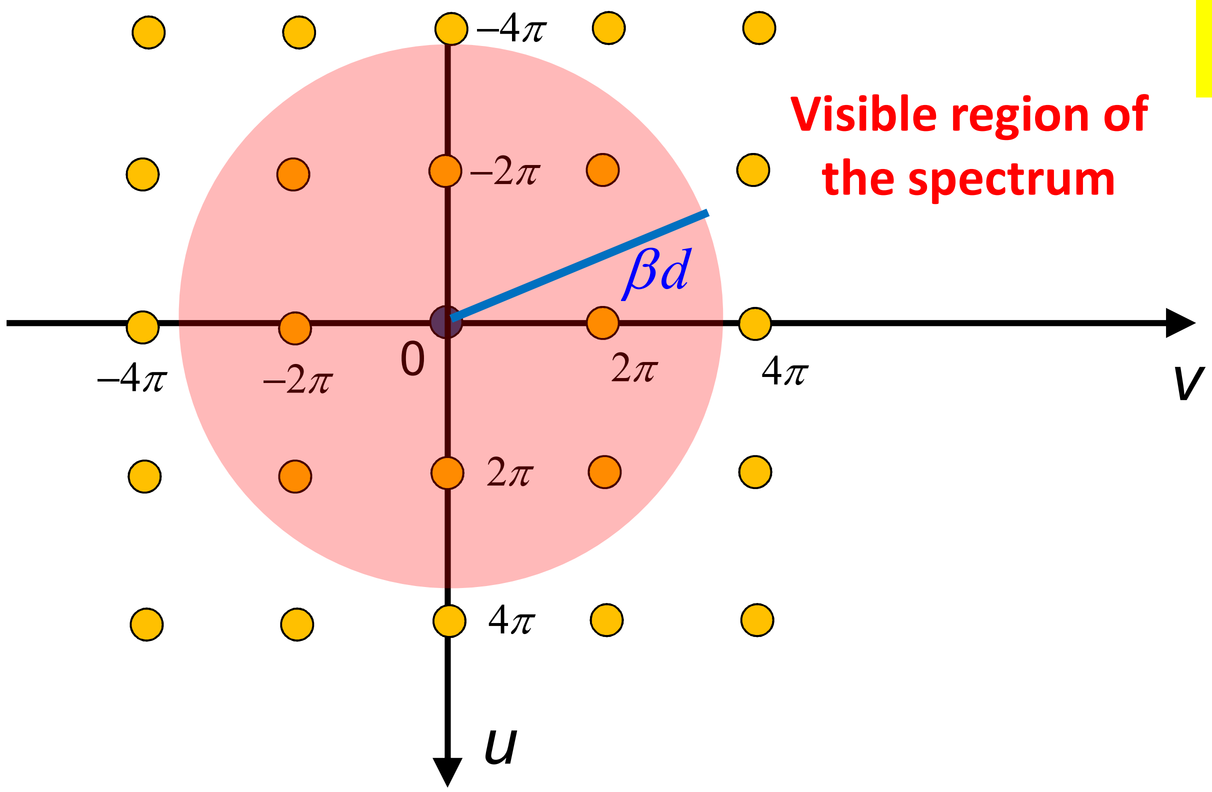
Visible region

Periodic Planar Arrays: Visible Region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{I}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v) \quad \begin{cases} u = -\beta d_x \sin\vartheta\cos\varphi \\ v = -\beta d_y \sin\vartheta\sin\varphi \end{cases}$$

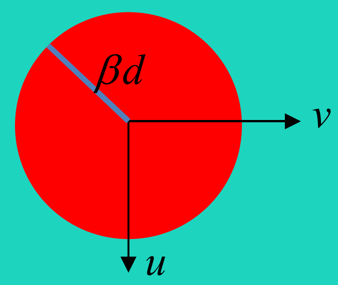
$$F(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)\exp(-jmv)$$



$$d_x = d_y = d \;\Rightarrow\; \begin{matrix} u = -\beta d\sin\vartheta\cos\varphi \\ v = -\beta d\sin\vartheta\sin\varphi \end{matrix}$$

$$\Rightarrow\; u^2 + v^2 = (\beta d)^2\sin^2\vartheta\left(\cos^2\varphi + \sin^2\varphi\right) = (\beta d)^2\sin^2\vartheta$$

$$\Rightarrow\; u^2 + v^2 \le (\beta d)^2$$

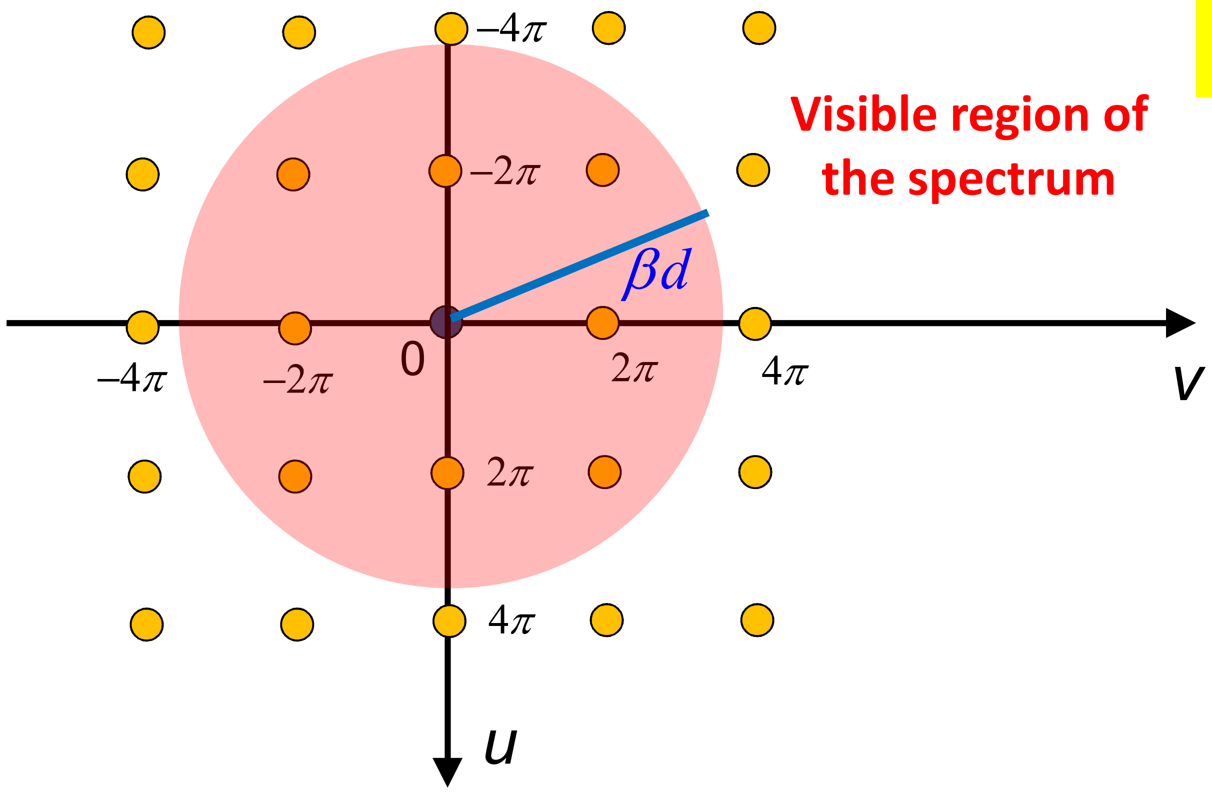


Periodic Planar Arrays: Visible Region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{I}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v)\Big|\begin{array}{l} u = -\beta d_x \sin\vartheta\cos\varphi \\ v = -\beta d_y \sin\vartheta\sin\varphi \end{array}$$

$$F(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)\exp(-jmv)$$



Can we circumvent the presence of the grating lobes?

Let's reduce the width of the visible region!

Periodic Planar Arrays: Visible Region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)\; F(\vartheta,\varphi)$$

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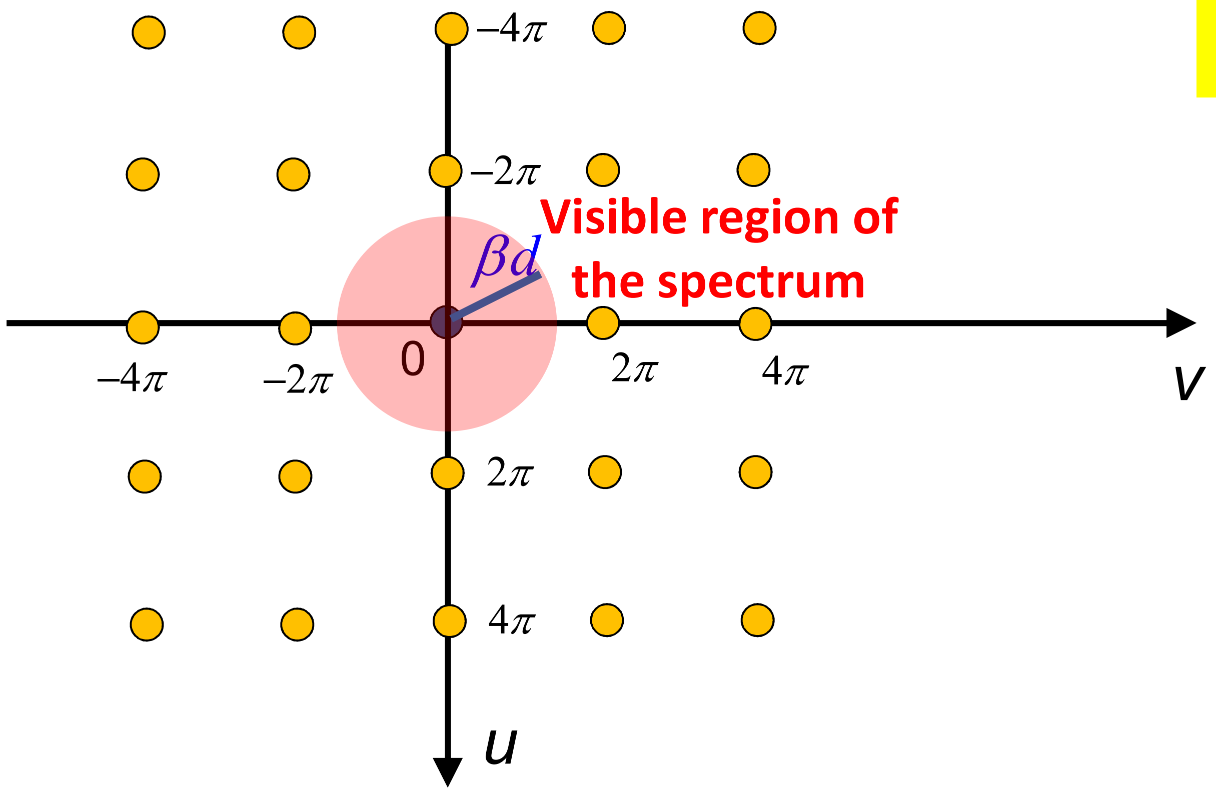

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Periodic Planar Arrays: Visible Region

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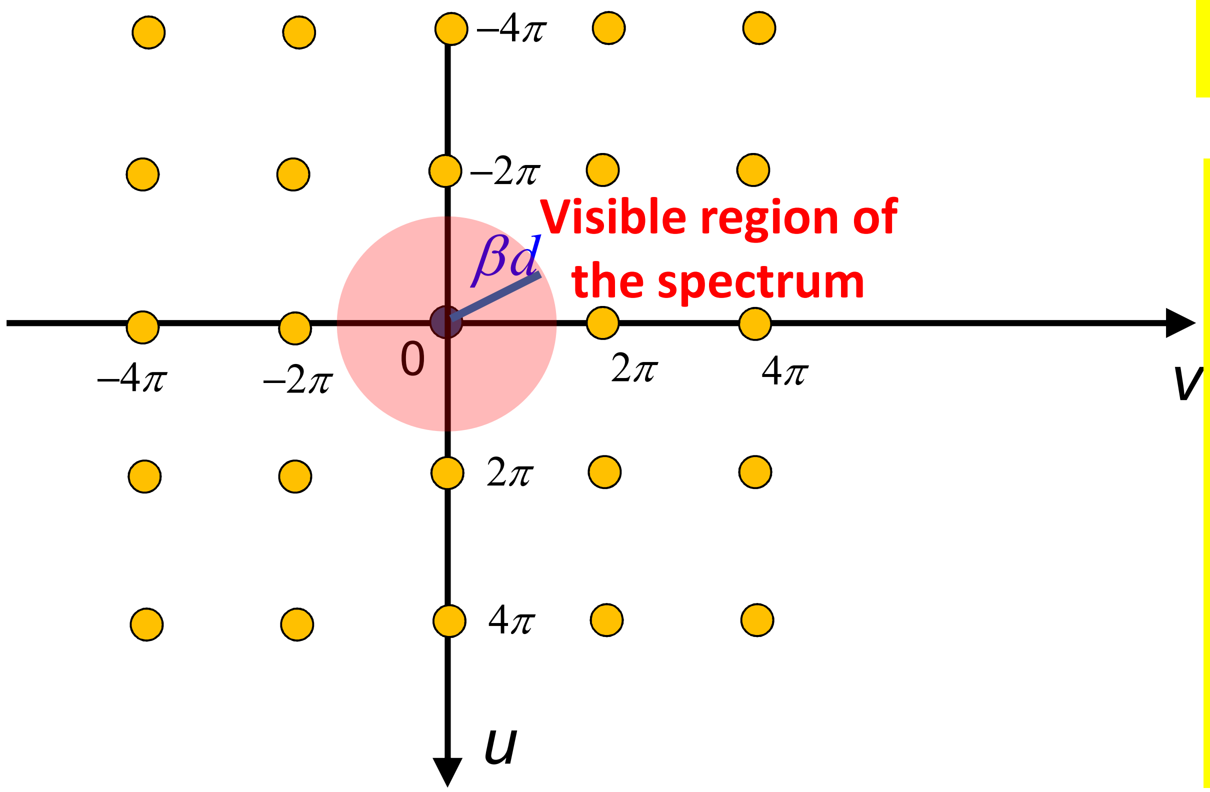

The condition

$$\beta d \le \pi \;\Rightarrow\; \frac{2\pi}{\lambda}d \le \pi \;\Rightarrow\; d \le \frac{\lambda}{2}$$

guarantees (with a safety margin) absence of grating lobes.

To avoid the presence of grating lobes the inter-element distance must be thus subject to an upper limit, on the order of half wavelength

Periodic Planar Arrays: Visible Region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{I}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v)\quad \begin{cases} u = -\beta d_x \sin\vartheta\cos\varphi \\ v = -\beta d_y \sin\vartheta\sin\varphi \end{cases}$$

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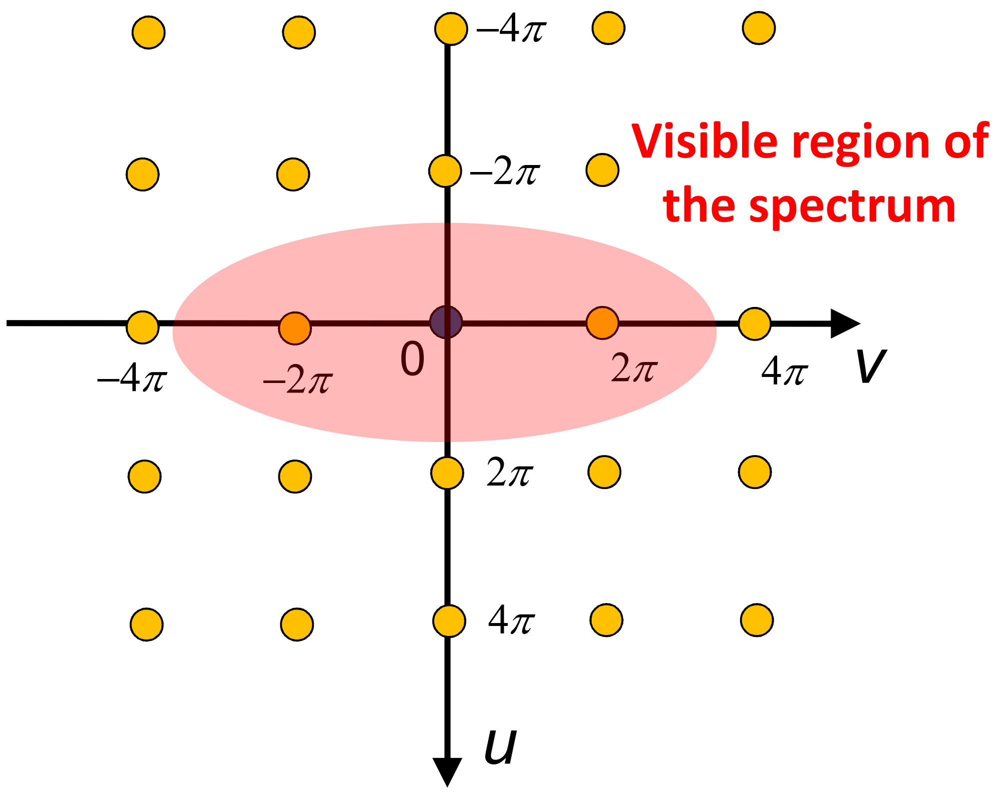
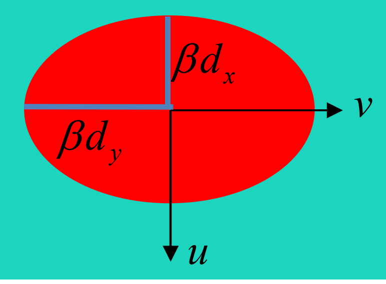


$$d_x \neq d_y \;\Rightarrow\; \frac{u^2}{(\beta d_x)^2} = \cos^2\varphi\sin^2\vartheta \qquad \frac{v^2}{(\beta d_y)^2} = \sin^2\varphi\sin^2\vartheta$$

$$\Rightarrow\; \frac{u^2}{(\beta d_x)^2} + \frac{v^2}{(\beta d_x)^2} = \left(\cos^2\varphi + \sin^2\varphi\right)\sin^2\vartheta \;= \sin^2\vartheta$$

$$\Rightarrow\; \frac{u^2}{(\beta d_x)^2} + \frac{v^2}{(\beta d_y)^2} \le 1 \;\Rightarrow$$



Periodic Planar Arrays

Uniform input excitations

Beam scanning

Periodic Planar Arrays: Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v)\Big|_{\substack{u=-\beta d_x \sin\vartheta\cos\varphi \\ v=-\beta d_y \sin\vartheta\sin\varphi}}$$

$$I_{nm} = I$$

$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I_{nm}\exp(-jnu)\exp(-jmv)$$

$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I_{nm}\exp(-jnu)\exp(-jmv) = I\sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\exp(-jnu)\exp(-jmv) = I\left[\sum_{n=0}^{N-1}\exp(-jnu)\right]\left[\sum_{m=0}^{M-1}\exp(-jmv)\right]$$

$$\Rightarrow \quad |F(u,v)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|\left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

MEMO (from uniform linear arrays)

$$\sum_{n=0}^{N-1}\exp(-jnu) = e^{-j\frac{(N-1)u}{2}}\frac{\sin(Nu/2)}{\sin(u/2)}$$

Periodic Planar Arrays: Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$I_{nm} = I \Rightarrow \left|F(u,v)\right| = \left|I\right|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|\left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

$$F(\vartheta,\varphi) = F(u,v)\Big|_{\substack{u=-\beta d_x \sin\vartheta\cos\varphi \\ v=-\beta d_y \sin\vartheta\sin\varphi}}$$

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Periodic Planar Arrays: Uniform Excitations

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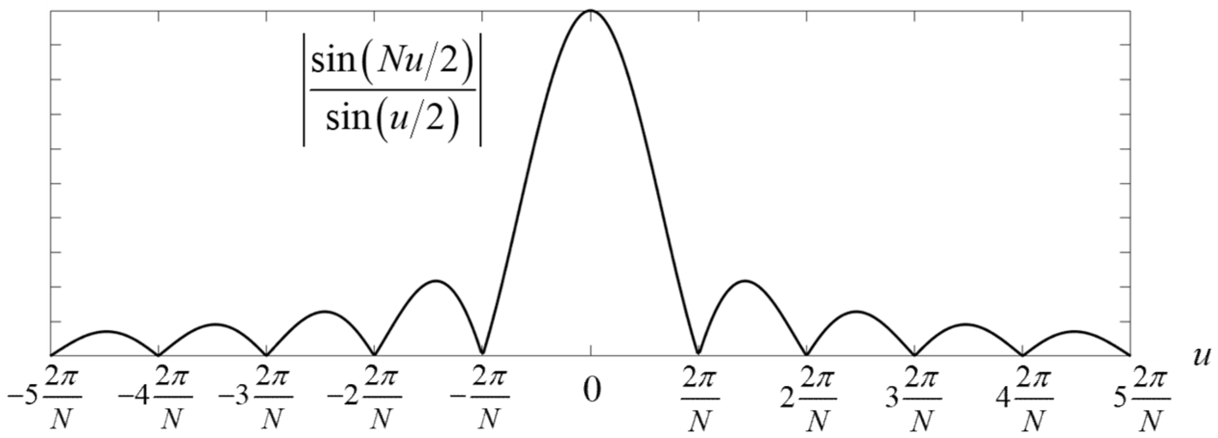
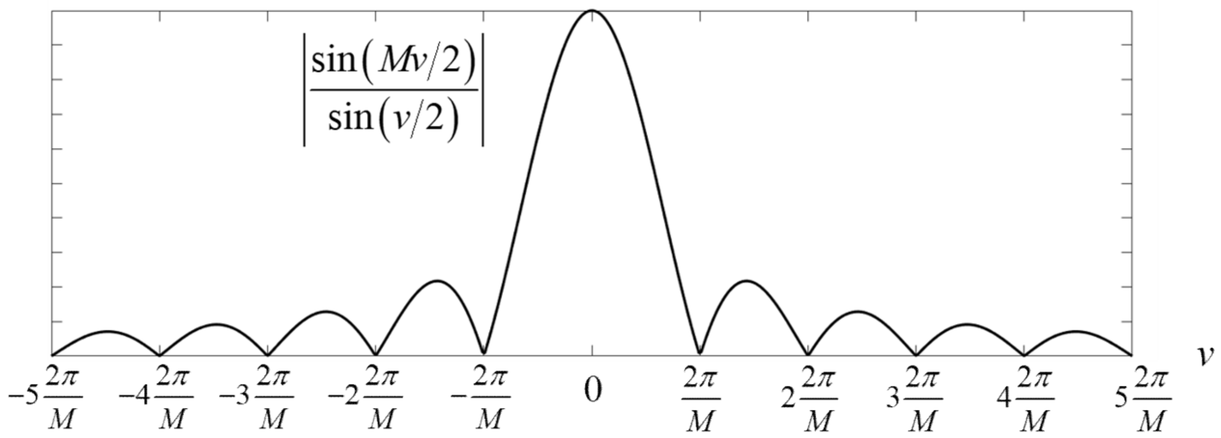
$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I_{nm}\exp(-jnu)\exp(-jmv)$$





Periodic Planar Arrays: Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

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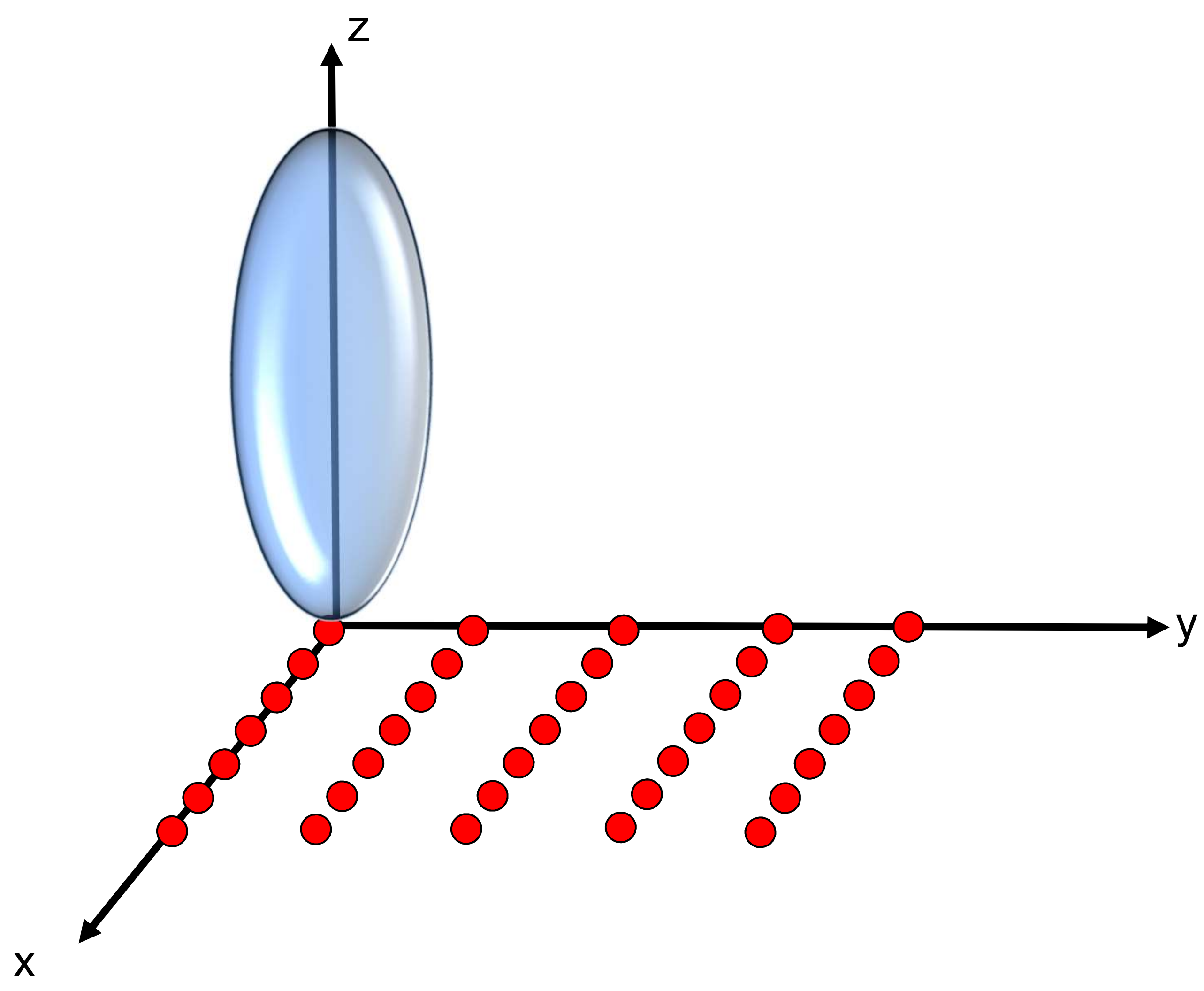
$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I_{nm}\exp(-jnu)\exp(-jmv)$$

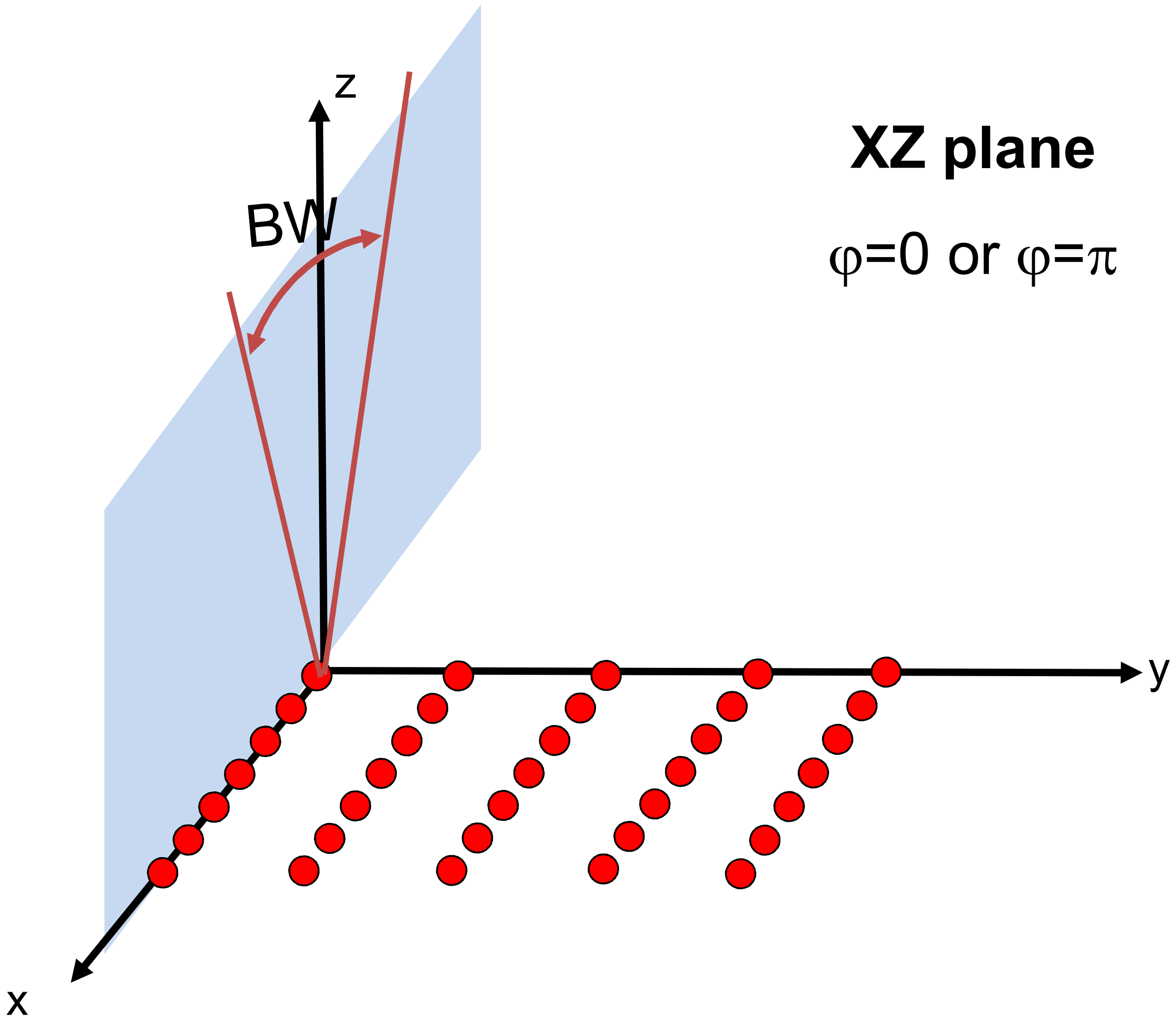
Let's jump from *(u,v)* to (ϑ,φ) and calculate:

- ✓ **The direction of the Main Lobe** $\vartheta_{MB} = 0$
- **The NNBW / HPBW**
- **The SLL**

Periodic Planar Arrays: Uniform Excitations



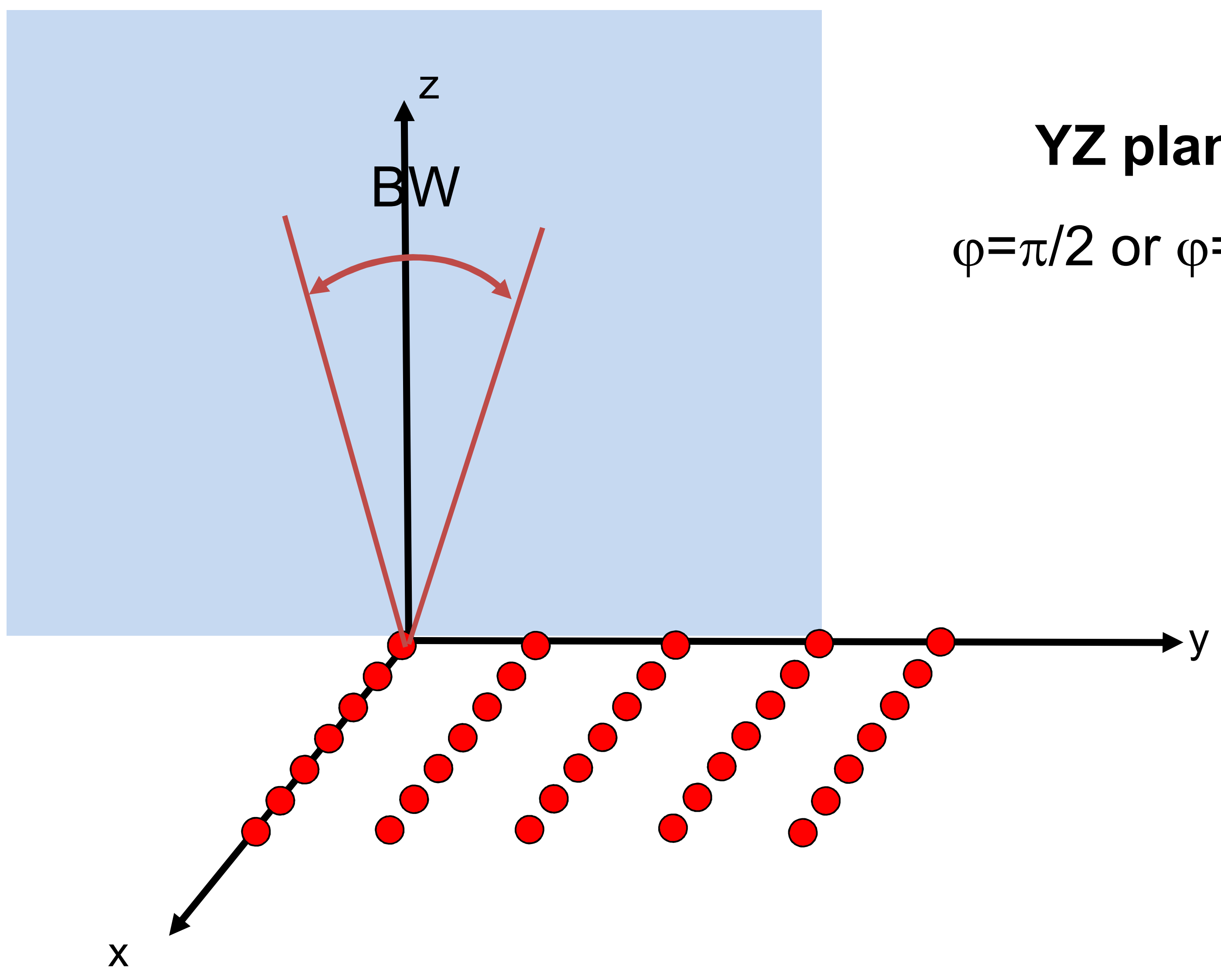
Periodic Planar Arrays: Uniform Excitations



XZ plane

$\varphi=0$ or $\varphi=\pi$

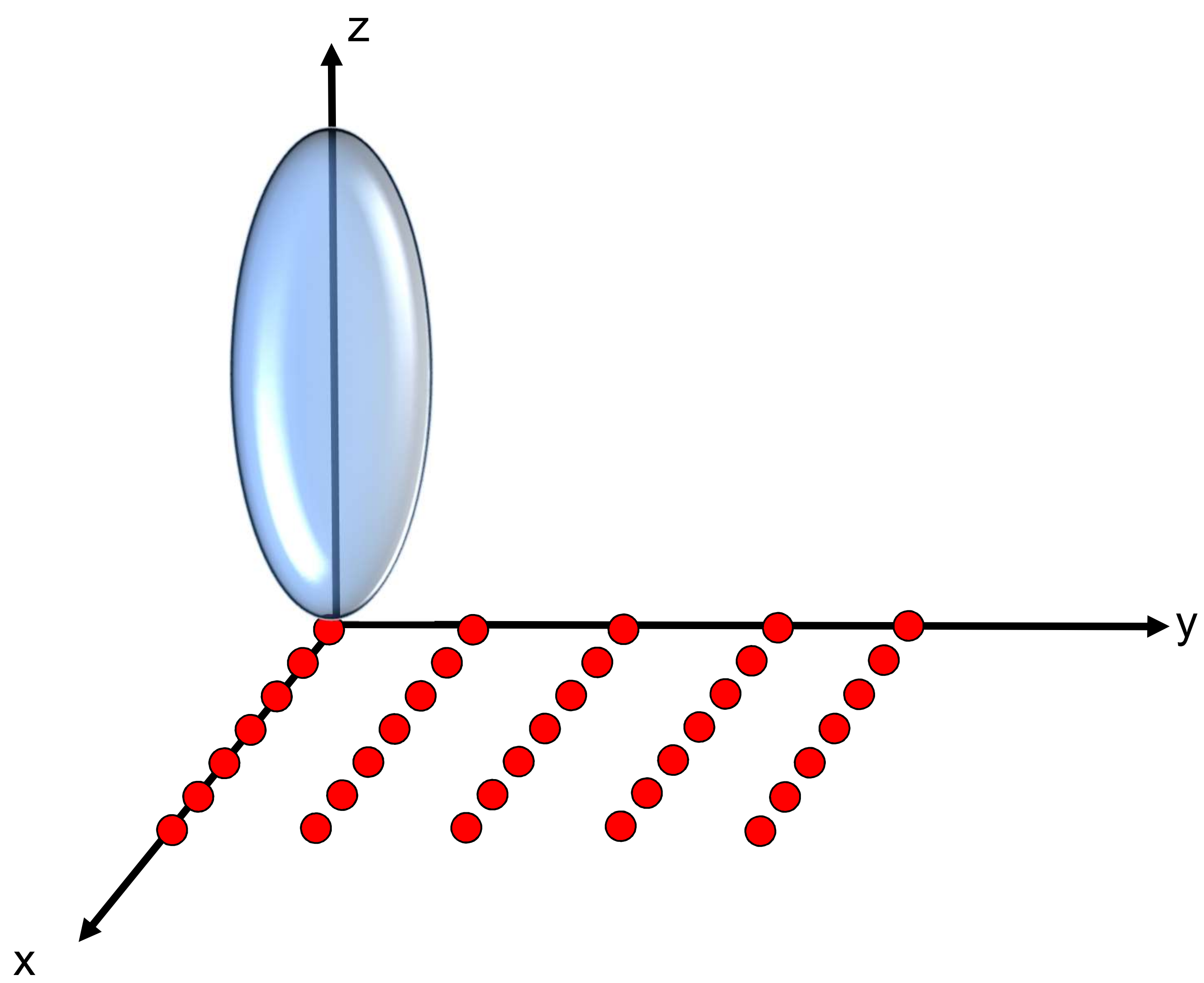
Periodic Planar Arrays: Uniform Excitations



YZ plane

$\varphi=\pi/2$ or $\varphi=3\pi/2$

Periodic Planar Arrays: Uniform Excitations



Periodic Planar Arrays: Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$I_{nm} = I \Rightarrow |F(u,v)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|\left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

$$F(\vartheta,\varphi) = F(u,v)\Big|_{\substack{u=-\beta d_x \sin\vartheta\cos\varphi \\ v=-\beta d_y \sin\vartheta\sin\varphi}}$$

$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I_{nm}\exp(-jnu)\exp(-jmv)$$

Let's jump from (*u,v*) to (ϑ,φ) and calculate:

- ✓ **The direction of the Main Lobe** $\vartheta_{MB} = 0$
- **The NNBW / HPBW**
  - **in the YZ plane** ($\varphi=\pi/2$)
  - **in the XZ plane** ($\varphi=0$)
- **The SLL**

Periodic Planar Arrays: Uniform Excitations

$$\left|F(u,v)\right| = |I| \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right| \left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

$$u = -\beta d_x \sin\vartheta \cos\varphi$$
$$v = -\beta d_y \sin\vartheta \sin\varphi$$

XZ Plane $\varphi = \pi$

$$\varphi = \pi \Rightarrow \begin{cases} \cos\varphi = -1 \\ \sin\varphi = 0 \end{cases} \Rightarrow \begin{cases} u = \beta d_x \sin\vartheta \\ v = 0 \end{cases}$$

$$|F| = |I| M \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

YZ Plane $\varphi = \frac{3\pi}{2}$

Periodic Planar Arrays: Uniform Excitations

u-domain

$$\frac{|F|}{|I|M} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$u_{FN} = \frac{2\pi}{N}$$

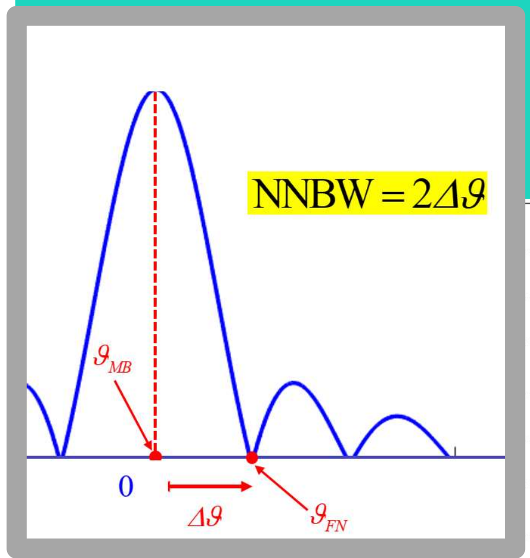
ϑ -domain

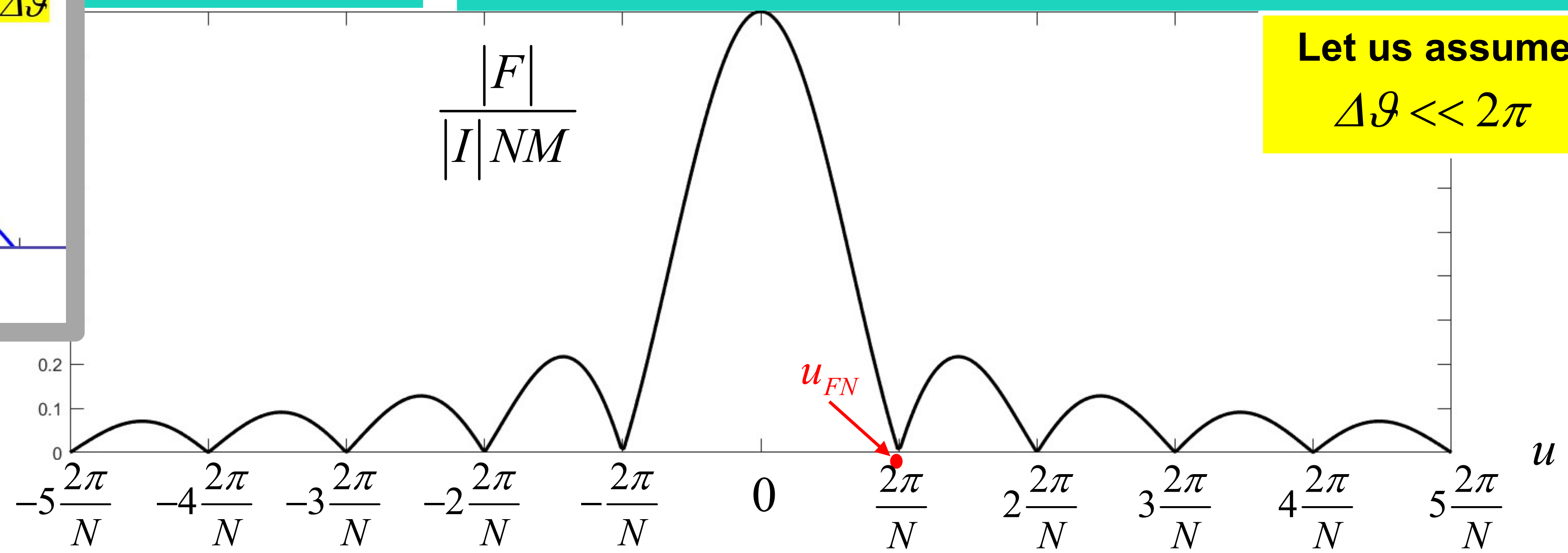
$$u = \beta d_x \sin\vartheta$$

$$u_{FN} = \beta d_x \sin\vartheta_{FN} = \beta d_x \sin(\vartheta_{MB} + \Delta\vartheta)$$

$$\frac{2\pi}{N} = \frac{2\pi}{\lambda} d_x \sin(\Delta\vartheta) \approx \frac{2\pi}{\lambda} d_x \Delta\vartheta \Rightarrow \frac{2\pi}{N} \approx \frac{2\pi}{\lambda} d_x \Delta\vartheta$$

$$\Rightarrow \Delta\vartheta \approx \frac{\lambda}{Nd_x} \Rightarrow \text{NNBW} = 2\Delta\vartheta \approx 2\frac{\lambda}{Nd_x}$$





Let us assume

$$\Delta\vartheta << 2\pi$$

$$\vartheta_{MB} = 0$$

$$u_{MB} = 0$$

$$v_{MB} = 0$$

Periodic Planar Arrays: Uniform Excitations

$$|F(u,v)| = |I| \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right| \left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

$$u = -\beta d_x \sin\vartheta \cos\varphi$$
$$v = -\beta d_y \sin\vartheta \sin\varphi$$

XZ Plane $\varphi = \pi$

$$\varphi = \pi \Rightarrow \begin{cases} \cos\varphi = -1 \\ \sin\varphi = 0 \end{cases} \Rightarrow \begin{cases} u = \beta d_x \sin\vartheta \\ v = 0 \end{cases}$$

$$|F| = |I| M \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$\text{NNBW} \approx 2\frac{\lambda}{Nd_x} \qquad \text{HPBW} \approx 0.88\frac{\lambda}{Nd_x}$$

YZ Plane $\varphi = \frac{3\pi}{2}$

$$\varphi = \frac{3\pi}{2} \Rightarrow \begin{cases} \cos\varphi = 0 \\ \sin\varphi = -1 \end{cases} \Rightarrow \begin{cases} u = 0 \\ v = \beta d_y \sin\vartheta \end{cases}$$

$$|F| = |I| N \left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

$$\text{NNBW} \approx 2\frac{\lambda}{Md_y} \qquad \text{HPBW} \approx 0.88\frac{\lambda}{Md_y}$$

Periodic Planar Arrays: Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$I_{nm} = I \Rightarrow |F(u,v)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|\left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

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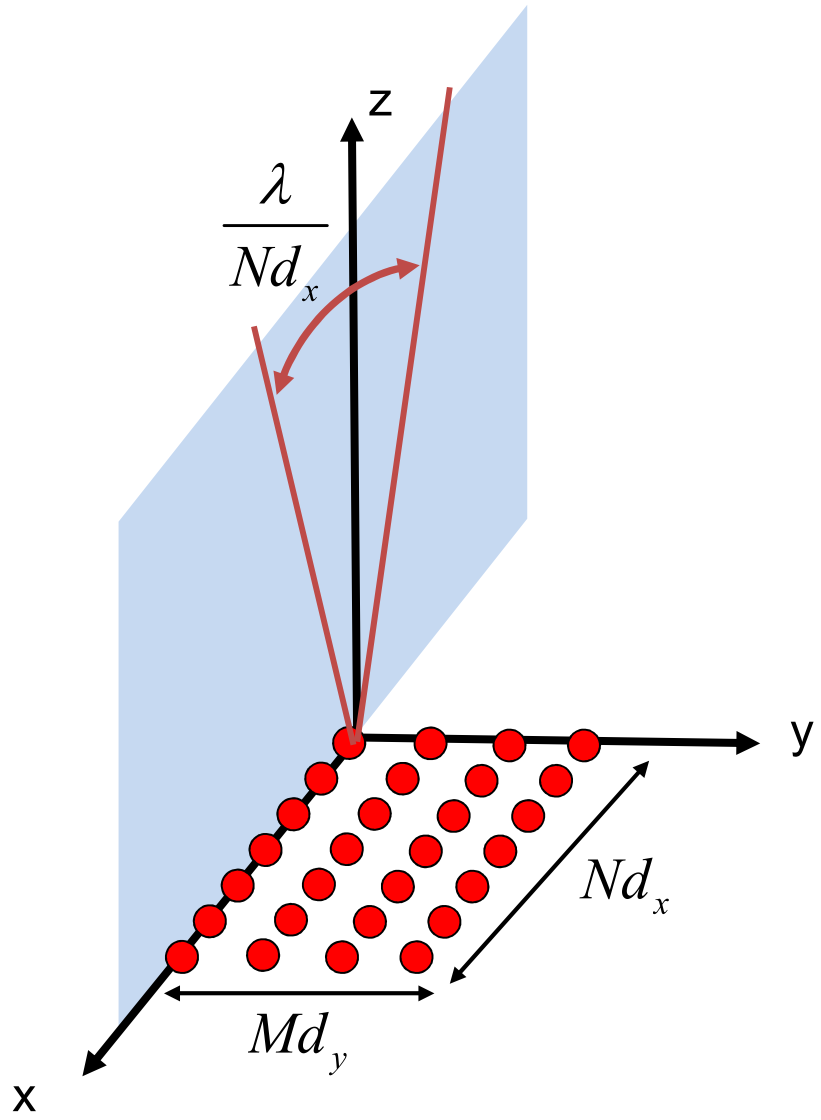
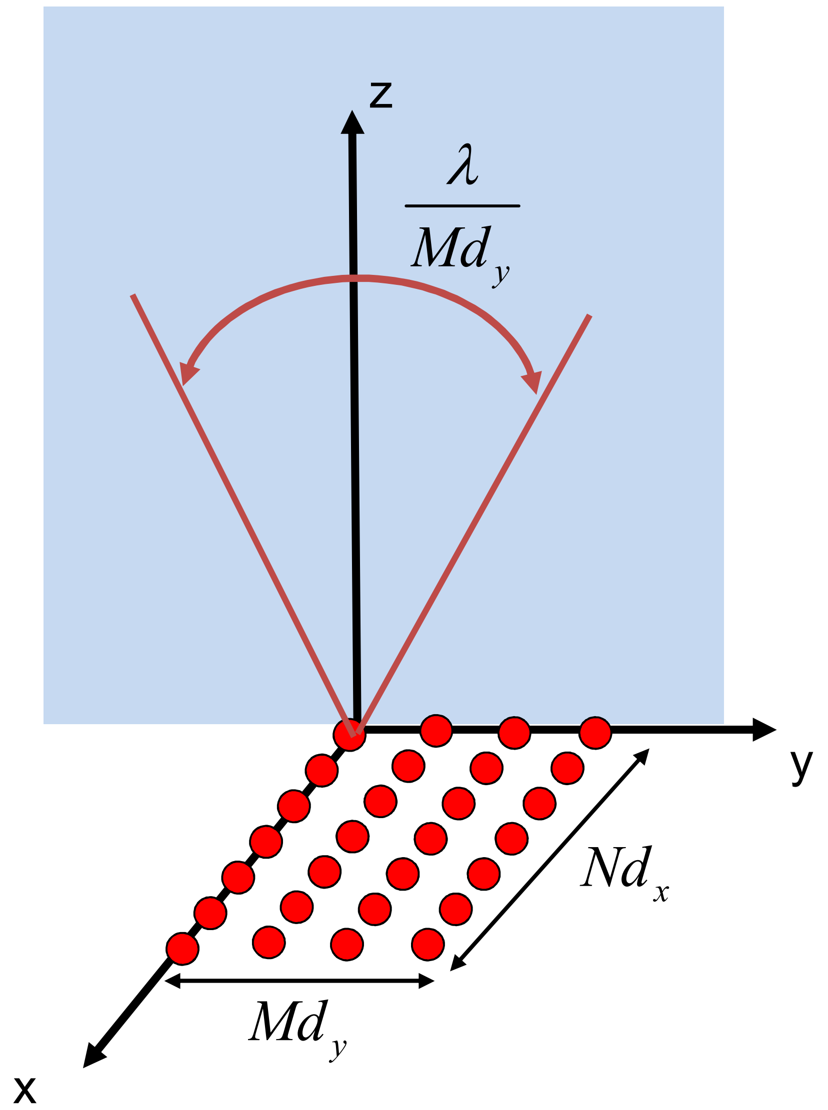
Let's jump from *(u,v)* to *(ϑ,φ)* and calculate:

- ✓ **The direction of the Main Lobe** $\vartheta_{MB} = 0$
- ✓ **The NNBW / HPBW**
  - **in the YZ plane** (*$\varphi=\pi/2$*): $\text{NNBW} \approx 2\frac{\lambda}{Md_y}$, $\text{HPBW} \approx 0.88\frac{\lambda}{Md_y}$
  - **in the XZ plane** (*$\varphi=0$*): $\text{NNBW} \approx 2\frac{\lambda}{Nd_x}$, $\text{HPBW} \approx 0.88\frac{\lambda}{Nd_x}$
- **The SLL**

Periodic Planar Arrays: Uniform Excitations





Periodic Planar Arrays: Uniform Excitations

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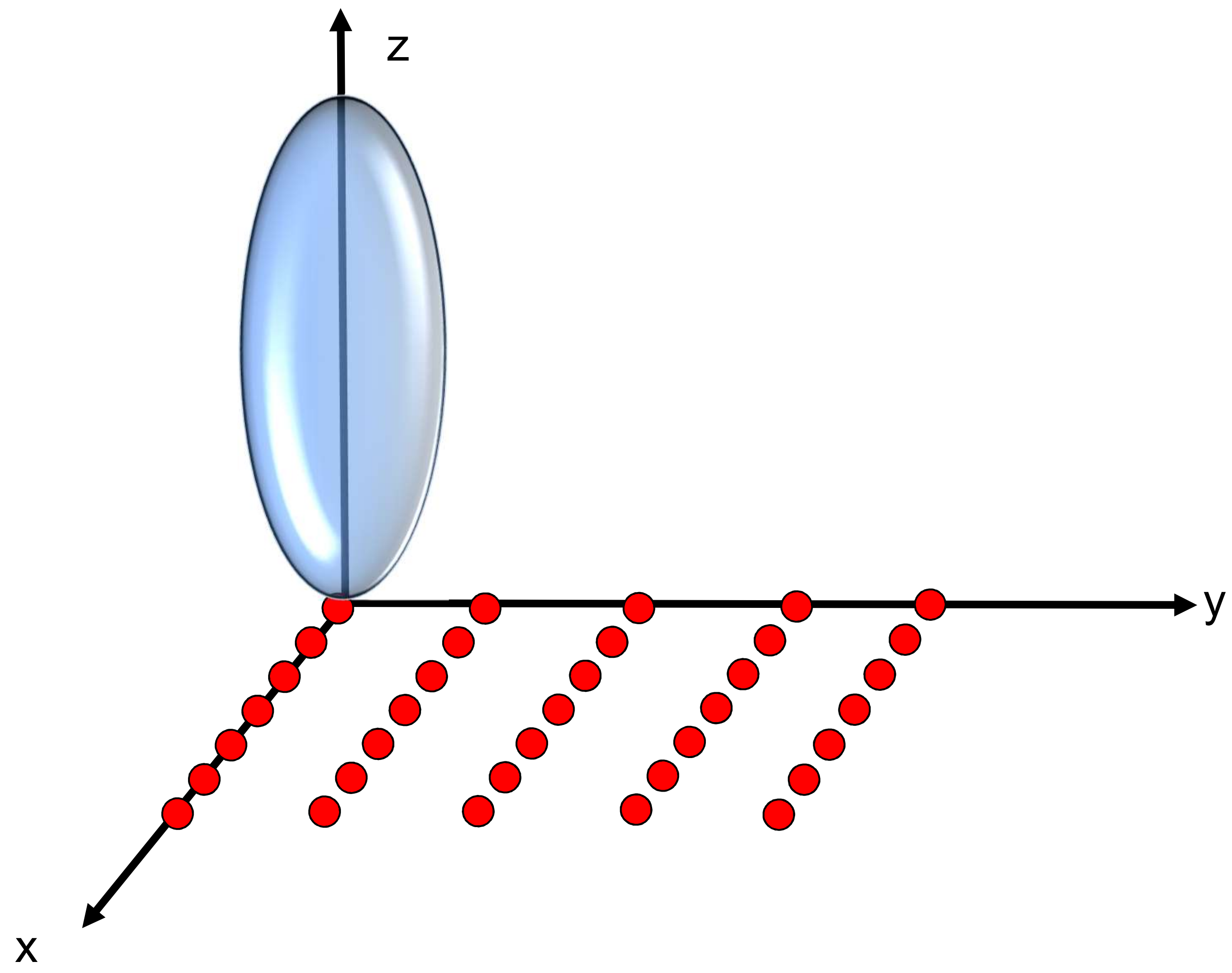
- ✓ **The direction of the Main Lobe** $\vartheta_{MB} = 0$
- ✓ **The NNBW / HPBW**
  - **in the YZ plane** ($\varphi=\pi/2$) $\quad \text{NNBW} \approx 2\frac{\lambda}{Md_y} \quad \text{HPBW} \approx 0.88\frac{\lambda}{Md_y}$
  - **in the XZ plane** ($\varphi=0$) $\quad \text{NNBW} \approx 2\frac{\lambda}{Nd_x} \quad \text{HPBW} \approx 0.88\frac{\lambda}{Nd_x}$
- ✓ **The SLL** $\quad \text{SLL} = -13.46\,dB$

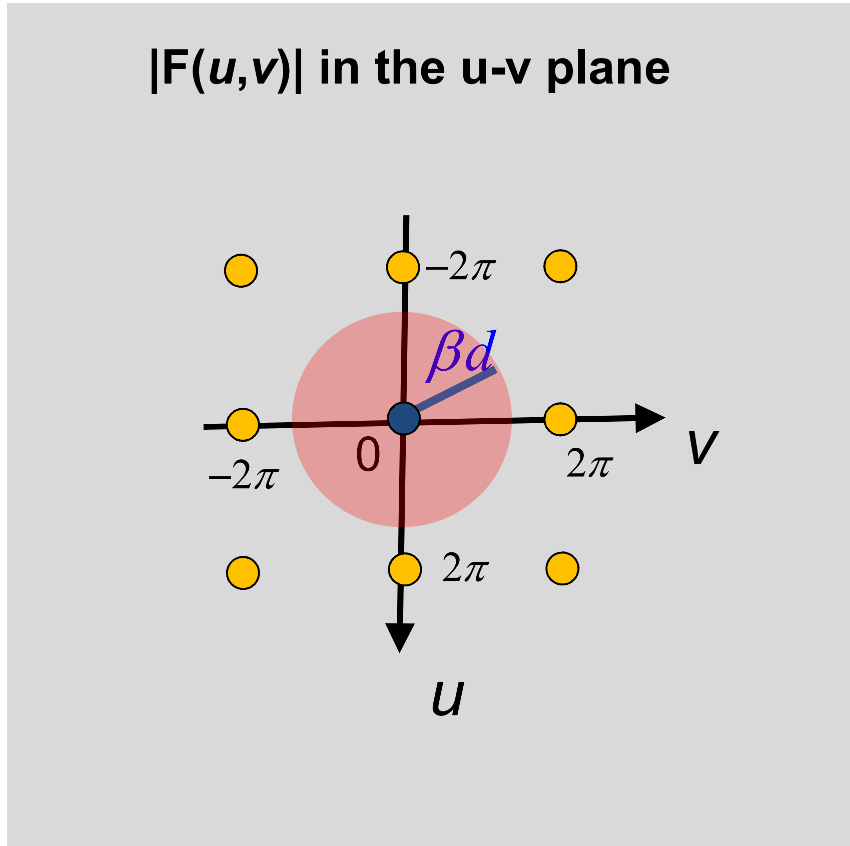
Periodic Planar Arrays

Uniform input excitations

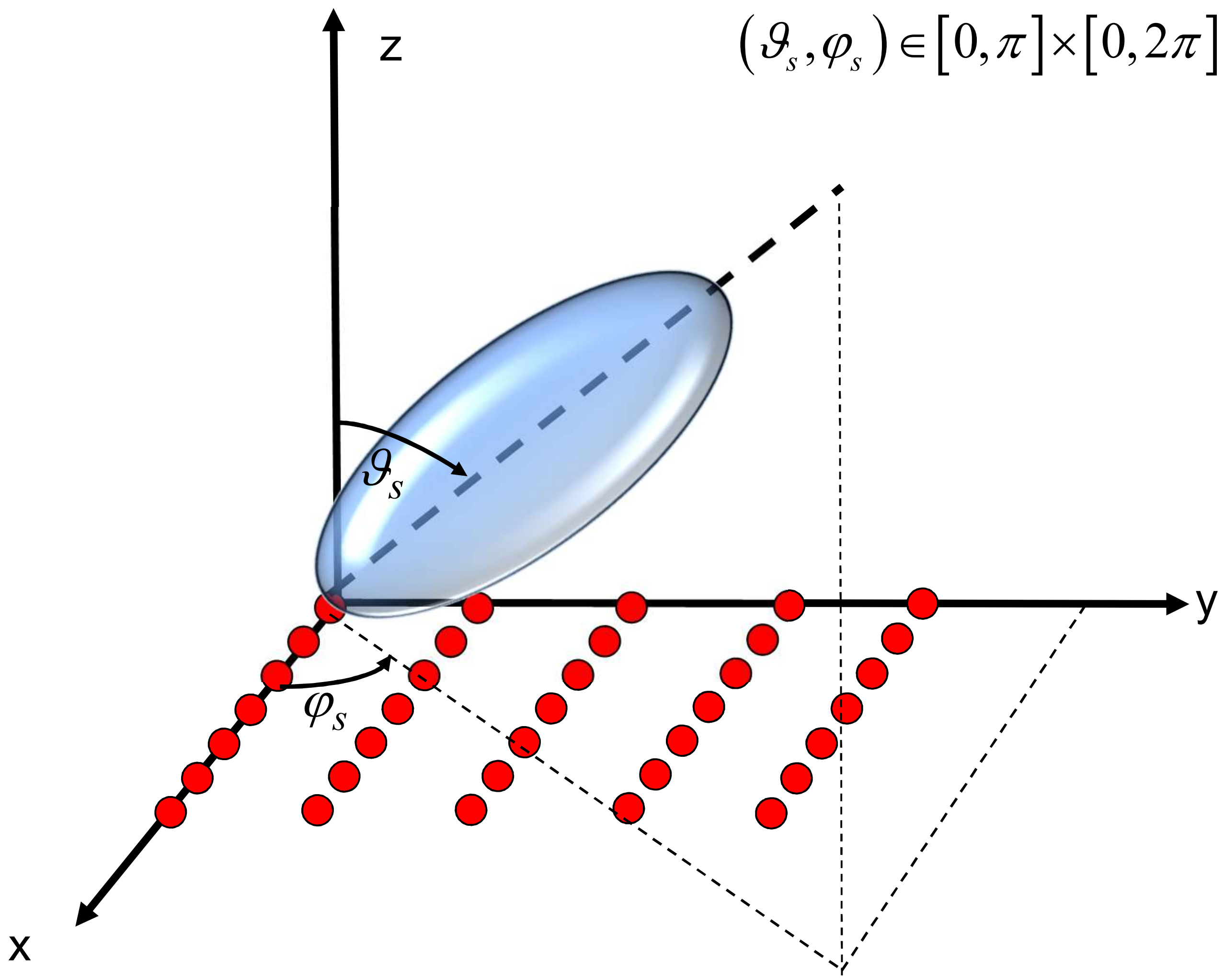
Beam scanning

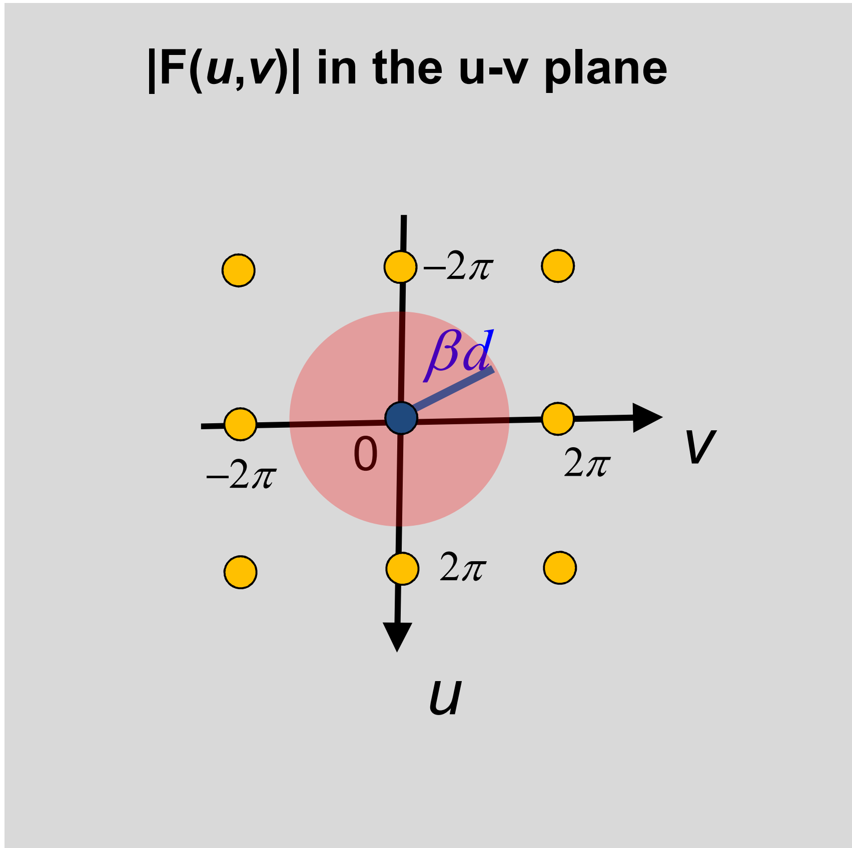
Periodic Planar Arrays: Beam Scanning



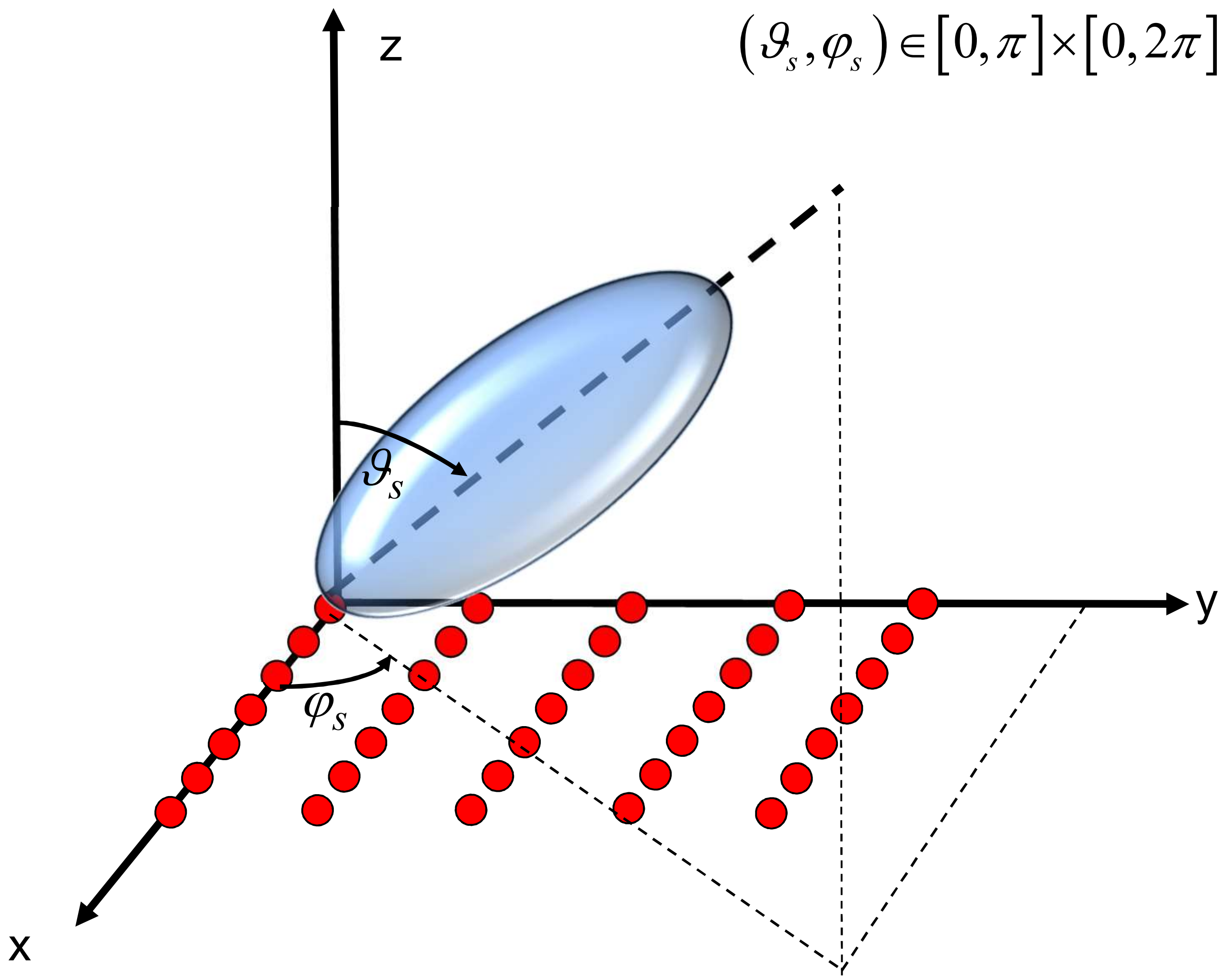


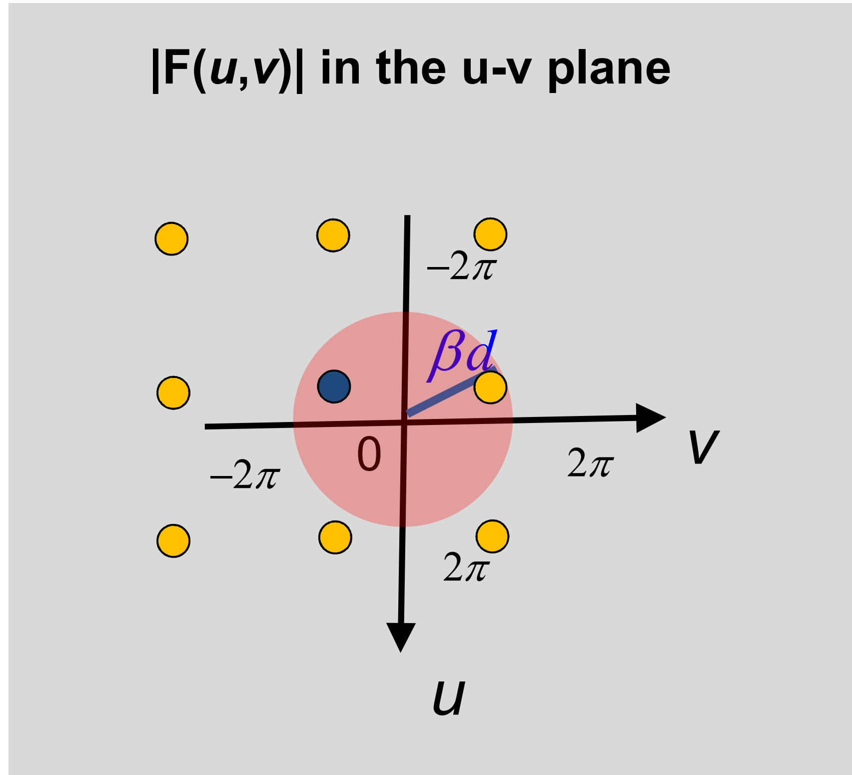
Periodic Planar Arrays: Beam Scanning





Periodic Planar Arrays: Beam Scanning





$$F(u,v) \rightarrow F(u-u_s, v-v_s)$$

Periodic Planar Arrays: Beam Scanning

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v)\Big|_{\substack{u=-\beta d_x \sin\vartheta\cos\varphi \\ v=-\beta d_y \sin\vartheta\sin\varphi}}$$

$$I_{nm} = I \quad \Rightarrow \quad |F(u,v)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|\left|\frac{\sin(Mv/2)}{\sin(v/2)}\right|$$

$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I_{nm}\exp(-jnu)\exp(-jmv)$$

$$F(u,v) \rightarrow F(u-u_s, v-v_s)$$

1) $(\vartheta_s,\varphi_s) \in [0,\pi]\times[0,2\pi]$

2) $u_s = -\beta d_x \sin\vartheta_s \cos\varphi_s$

$v_s = -\beta d_y \sin\vartheta_s \sin\varphi_s$

3) $I \rightarrow I e^{jnu_s} e^{jmv_s}$

$$F(u,v) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I e^{-jnu} e^{-jmv} \rightarrow \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I e^{jnu_s} e^{jmv_s} e^{-jnu} e^{-jmv} = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} I e^{-jn(u-u_s)} e^{-jm(v-v_s)} = F(u-u_s, v-v_s)$$