# Arrays

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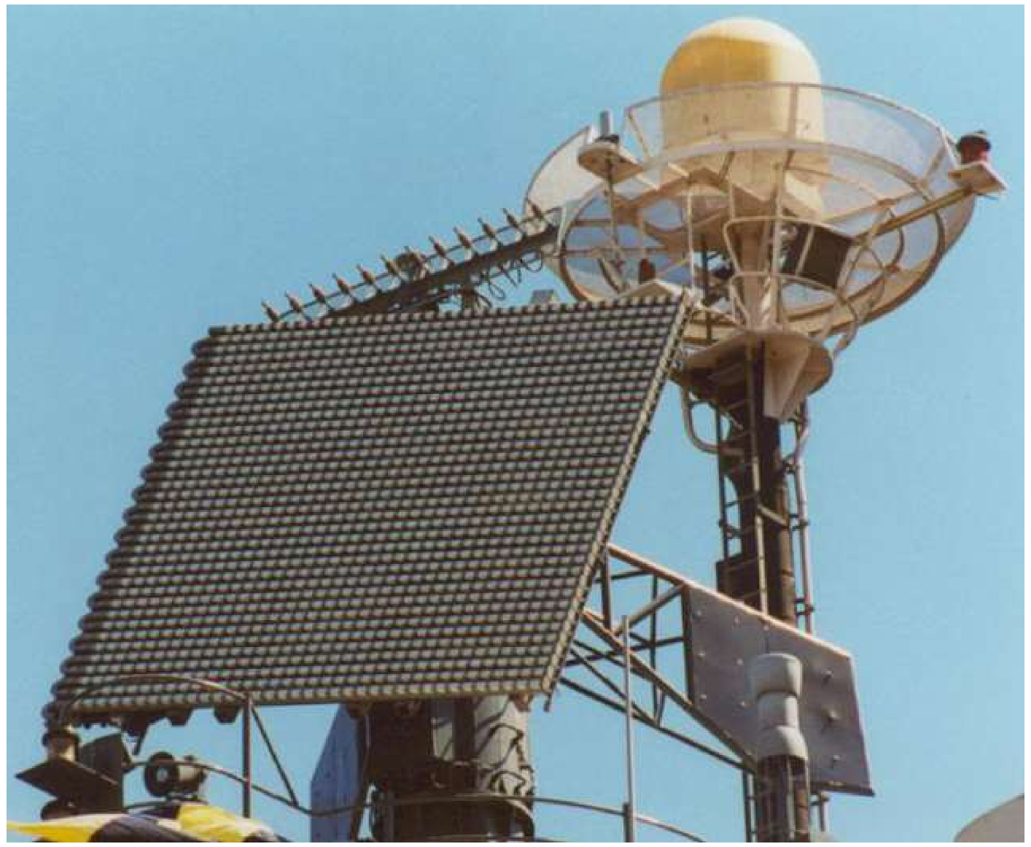


# Color legend

New formulas, important considerations,
important formulas, important concepts

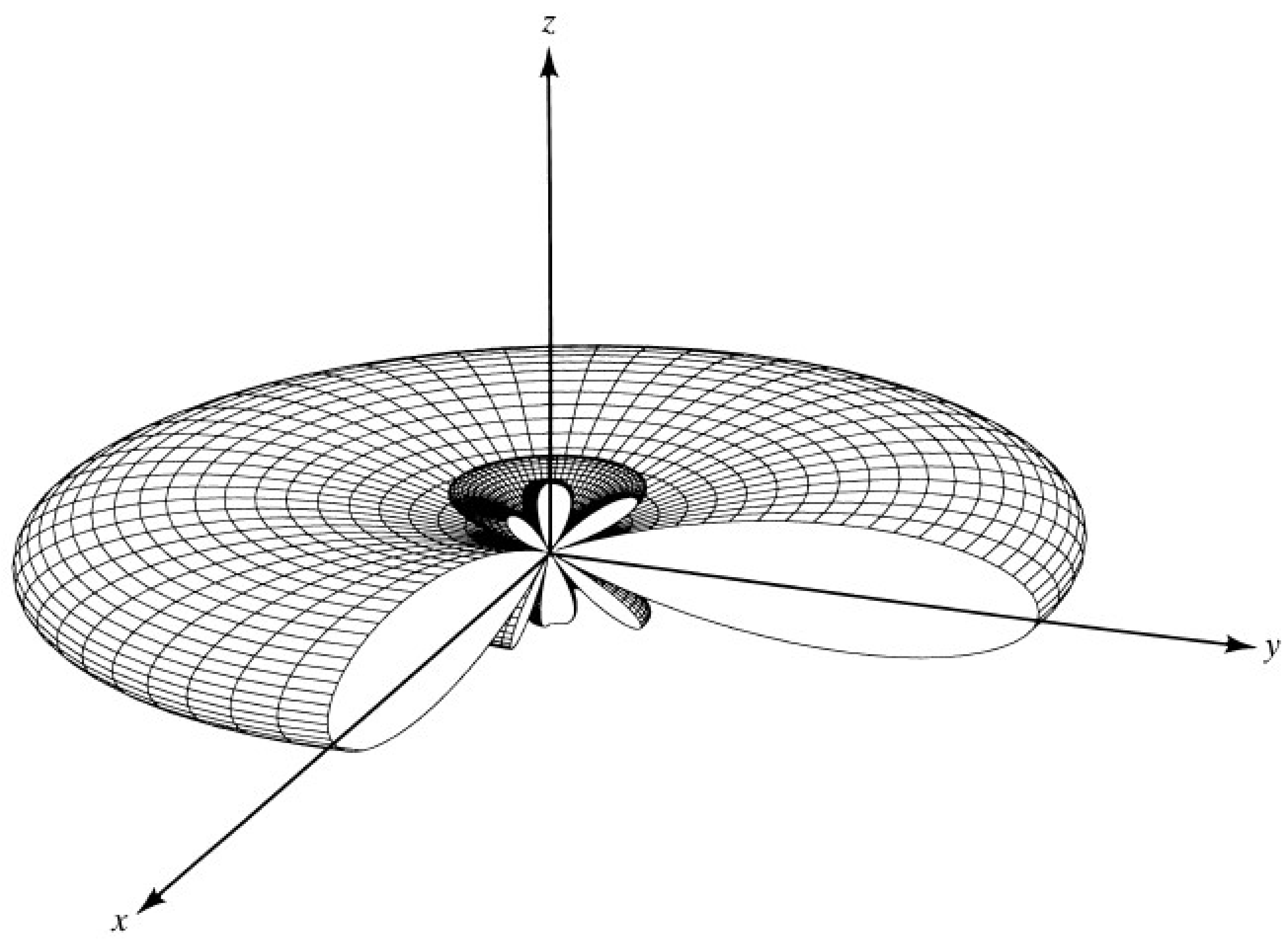
Very important for the discussion

Memo

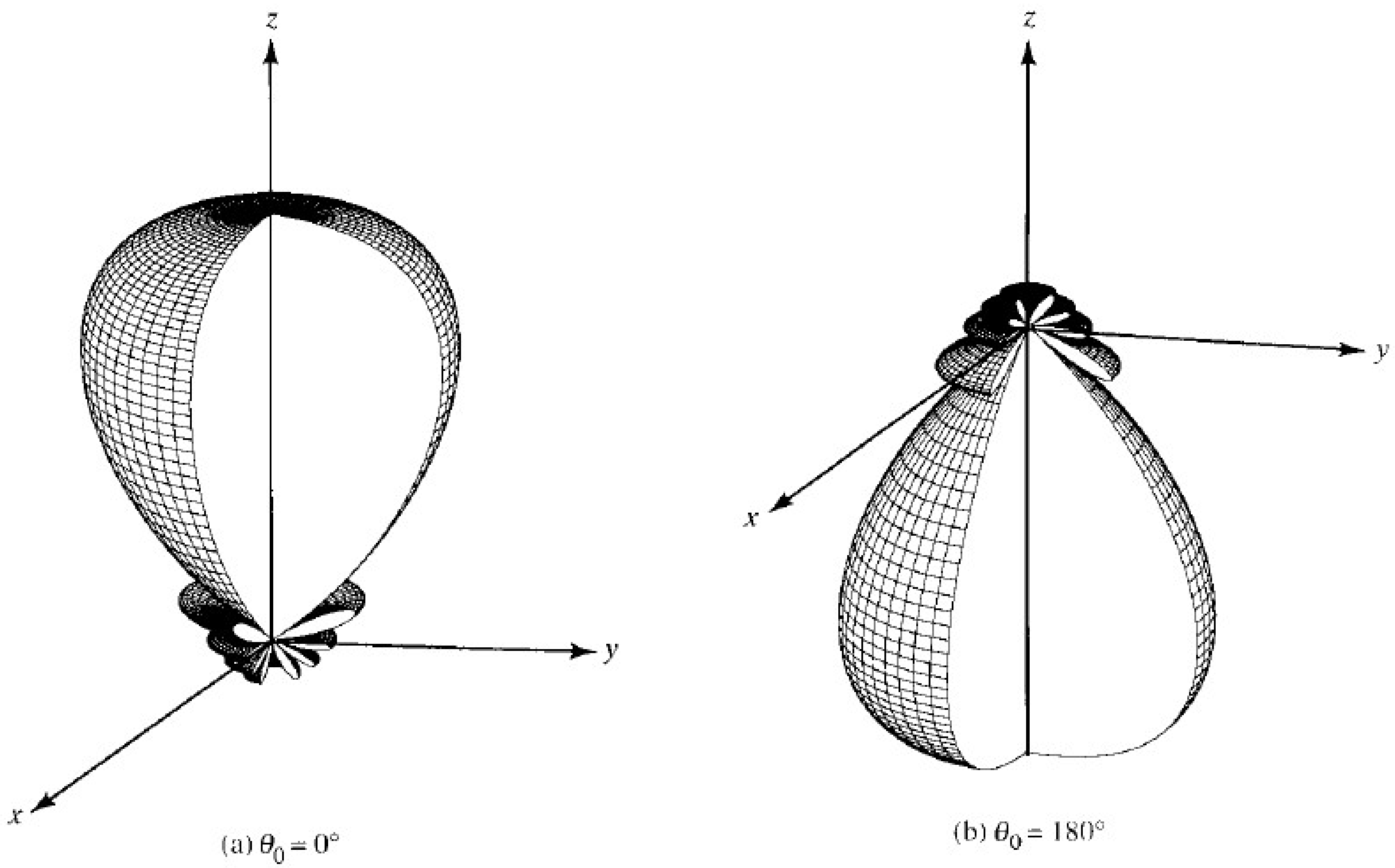
Mathematical tools to be exploited

Mathematics

# Broadside Arrays



# Endfire Arrays



# Linear Arrays

**P is located in the Fraunhofer Region relevant to the each antenna of the considered array**

**P is located in the Fraunhofer Region relevant to the overall array antenna**

**The antennas of the considered array are equal**

**The antennas of the considered array are deployed along one axis (Linear Arrays)**

$$\vec{\mathbf{r}}'_n = d_n \hat{i}_z \quad \Rightarrow \vec{\mathbf{r}}'_n \cdot \hat{i}_r = d_n \hat{i}_z \cdot \hat{i}_r = d_n \cos\vartheta$$

$$\vec{\mathbf{r}}'_n = d_n \hat{i}_y \quad \Rightarrow \vec{\mathbf{r}}'_n \cdot \hat{i}_r = d_n \hat{i}_y \cdot \hat{i}_r = d_n \sin\vartheta \sin\varphi$$

$$\vec{\mathbf{r}}'_n = d_n \hat{i}_x \quad \Rightarrow \vec{\mathbf{r}}'_n \cdot \hat{i}_r = d_n \hat{i}_x \cdot \hat{i}_r = d_n \sin\vartheta \cos\varphi$$

$$\hat{i}_r = \sin\vartheta\cos\varphi \hat{i}_x + \sin\vartheta\sin\varphi \hat{i}_y + \cos\vartheta \hat{i}_z$$

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)$$

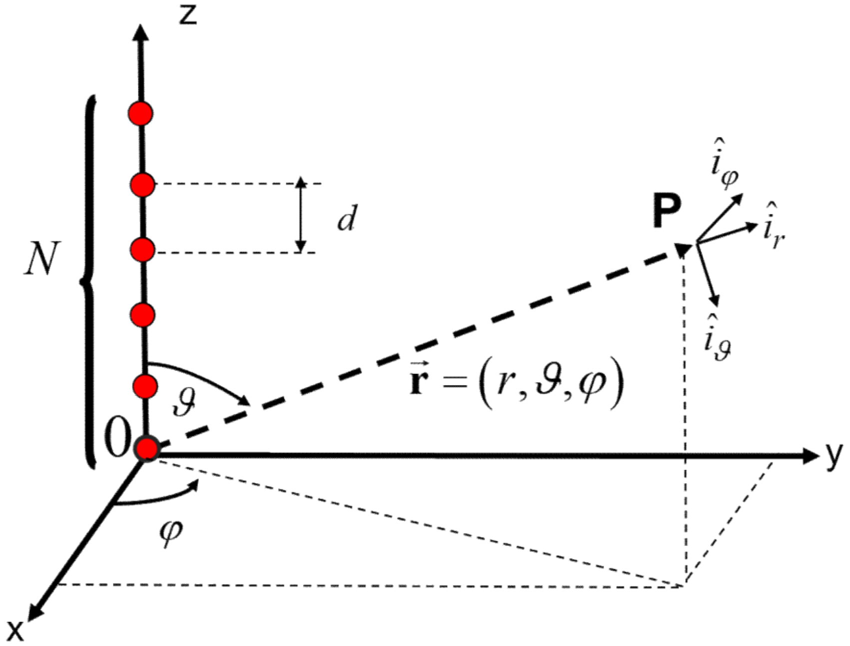
# Periodic Linear Arrays (z-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

**For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule**

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$




$$F(\vartheta) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\cos\vartheta)$$

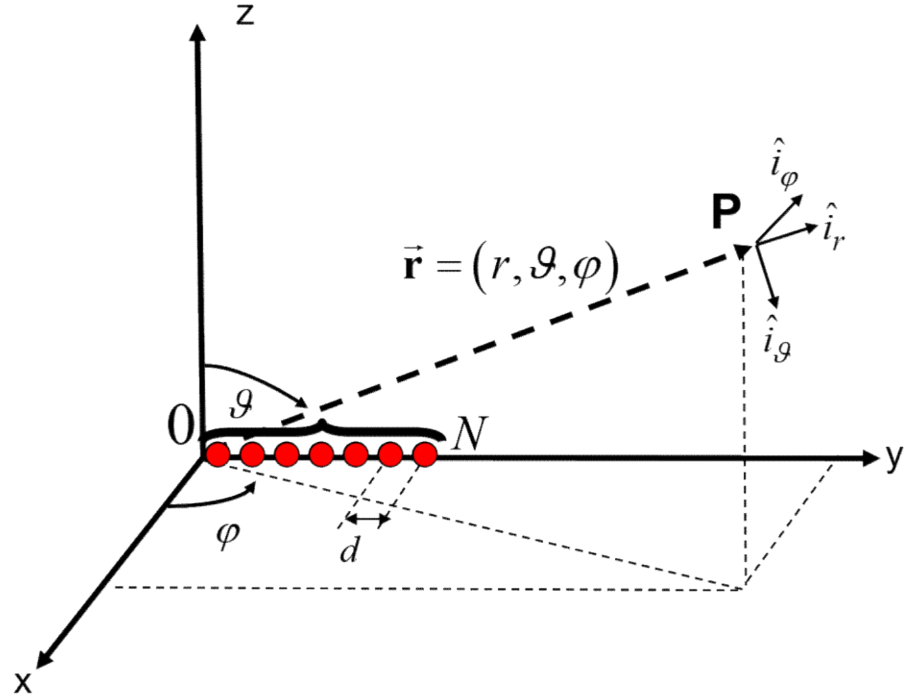
# Periodic Linear Arrays (y-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

**For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule**

$$F(\vartheta,\varphi) = F(u)\Big|_{u=-\beta d \sin\vartheta \sin\varphi}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$




$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd \sin\vartheta \sin\varphi)$$

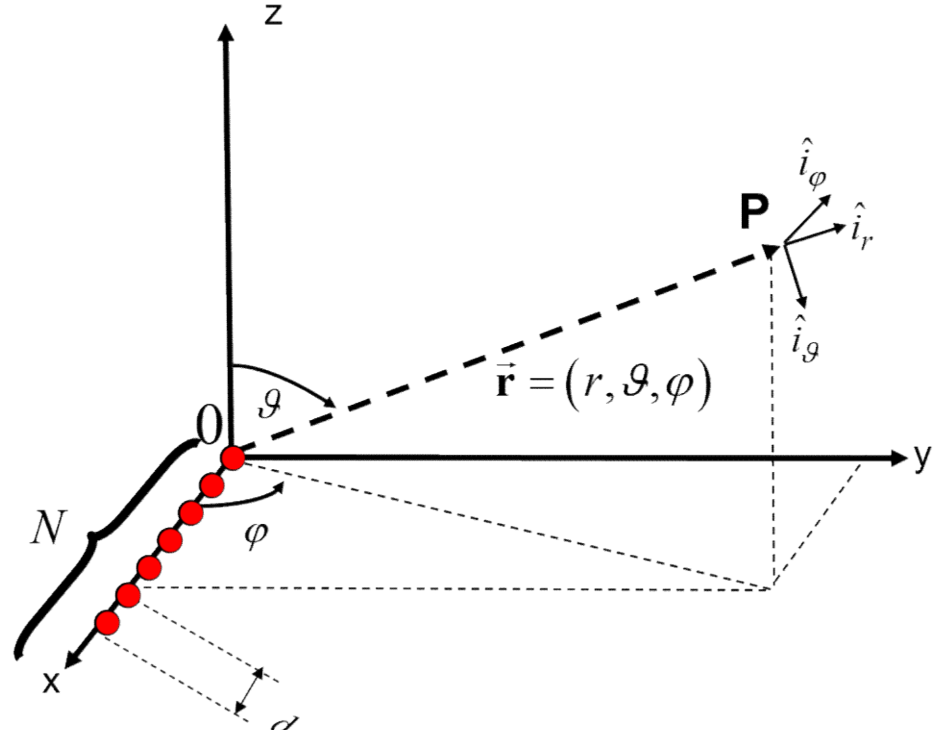
# Periodic Linear Arrays (x-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

**For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule**

$$F(\vartheta,\varphi) = F(u)\Big|_{u=-\beta d\sin\vartheta\cos\varphi}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$




$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\sin\vartheta\cos\varphi)$$

# Planar Arrays

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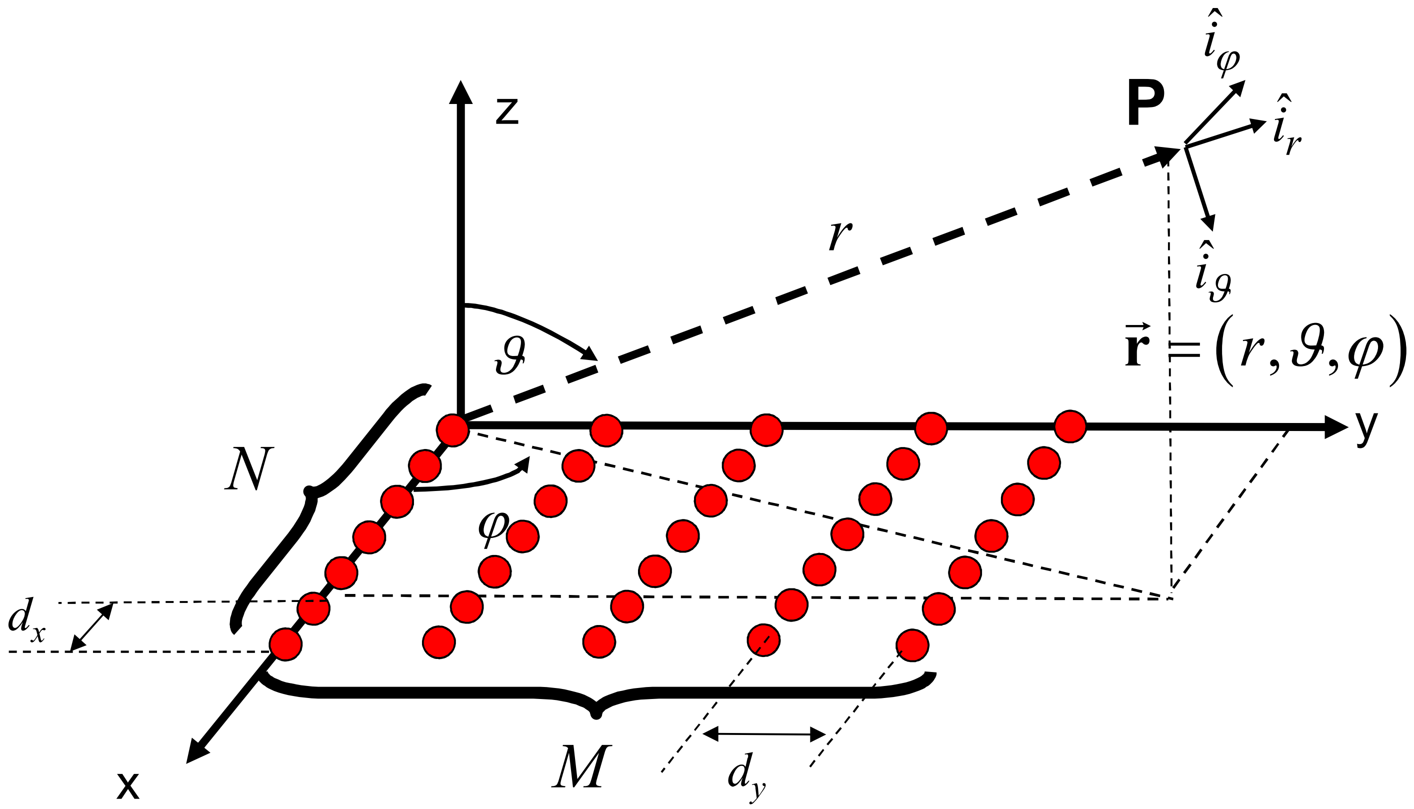
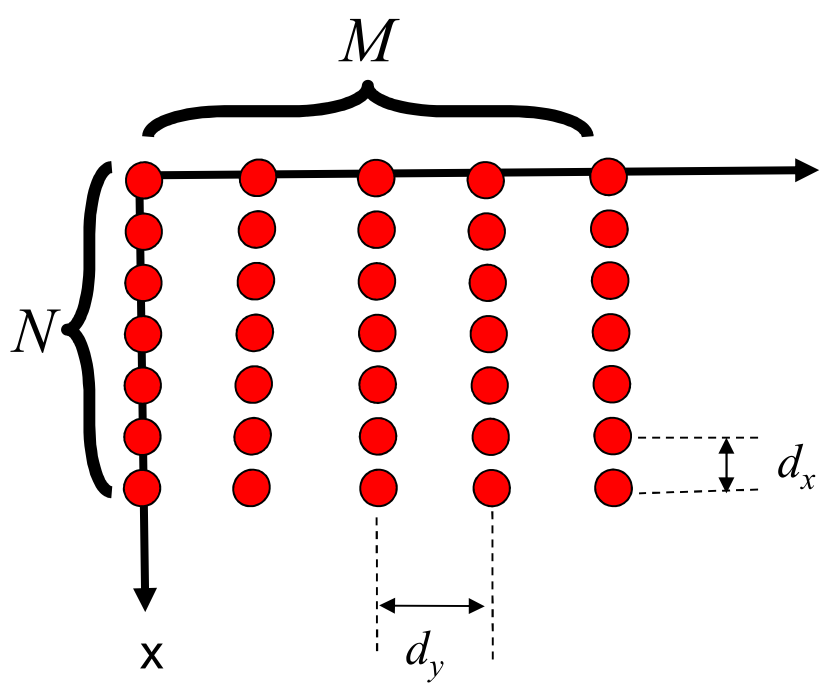


# Arrays

**P is located in the Fraunhofer Region relevant to the each antenna of the considered array**

**P is located in the Fraunhofer Region relevant to the overall array antenna**

**The antennas of the considered array are equal**

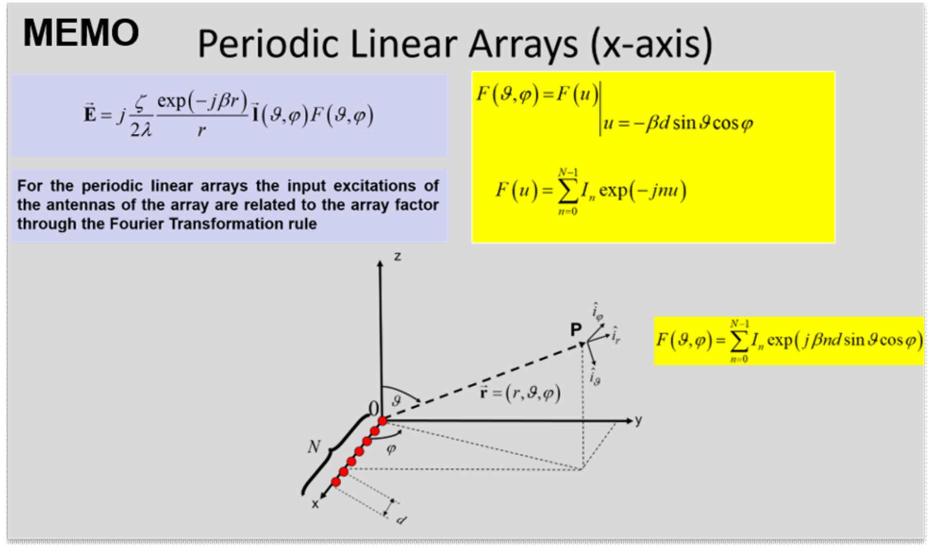
$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{l}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{k=0}^{(N\times M)-1} I_k \exp\left(j\beta\vec{\mathbf{r}}_k'\cdot\hat{i}_r\right)$$

**Principle of pattern multiplication**

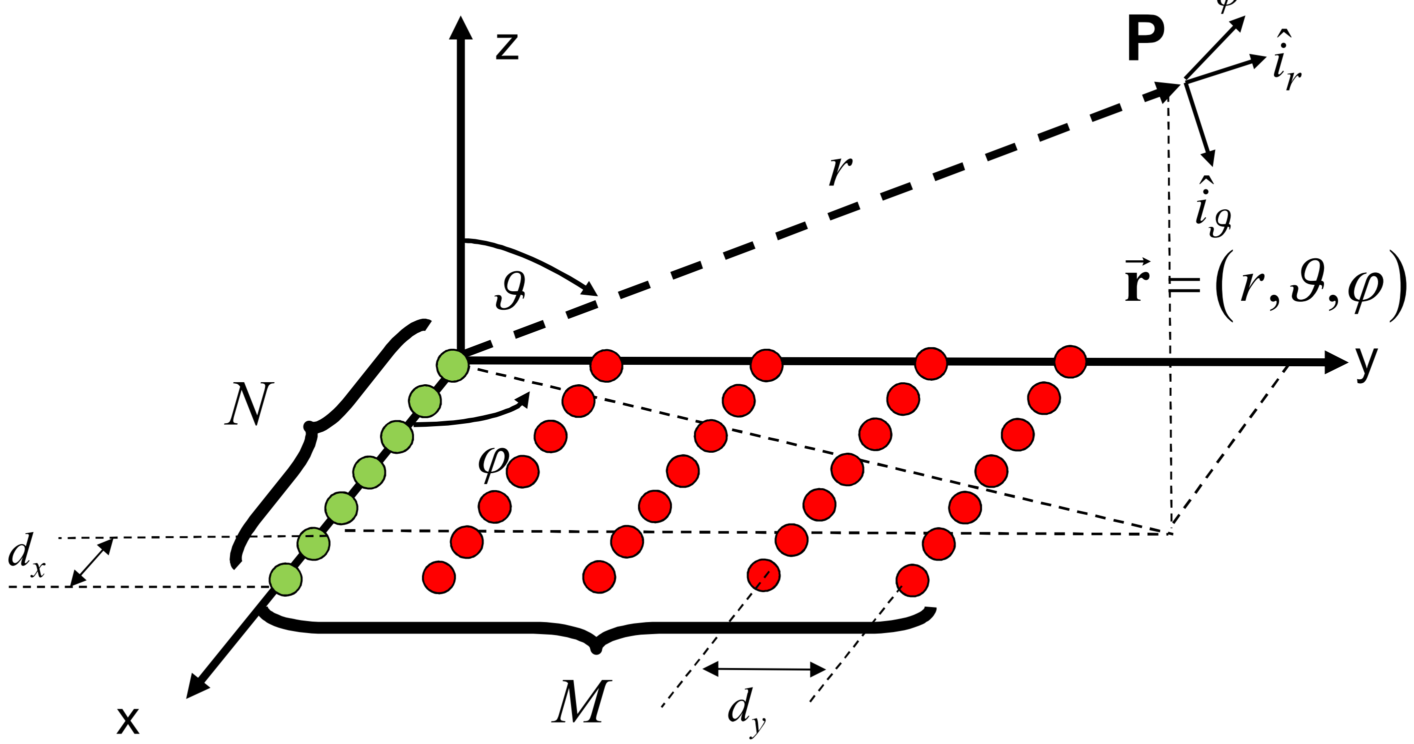
**The antennas are deployed on the xy plane (planar array)**

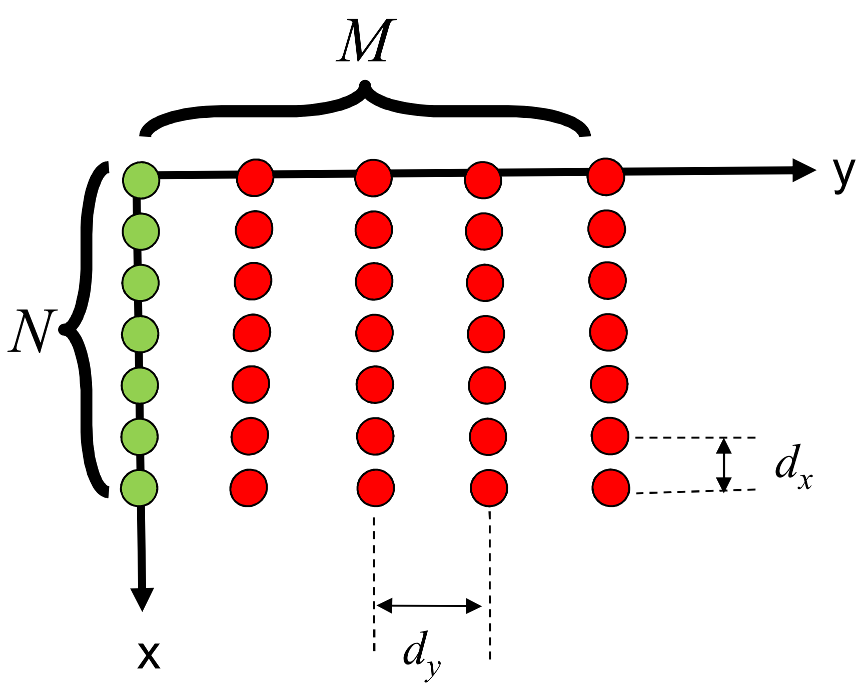
**The antennas are equispaced along both the x and y directions (periodic array)**

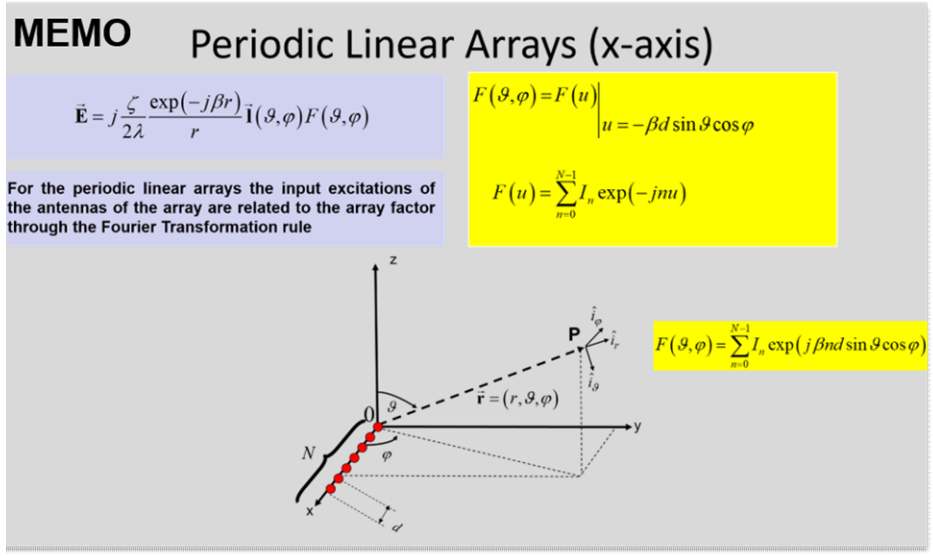



# Planar Arrays

**The antennas are equispaced along the x direction**

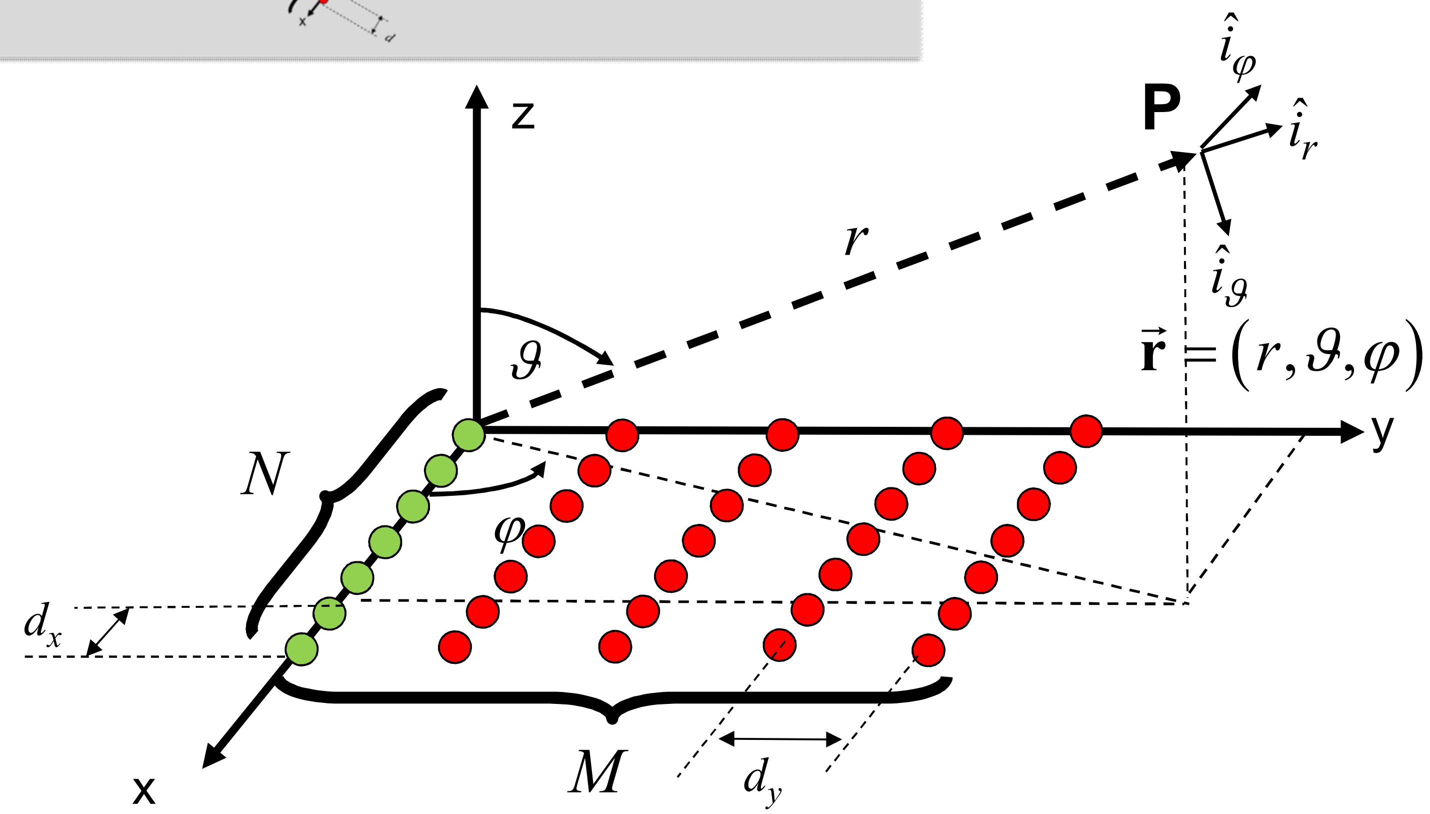


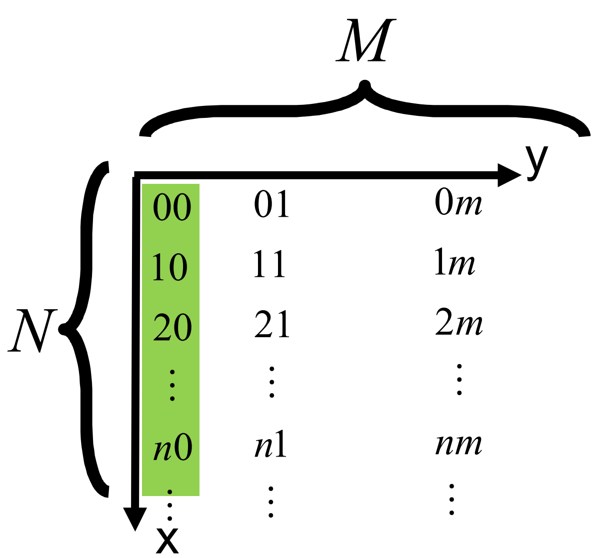






# Planar Arrays

$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$



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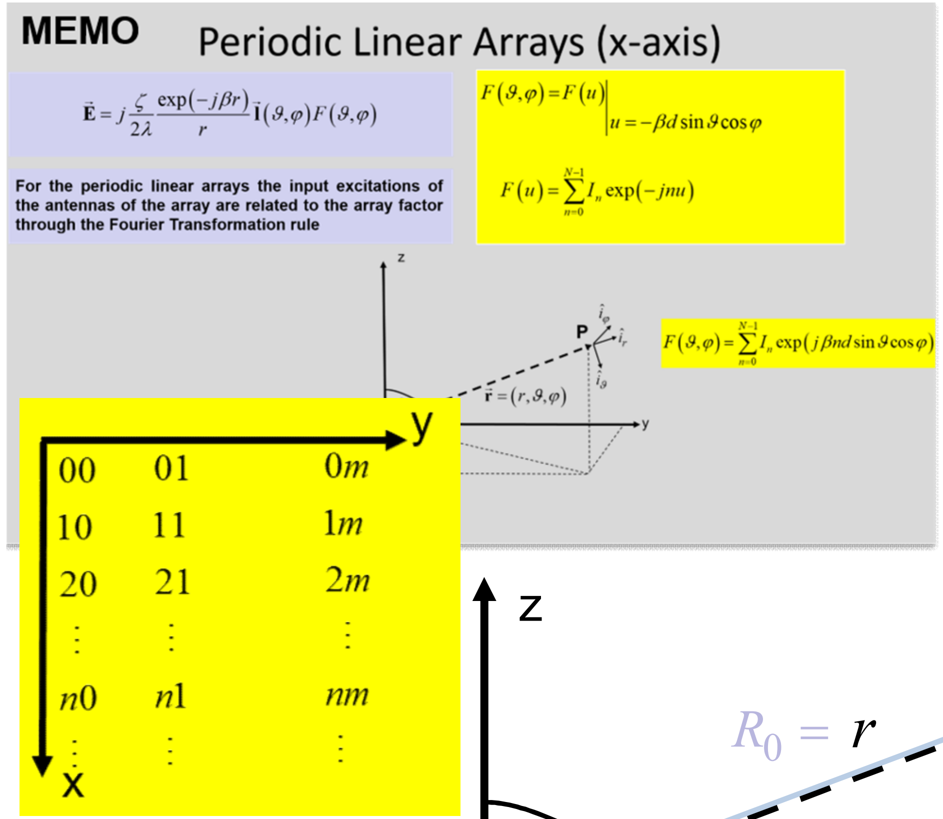
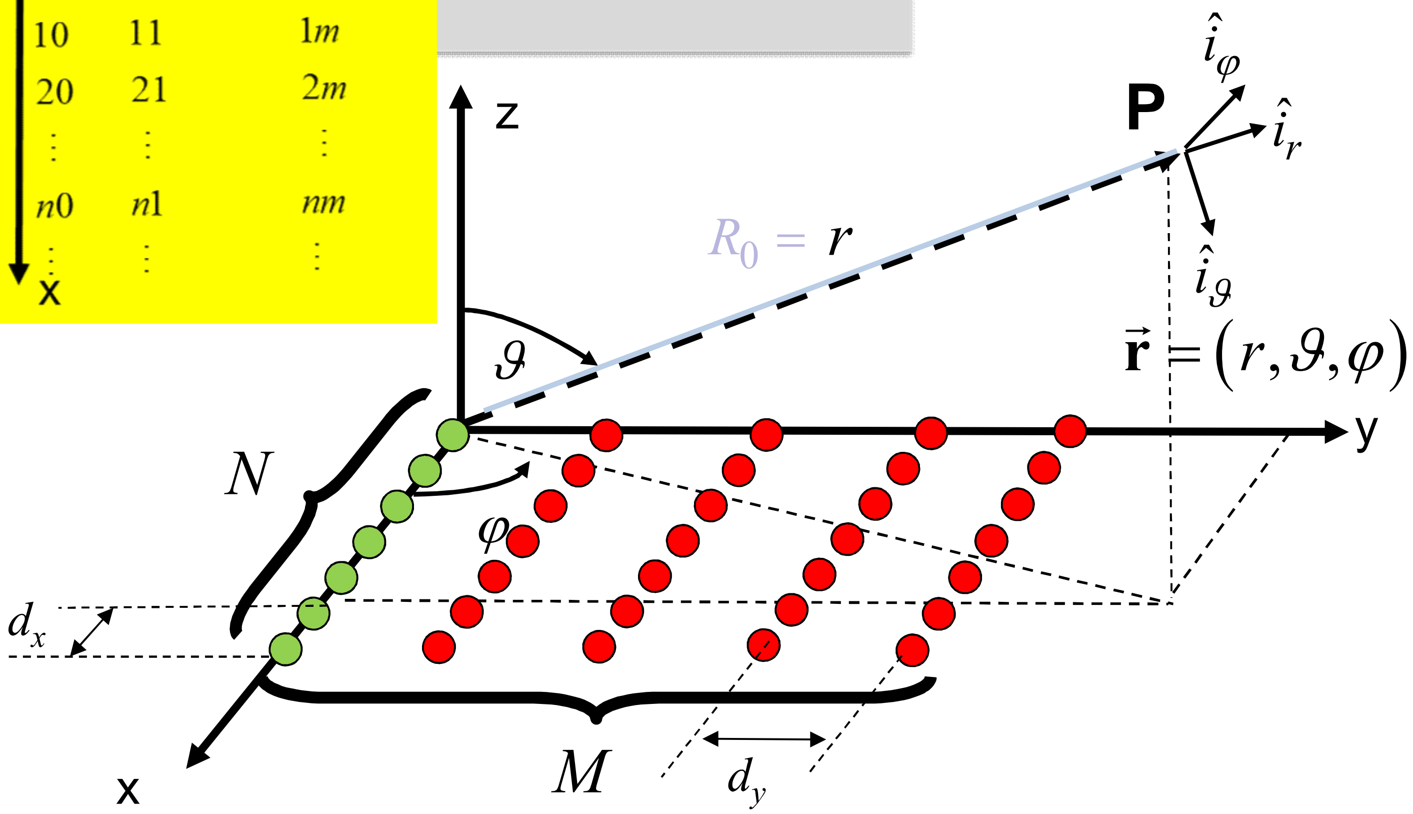
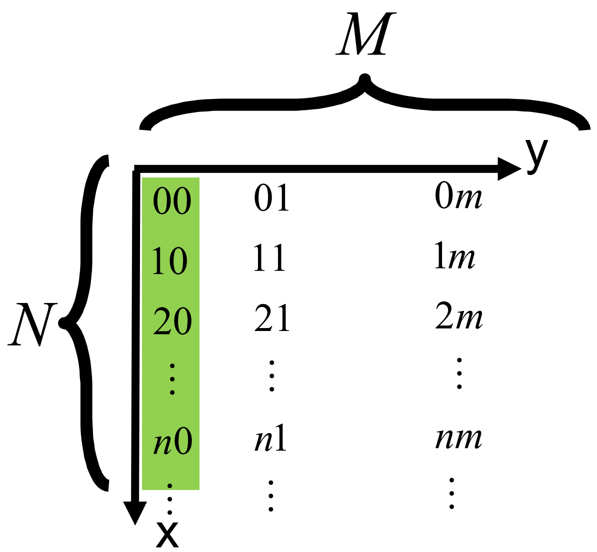



$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$



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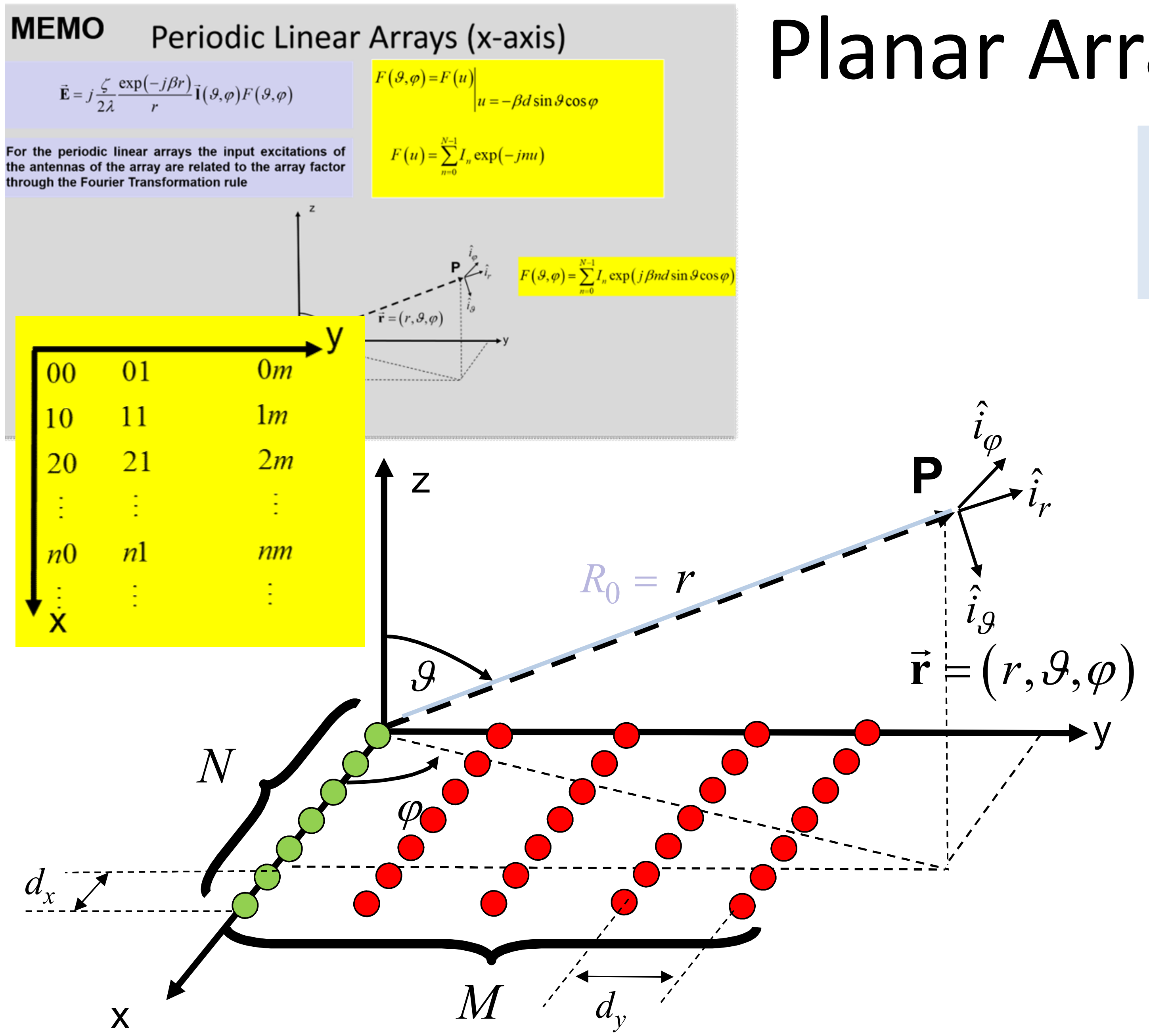

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$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

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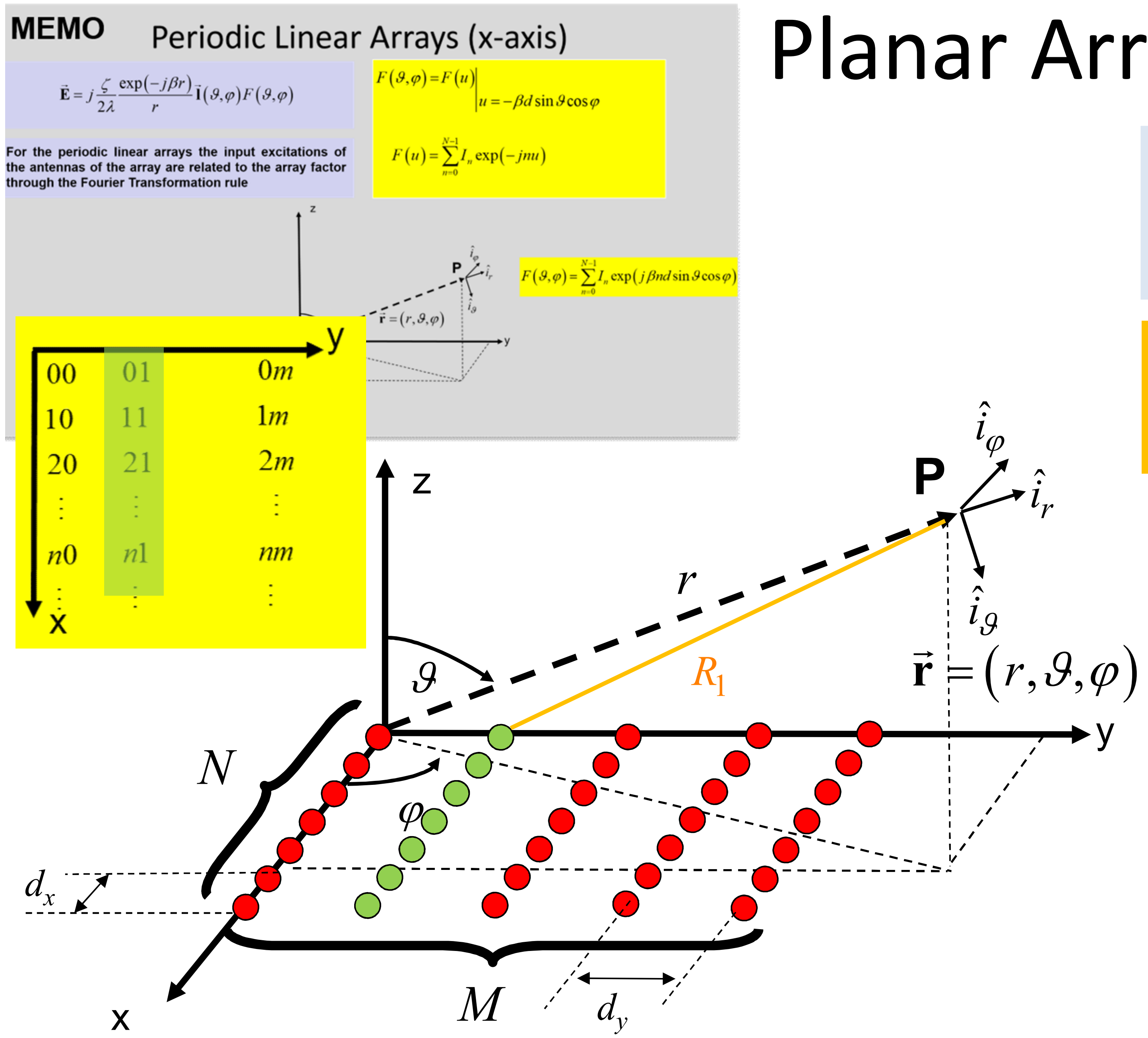

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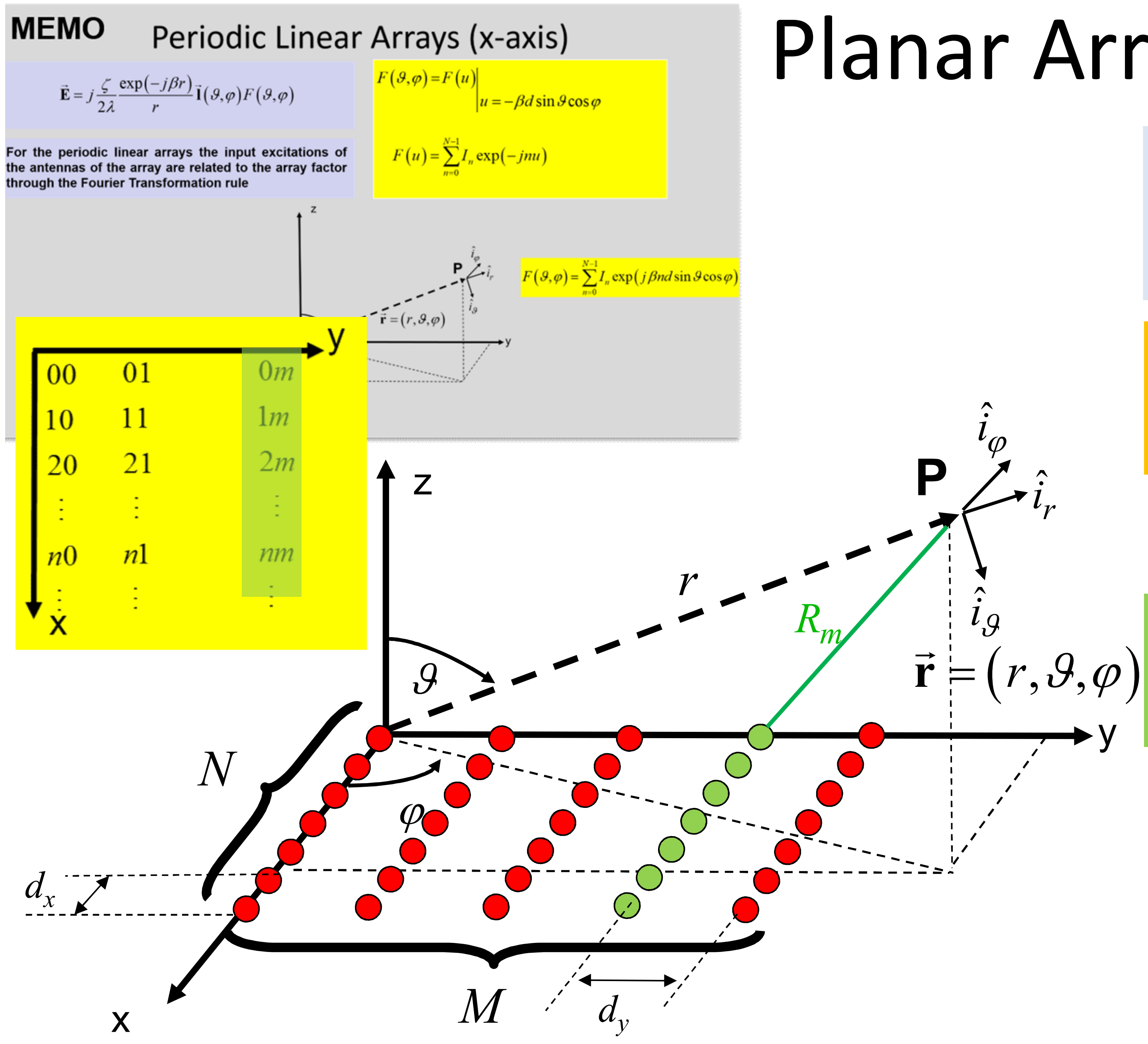


$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

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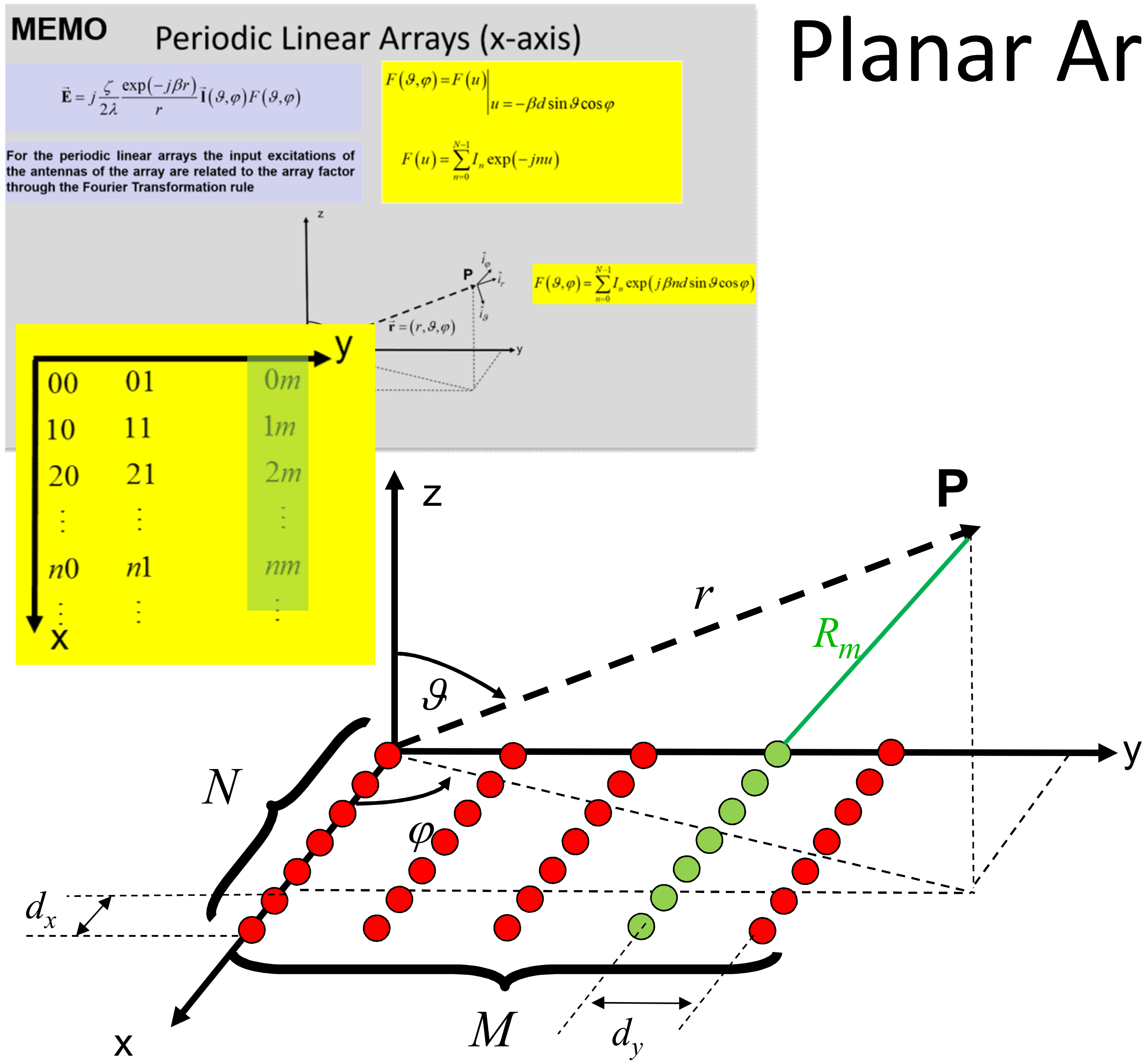
$\vdots$

$$\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_m)}{R_m}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

# Planar Arrays




$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

$$\vec{\mathbf{E}}_1 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_1)}{R_1}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n1}\exp(-jnu)$$

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# Planar Arrays

**MEMO** Periodic Linear Arrays (x-axis)

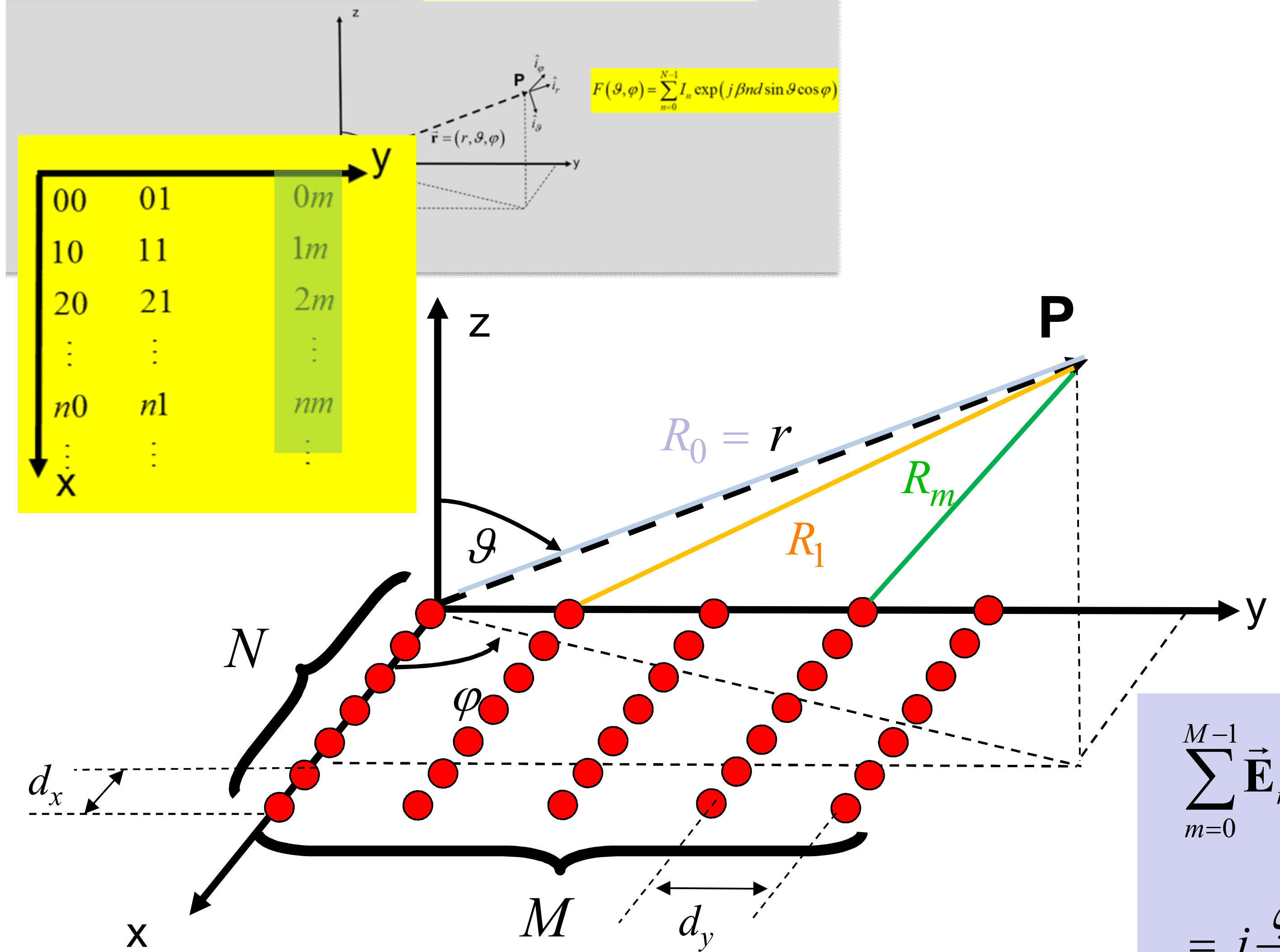
$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{i}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi)=F(u)\Big|_{u=-\beta d\sin\vartheta\cos\varphi}$$

$$F(u)=\sum_{n=0}^{N-1} I_n \exp(-jnu)$$

**For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule**

$$F(\vartheta,\varphi)=\sum_{n=0}^{N-1} I_n \exp(j\beta nd\sin\vartheta\cos\varphi)$$




$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

$$\vec{\mathbf{E}}_1 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_1)}{R_1}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n1}\exp(-jnu)$$

$\vdots$

$$\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_m)}{R_m}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

$$\sum_{m=0}^{M-1}\vec{\mathbf{E}}_m = \sum_{m=0}^{M-1}\left[ j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_m)}{R_m}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)\right]$$

$$= j\frac{\zeta}{2\lambda}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

**MEMO** Periodic Linear Arrays (x-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{i}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u)\Big|_{u=-\beta d\sin\vartheta\cos\varphi}$$

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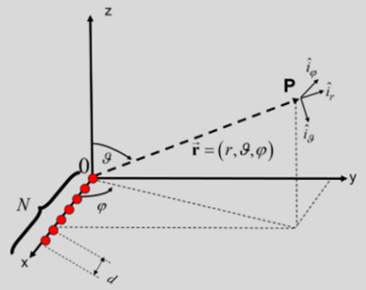
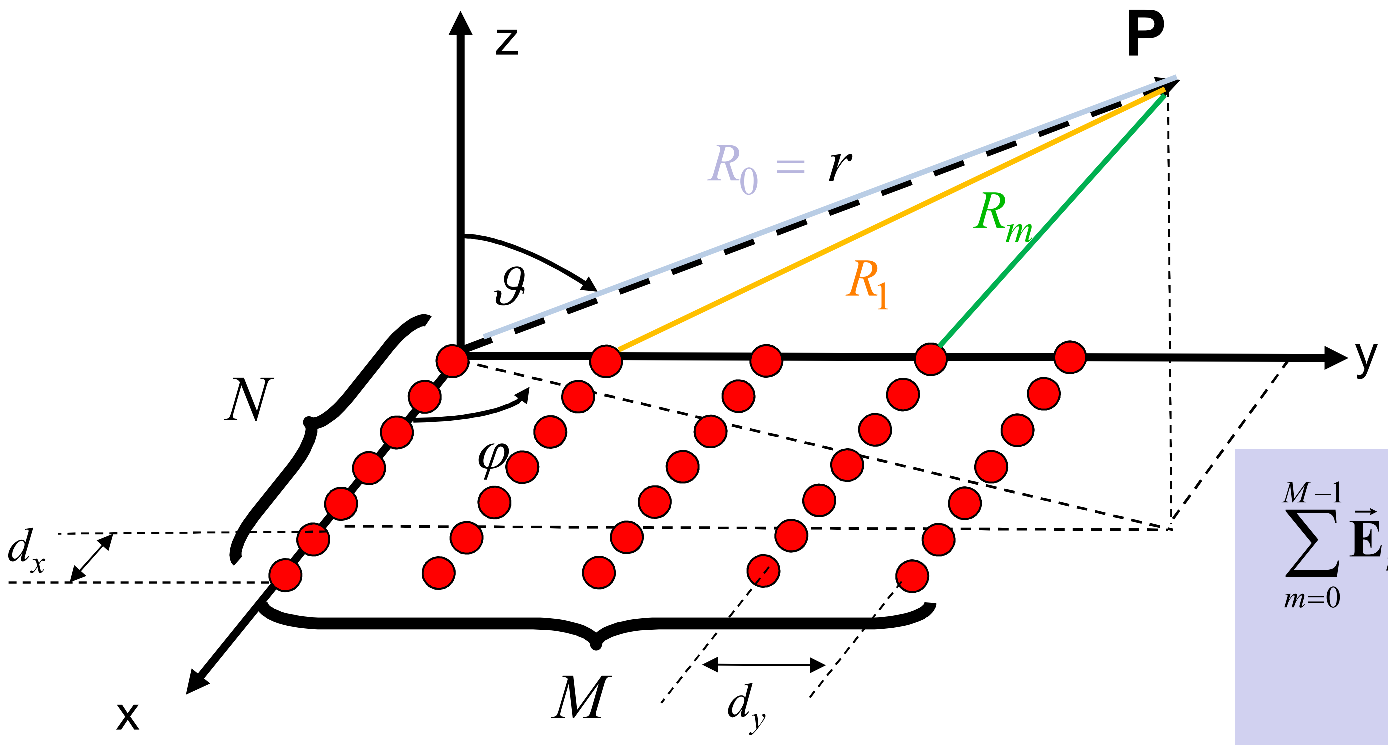
**For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule**




$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\sin\vartheta\cos\varphi)$$

# Planar Arrays




$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

$$\vec{\mathbf{E}}_1 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_1)}{R_1}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n1}\exp(-jnu)$$

$\vdots$

$$\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_m)}{R_m}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

$$\sum_{m=0}^{M-1}\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\vec{\mathbf{i}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

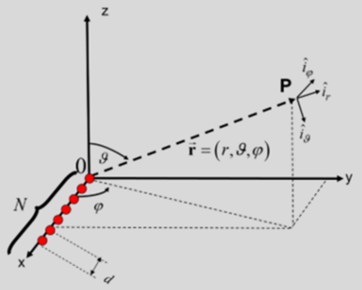
**MEMO** Periodic Linear Arrays (x-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u)\Big|_{u=-\beta d\sin\vartheta\cos\varphi}$$

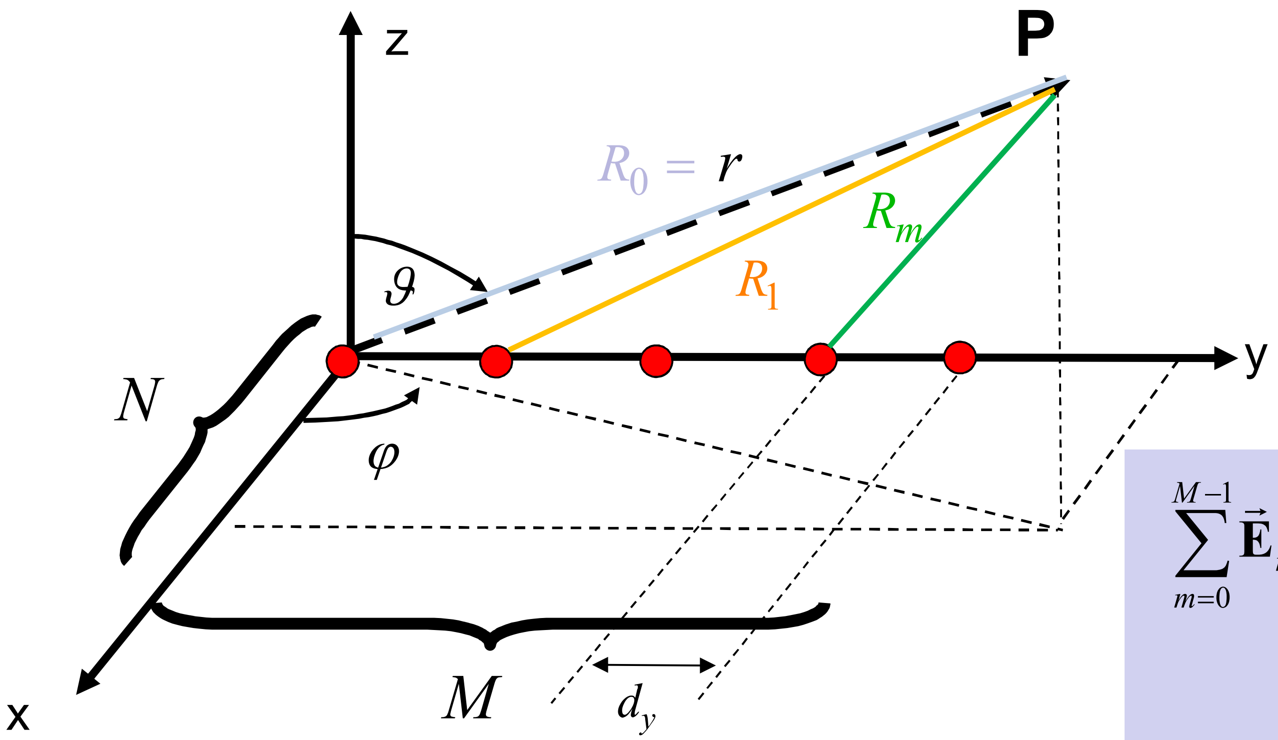
$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule



$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp(j\beta n d\sin\vartheta\cos\varphi)$$

# Planar Arrays




$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

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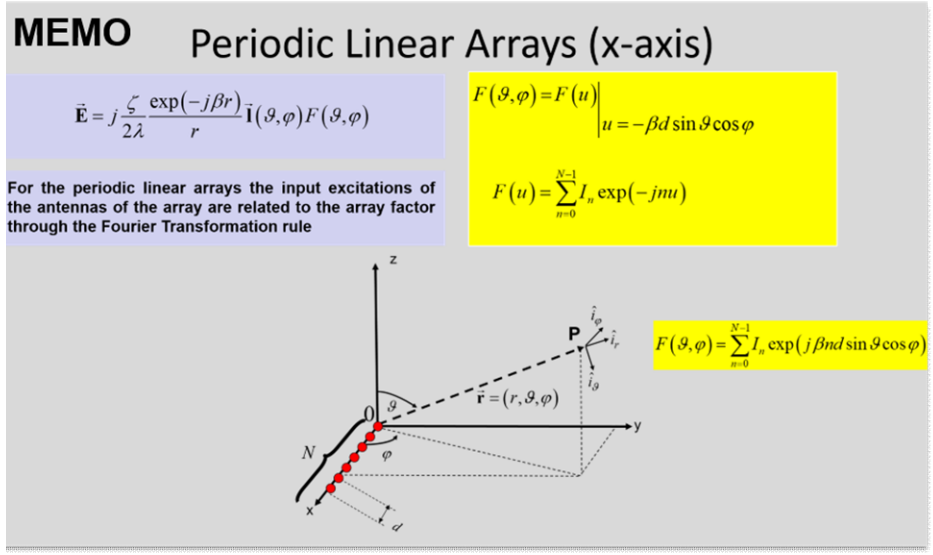
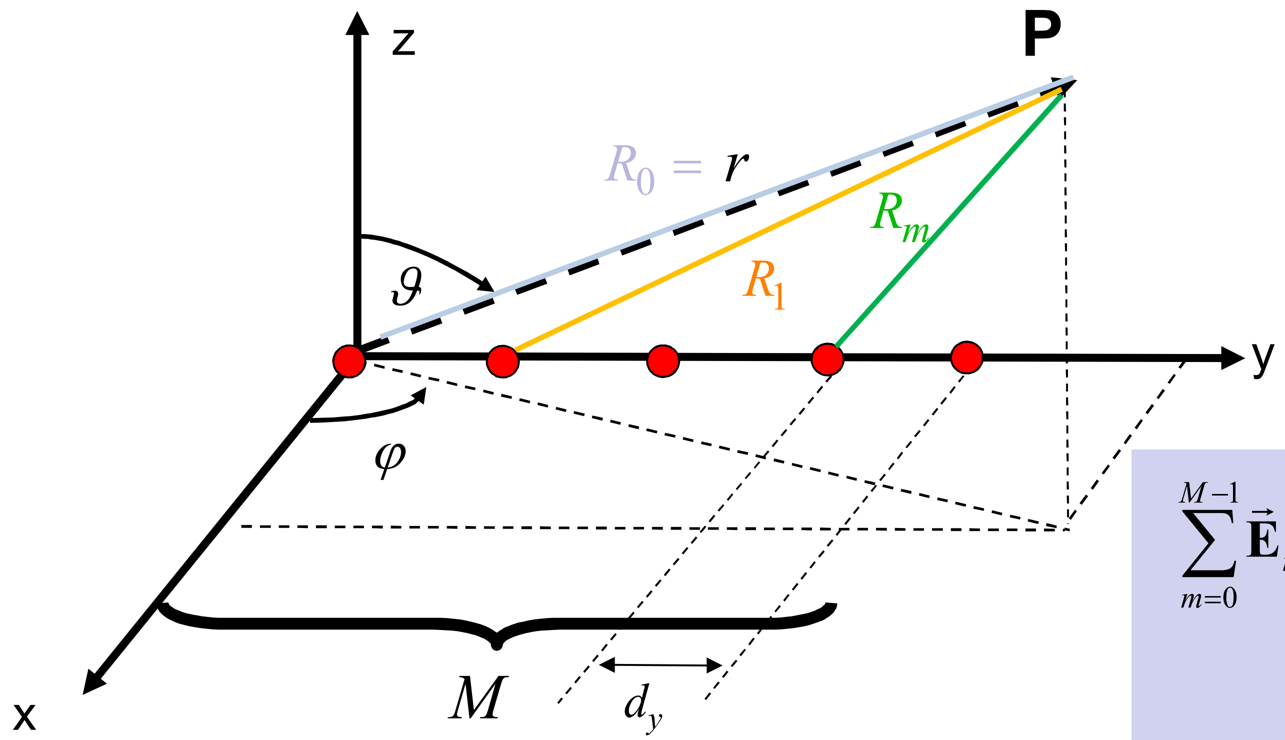
⋮

$$\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_m)}{R_m}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

$$\sum_{m=0}^{M-1}\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

# Planar Arrays




$$\vec{\mathbf{E}}_0 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n0}\exp(-jnu)$$

$$\vec{\mathbf{E}}_1 = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_1)}{R_1}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{n1}\exp(-jnu)$$

$$\vdots$$

$$\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta R_m)}{R_m}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

$$\sum_{m=0}^{M-1}\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

# MEMO: Fraunhofer Region

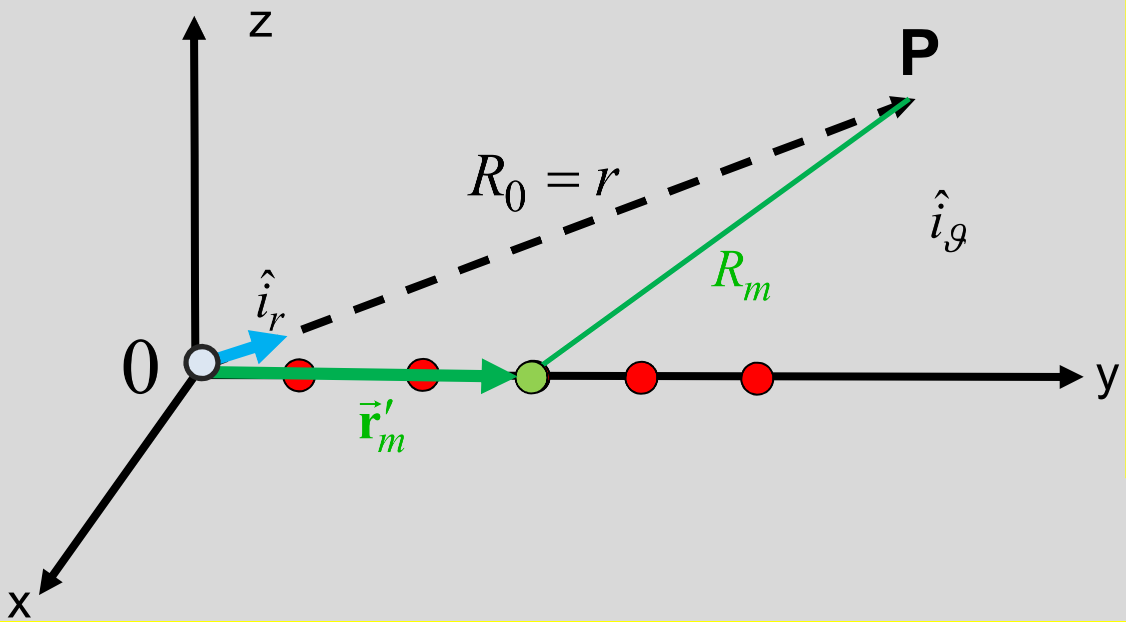
**P is located in the Fraunhofer Region relevant to the overall array antenna**

$$\hat{i}_r = \sin\vartheta\cos\varphi\,\hat{i}_x + \sin\vartheta\sin\varphi\,\hat{i}_y + \cos\vartheta\,\hat{i}_z$$

$$\sum_{m=0}^{M-1}\vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1}I_{nm}\exp(-jnu)$$

$$u = -\beta d_x \sin\vartheta\cos\varphi$$

- $R_m \approx r - \vec{\mathbf{r}}'_m \cdot \hat{i}_r = r - md_y \sin\vartheta\sin\varphi$




**The antennas of the considered array are deployed along the y axis (Linear Arrays)**

$$\vec{\mathbf{r}}'_m = d_m\hat{i}_y \Rightarrow \vec{\mathbf{r}}'_m \cdot \hat{i}_r = d_m\hat{i}_y \cdot \hat{i}_r = d_m \sin\vartheta\sin\varphi$$

**also, they are equispaced**

$$\vec{\mathbf{r}}'_m = md_y\hat{i}_y \Rightarrow \vec{\mathbf{r}}'_m \cdot \hat{i}_r = md_y\hat{i}_y \cdot \hat{i}_r = md_y \sin\vartheta\sin\varphi$$

$$\Rightarrow \frac{\exp(-j\beta R_m)}{R_m} \approx \frac{\exp(-j\beta r)\exp(j\beta md_y\sin\vartheta\sin\varphi)}{r} = \frac{\exp(-j\beta r)\exp(-jmv)}{r}$$

$$v = -\beta d_y \sin\vartheta\sin\varphi$$

# MEMO: Fraunhofer Region

**P is located in the Fraunhofer Region relevant to the overall array antenna**

$$\sum_{m=0}^{M-1} \vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

$$u = -\beta d_x \sin\vartheta\cos\varphi$$

$$\Rightarrow \frac{\exp(-j\beta R_m)}{R_m} \approx \frac{\exp(-j\beta r)\exp\left(j\beta m d_y \sin\vartheta\sin\varphi\right)}{r} = \frac{\exp(-j\beta r)\exp(-jmv)}{r}$$

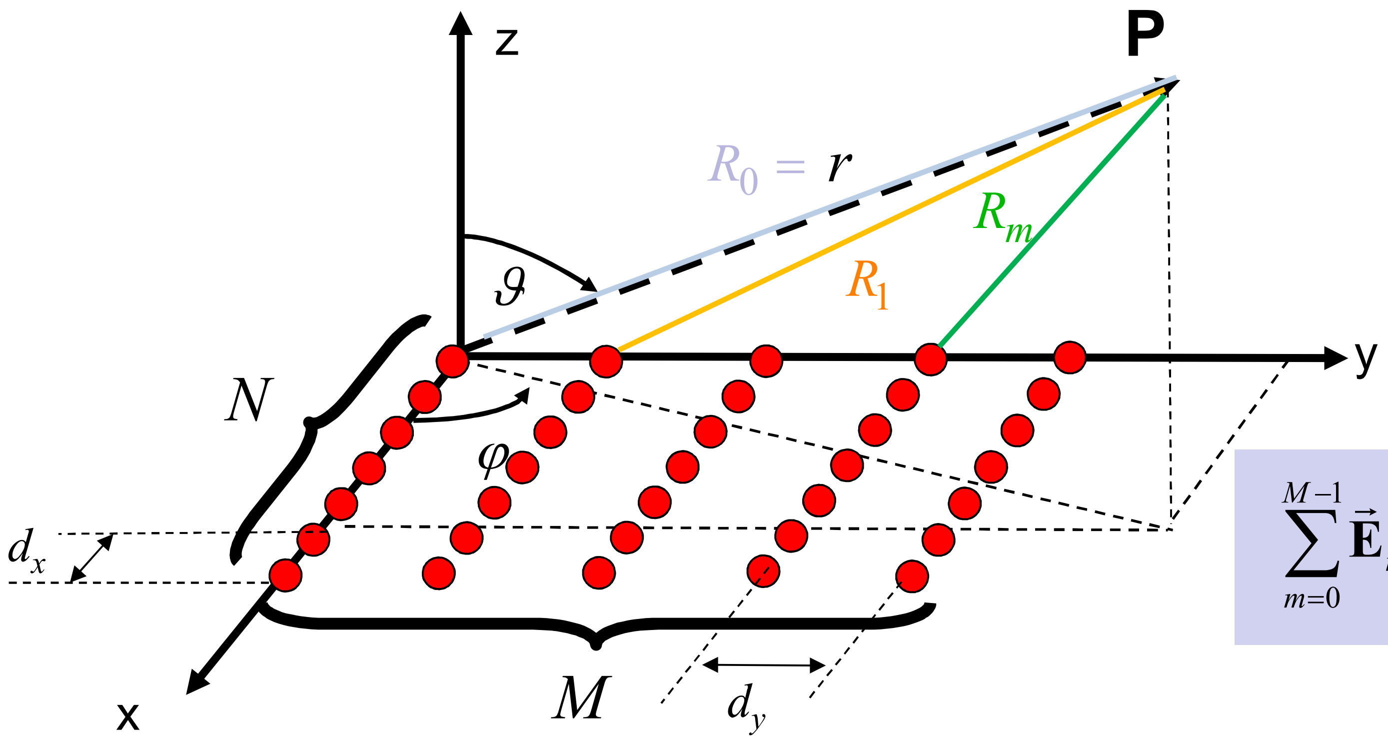
$$v = -\beta d_y \sin\vartheta\sin\varphi$$

# Planar Arrays

$$v = -\beta d_y \sin\vartheta \sin\varphi$$

$$\frac{\exp(-j\beta R_m)}{R_m} \approx \frac{\exp(-j\beta r)\exp(-jmv)}{r}$$

$$u = -\beta d_x \sin\vartheta \cos\varphi$$




$$\sum_{m=0}^{M-1} \vec{\mathbf{E}}_m = j\frac{\zeta}{2\lambda}\vec{\mathbf{I}}(\vartheta,\varphi)\sum_{m=0}^{M-1}\frac{\exp(-j\beta R_m)}{R_m}\sum_{n=0}^{N-1} I_{nm}\exp(-jnu)$$

# Planar Arrays

$$v = -\beta d_y \sin\vartheta \sin\varphi$$

$$\frac{\exp(-j\beta R_m)}{R_m} \approx \frac{\exp(-j\beta r)\exp(-jmv)}{r}$$

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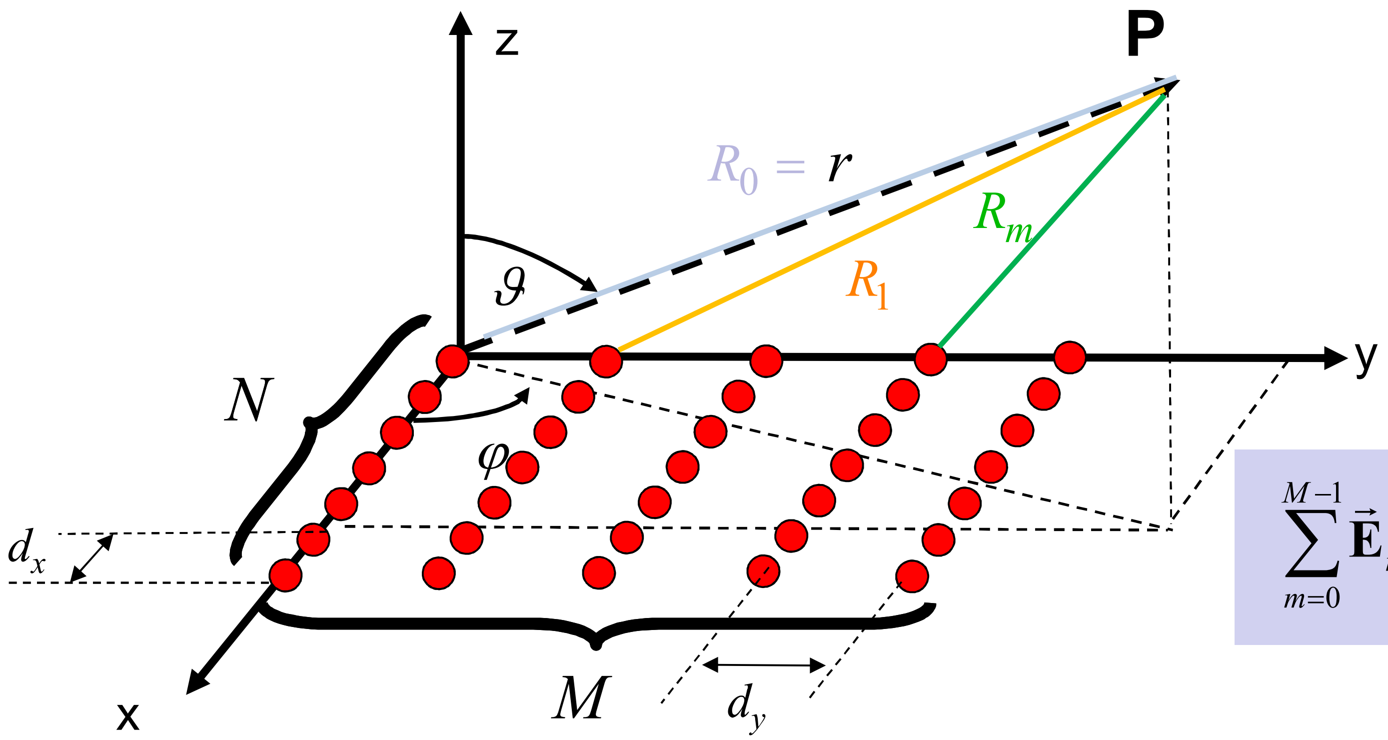


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# Planar Arrays

$$v = -\beta d_y \sin\vartheta \sin\varphi$$

$$\frac{\exp(-j\beta R_m)}{R_m} \approx \frac{\exp(-j\beta r)\exp(-jmv)}{r}$$

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# Periodic Planar Arrays

**P is located in the Fraunhofer Region relevant to the each antenna of the considered array**

**P is located in the Fraunhofer Region relevant to the overall array antenna**

**The antennas of the considered array are equal**

**The antennas are deployed on the xy plane (planar array)**

**The antennas are equispaced along both the x and y directions (periodic array)**

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\ F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{k=0}^{(N\times M)-1} I_k \exp\left(j\beta\vec{\mathbf{r}}'_k\cdot\hat{i}_r\right)$$

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**For the periodic planar arrays the input excitations of the antennas of the array are related to the array factor through the Two Dimensional (2D) Fourier Transformation rule**

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**The properties of the Fourier Transformation suggest some interesting considerations**

- **Arrays's size and beamwidth**
- **Scanning of the pattern**
- **Synthesis of the pattern**

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{I}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

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# Periodic Planar Arrays

**Grating lobes**

**Visible region**

# Periodic Planar Arrays: Grating Lobes

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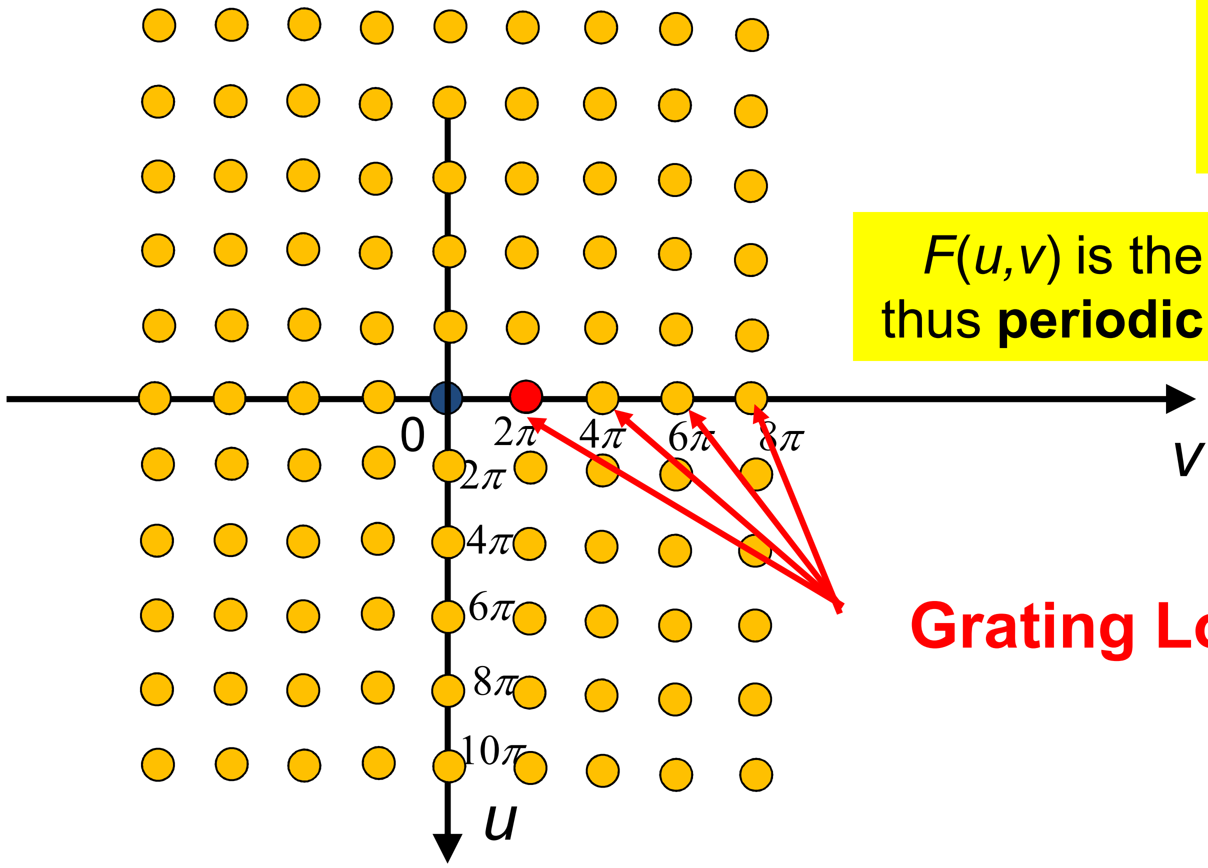


*F*(*u,v*) is the 2D Fourier Transform of a discrete sequence: it is thus **periodic** in both directions (*u* and *v*) with **period equal to 2π**

**Grating Lobes: undesired**

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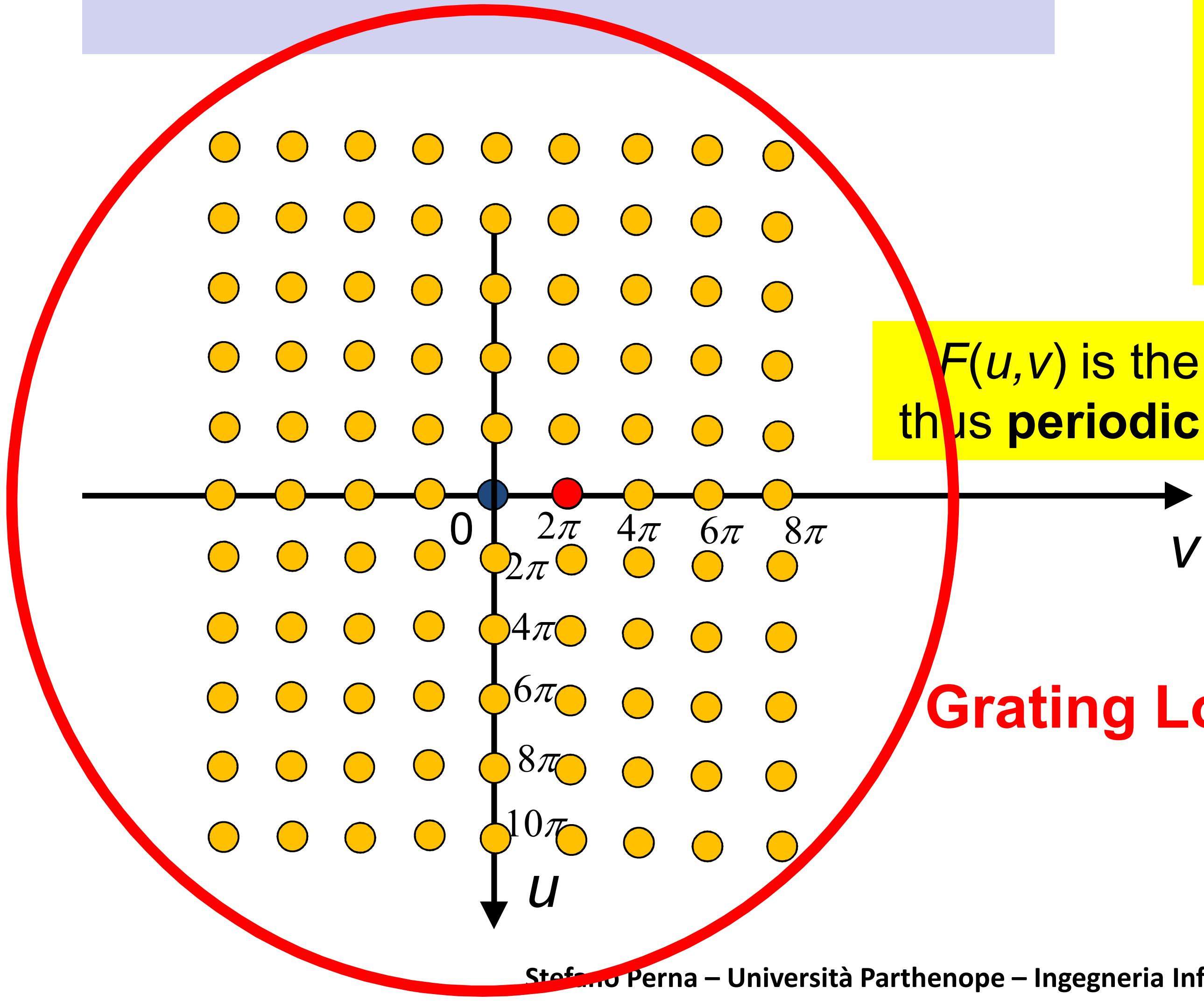


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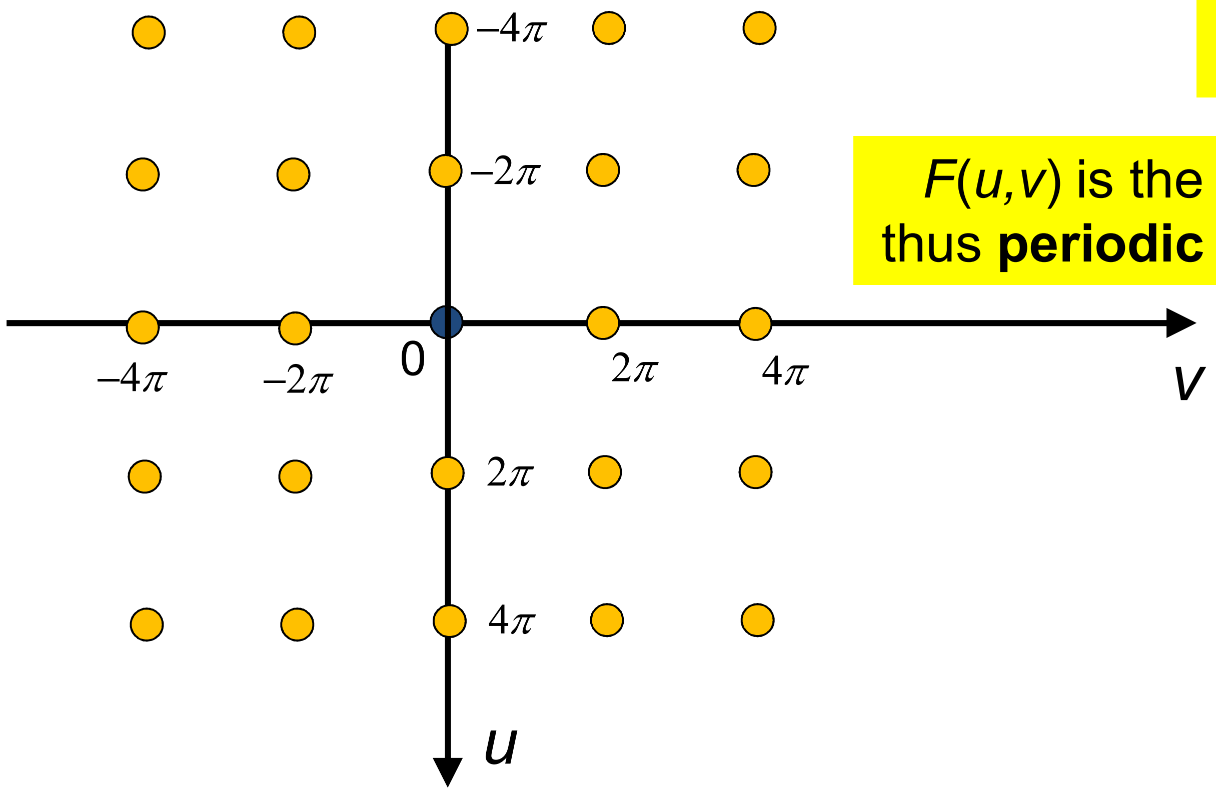

# Periodic Planar Arrays: Visible Region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\,\vec{\mathbf{I}}(\vartheta,\varphi)\;F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = F(u,v)\;\left|\begin{array}{l} u = -\beta d_x \sin\vartheta\cos\varphi \\ v = -\beta d_y \sin\vartheta\sin\varphi \end{array}\right.$$

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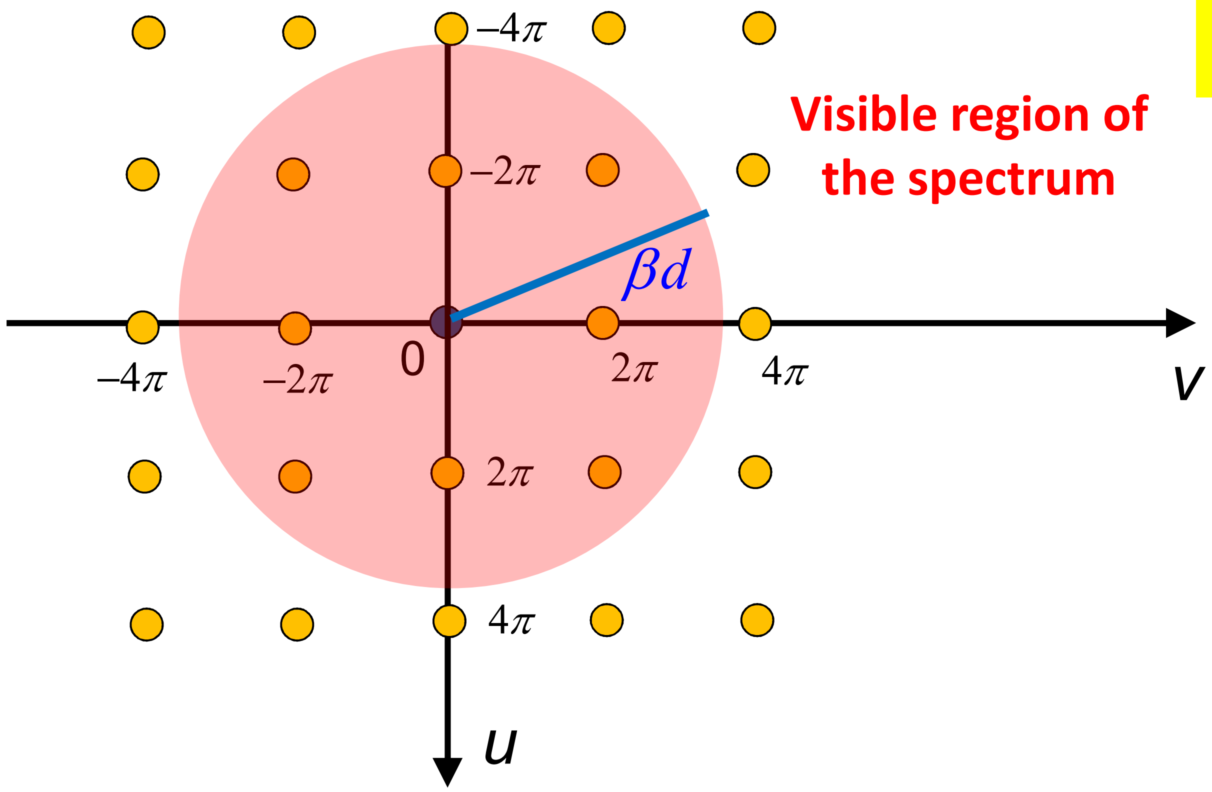
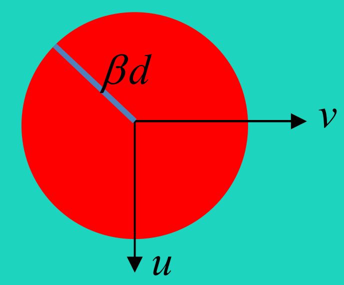



$$d_x = d_y = d \;\Rightarrow\; \begin{array}{l} u = -\beta d\sin\vartheta\cos\varphi \\ v = -\beta d\sin\vartheta\sin\varphi \end{array}$$

$$\Rightarrow\; u^2+v^2 = (\beta d)^2\sin^2\vartheta\left(\cos^2\varphi+\sin^2\varphi\right) = (\beta d)^2\sin^2\vartheta$$

$$\Rightarrow\; u^2+v^2 \le (\beta d)^2$$



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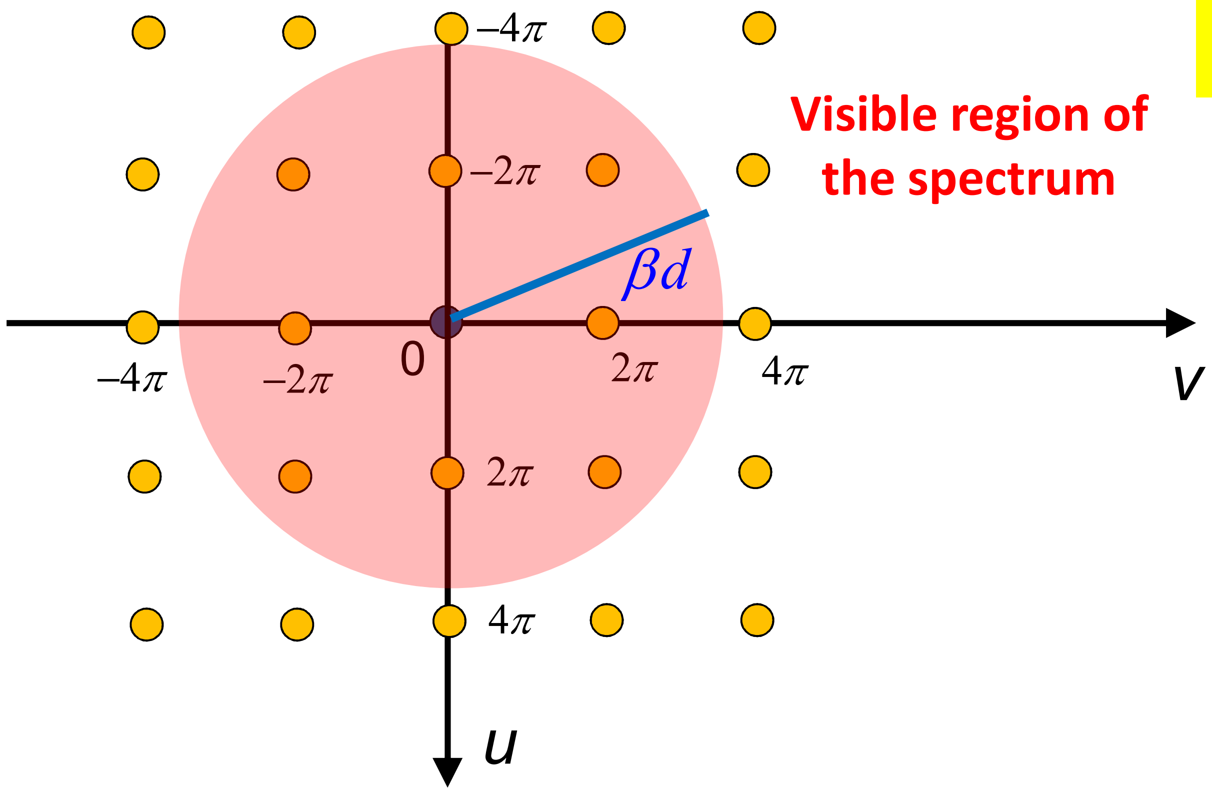


**Can we circumvent the presence of the grating lobes?**

**Let's reduce the width of the visible region!**

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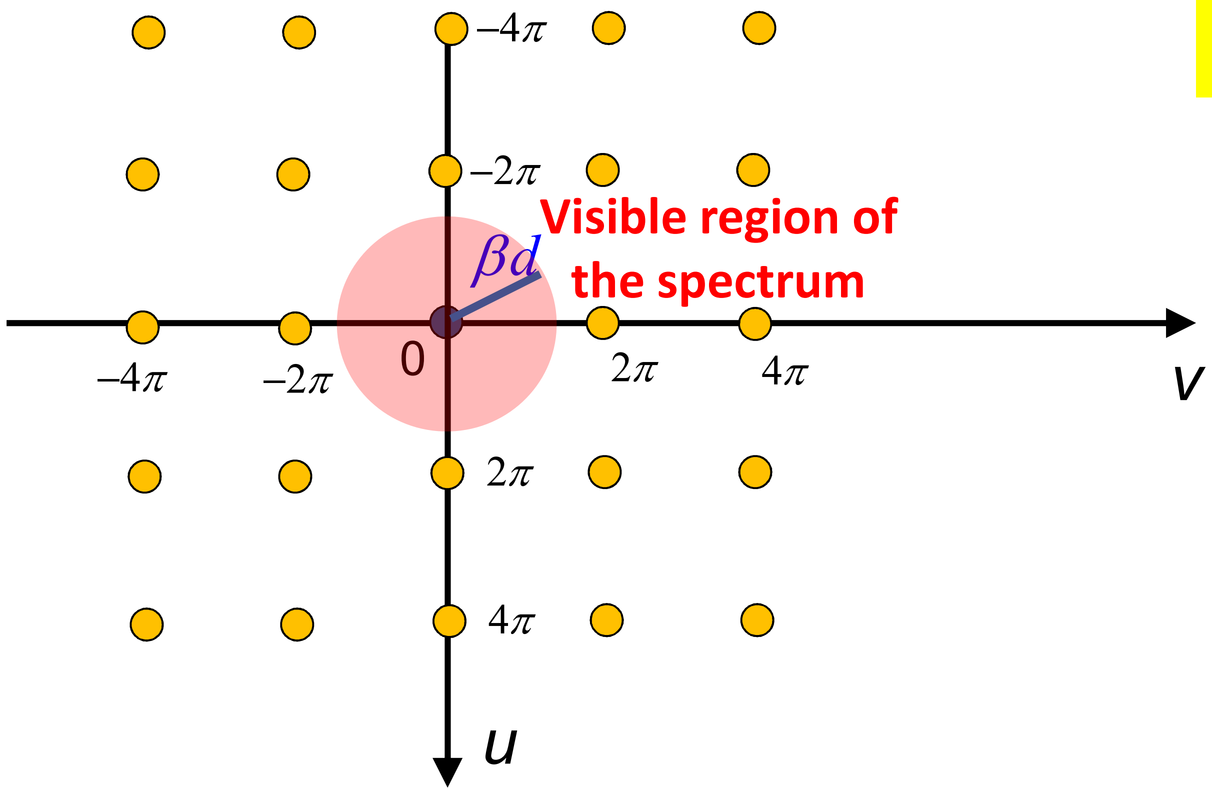


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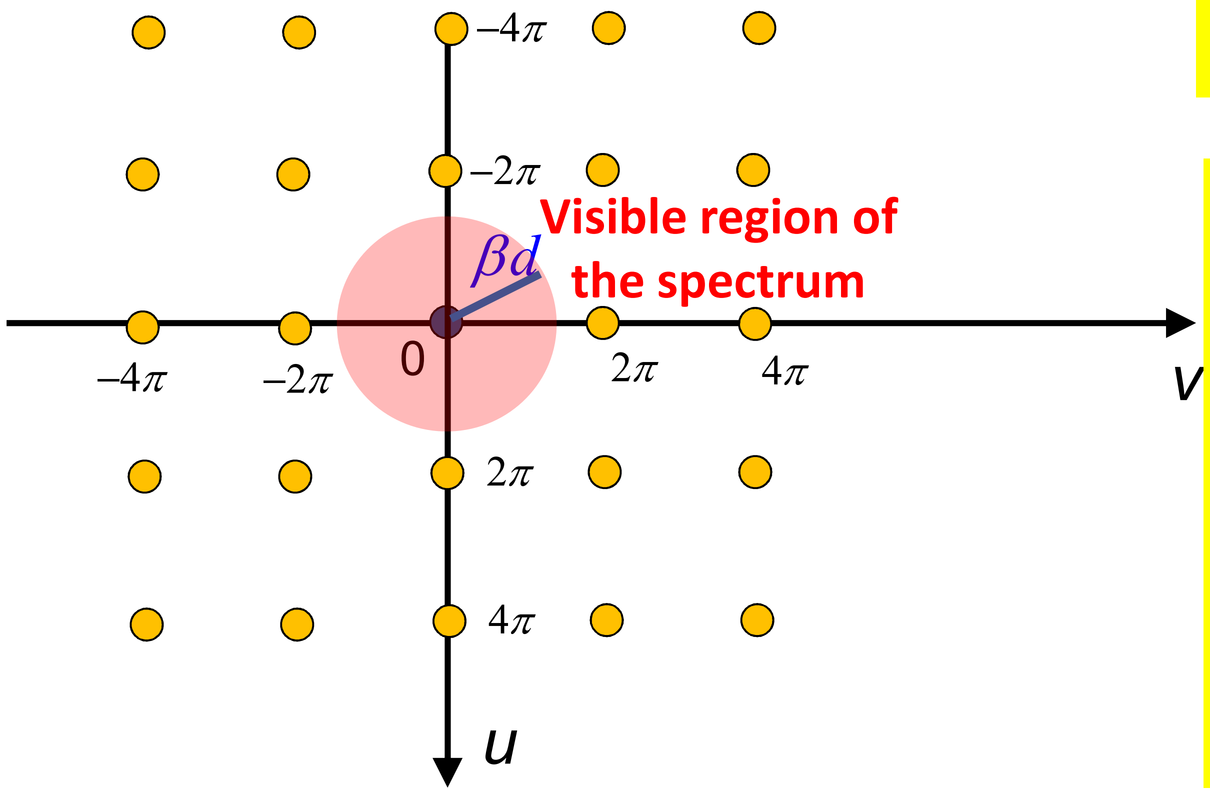


**The condition**

$$\beta d \le \pi \quad\Rightarrow\quad \frac{2\pi}{\lambda}d \le \pi \quad\Rightarrow\quad d \le \frac{\lambda}{2}$$

**guarantees (with a safety margin) absence of grating lobes.**

**To avoid the presence of grating lobes the inter-element distance must be thus subject to an upper limit, on the order of half wavelength**

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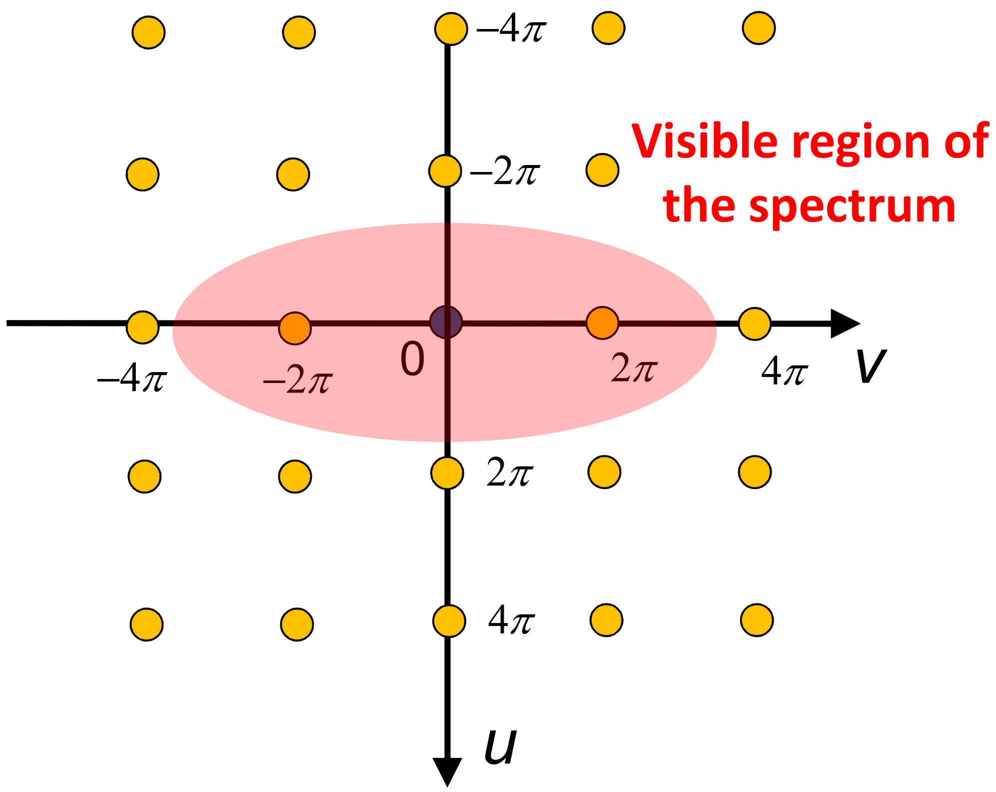


$$d_x \neq d_y \;\Rightarrow\; \frac{u^2}{(\beta d_x)^2} = \cos^2\varphi\sin^2\vartheta \qquad \frac{v^2}{(\beta d_y)^2} = \sin^2\varphi\sin^2\vartheta$$

$$\Rightarrow\; \frac{u^2}{(\beta d_x)^2} + \frac{v^2}{(\beta d_x)^2} = \left(\cos^2\varphi + \sin^2\varphi\right)\sin^2\vartheta \;= \sin^2\vartheta$$

$$\Rightarrow\; \frac{u^2}{(\beta d_x)^2} + \frac{v^2}{(\beta d_y)^2} \leq 1 \;\Rightarrow$$