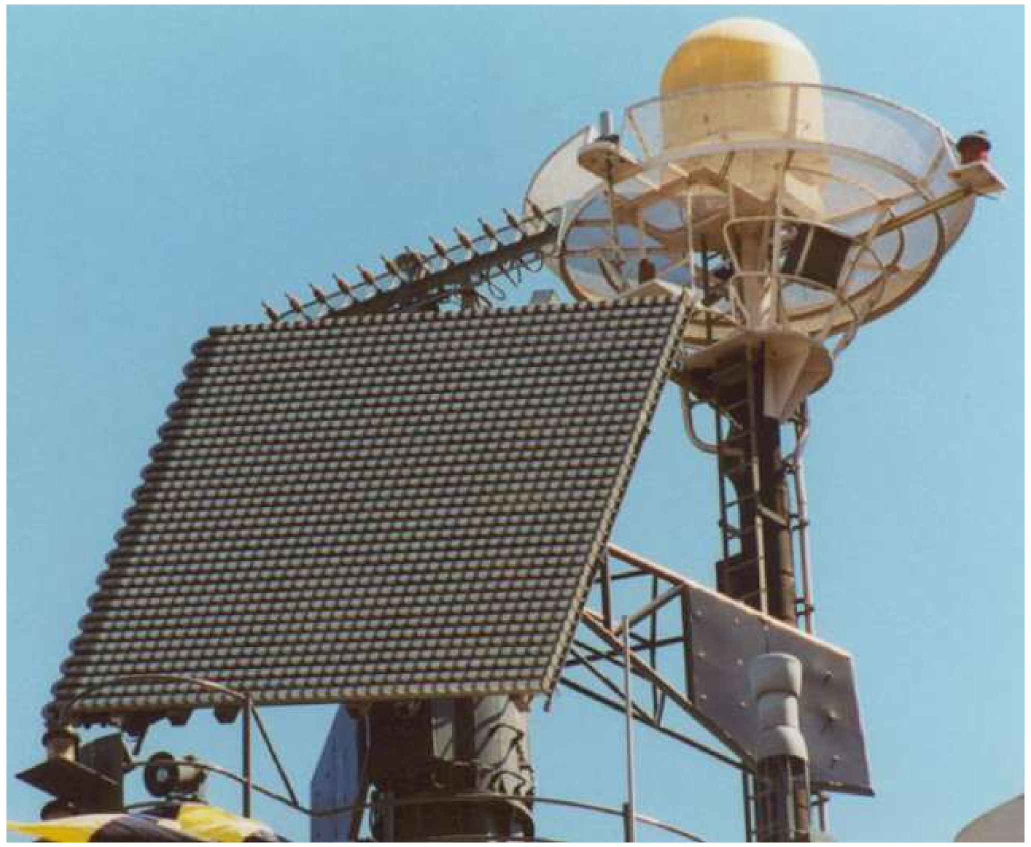
Arrays

Arrays



Color legend

New formulas, important considerations,
important formulas, important concepts

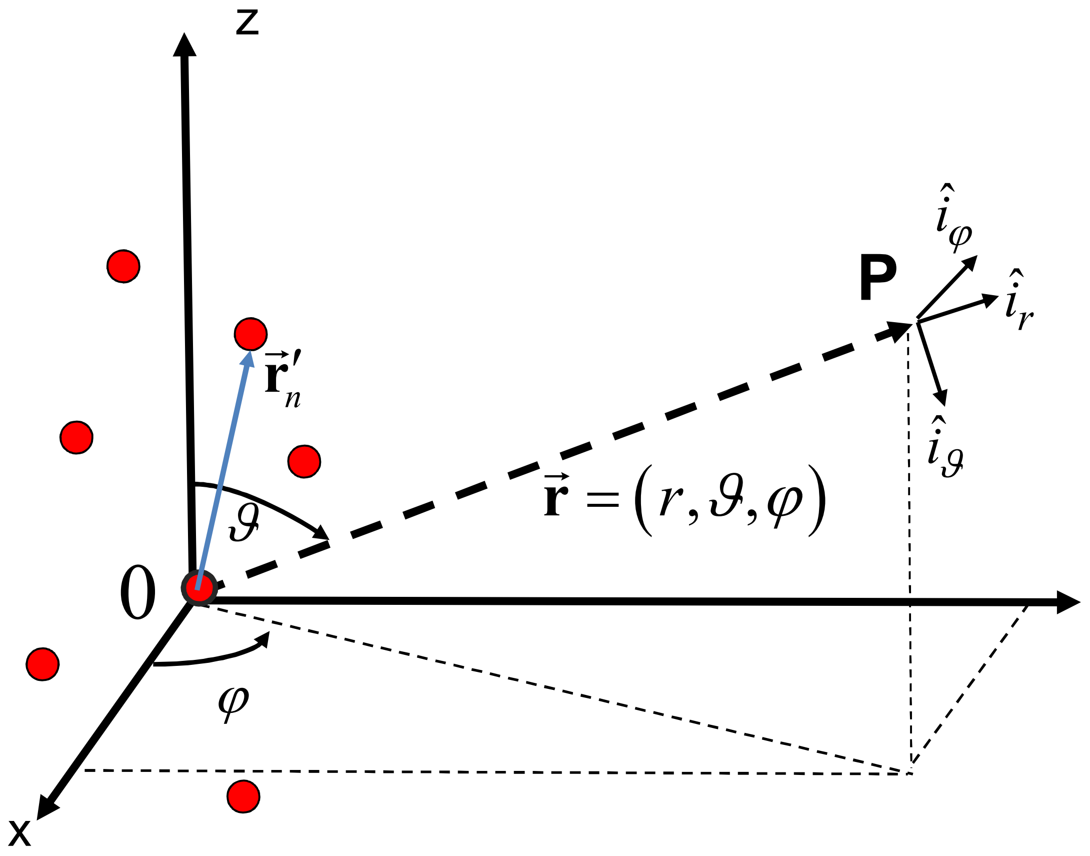
Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Arrays



z
P
$\hat{i}_\varphi$
$\hat{i}_r$
$\hat{i}_\vartheta$
$\vec{\mathbf{r}}'_n$
$\vec{\mathbf{r}} = (r, \vartheta, \varphi)$
ϑ
0
φ
x

Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\,F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta\vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)$$

Principle of pattern multiplication

Array Factor

Element Factor

The Array Factor $F(\vartheta,\varphi)$ is a function of the angular coordinates (ϑ,φ) and depends upon:

- **the array geometry (through N and $\vec{\mathbf{r}}'_n$)**
- **the input excitations of the antennas of the array itself (through I_n)**

Very interesting implications relevant to the synthesis of the pattern

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

✓ **1. Let's depict F(*u*)**

2. Let's jump from *u* to ϑ and calculate:

✓ **The direction of the Main Lobe** $\vartheta_{MB} = \frac{\pi}{2}$

✓ **The NNBW / HPBW** $\mathrm{NNBW} \approx 2\frac{\lambda}{Nd}$ $\mathrm{HPBW} \approx 0.88\frac{\lambda}{Nd}$

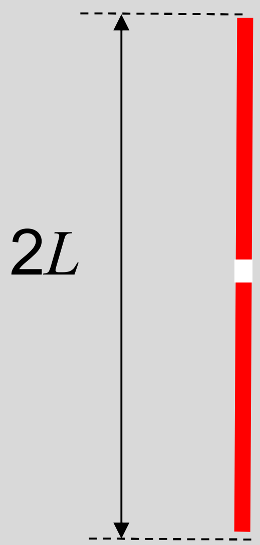
✓ **The SLL** $\mathrm{SLL} = -13.46\,dB$

BROADSIDE ARRAYS

Periodic Linear Arrays (z-axis): Uniform Excitations
vs.
Wire Antennas with Uniform Current Distribution

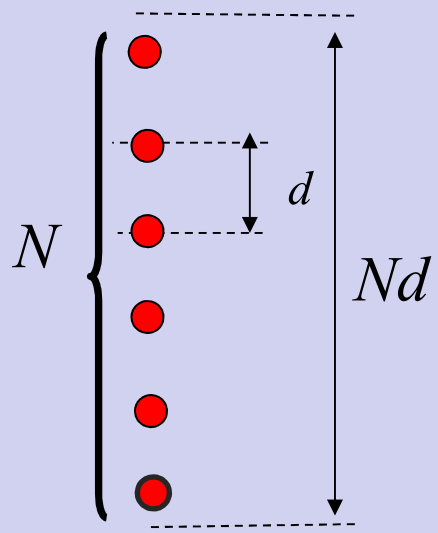
Wire Antennas

$$\text{HPBW} \approx 0.88\frac{\lambda}{2L}$$



Linear Arrays

$$\text{HPBW} \approx 0.88\frac{\lambda}{Nd}$$



Periodic Linear Arrays (z-axis)

Uniform input excitations (Broadside Array)

Beam scanning

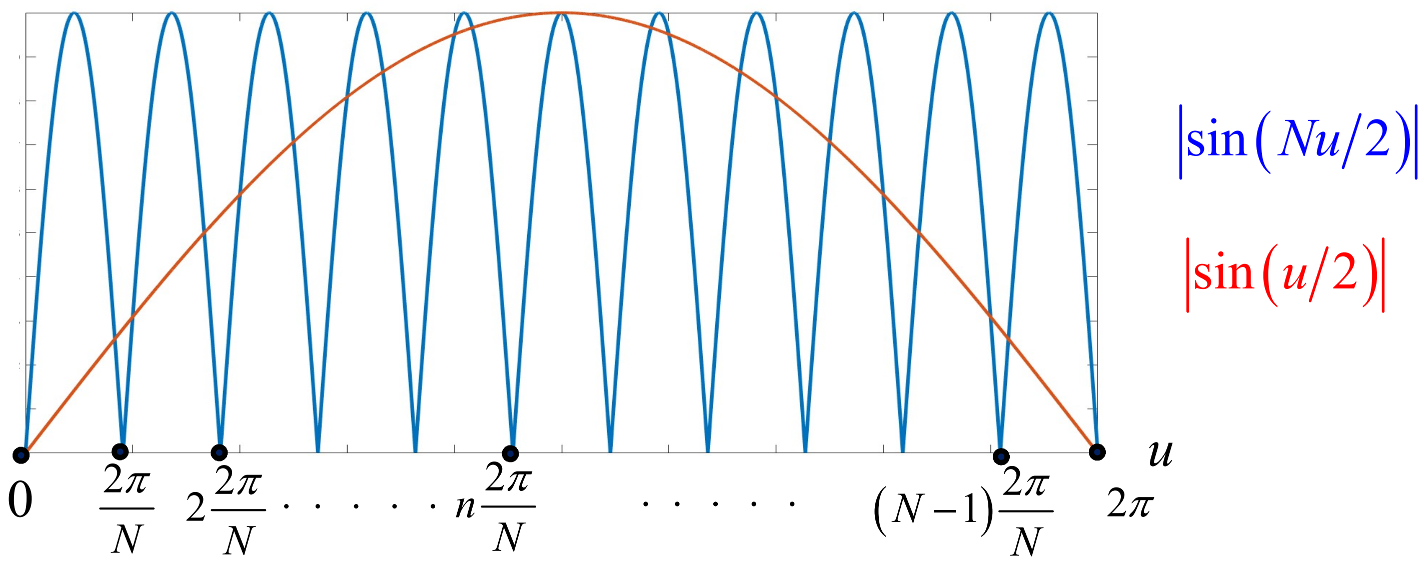
Endfire Array

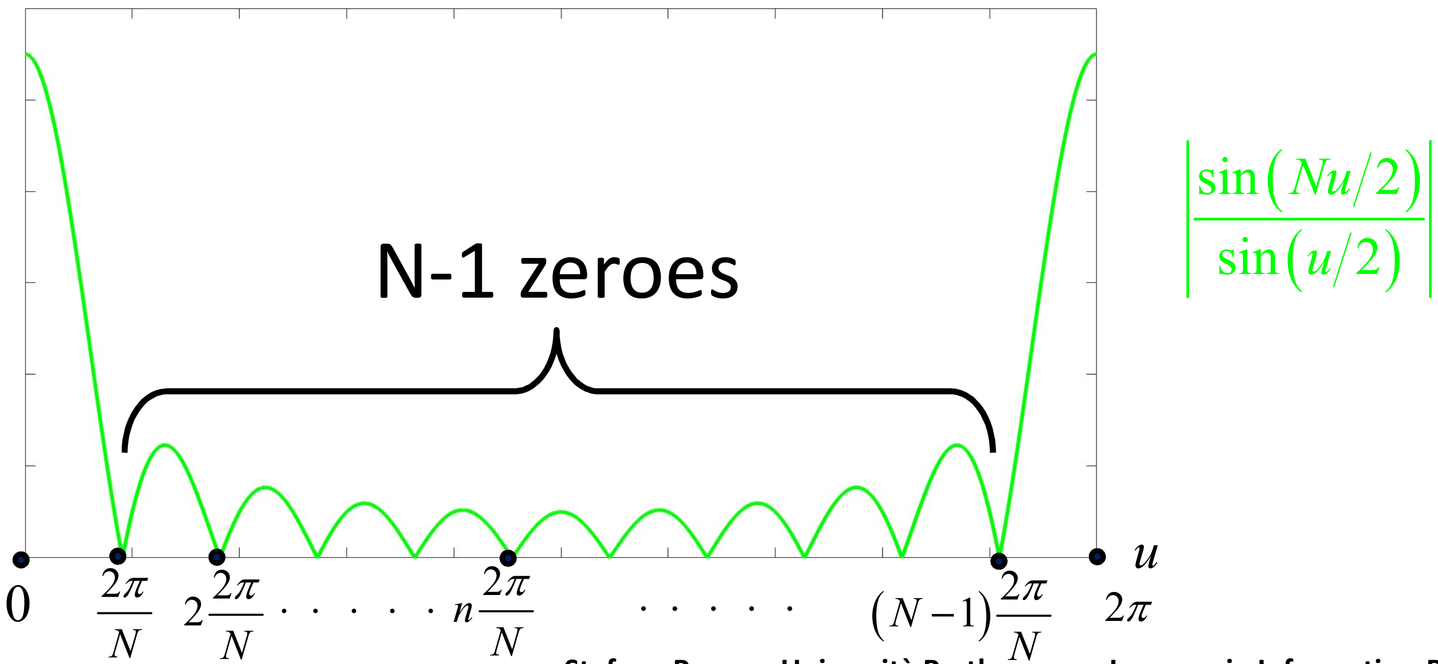
Beam scanning and grating lobes

Numerical Examples

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$



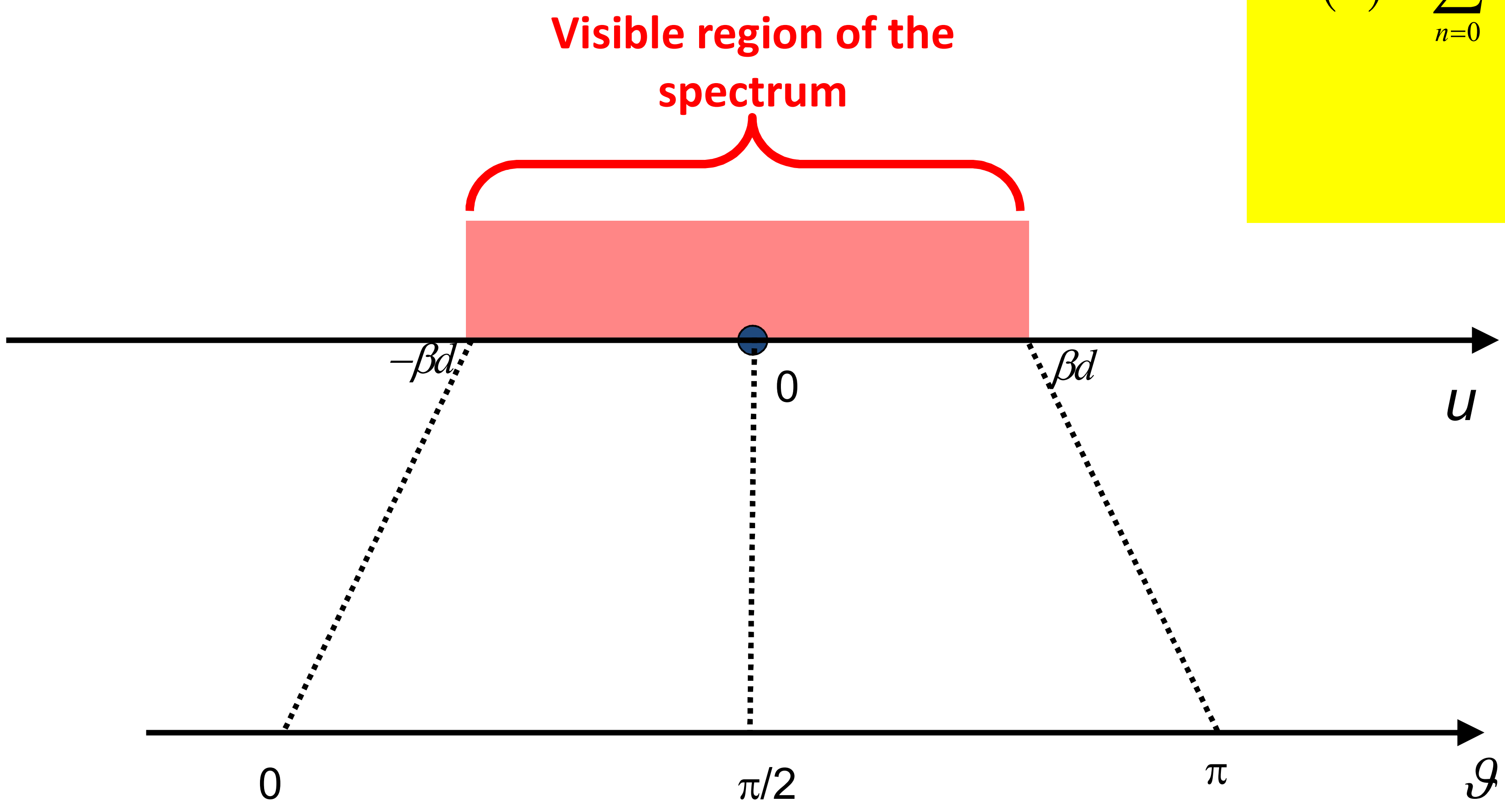


Periodic Linear Arrays (z-axis): visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

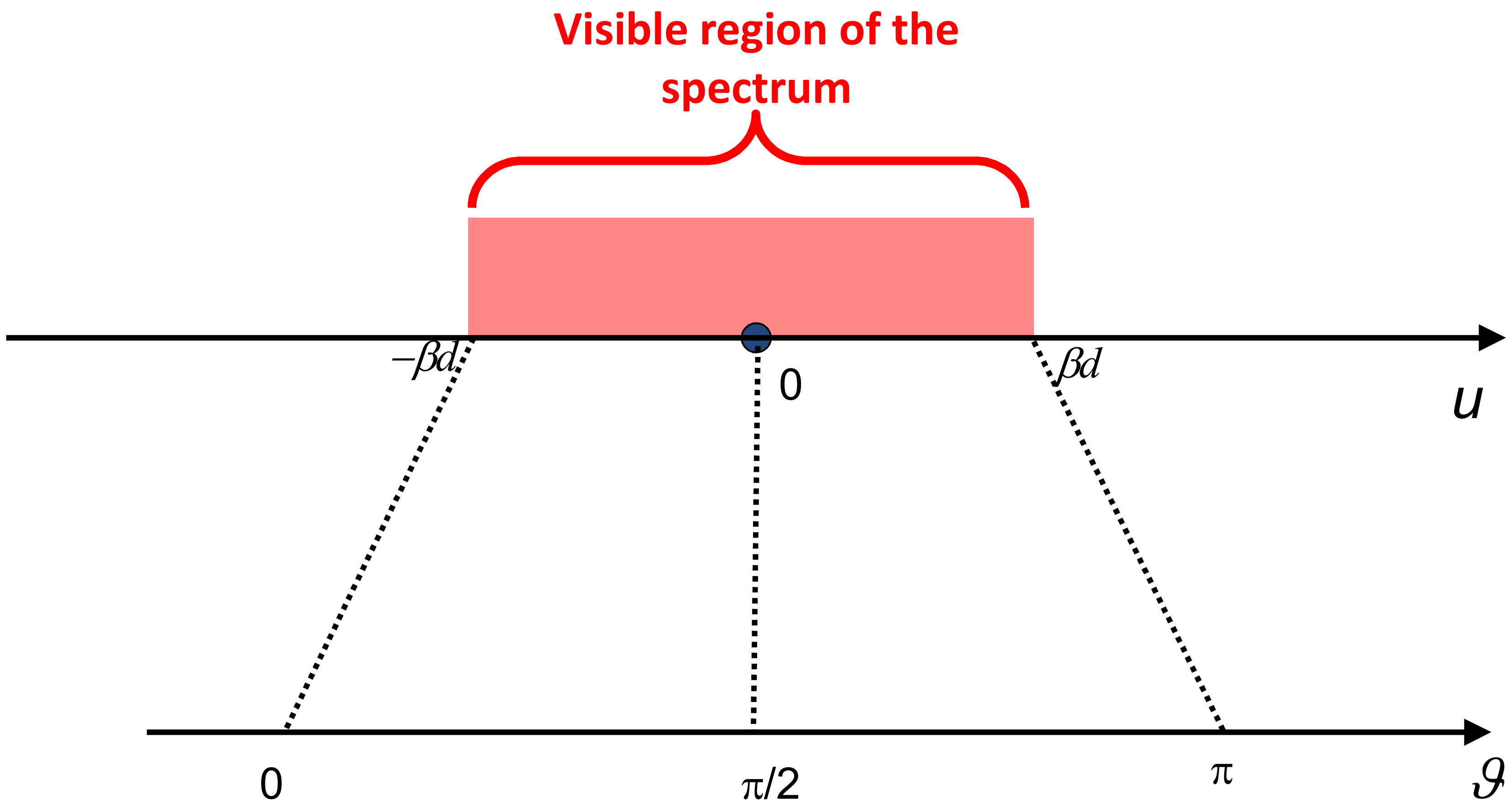
$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$



Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

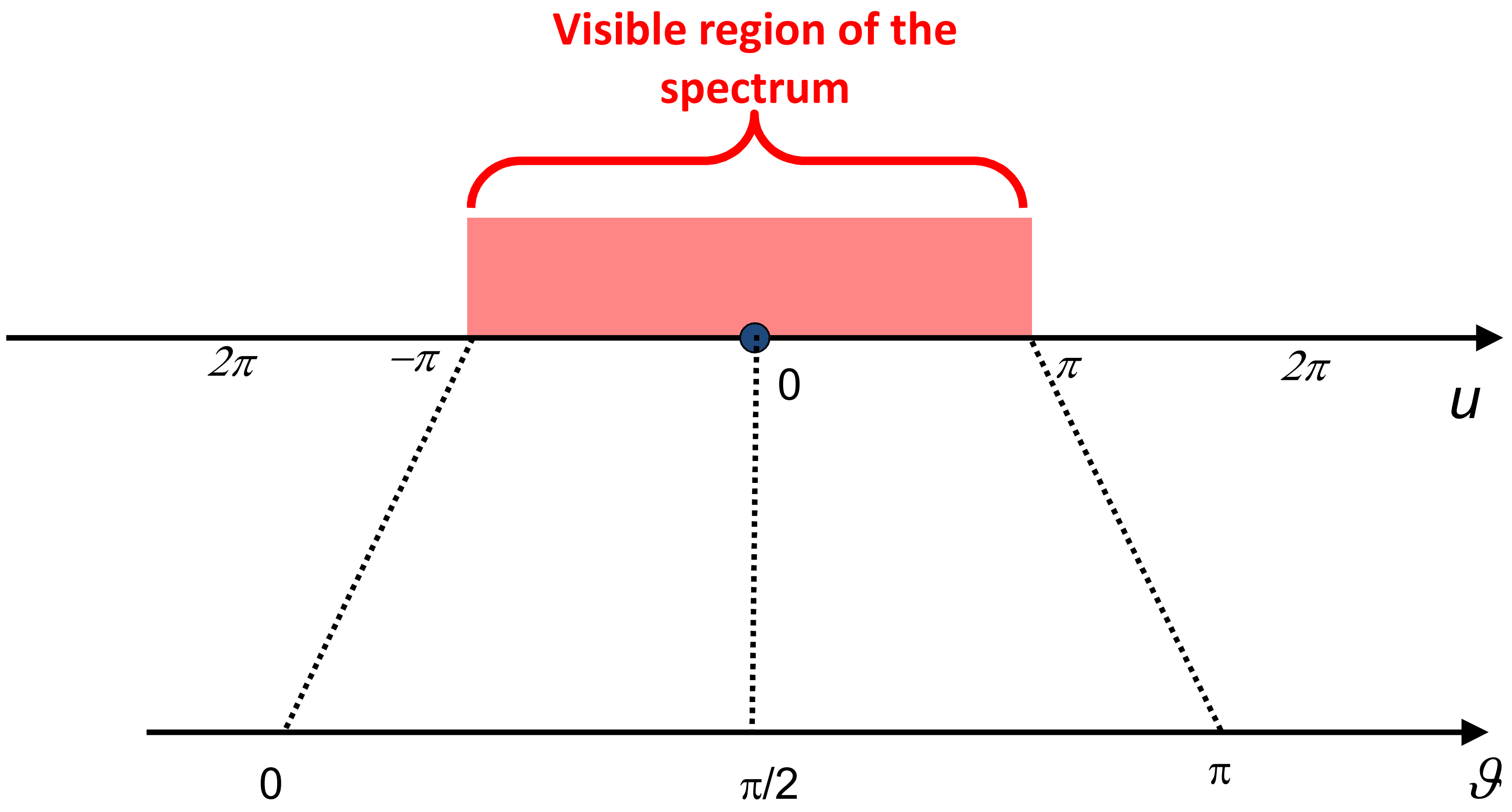
$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$



Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$

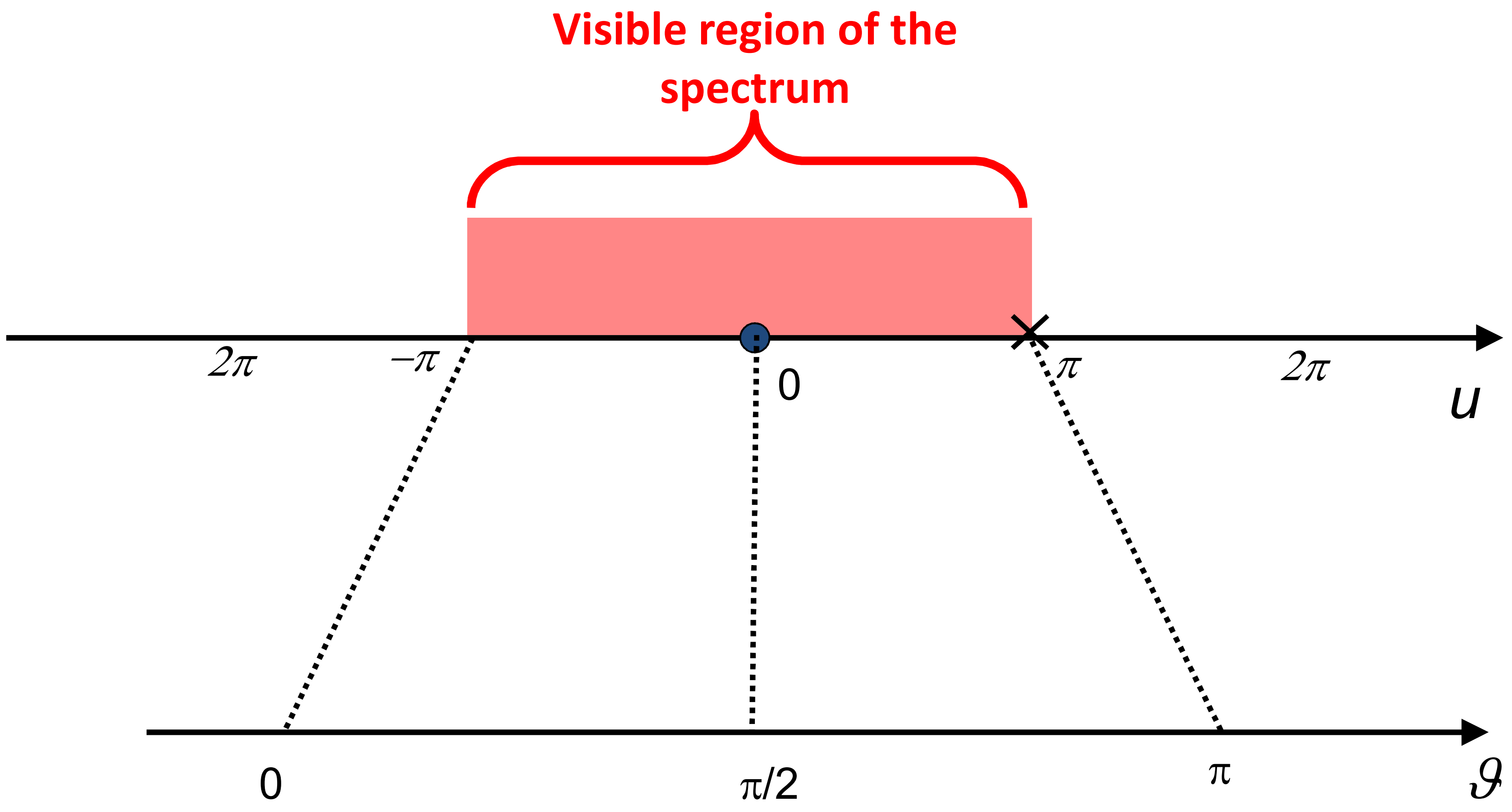


Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$

$$N = 2$$

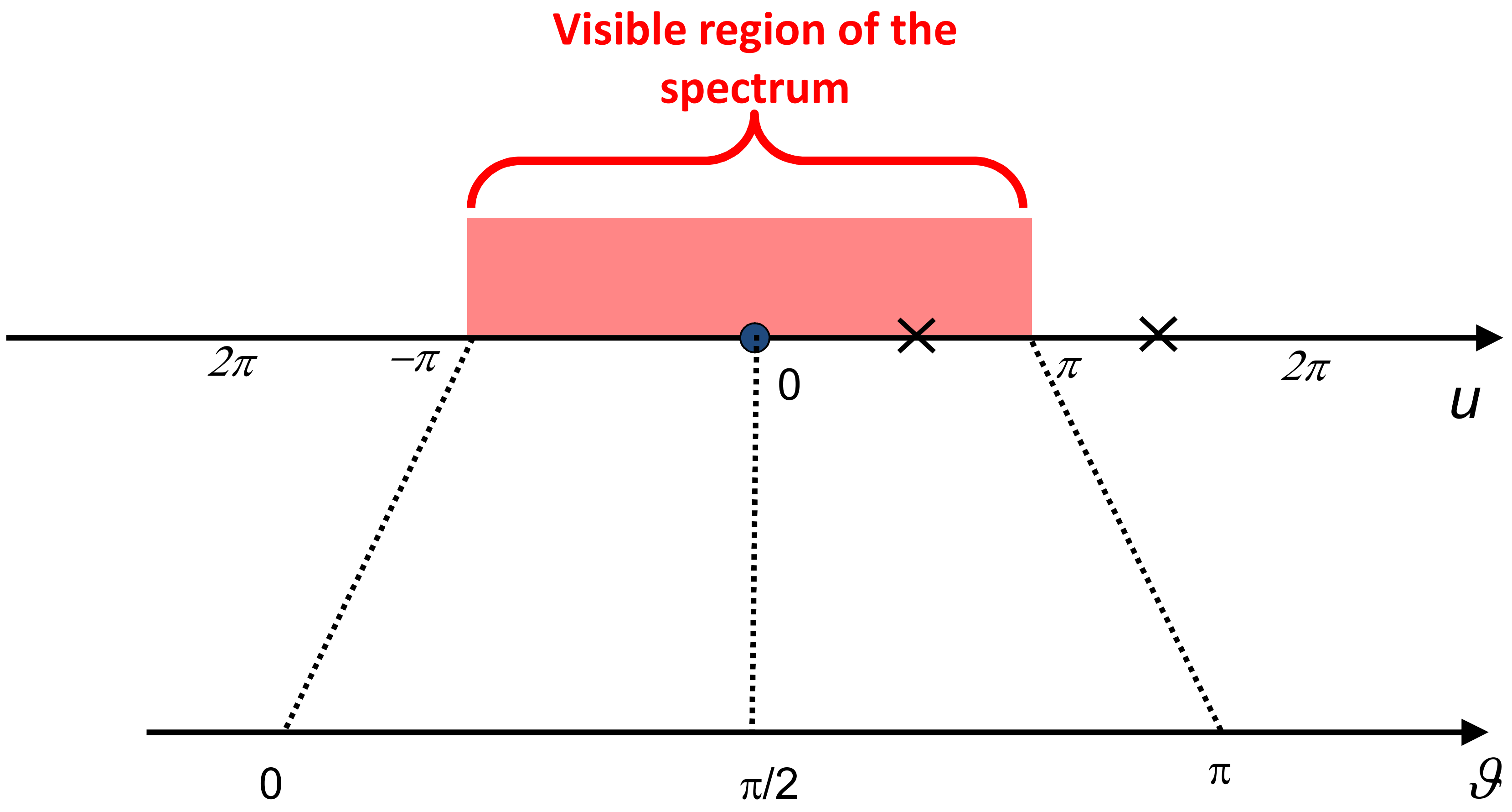


Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$

$$N = 3$$

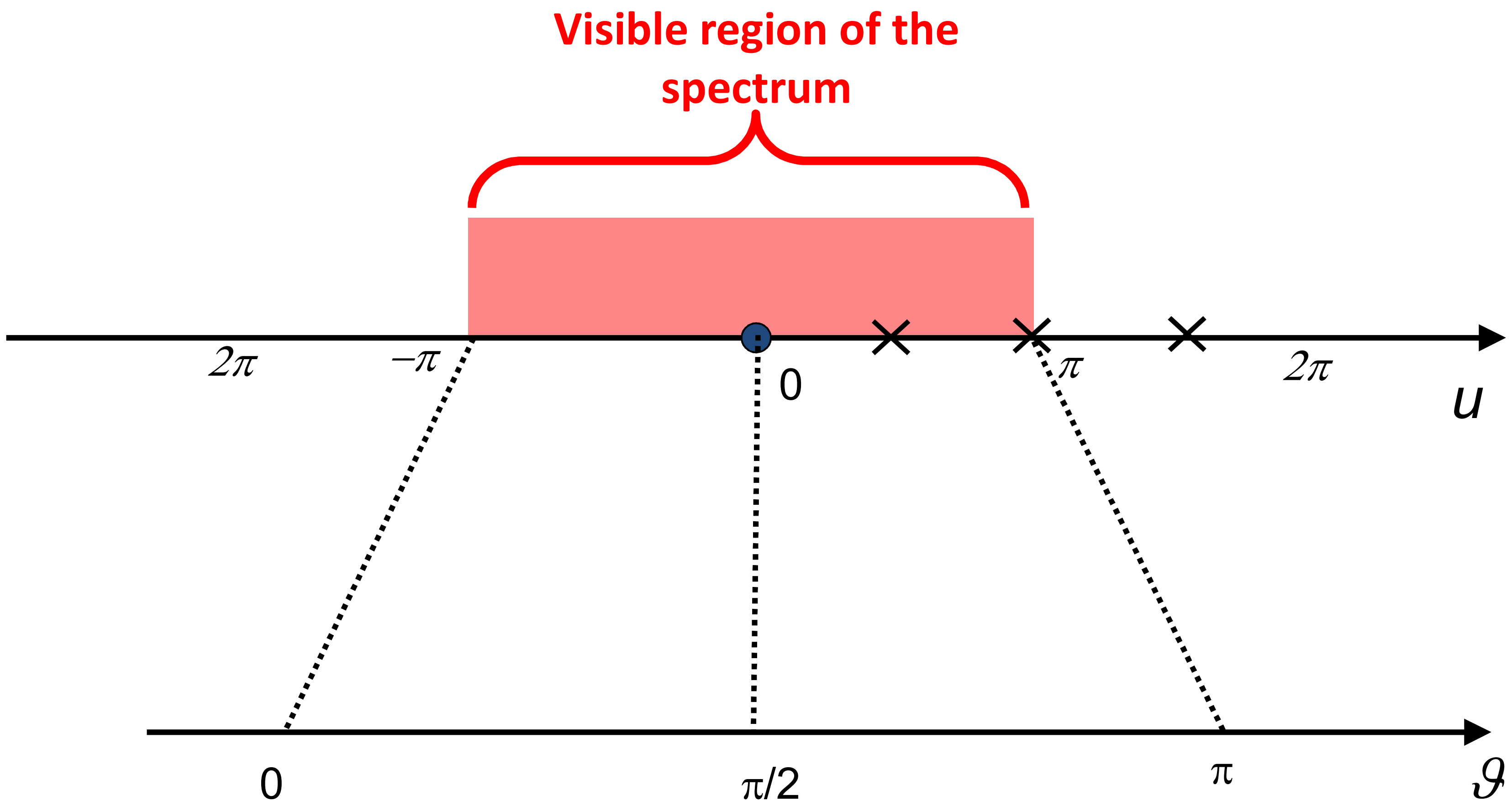


Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$

$$N = 4$$

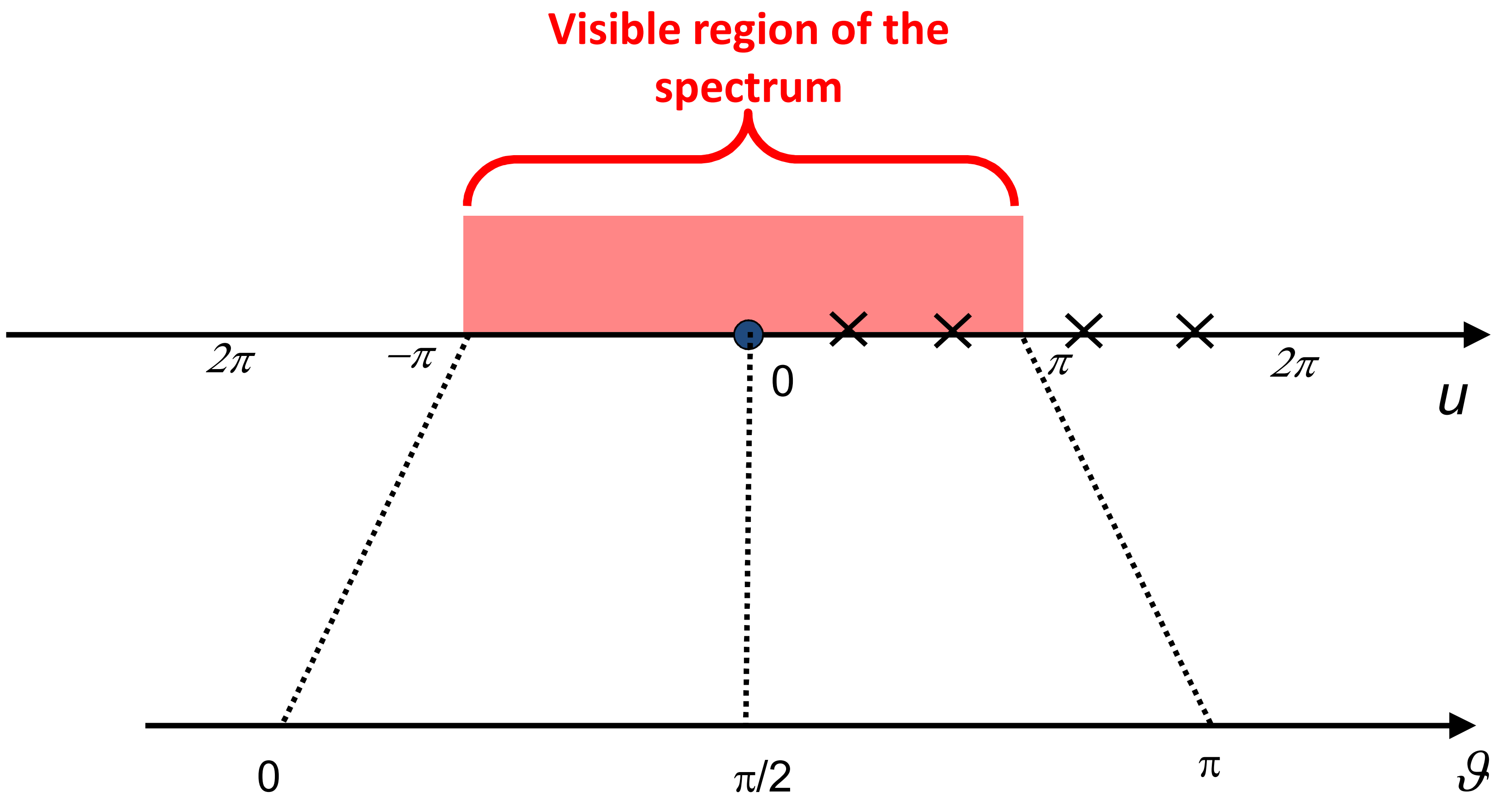


Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$

$$N = 5$$

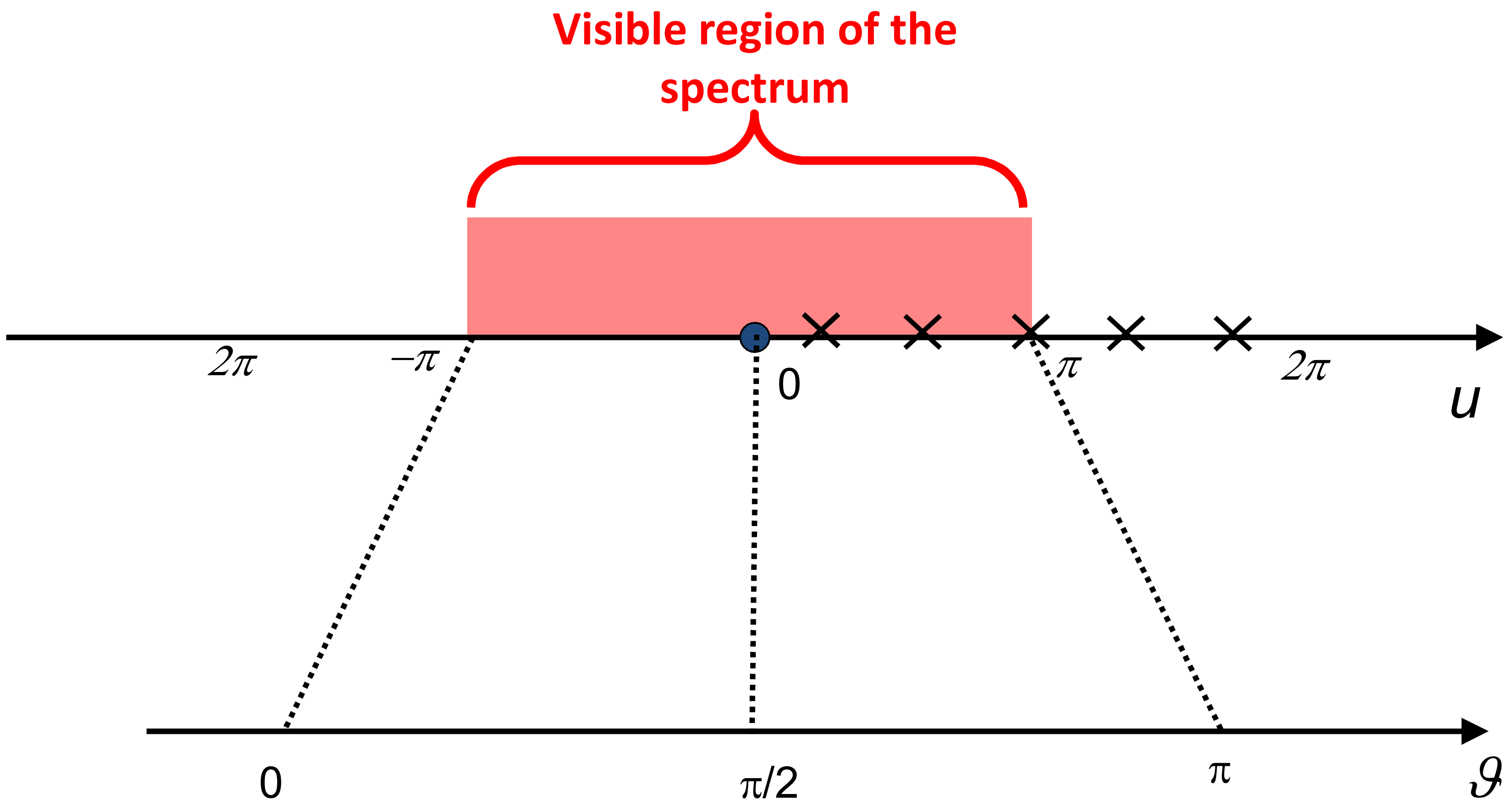


Exercise n.1

Periodic Linear Arrays (z-axis): Uniform Excitations

$$d = \frac{\lambda}{2} \quad \Rightarrow \beta d = \pi$$

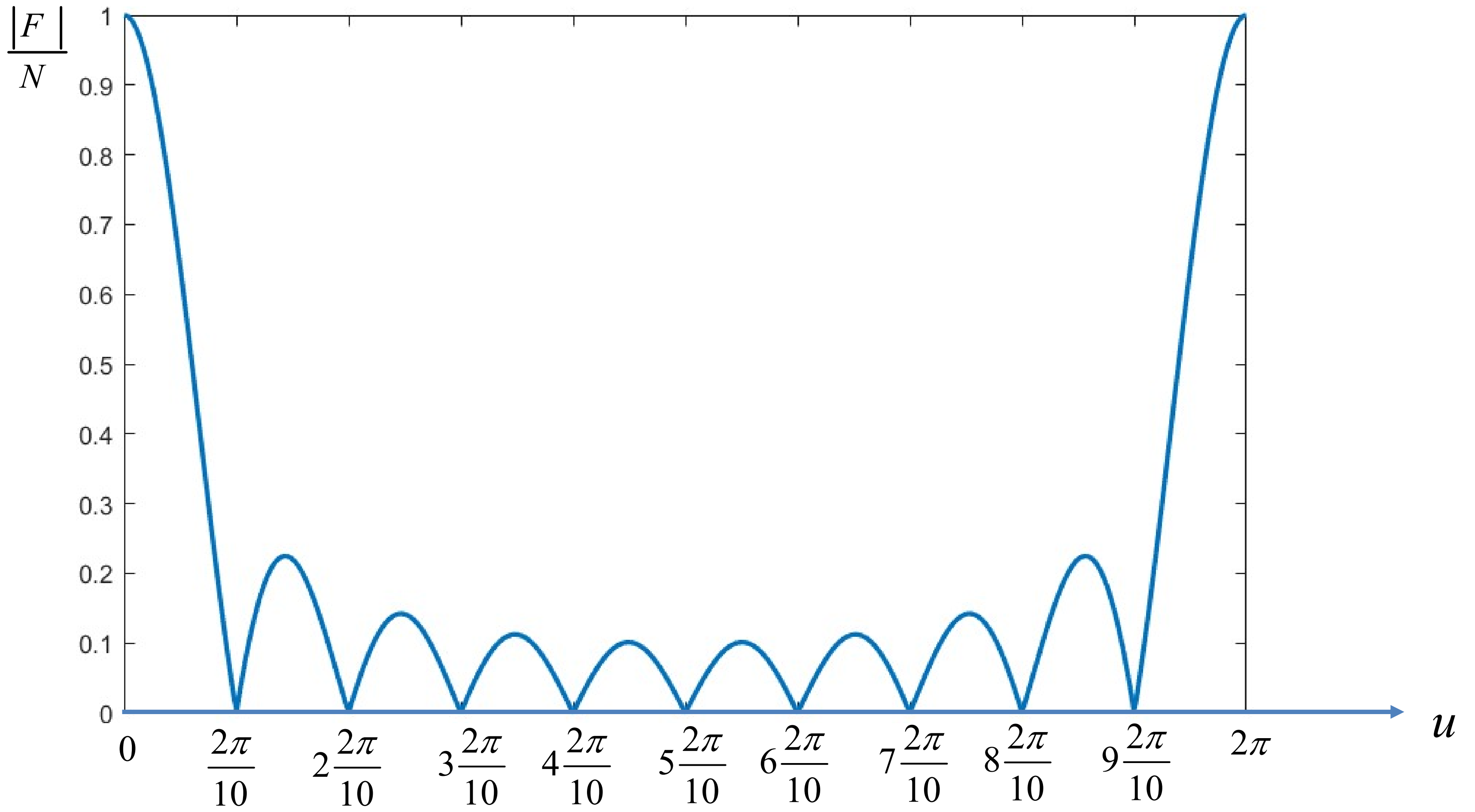
$$N = 6$$



Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

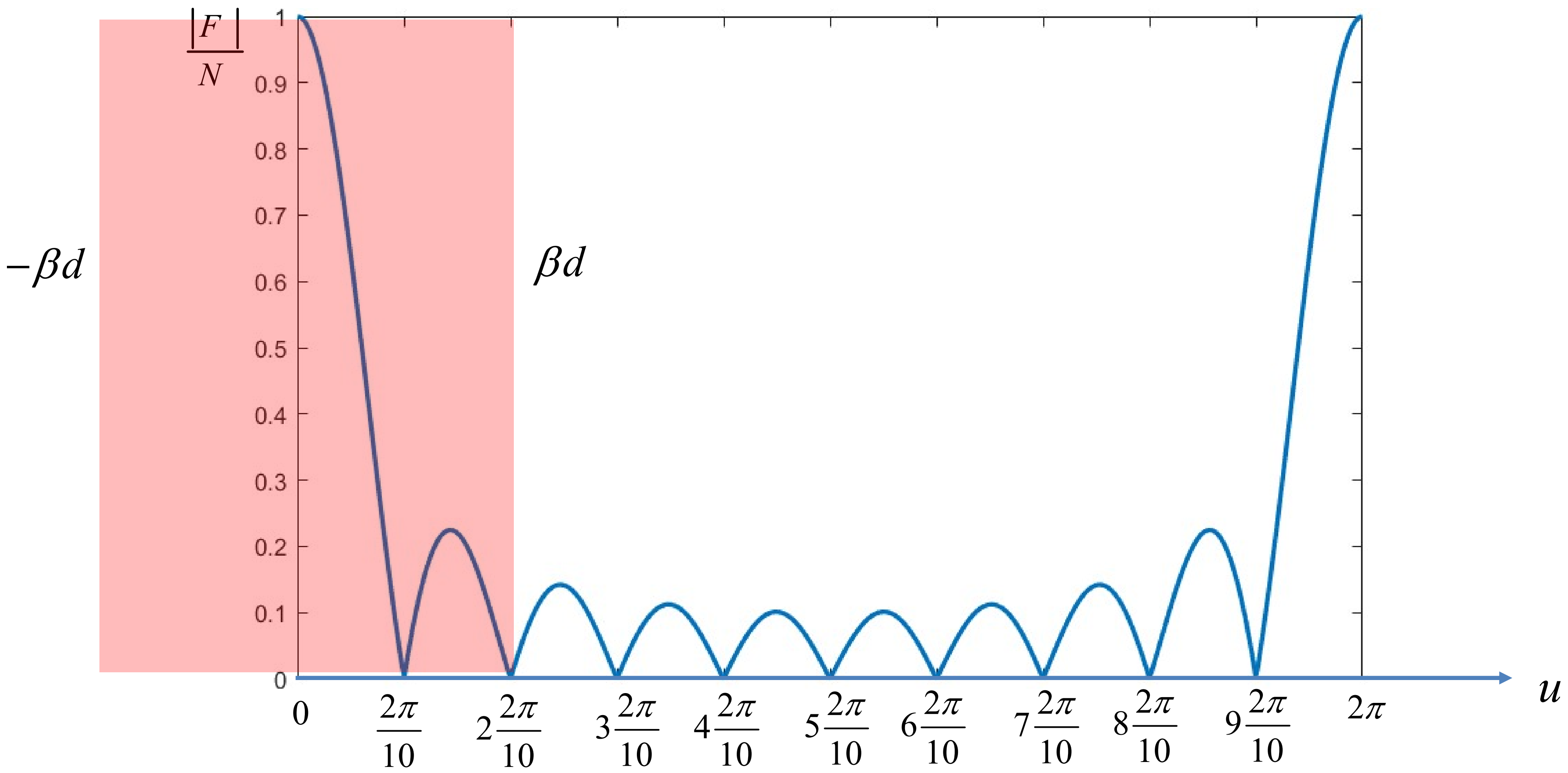


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 0.2\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda}\frac{2}{10}\lambda = 2\frac{2\pi}{10}$$

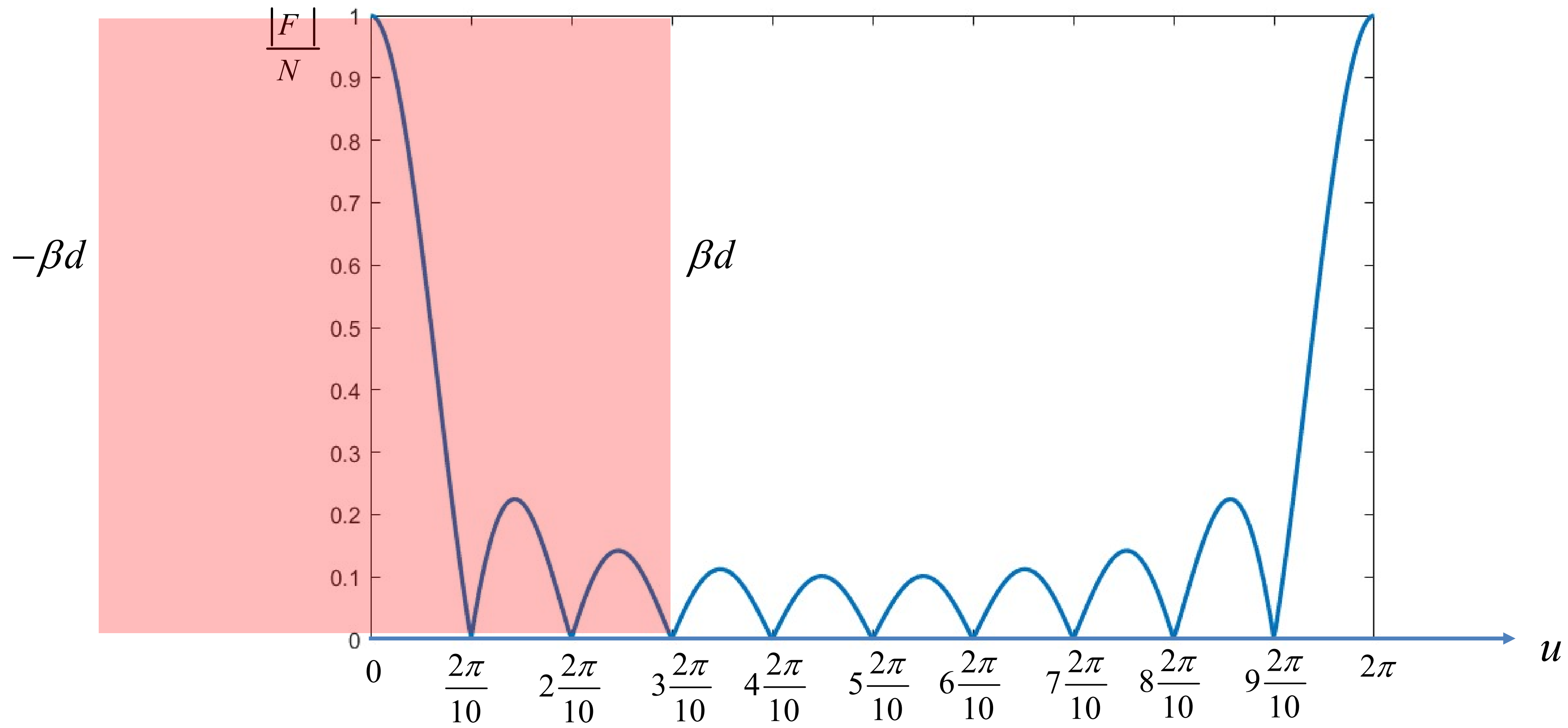


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 0.3\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda}\frac{3}{10}\lambda = 3\frac{2\pi}{10}$$

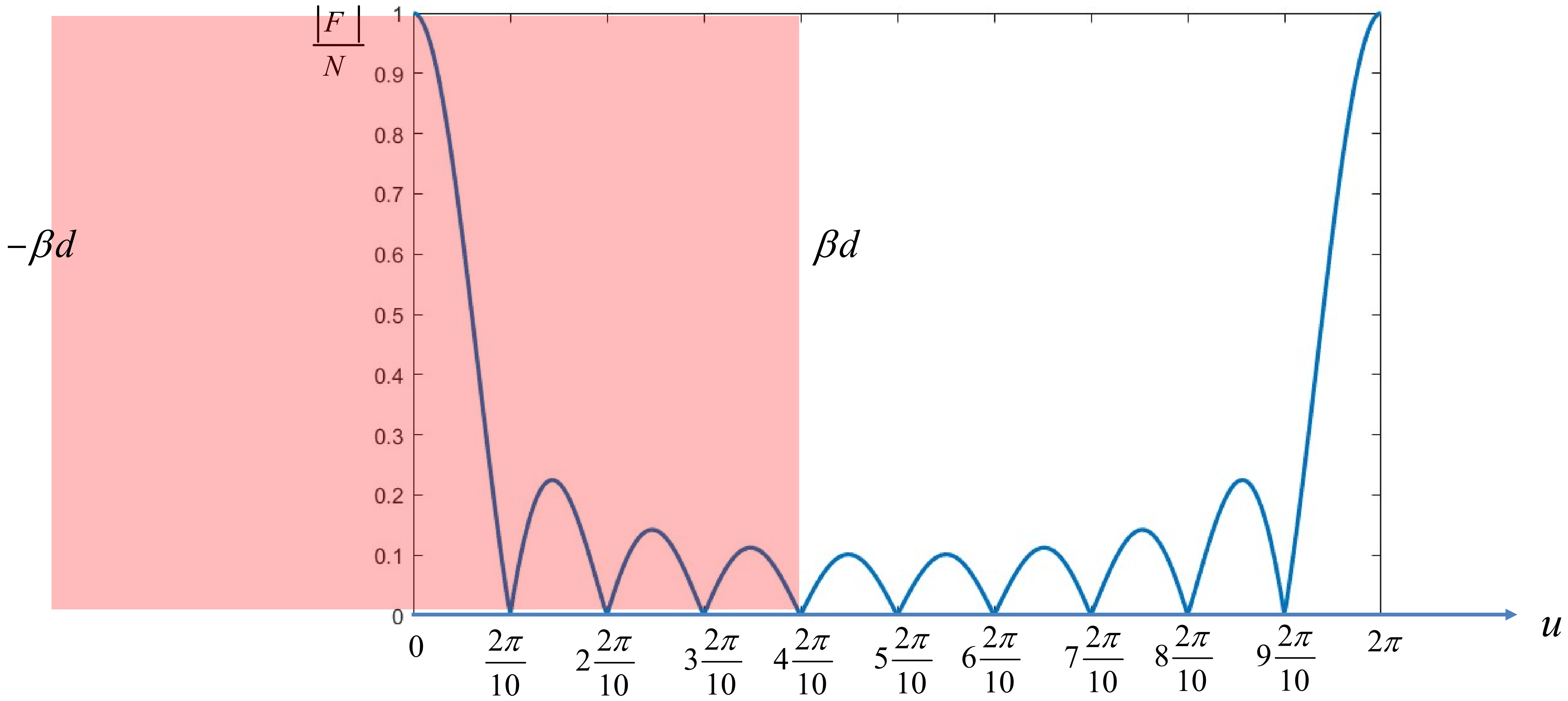


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.4\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda}\frac{4}{10}\lambda = 4\frac{2\pi}{10}$

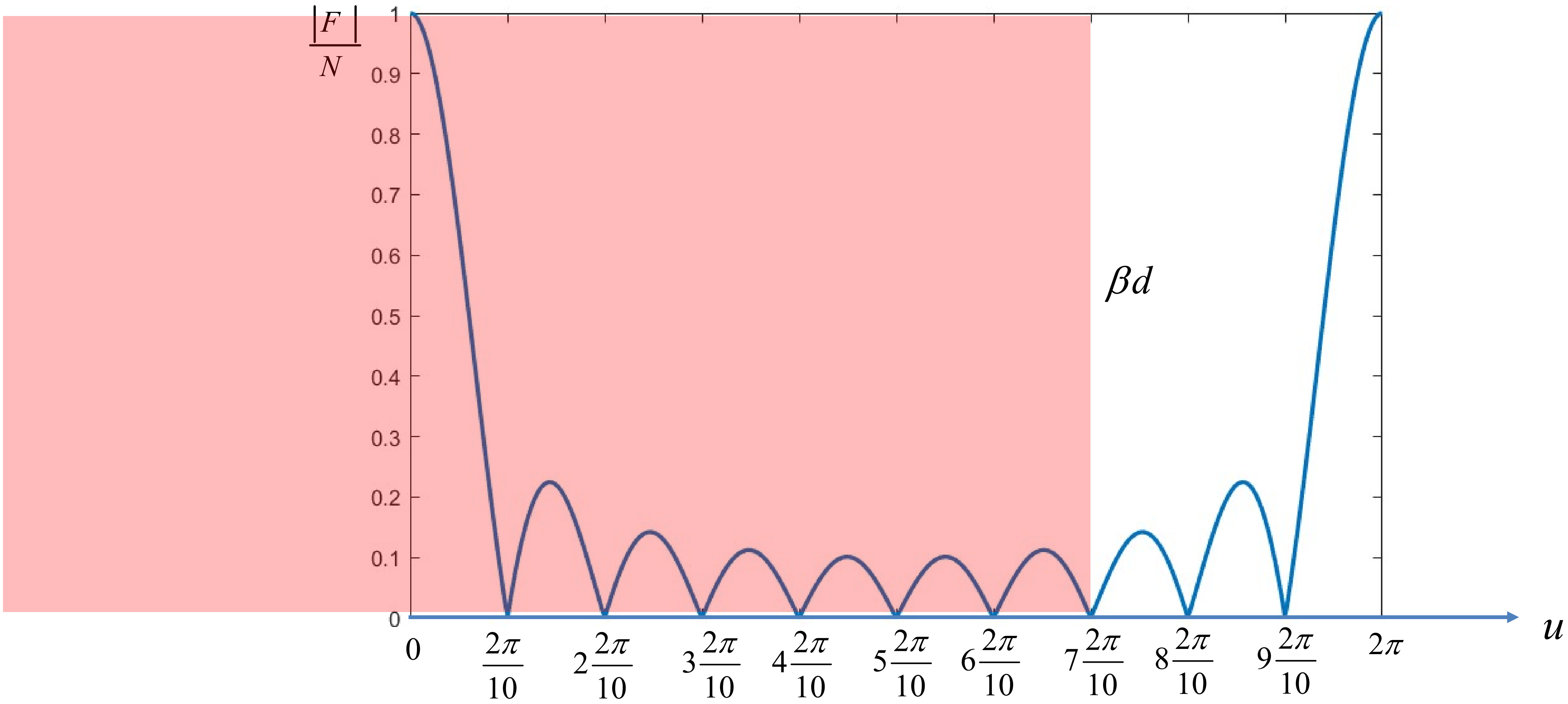


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 0.7\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda}\frac{7}{10}\lambda = 7\frac{2\pi}{10}$$

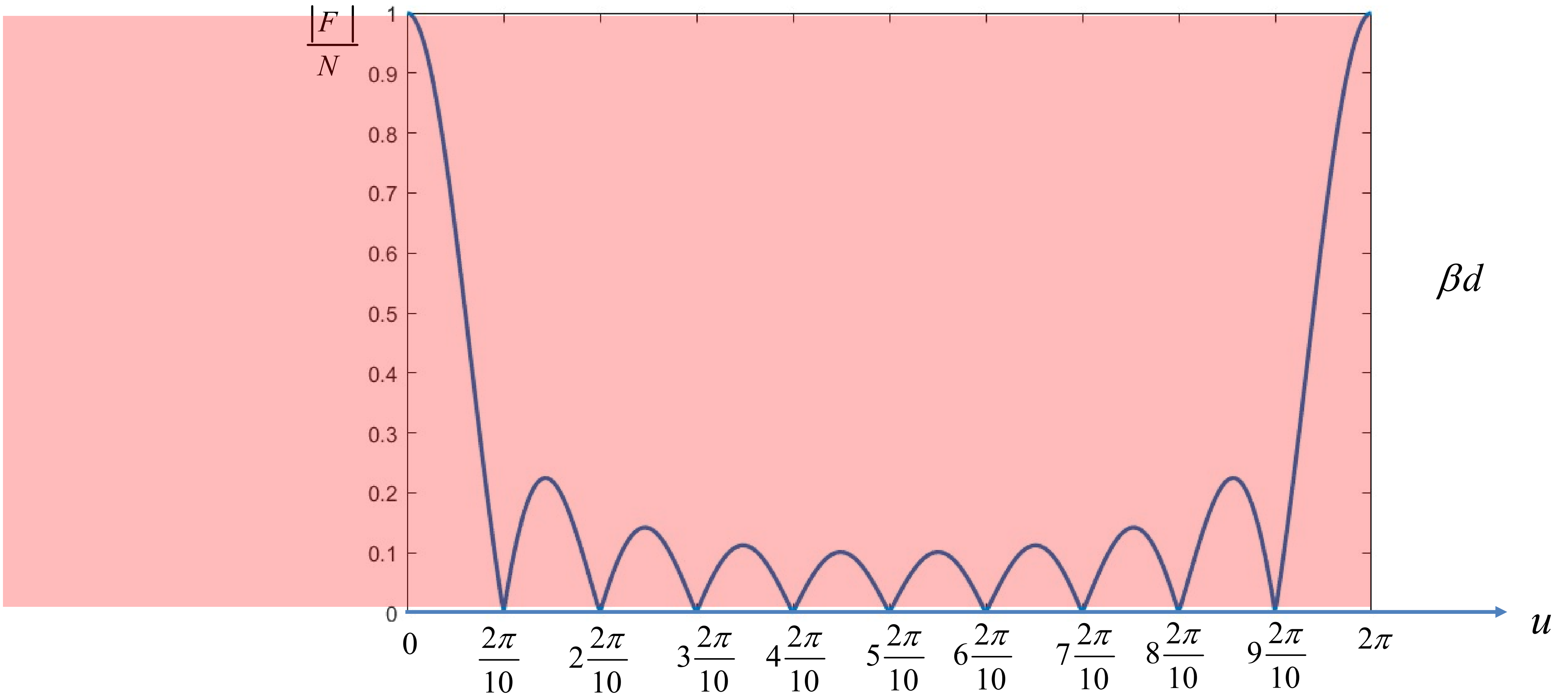


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = \lambda \Rightarrow \beta d = \frac{2\pi}{\lambda}\lambda = 2\pi$$

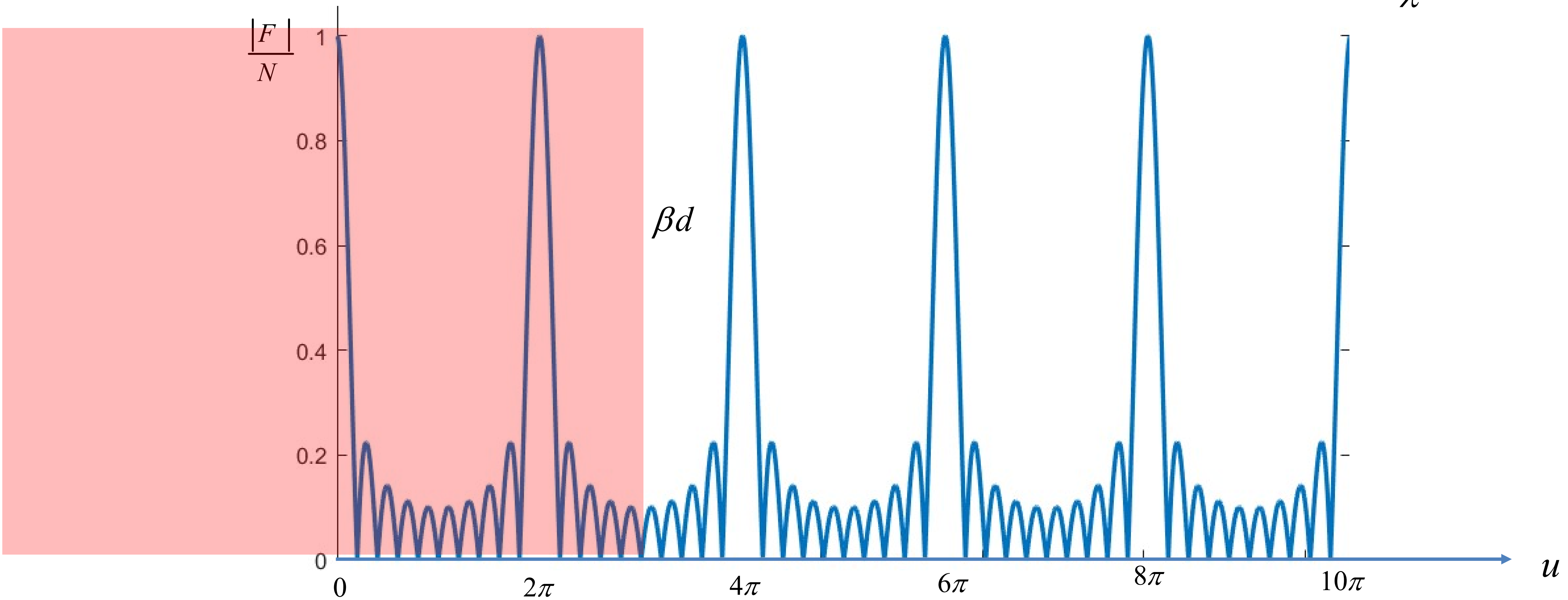


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 1.5\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda} 1.5\lambda = 3\pi$$

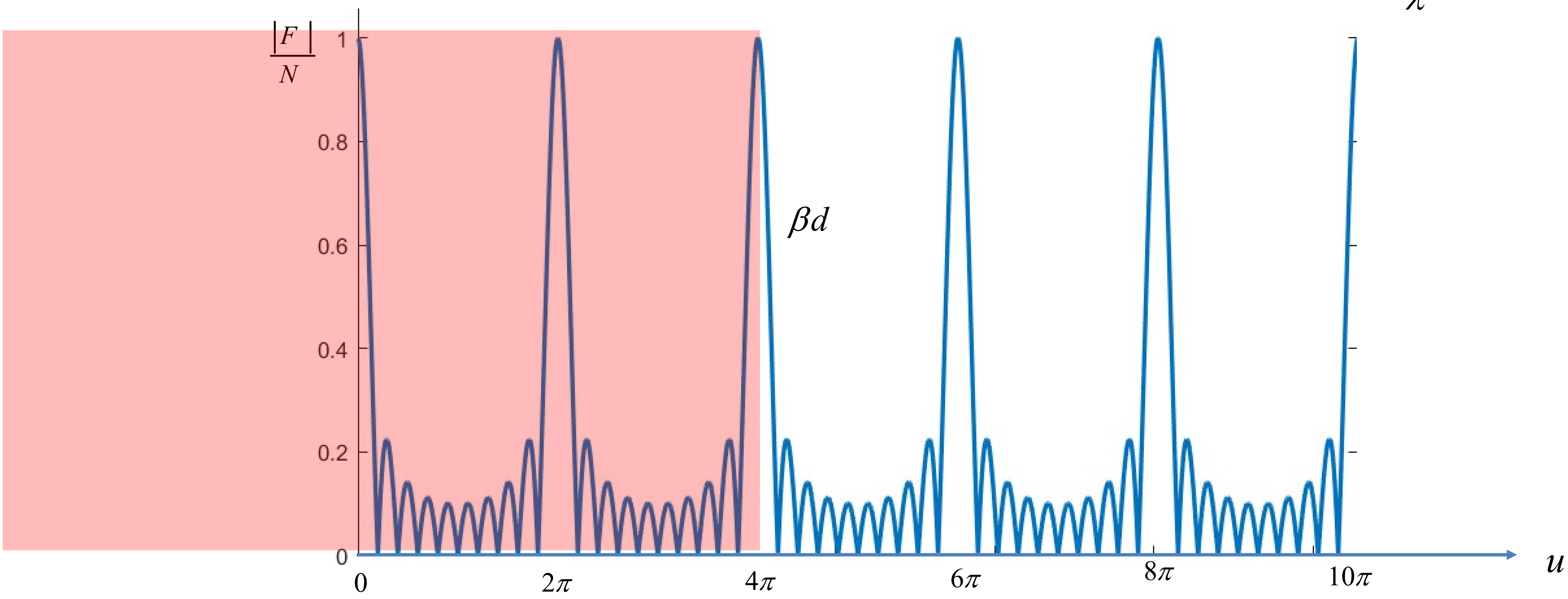


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 2\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda} 2\lambda = 4\pi$$

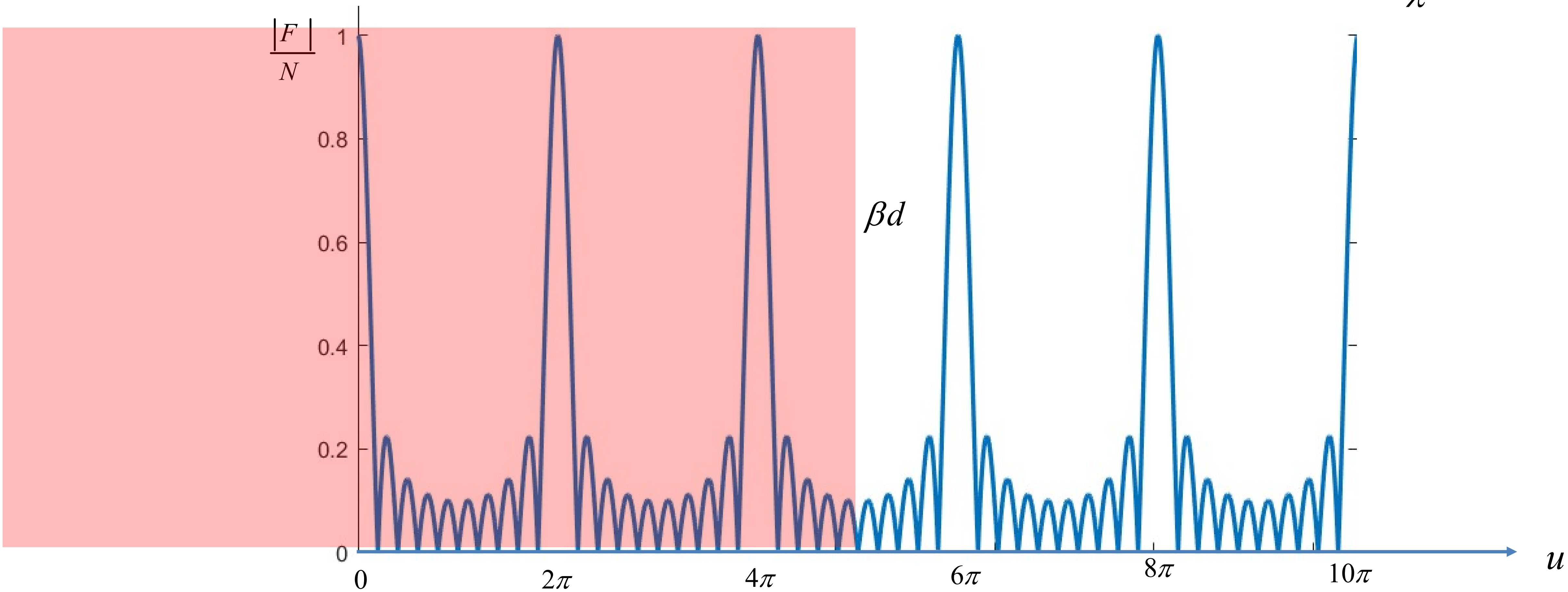


Exercise n.2

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 2.5\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda} 2.5\lambda = 5\pi$

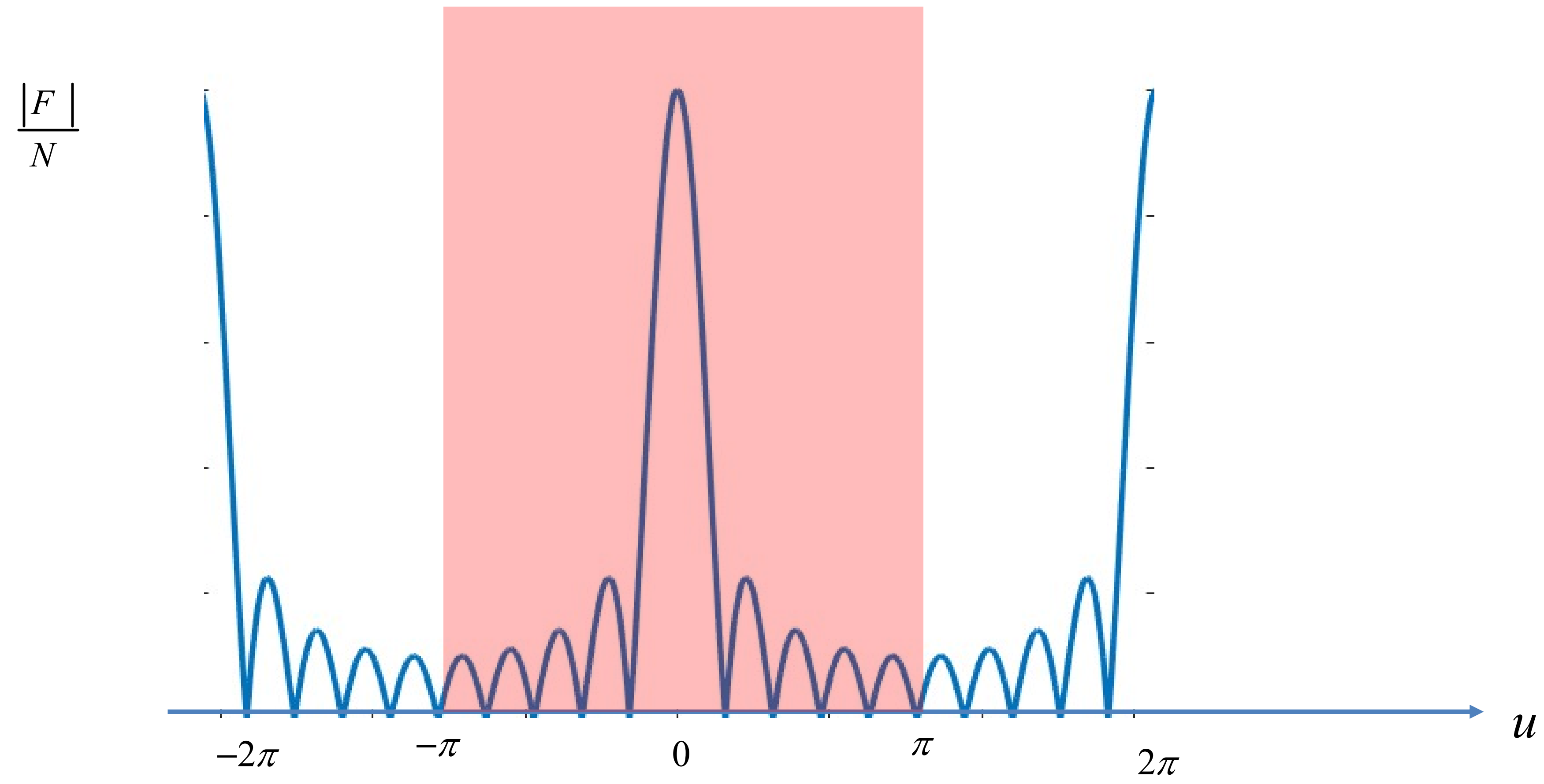


Exercise n.3

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

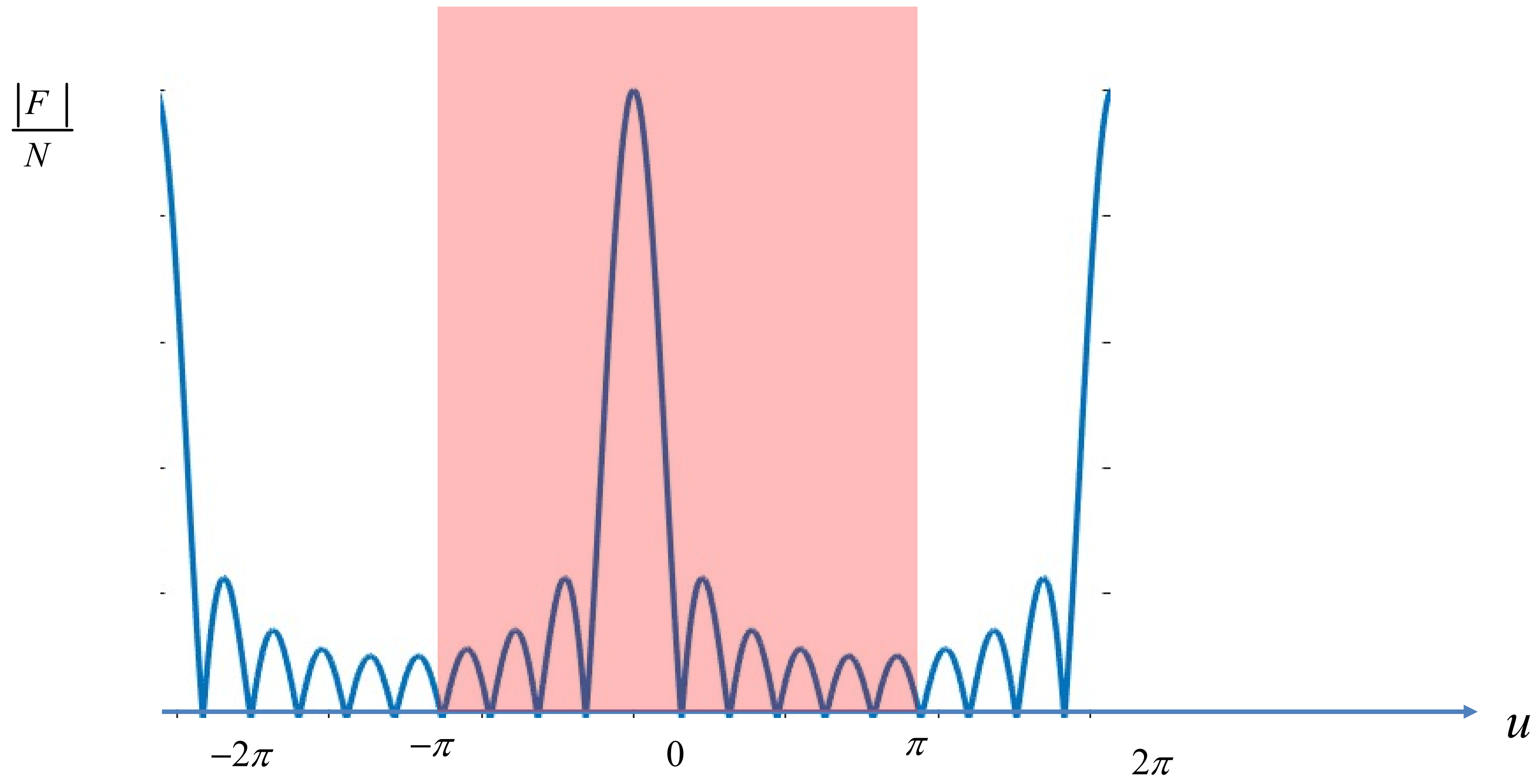


Exercise n.3

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

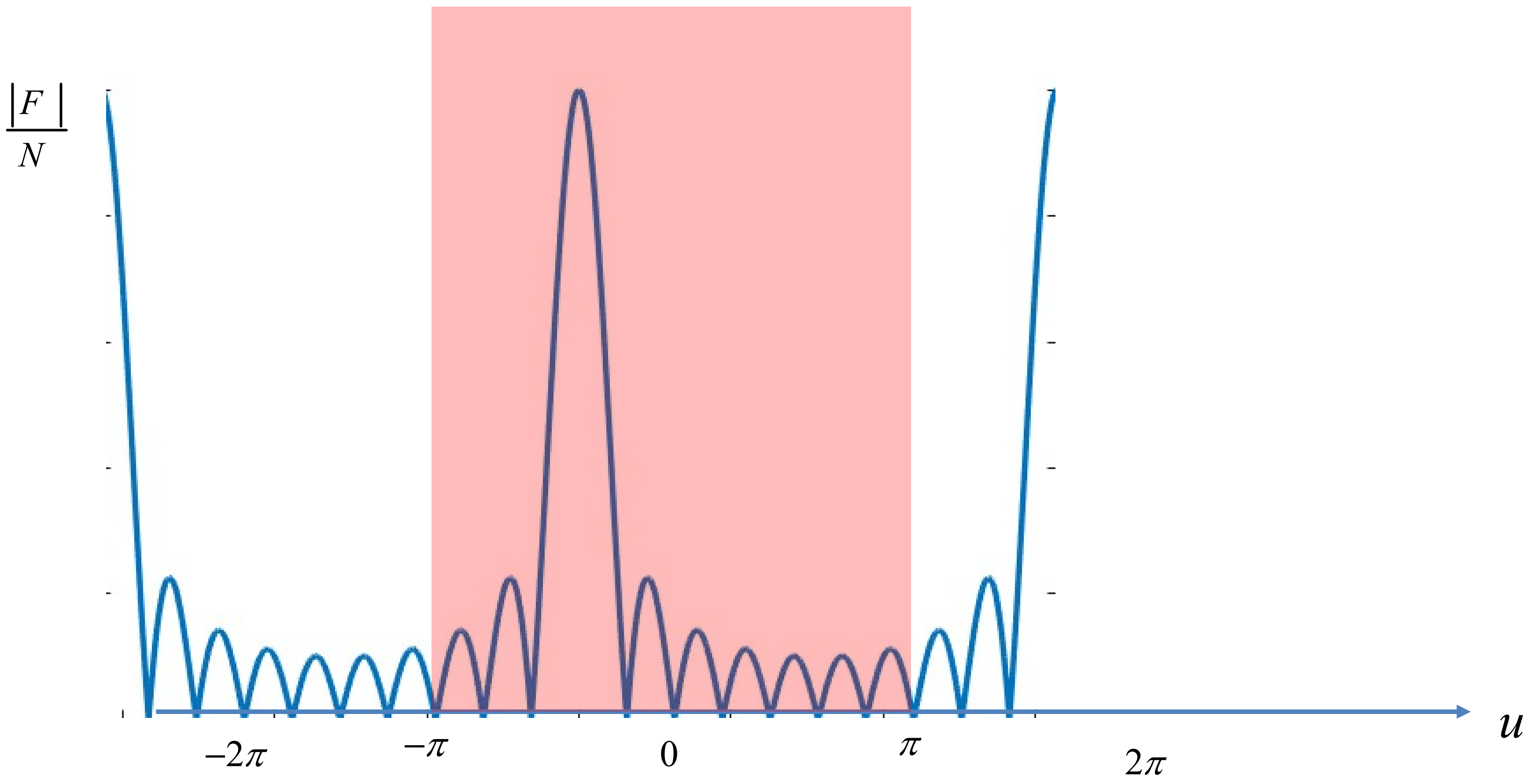


Exercise n.3

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

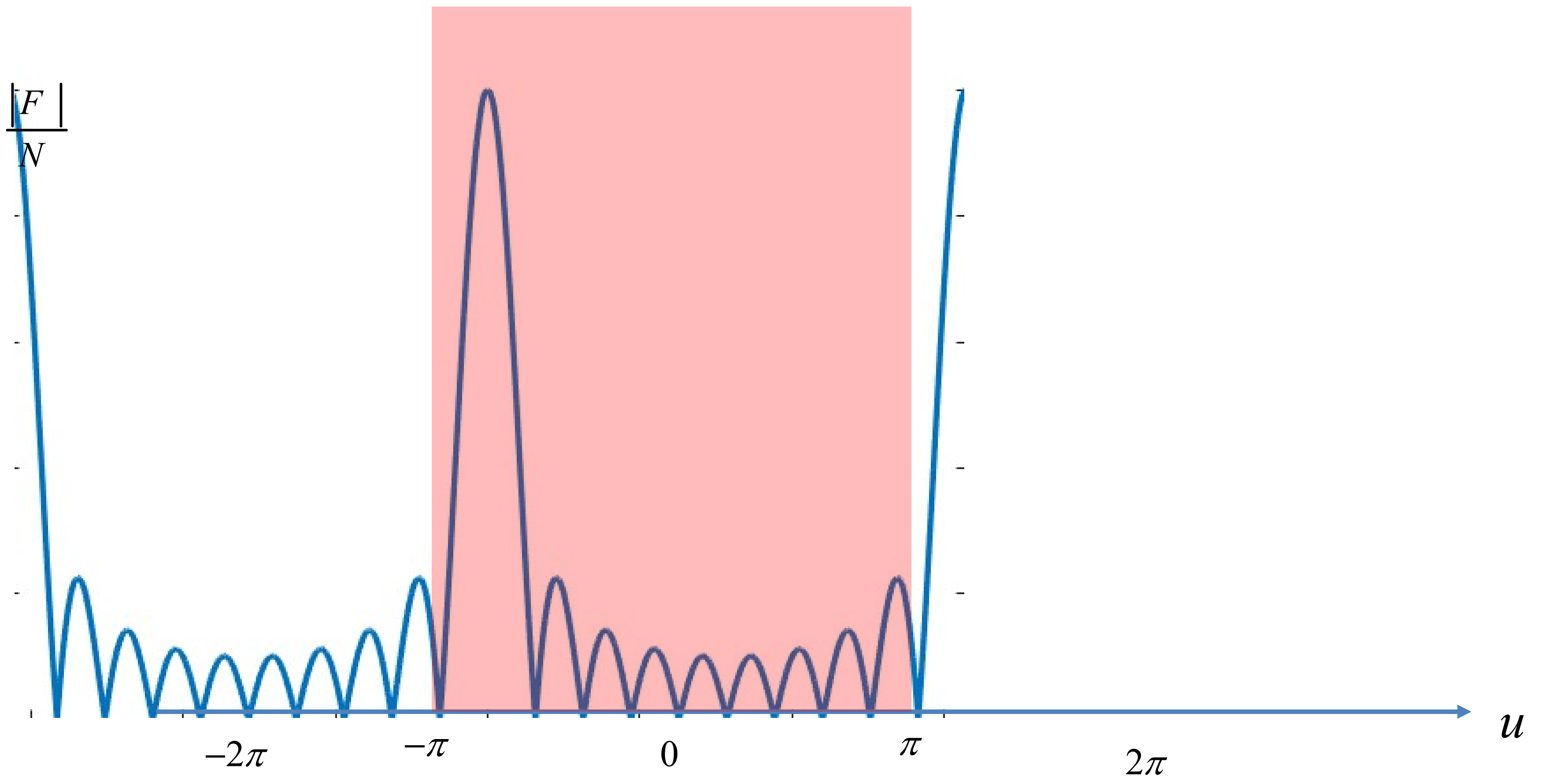


Exercise n.3

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

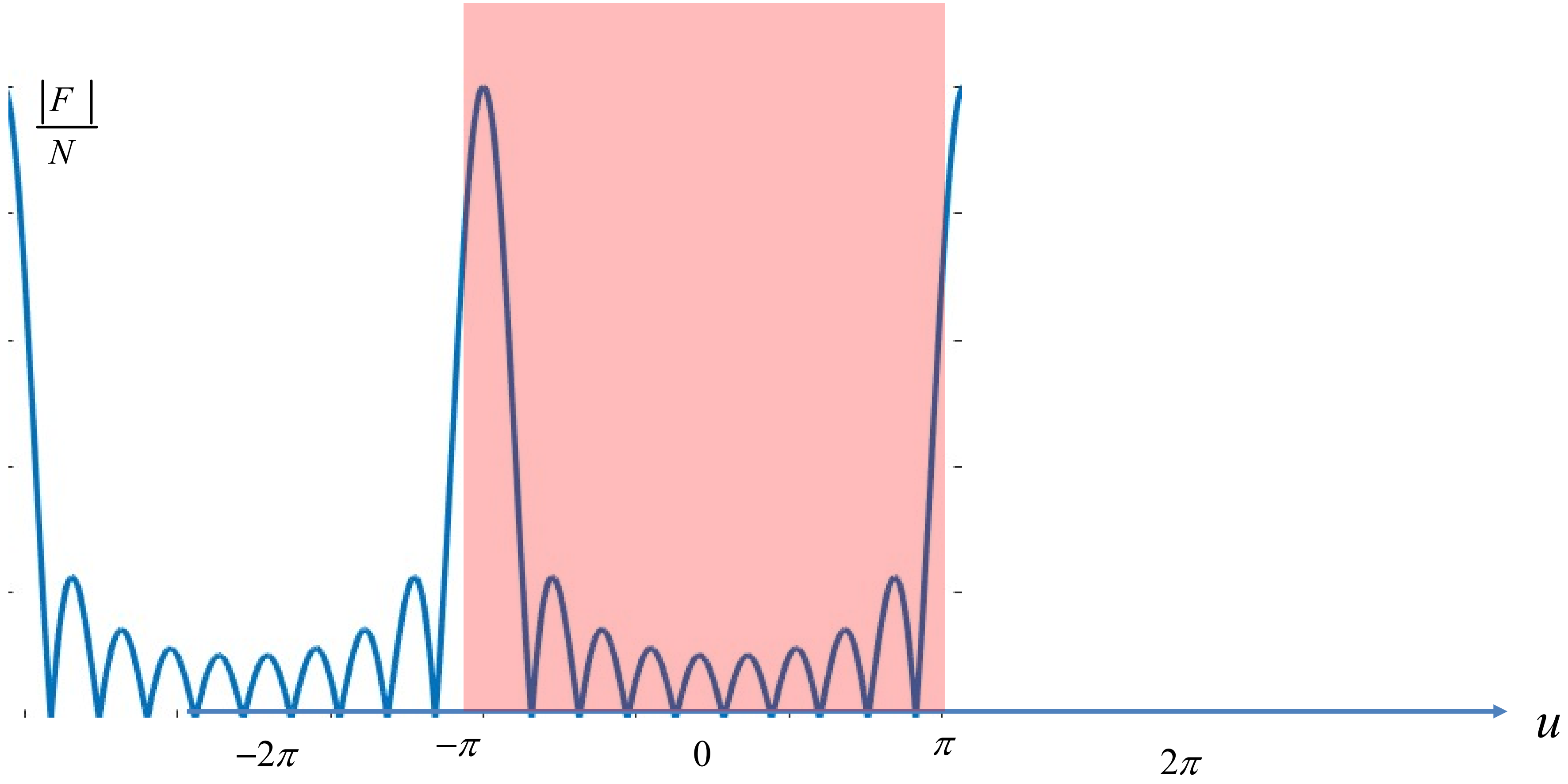


Exercise n.3

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

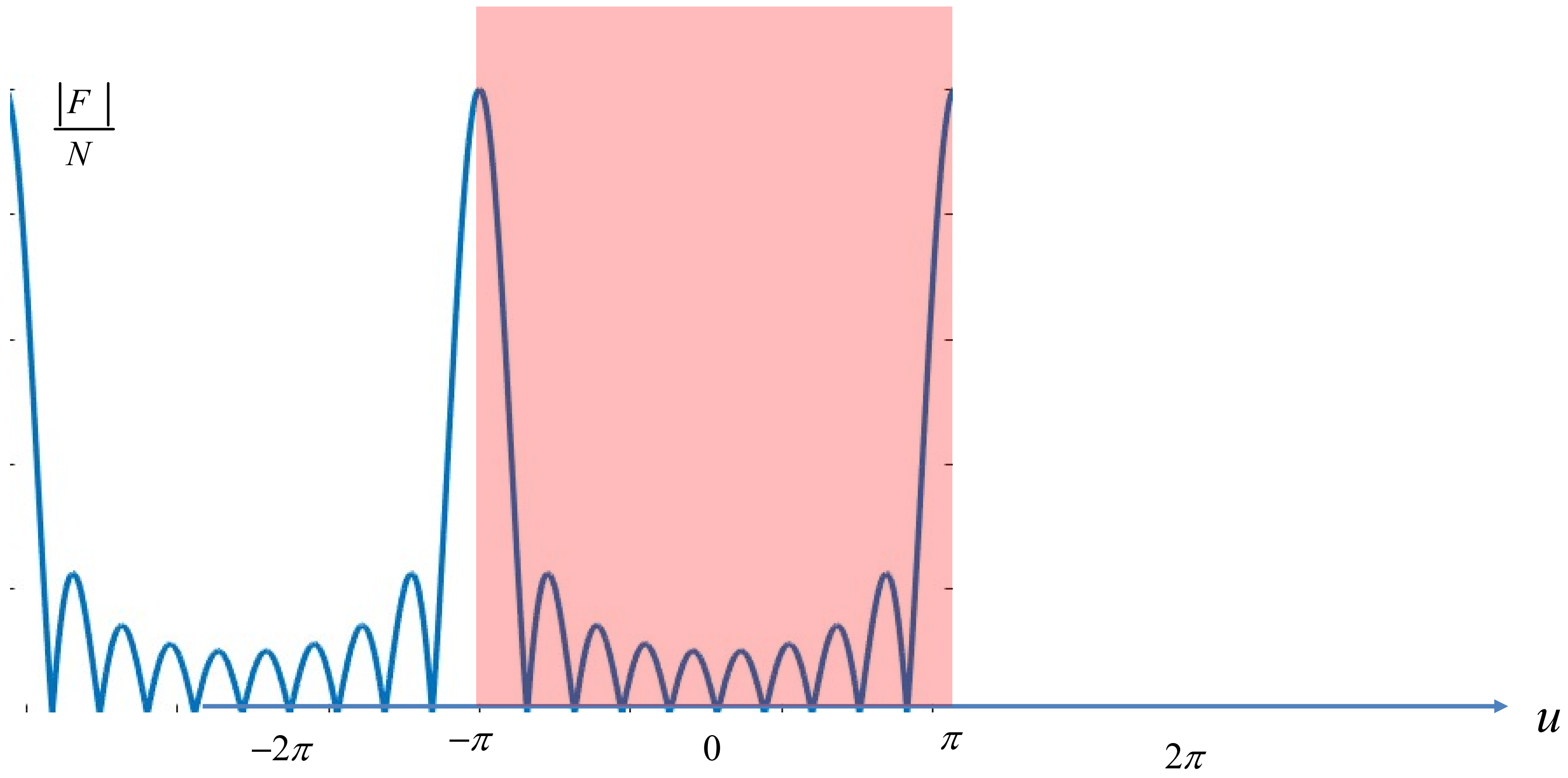


Exercise n.3

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

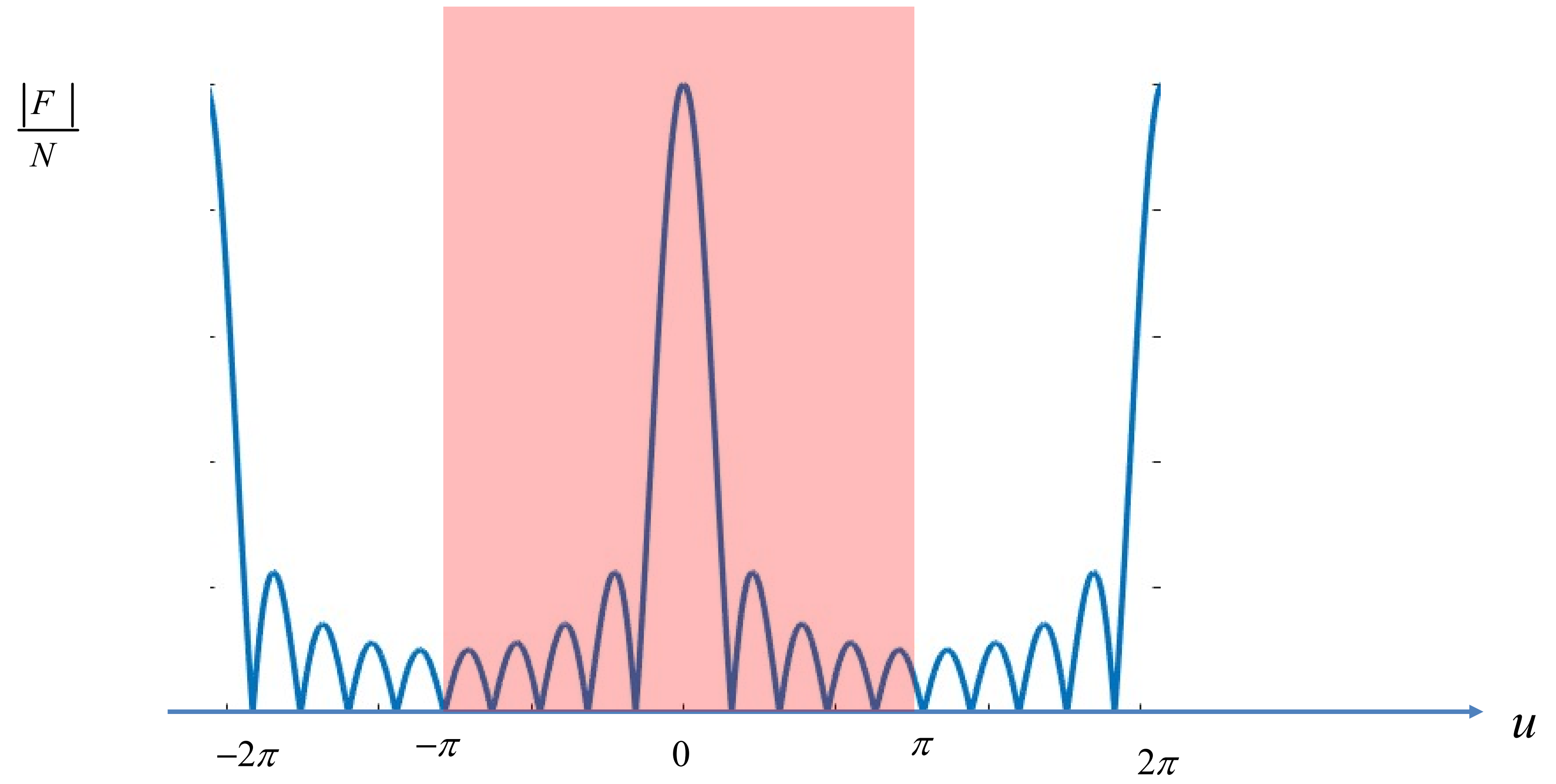


Exercise n.4

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

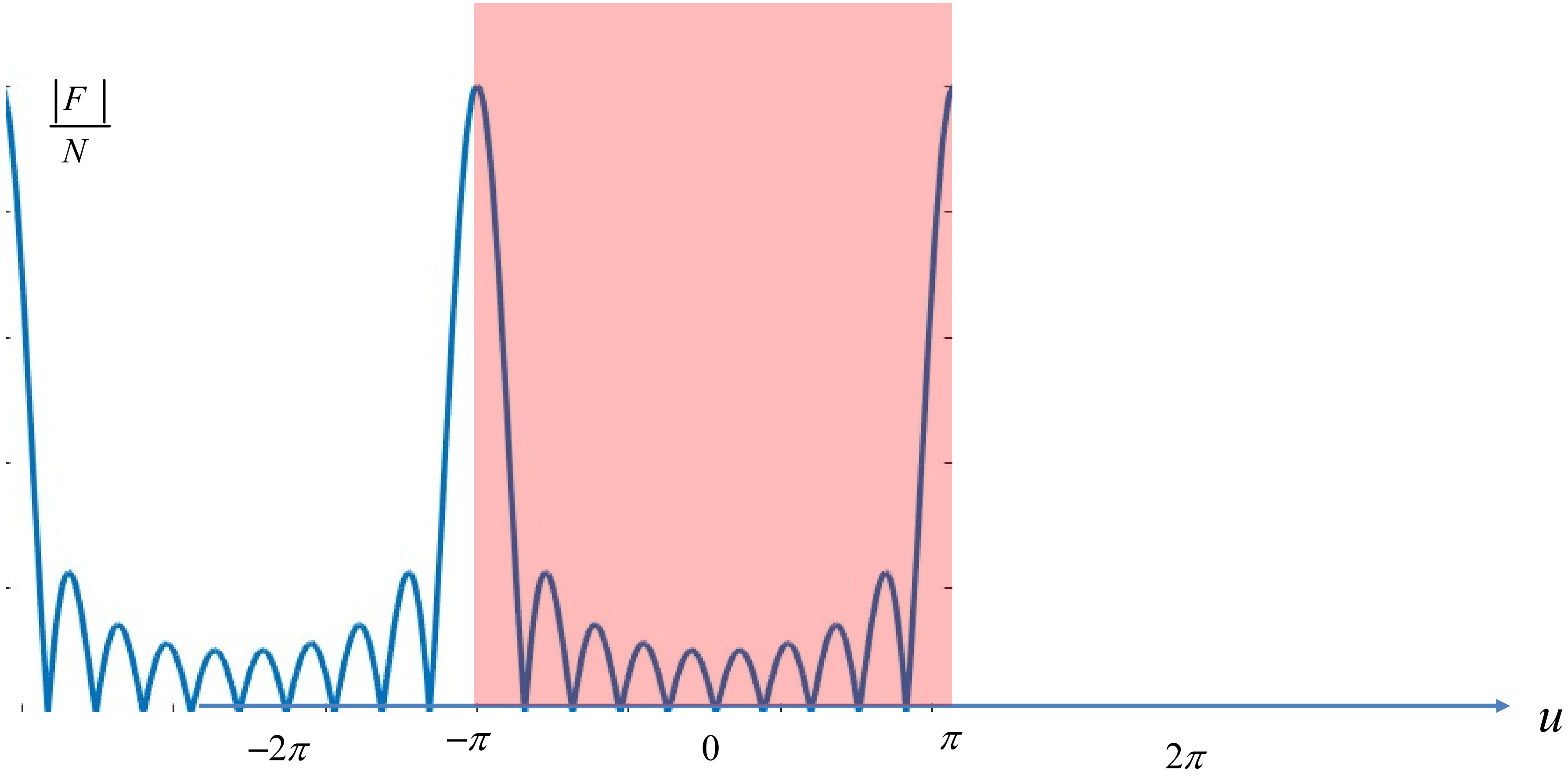


Exercise n.4

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$d = 0.5\lambda \Rightarrow \beta d = \pi$

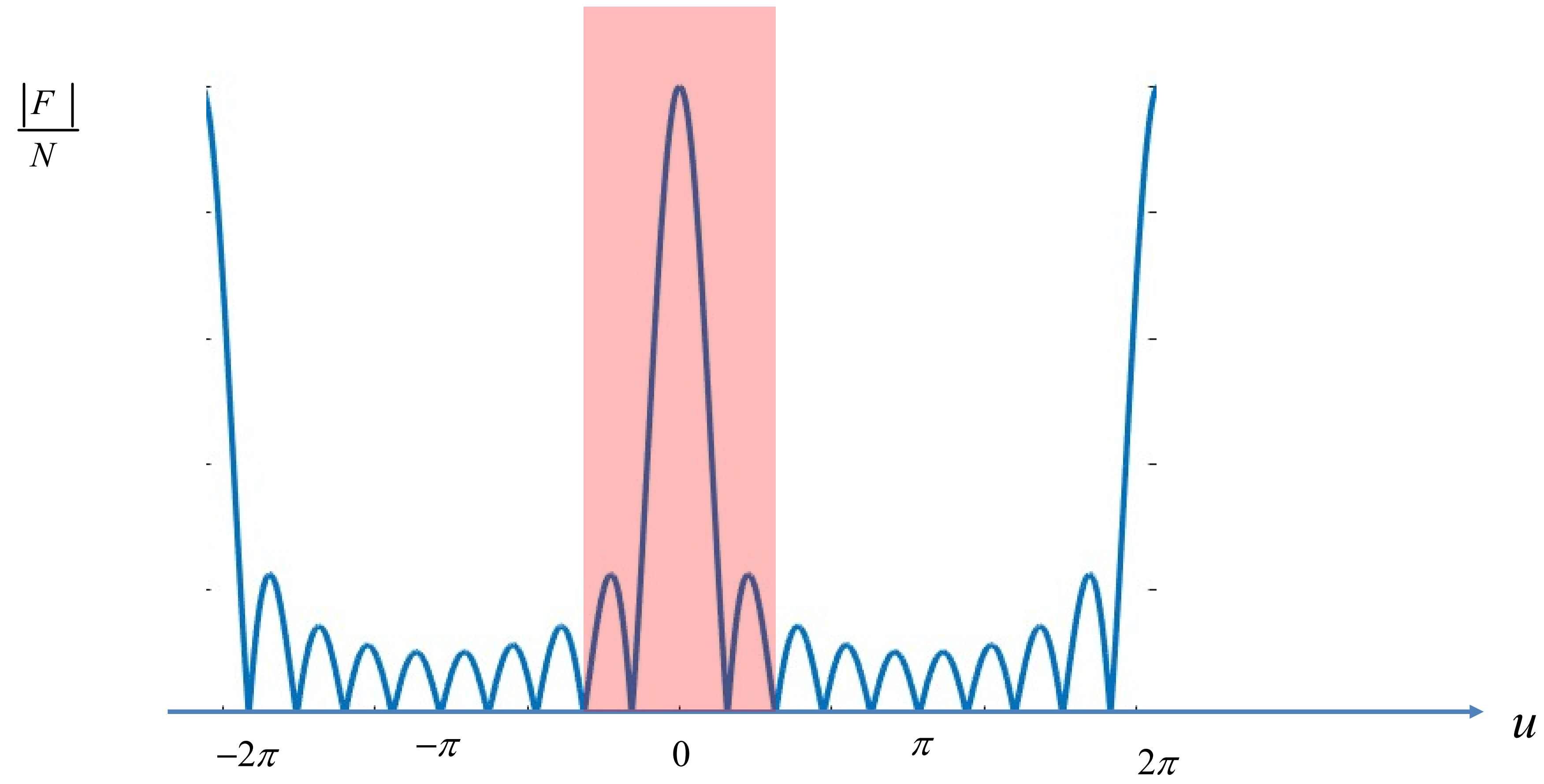


Exercise n.4

Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 0.2\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda}0.2\lambda = 2\frac{2\pi}{10}$$

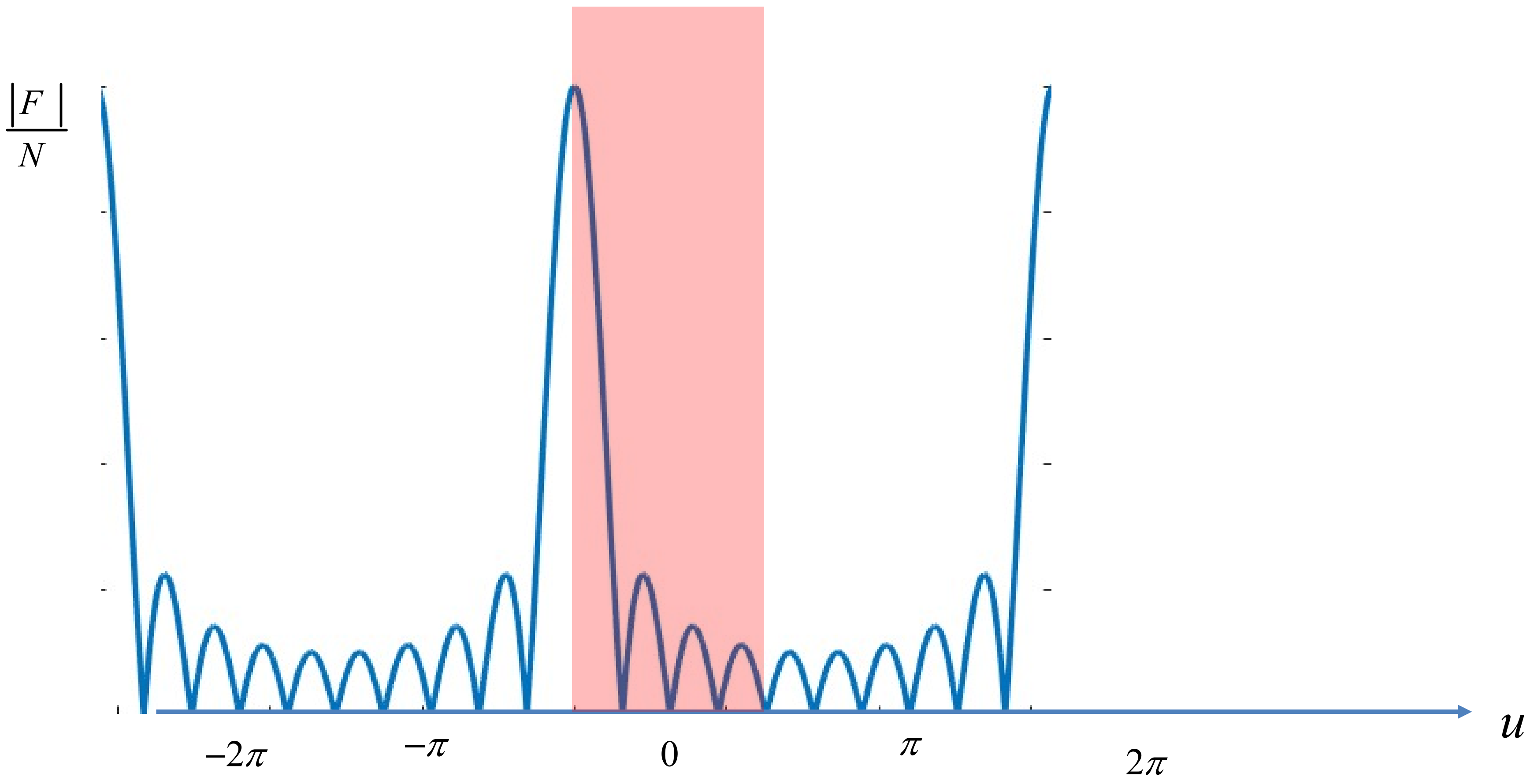


Exercise n.4

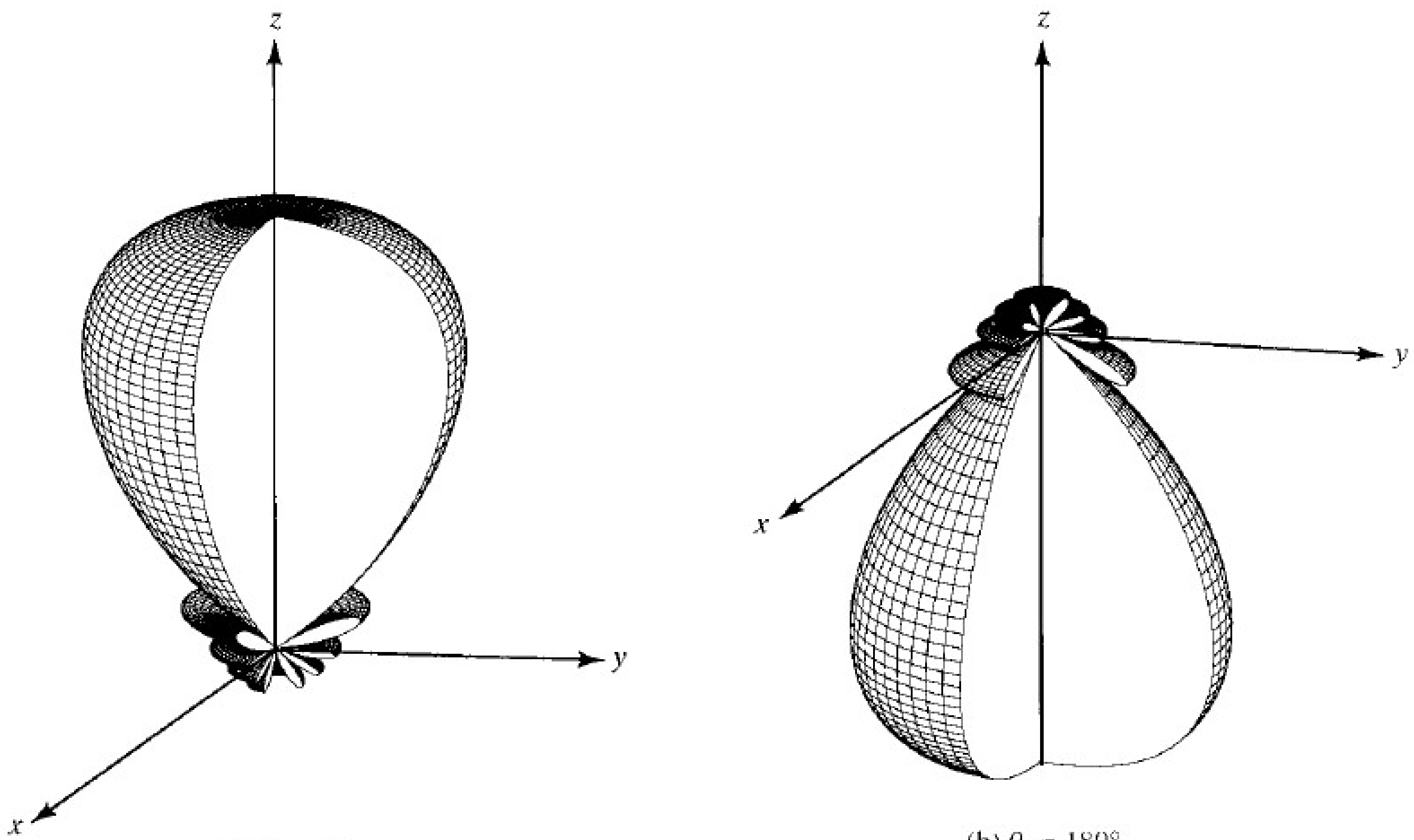
Periodic Linear Arrays (z-axis): Uniform Excitations

$N = 10$

$$d = 0.2\lambda \Rightarrow \beta d = \frac{2\pi}{\lambda} 0.2\lambda = 2\frac{2\pi}{10}$$



Endfire Arrays



(a) $\theta_0 = 0°$ (b) $\theta_0 = 180°$

Broadside Arrays