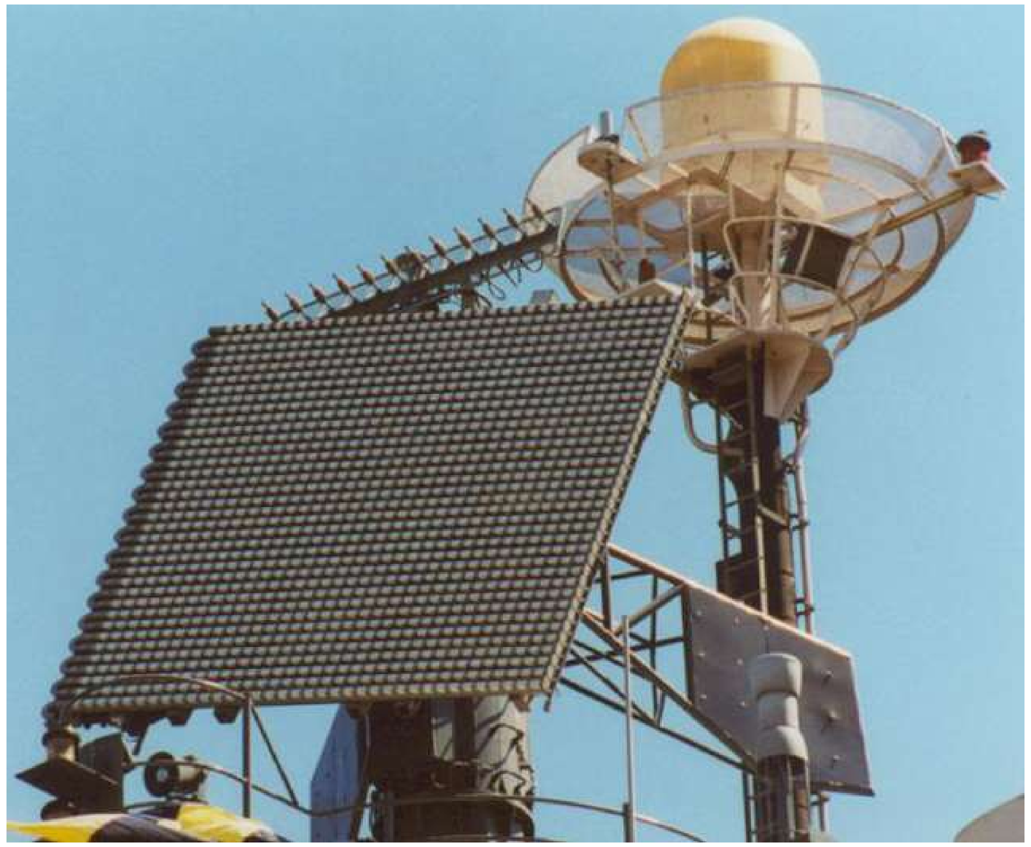
Arrays

Arrays



Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

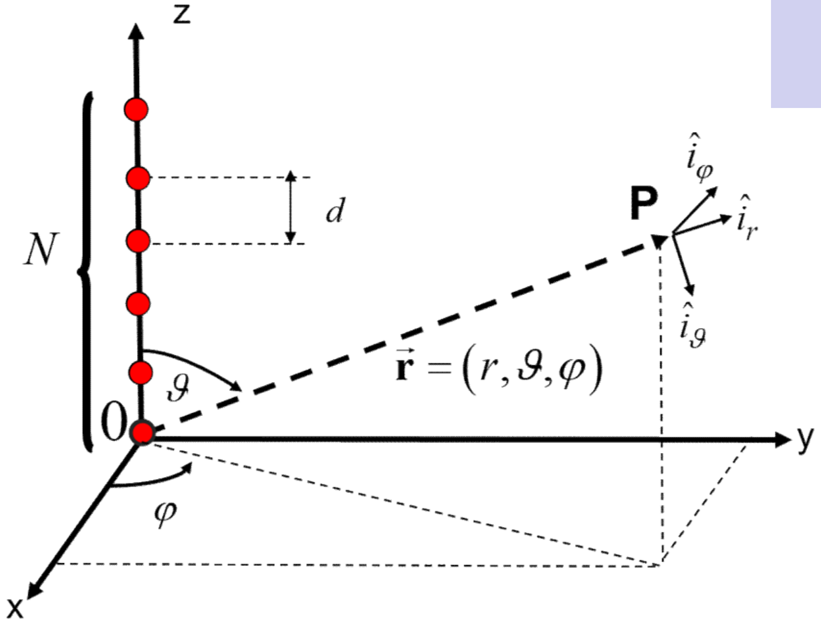
Mathematical tools to be exploited

Mathematics

Periodic Linear Arrays (z-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\cos\vartheta)$$



$$u = -\beta d\cos\vartheta$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

Periodic Linear Arrays (z-axis)

Uniform input excitations (Broadside Array)

Beam scanning

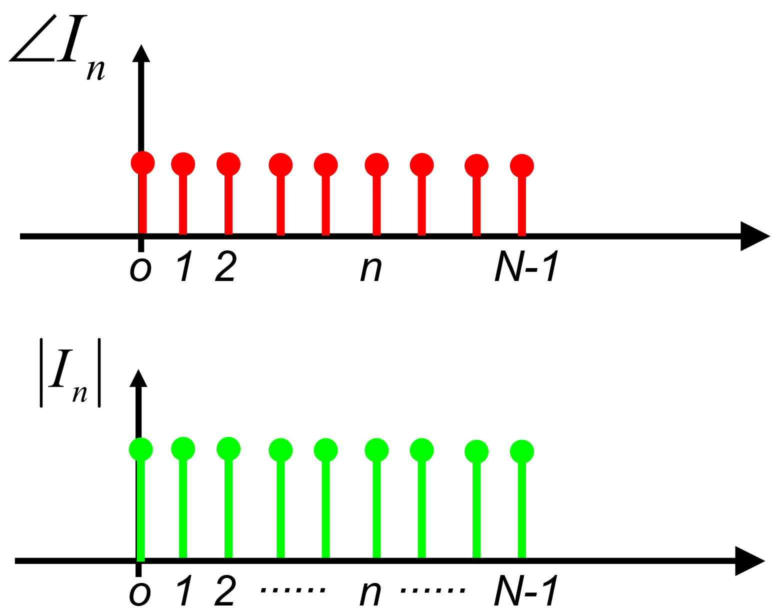
Endfire Array

Beam scanning and grating lobes

Periodic Linear Arrays (z-axis)

Uniform input excitations

$$I_n = I$$



Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

✓ **1. Let's depict F(*u*)**

2. Let's jump from *u* to ϑ and calculate:

✓ **The direction of the Main Lobe** $\vartheta_{MB} = \frac{\pi}{2}$

✓ **The NNBW / HPBW** $\mathrm{NNBW} \approx 2\frac{\lambda}{Nd}$ $\mathrm{HPBW} \approx 0.88\frac{\lambda}{Nd}$

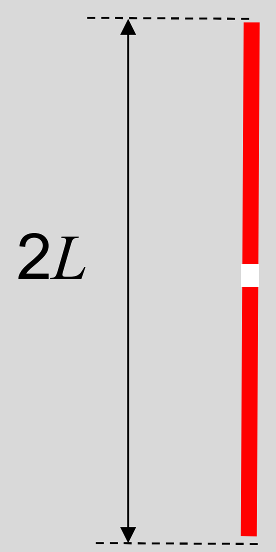
✓ **The SLL** $\mathrm{SLL} = -13.46\,dB$

BROADSIDE ARRAYS

Periodic Linear Arrays (z-axis): Uniform Excitations vs. Wire Antennas with Uniform Current Distribution

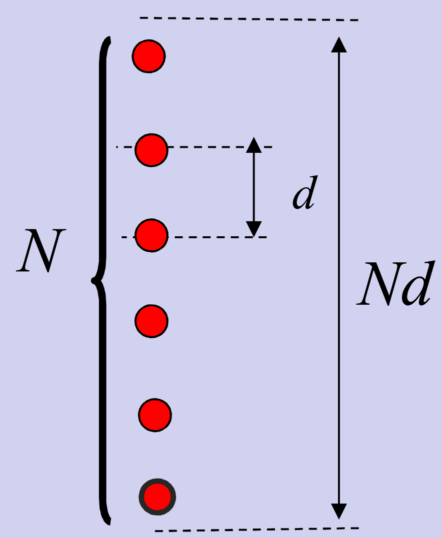
Wire Antennas

$$\text{HPBW} \approx 0.88\frac{\lambda}{2L}$$



Linear Arrays

$$\text{HPBW} \approx 0.88\frac{\lambda}{Nd}$$



Periodic Linear Arrays (z-axis)

Uniform input excitations (Broadside Array)

Beam scanning

Endfire Array

Beam scanning and grating lobes

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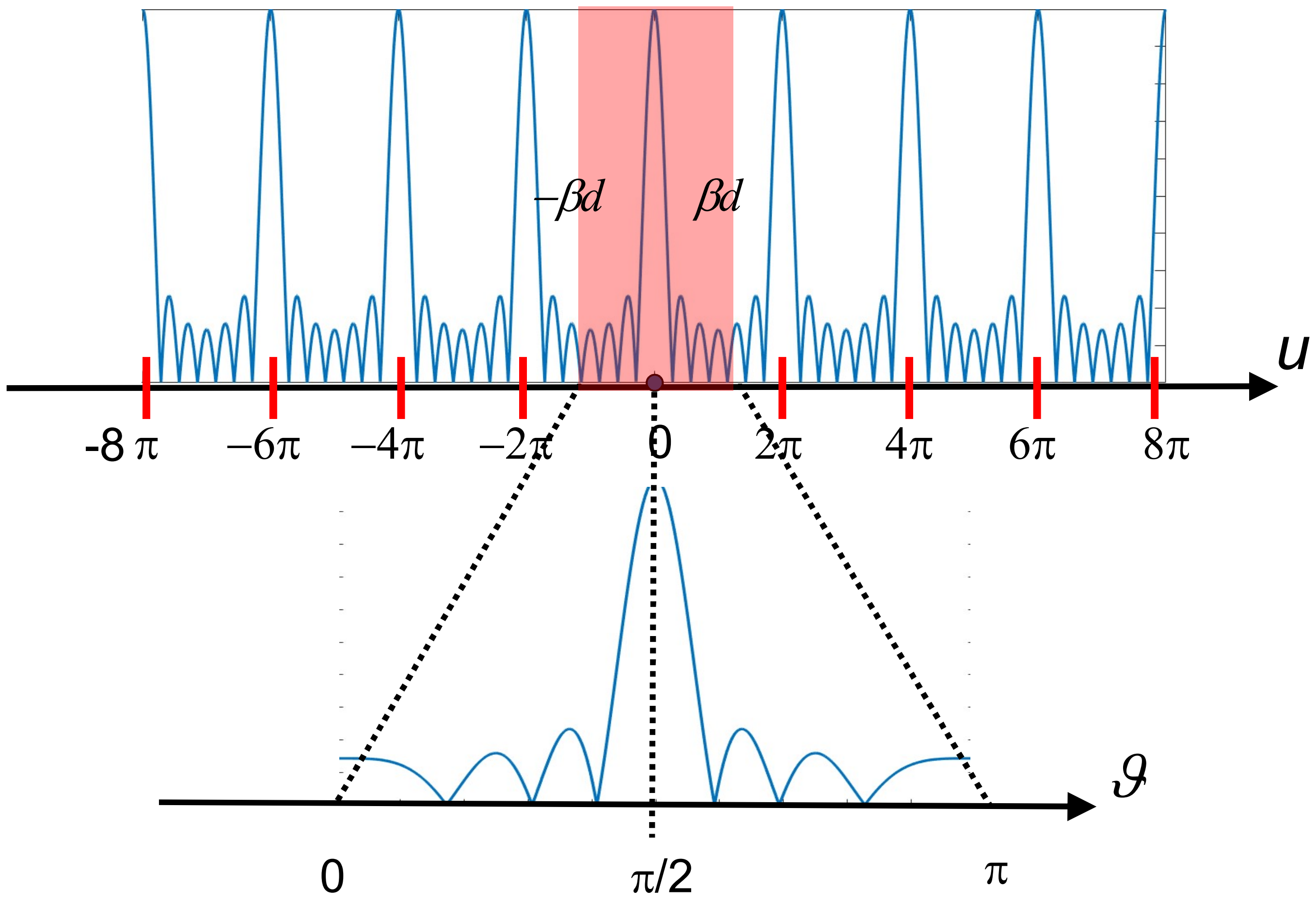
Periodic Linear Arrays (z-axis): Beam Scanning

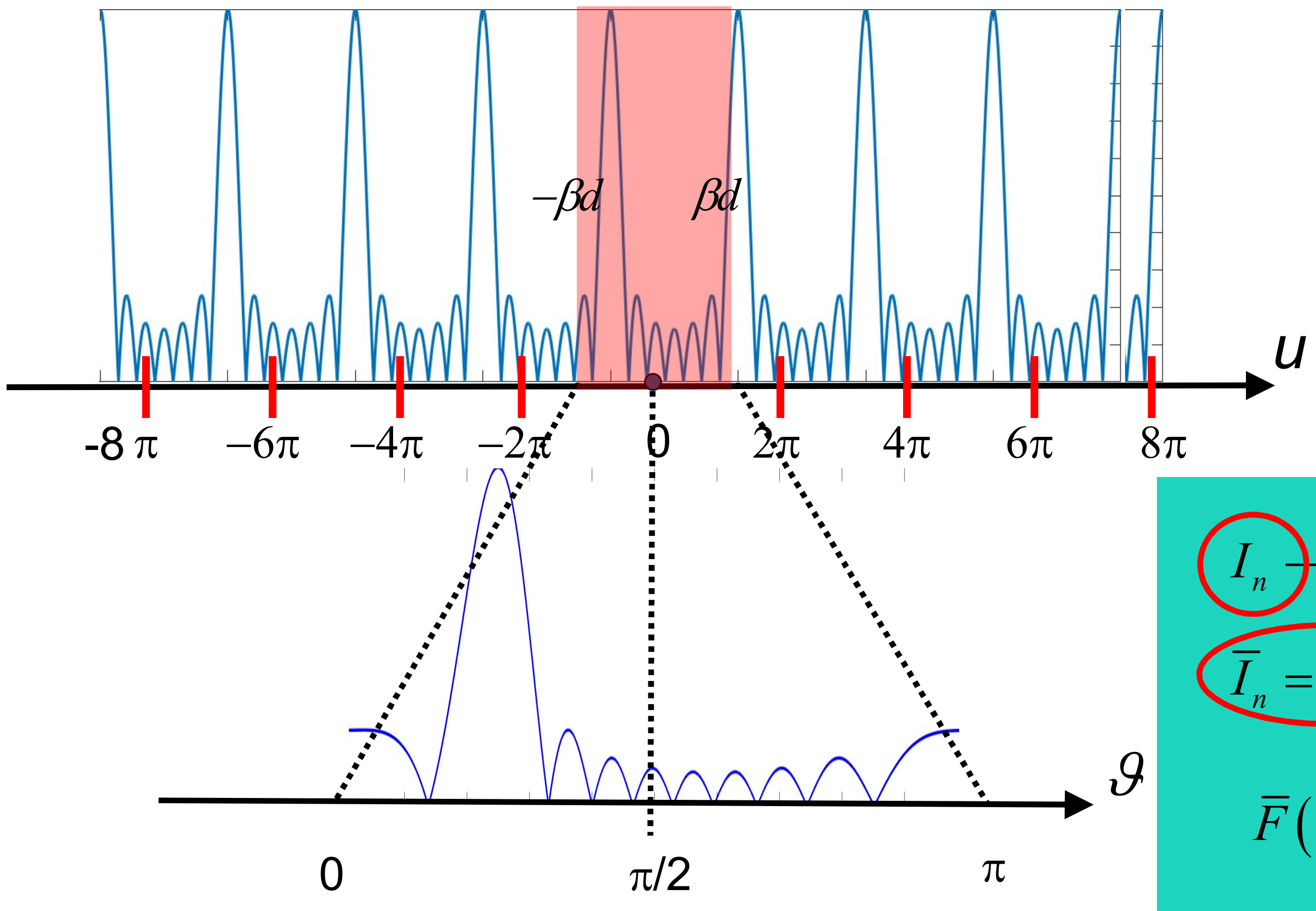
$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$



$$F(u) \rightarrow F(u - u_s)$$

Periodic Linear Arrays (z-axis): Beam Scanning

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$



$$F(u) \rightarrow F(u-u_s)$$

A rigid shift of *F*(u) produces a beam scanning.

The rigid shift of *F*(u) can be obtained electronically by applying a phase ramp to the input currents I_n.

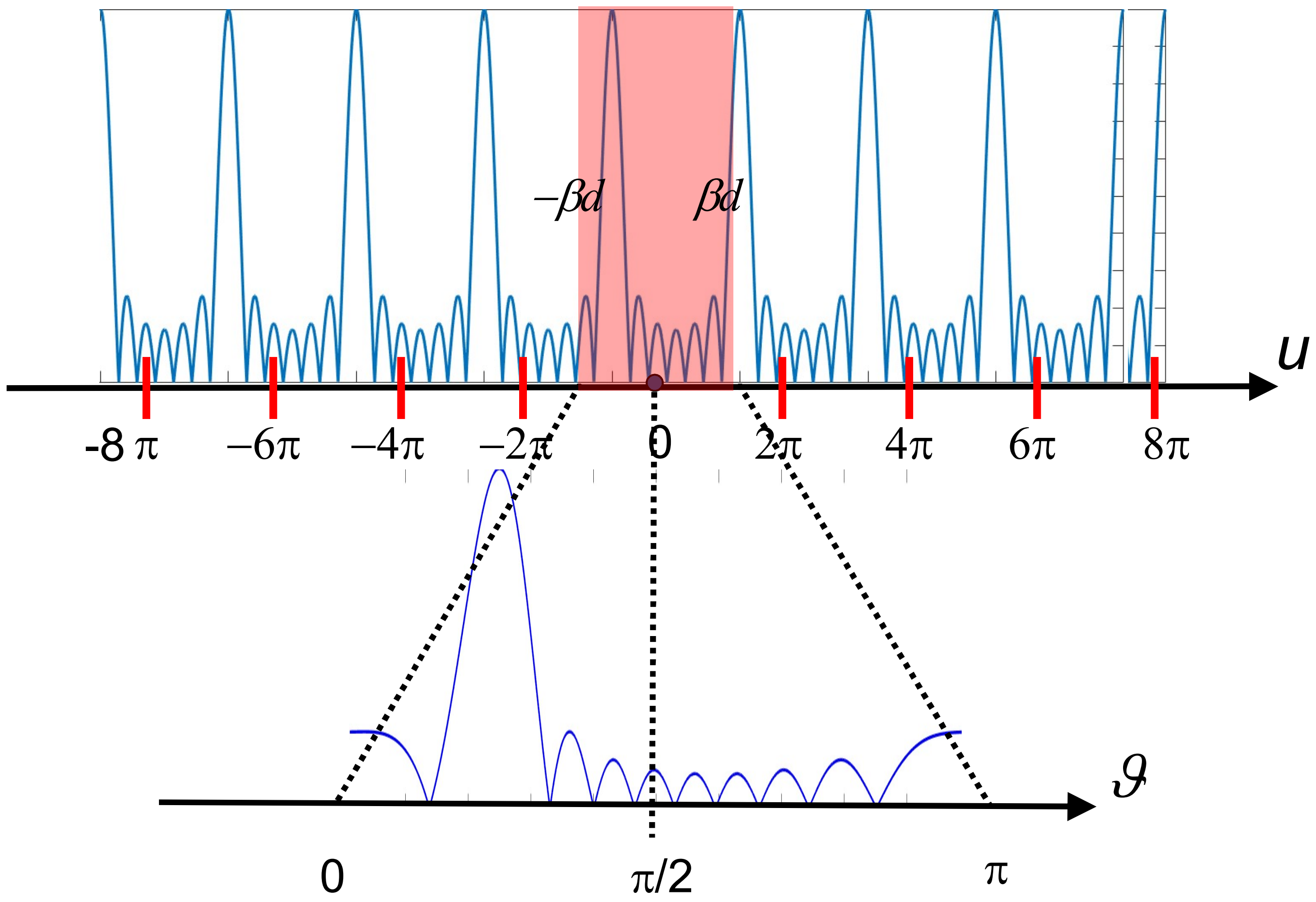
$$I_n \rightarrow F(u)$$

$$\bar{I}_n = I_n e^{jnu_s} \rightarrow \bar{F}(u) = F(u-u_s)$$

$$\bar{F}(u) = \sum_{n=0}^{N-1} I_n e^{jnu_s} e^{-jnu} = \sum_{n=0}^{N-1} I_n e^{-jn(u-u_s)} = F(u-u_s)$$

Periodic Linear Arrays (z-axis): Beam Scanning

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$



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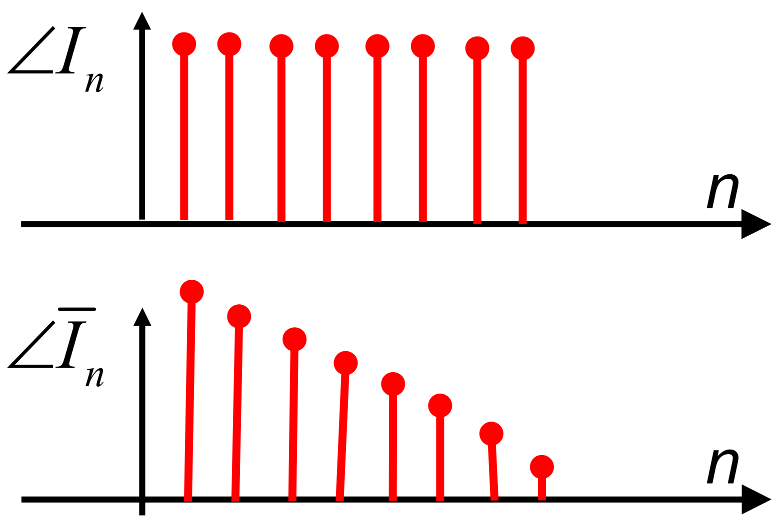
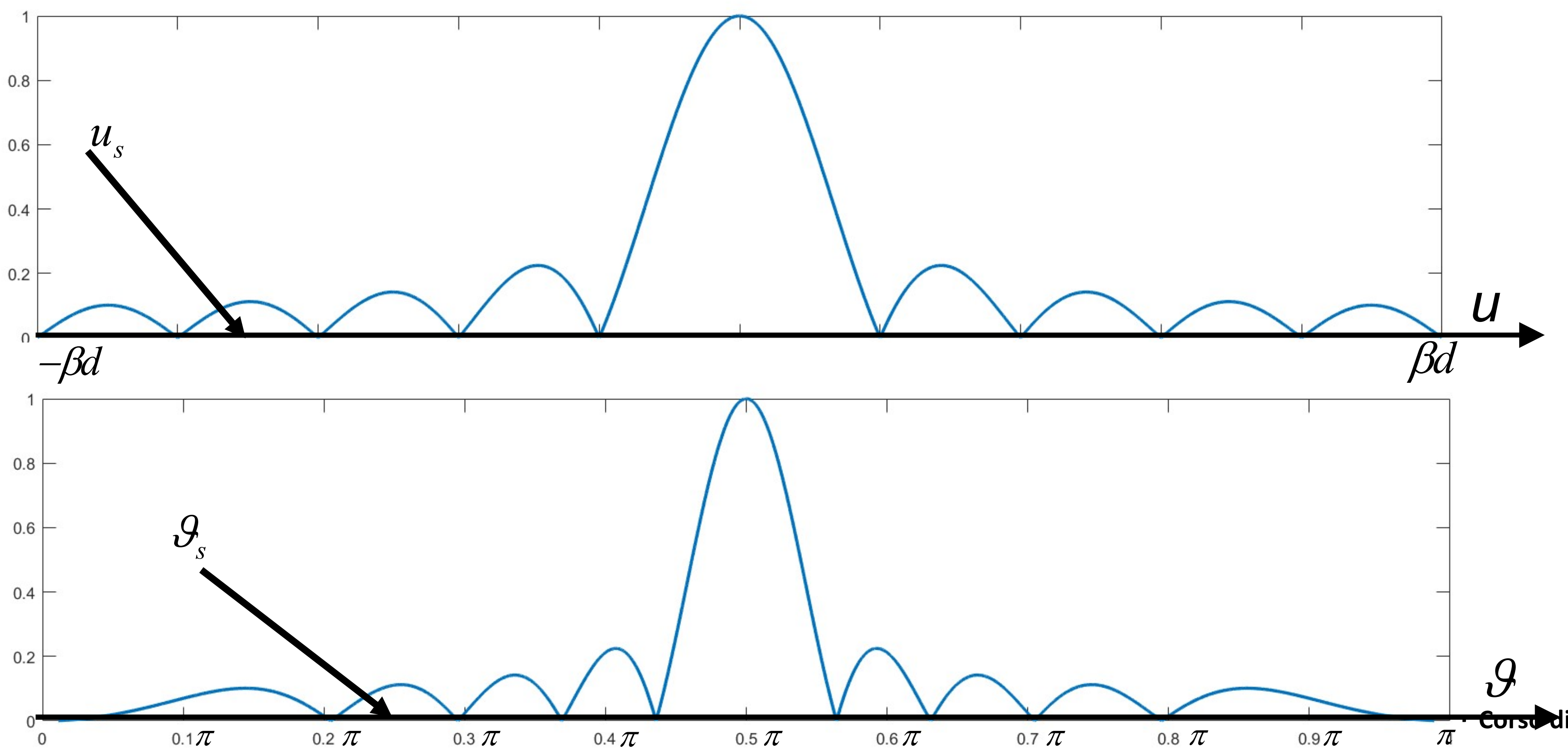


Periodic Linear Arrays (z-axis): Beam Scanning

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$u = -\beta d \cos \vartheta$$

$$F(u) \rightarrow F(u - u_s)$$



1) $\vartheta_s \in [0, \pi]$

2) $u_s = -\beta d \cos \vartheta_s$

3) $I_n \rightarrow I_n e^{jnu_s}$

Periodic Linear Arrays (z-axis): Beam Scanning

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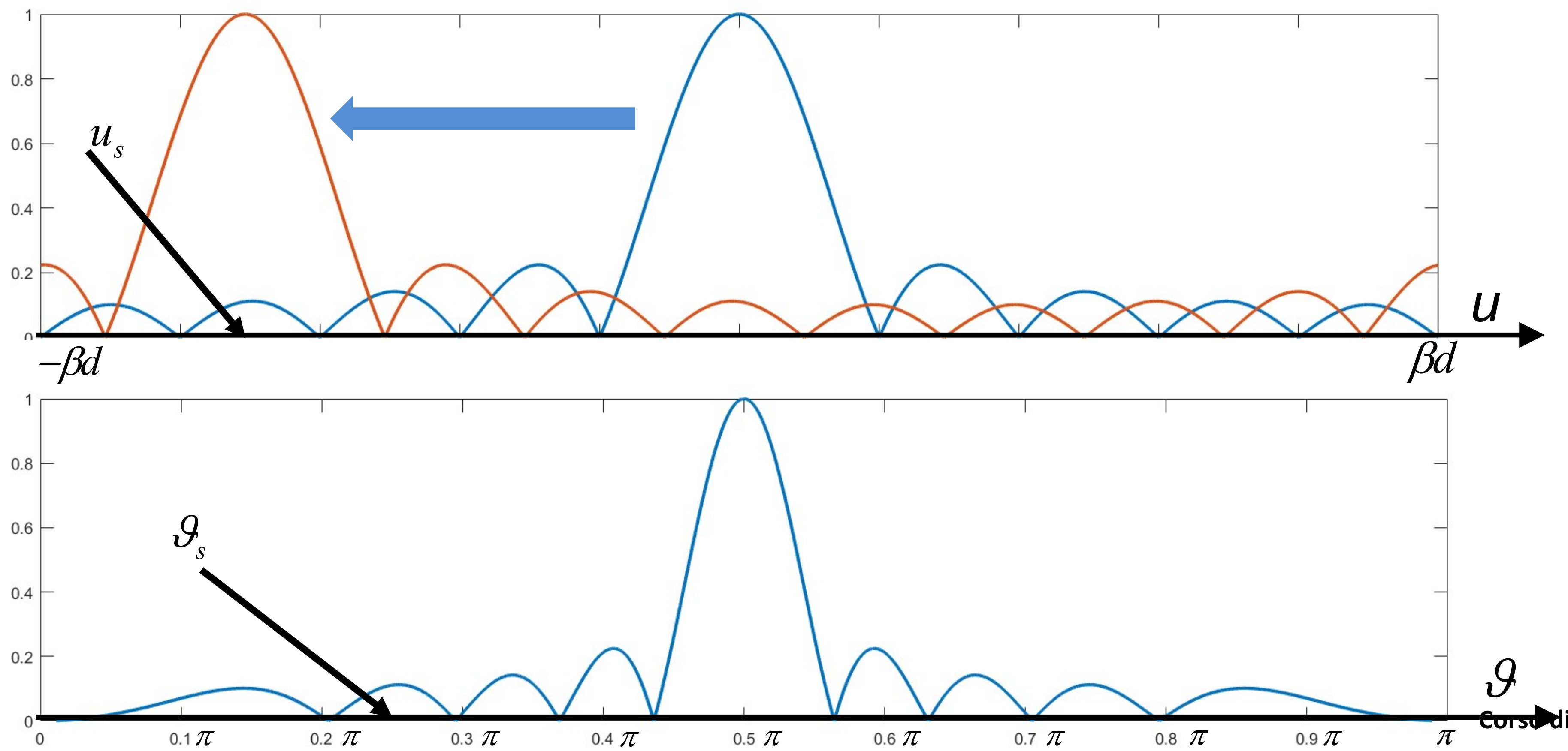

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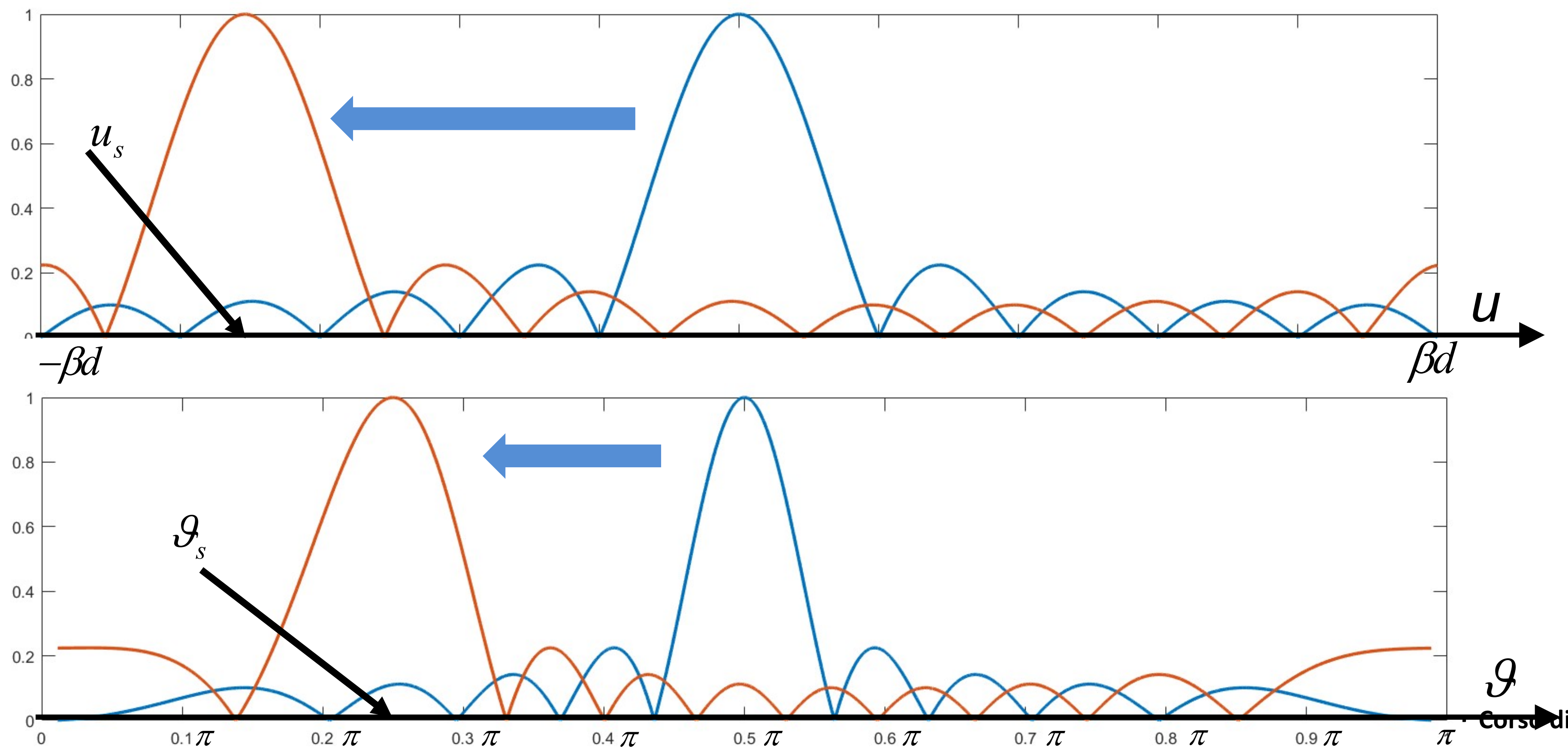

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Periodic Linear Arrays (z-axis)

Uniform input excitations (Broadside Array)

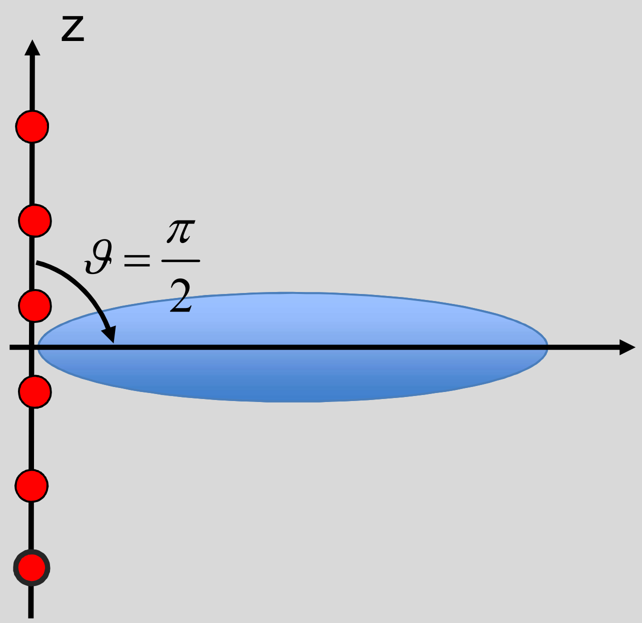
Beam scanning

Endfire Array

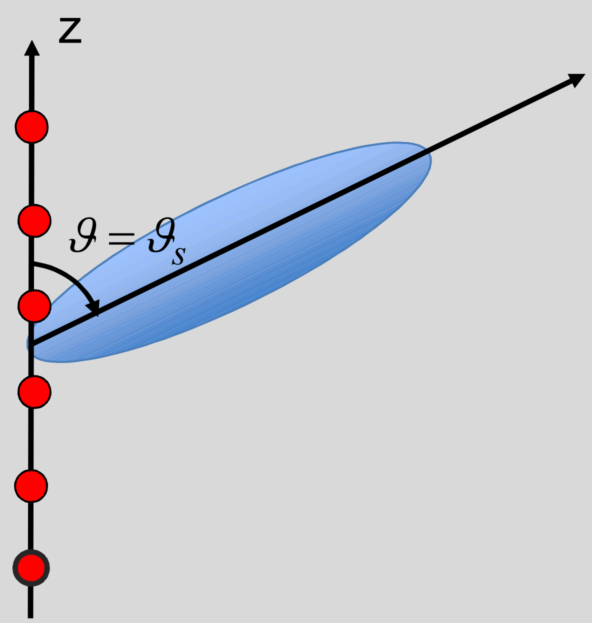
Beam scanning and grating lobes

Periodic Linear Arrays (z-axis): Endfire Arrays

Broadside array

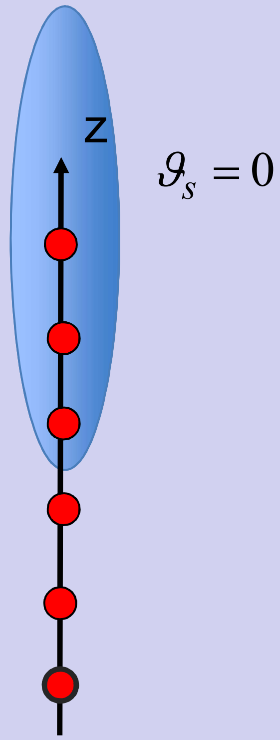


The beam is oriented along the direction orthogonal to the array axis. It can be achieved, for instance, with uniform excitations.



The beam scanning can be achieved electronically starting from a broadside array.

Endfire array



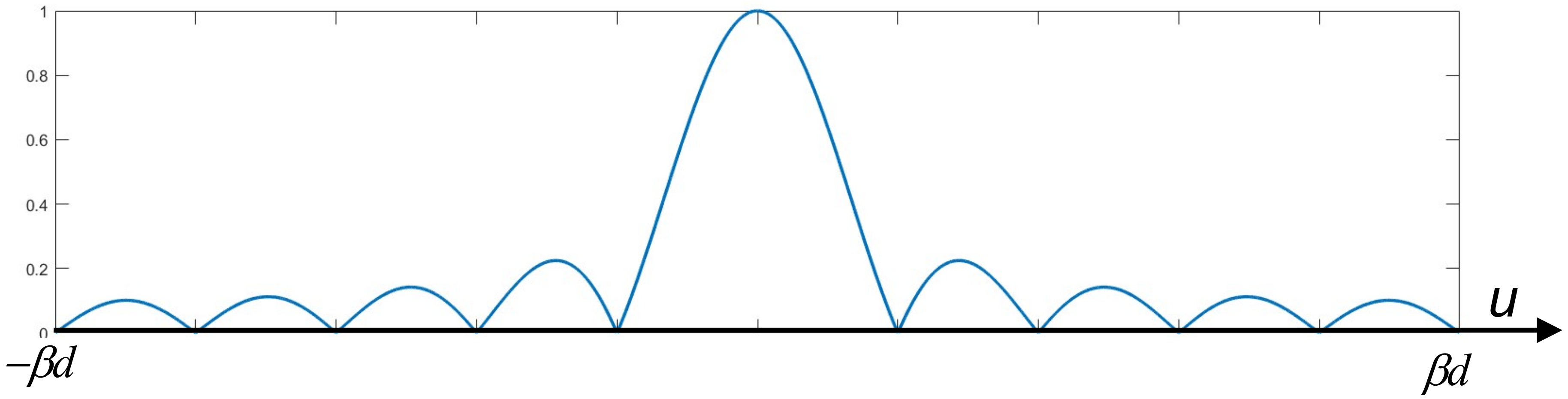
The beam is oriented along the array axis. It is a particular case of beam scanning, and can be thus achieved electronically starting from a broadside array.

Periodic Linear Arrays (z-axis): Beam Scanning

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$u = -\beta d \cos\vartheta$$

$$F(u) \rightarrow F(u - u_s)$$



1) $\vartheta_s \in [0, \pi]$

2) $u_s = -\beta d \cos\vartheta_s$

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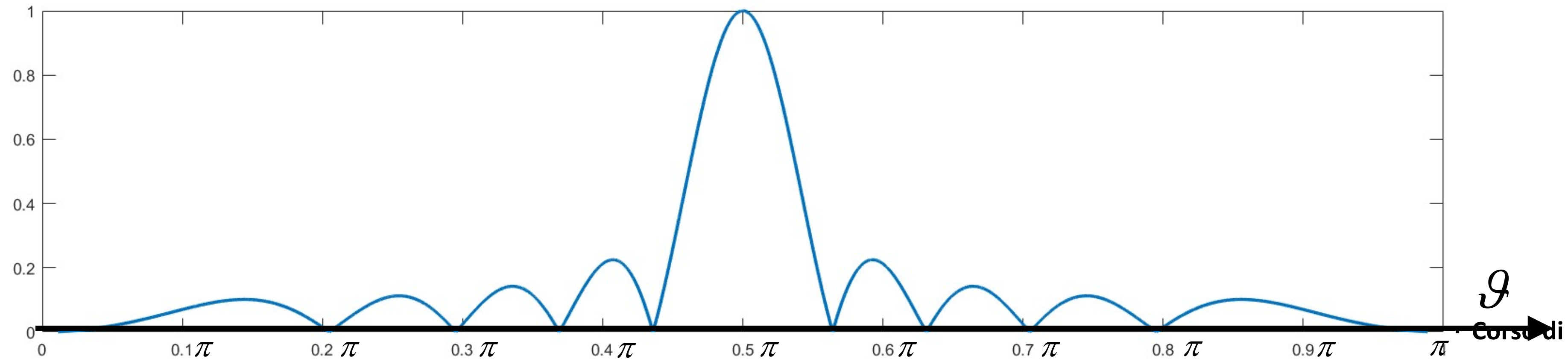

Endfire Arrays

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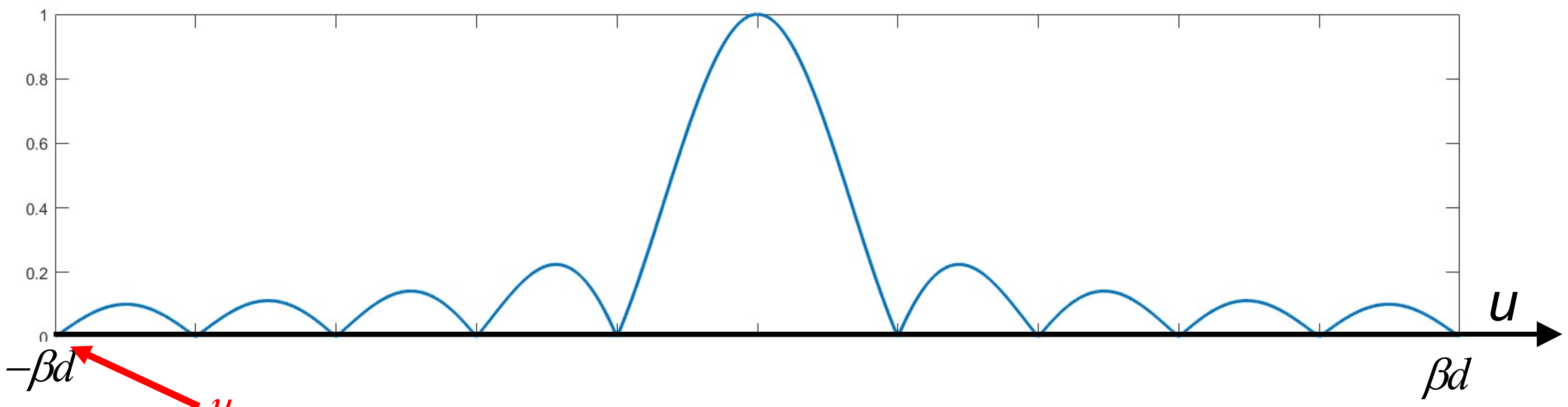
$$F(u)\to F(u-u_s)$$



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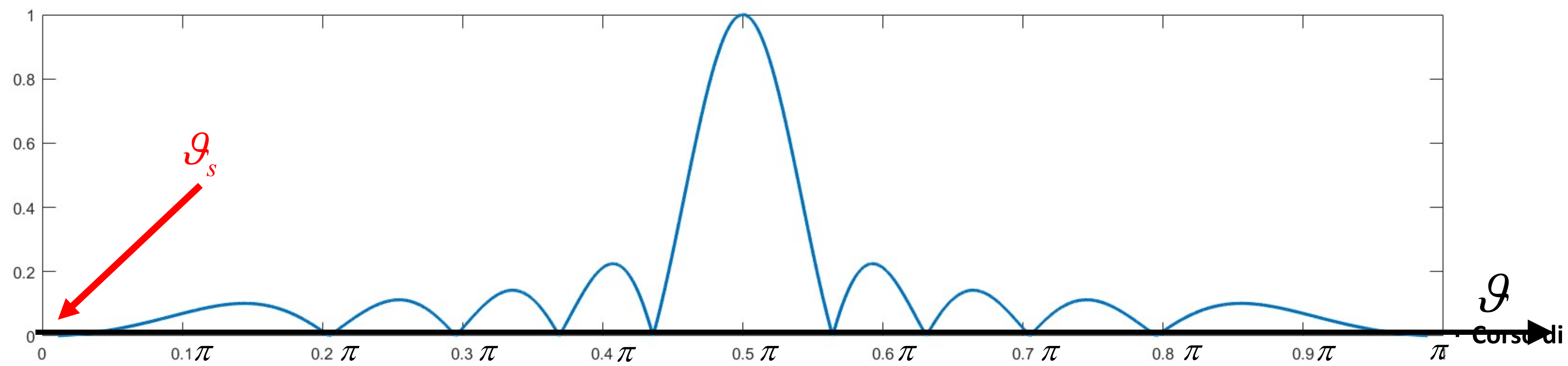

1) $\vartheta_s = \vartheta_{MB} = 0$

2) $u_s = -\beta d$

3) $I_n \to I_n e^{-jn\beta d}$

Endfire Arrays

$$|F(u)| = |I| \left| \frac{\sin(Nu/2)}{\sin(u/2)} \right|$$

$$u = -\beta d \cos\vartheta$$

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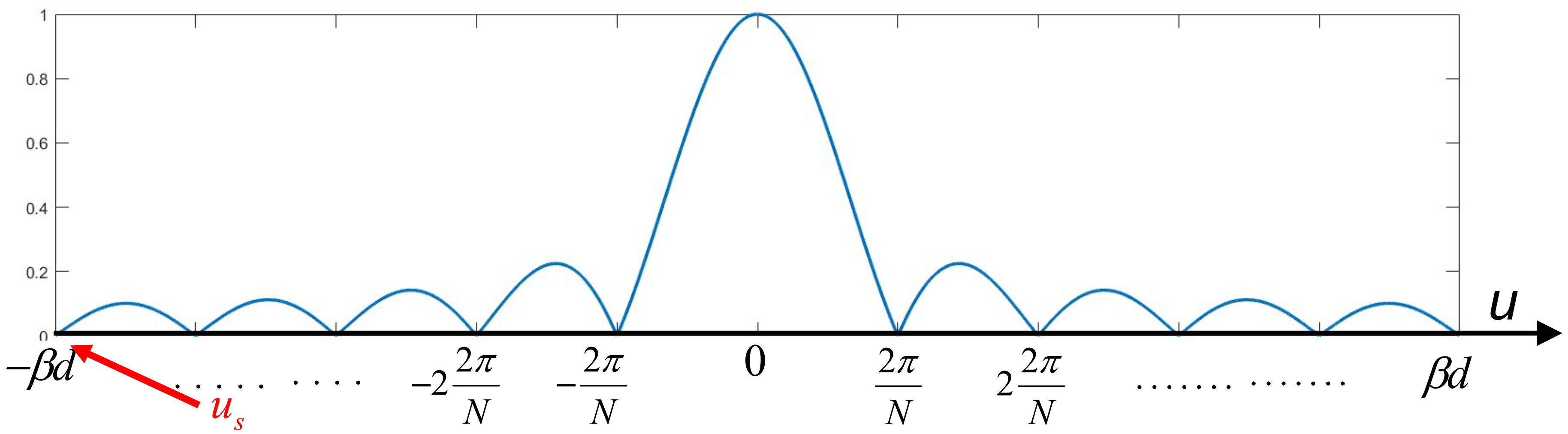

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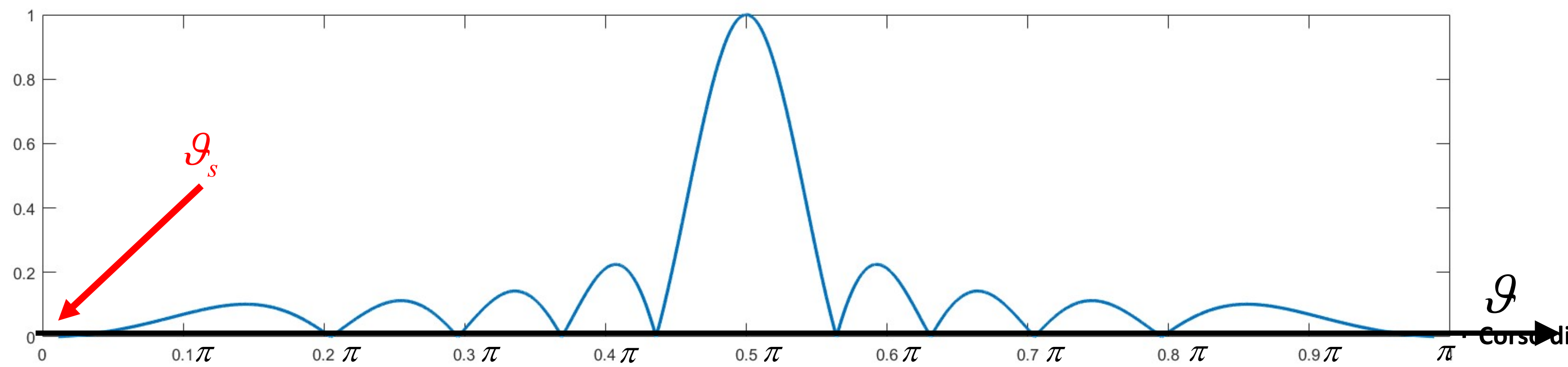

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Endfire Arrays

$$\left|F(u+\beta d)\right| = |I| \left| \frac{\sin\left(\frac{N(u+\beta d)}{2}\right)}{\sin\left(\frac{u+\beta d}{2}\right)} \right|$$

$$u = -\beta d \cos\vartheta$$

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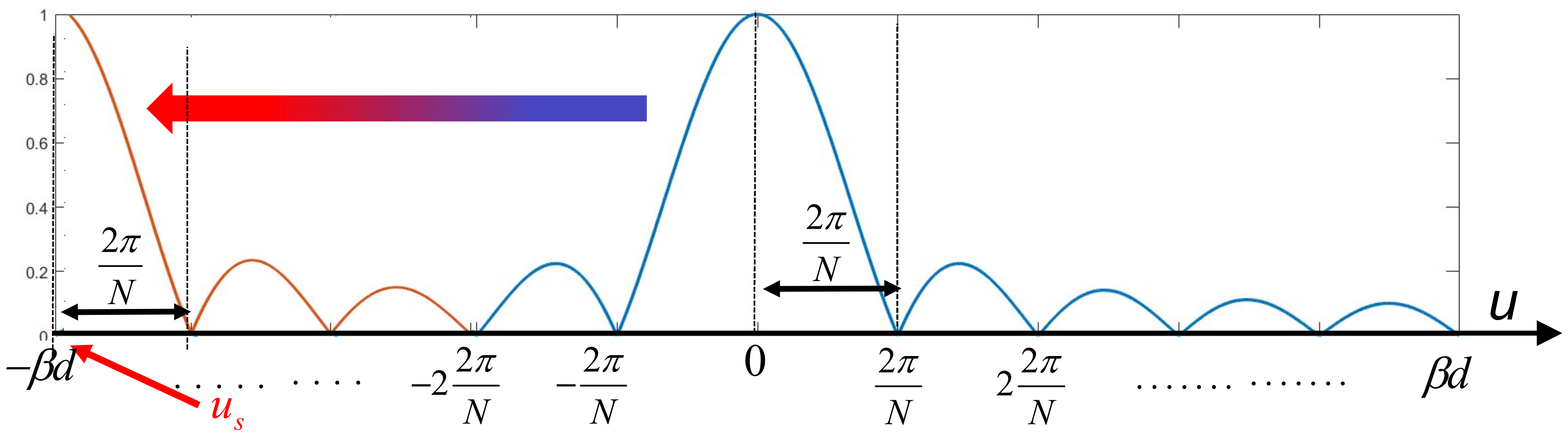

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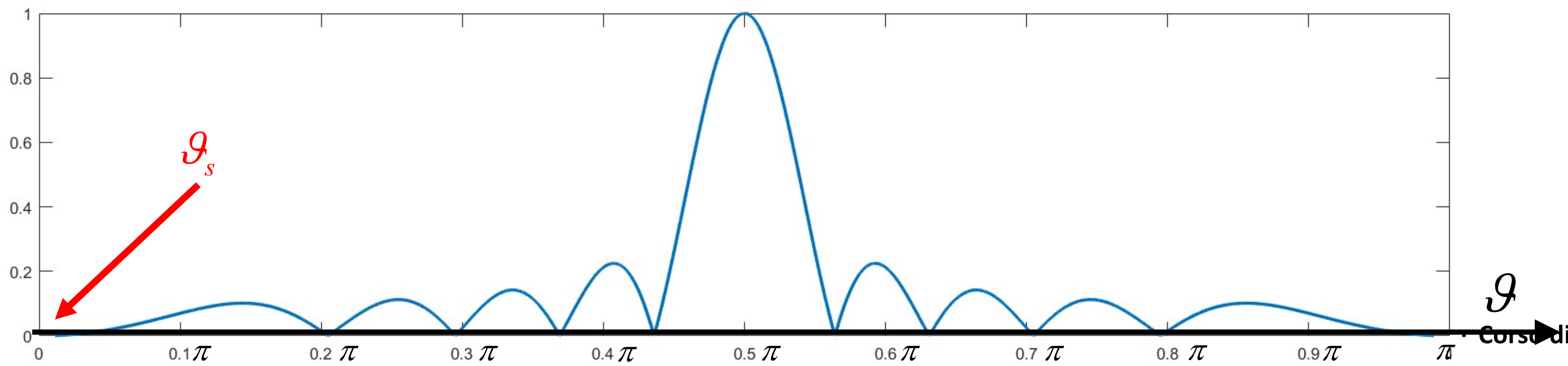

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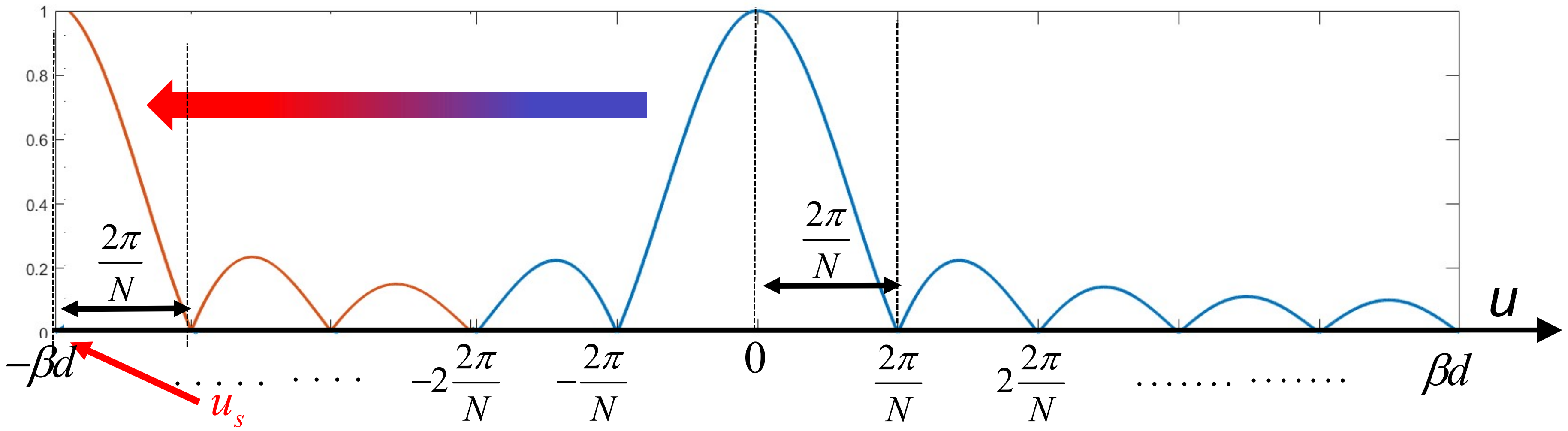

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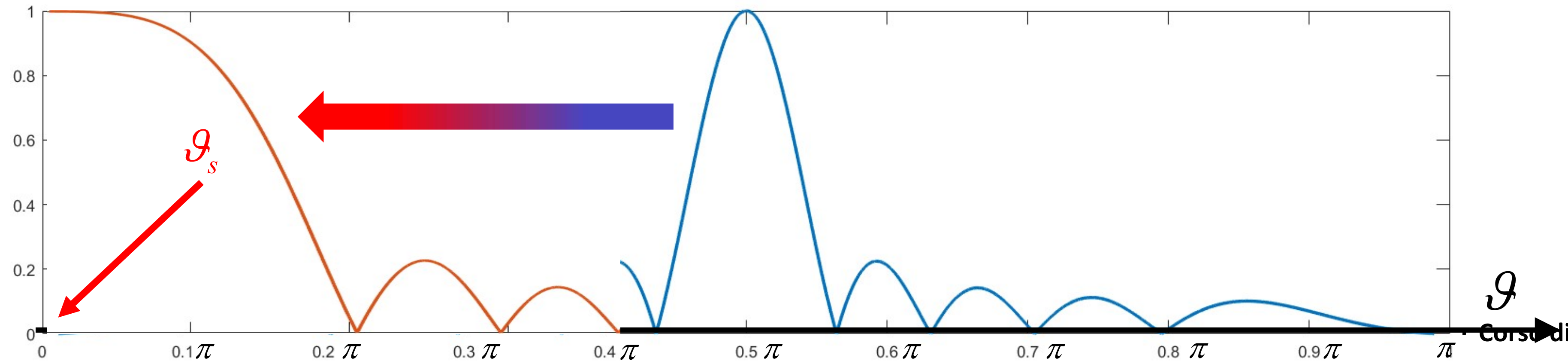

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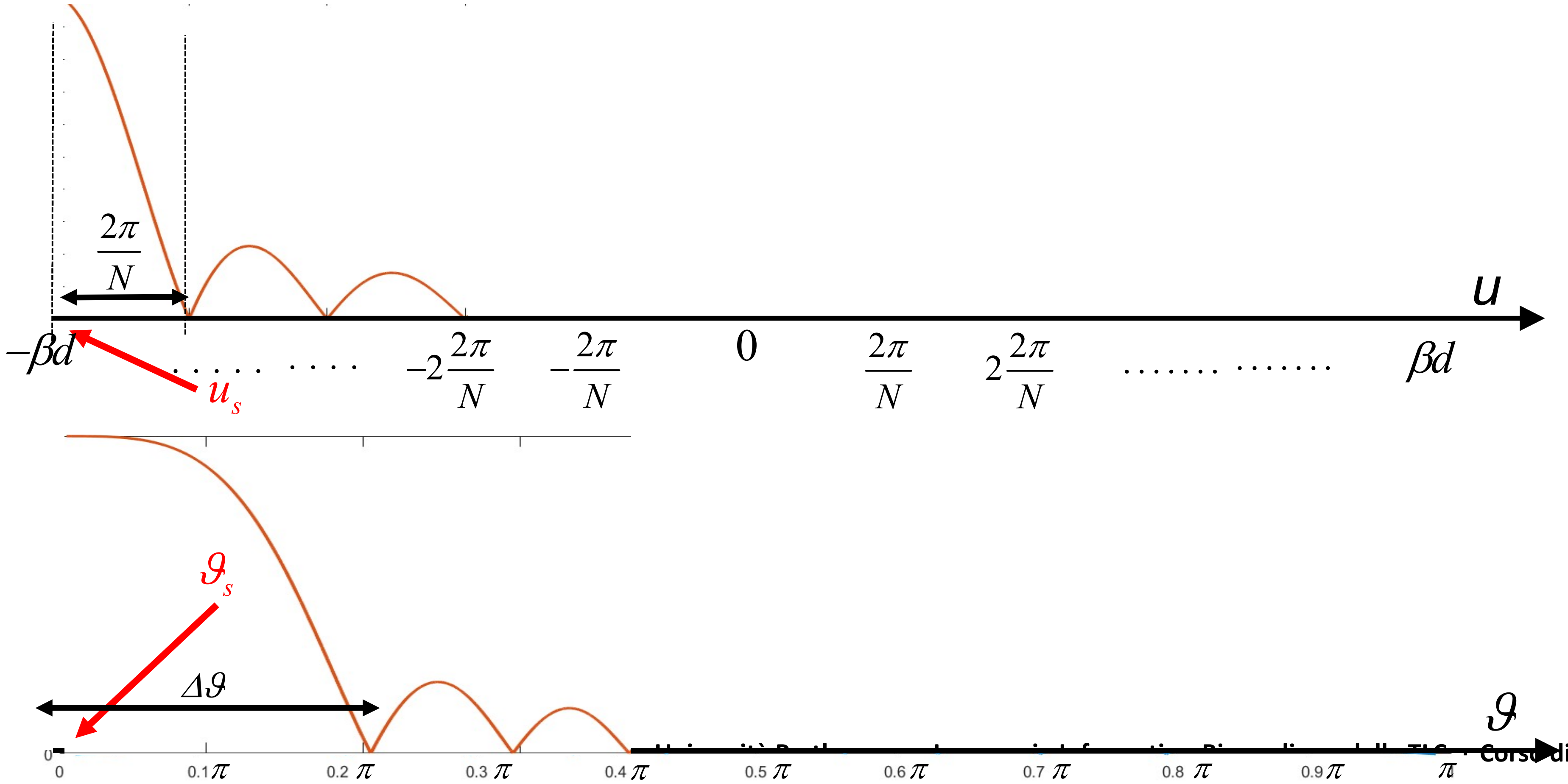

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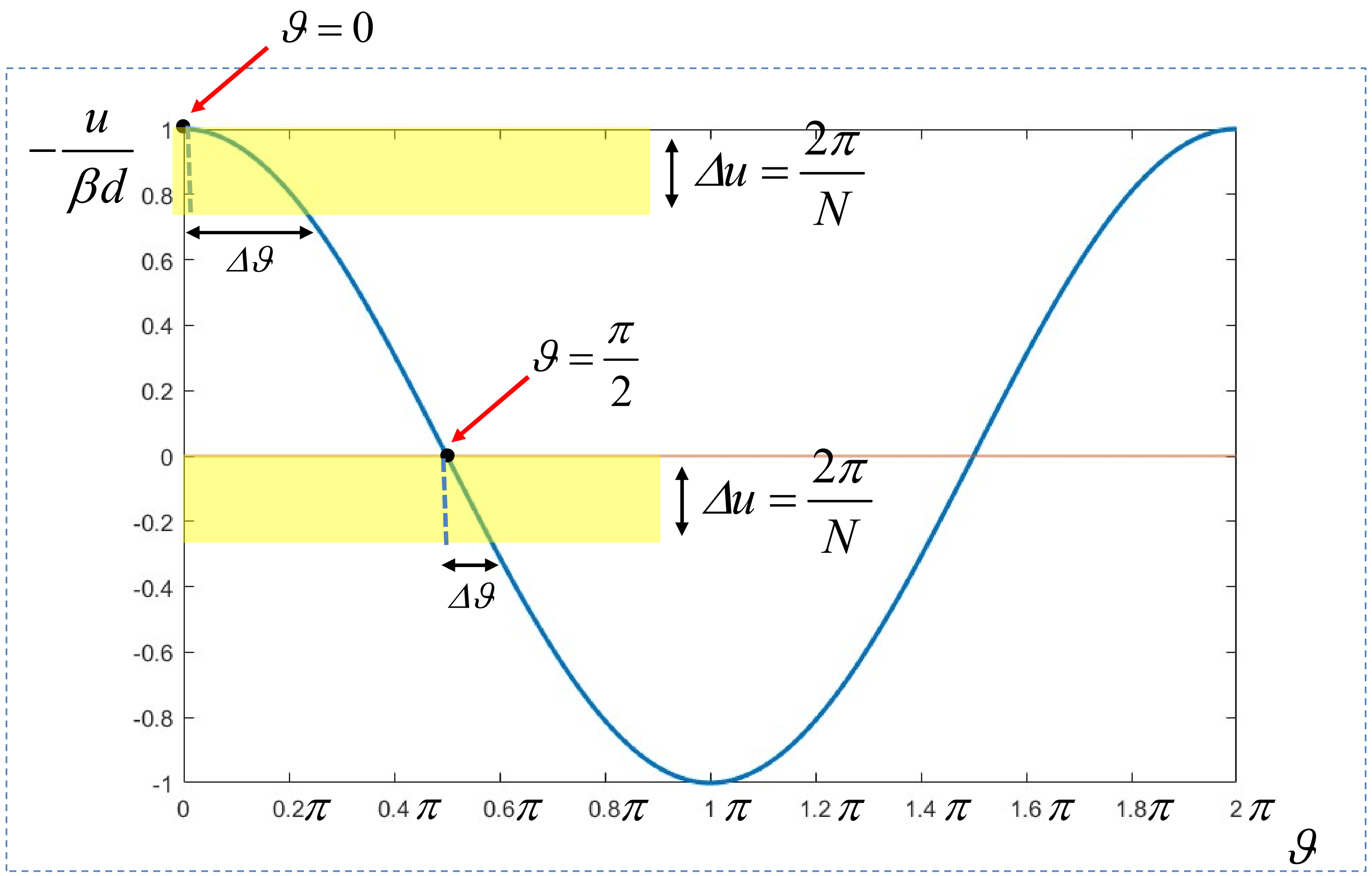

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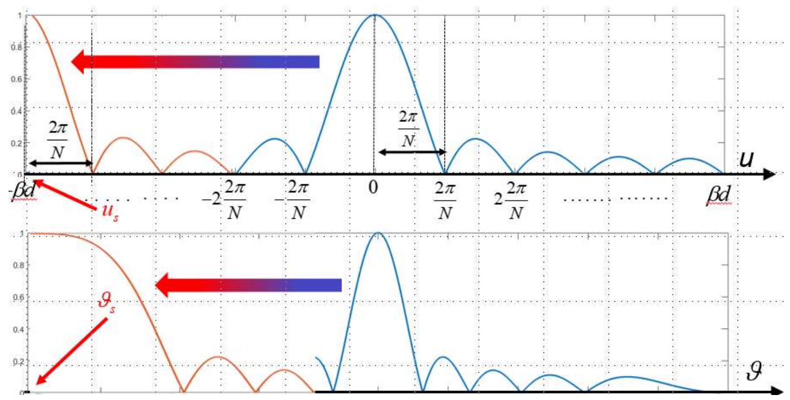


Periodic Linear Arrays (z-axis): Endfire Arrays

u-domain

$$|F(u-u_s)| = |I| \left| \frac{\sin\left[N\frac{(u+\beta d)}{2}\right]}{\sin\left[\frac{(u+\beta d)}{2}\right]} \right|$$

$$u_{FN} = -\beta d + \frac{2\pi}{N} = -\frac{2\pi}{\lambda}d + \frac{2\pi}{N}$$

ϑ -domain

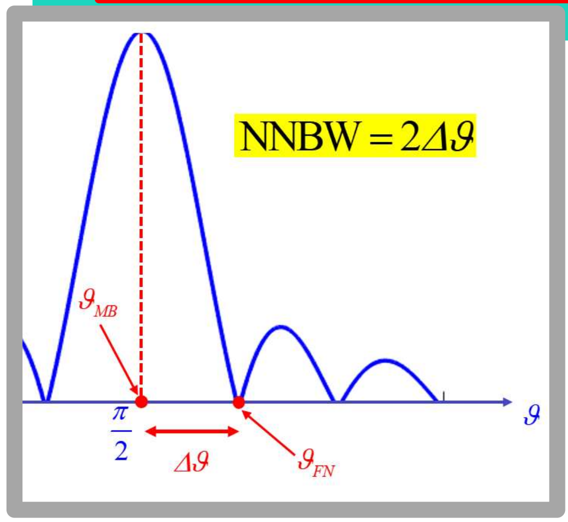
$$u = -\beta d \cos\vartheta$$

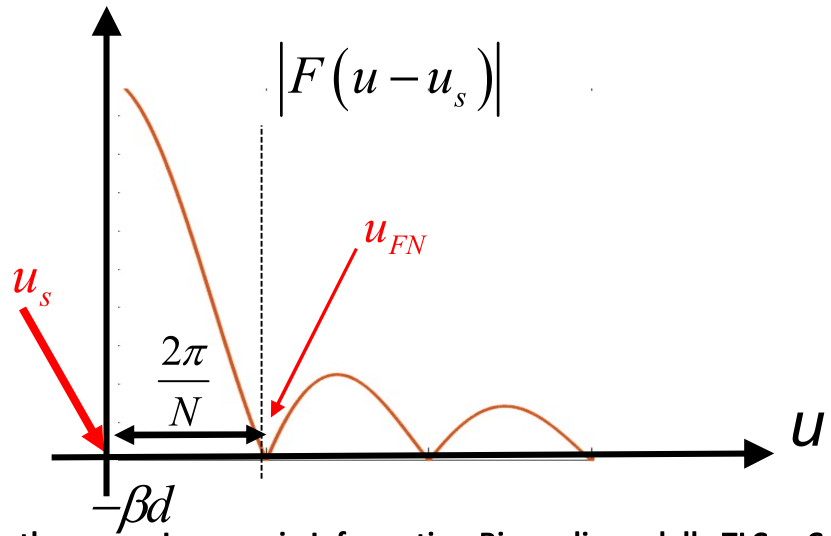
$$u_{FN} = -\beta d \cos\vartheta_{FN} = -\beta d \cos(\vartheta_{MB} + \Delta\vartheta)$$

$$-\frac{2\pi}{\lambda}d + \frac{2\pi}{N} = -\frac{2\pi}{\lambda}d\cos(\Delta\vartheta) \approx -\frac{2\pi}{\lambda}d\left(1-\frac{1}{2}\Delta\vartheta^2\right) \Rightarrow \frac{2\pi}{N} \approx \frac{2\pi}{\lambda}d\frac{\Delta\vartheta^2}{2}$$

$$\Rightarrow \Delta\vartheta^2 \approx \frac{2\lambda}{Nd} \Rightarrow \Delta\vartheta \approx \sqrt{\frac{2\lambda}{Nd}} \Rightarrow$$

$$\cos(x) = 1 - \frac{1}{2}x^2 + \cdots$$





Let us assume
$\Delta\vartheta << 2\pi$

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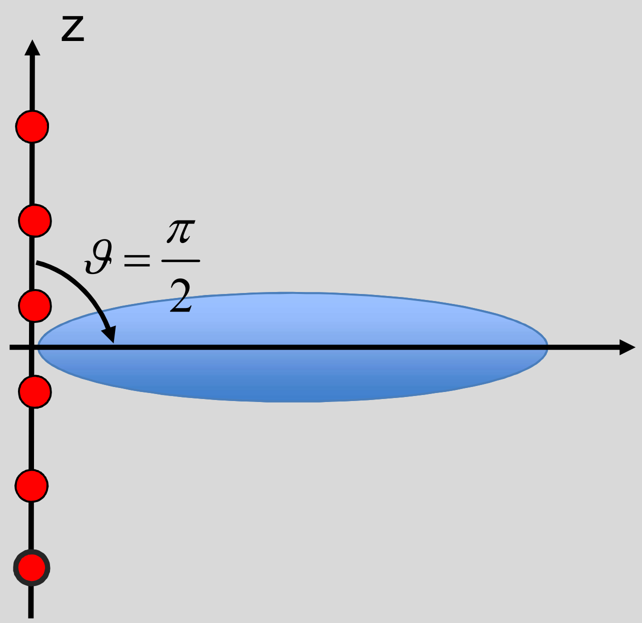
Periodic Linear Arrays (z-axis): Endfire Arrays

Broadside array



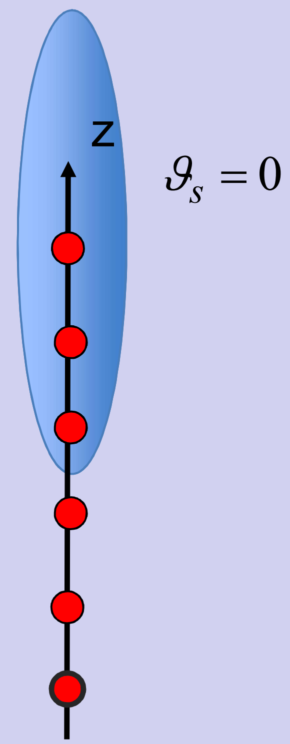
$$\text{NNBW} = \frac{2\lambda}{Nd} \quad (\text{when } Nd >> 2\lambda)$$

$$(\text{when } Nd >> 2\lambda)$$

$$\frac{\text{NNBW}_B}{\text{NNBW}_E} \approx \frac{1}{2}\sqrt{\frac{2\lambda}{Nd}} < 1$$

Jumping from the broadside to the endfire mode involves a price: the blurring of the beam

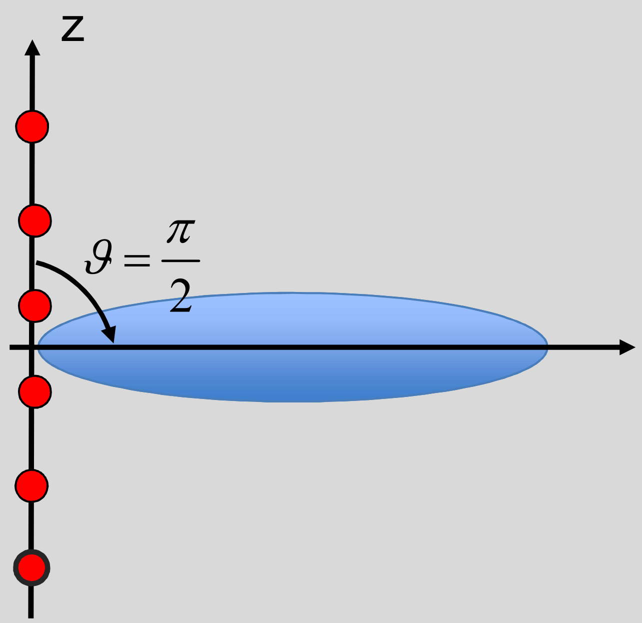
Endfire array



$$\text{NNBW} \approx 2\sqrt{\frac{2\lambda}{Nd}} \quad (\text{when } Nd >> 2\lambda)$$

Periodic Linear Arrays (z-axis): Endfire Arrays

Broadside array



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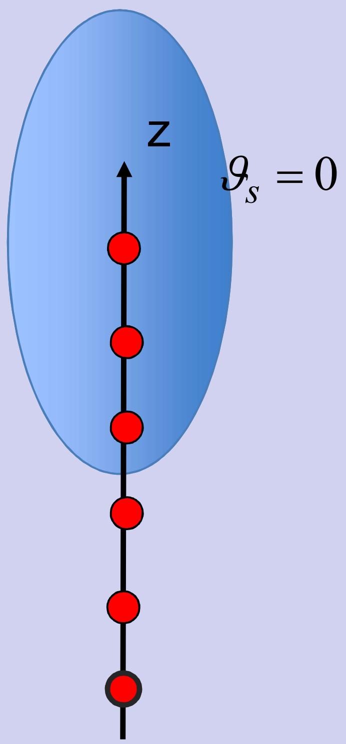

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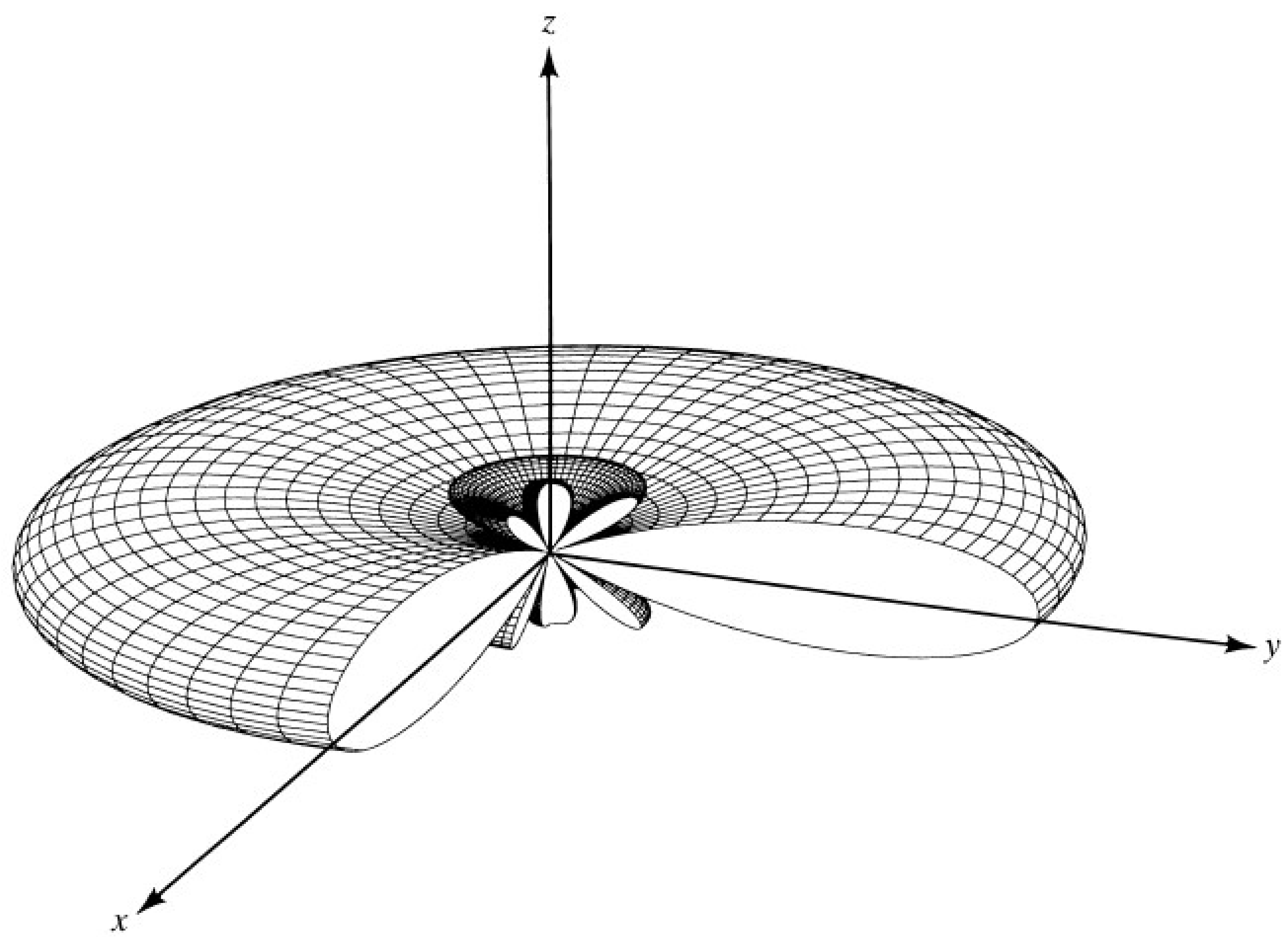
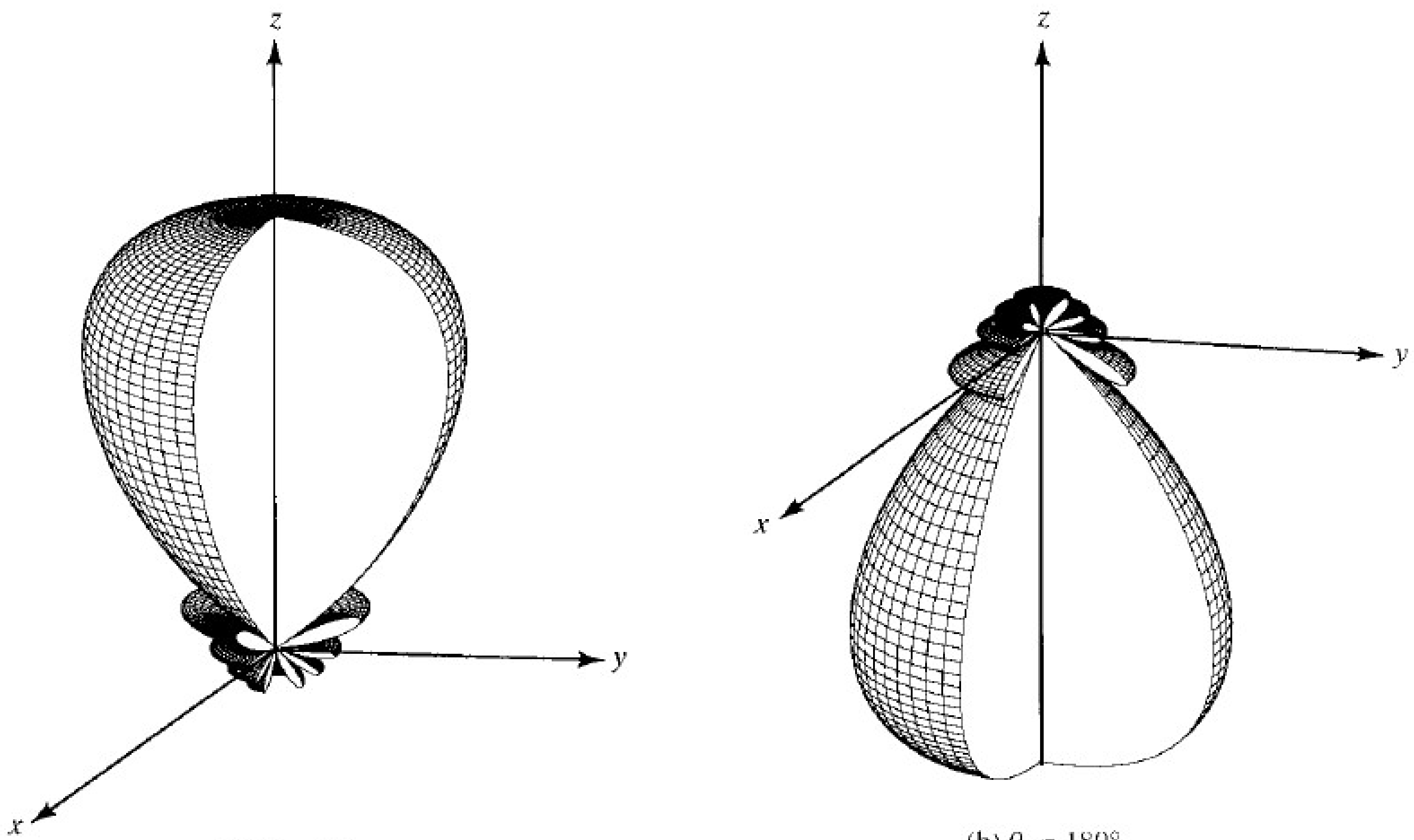
Broadside Arrays

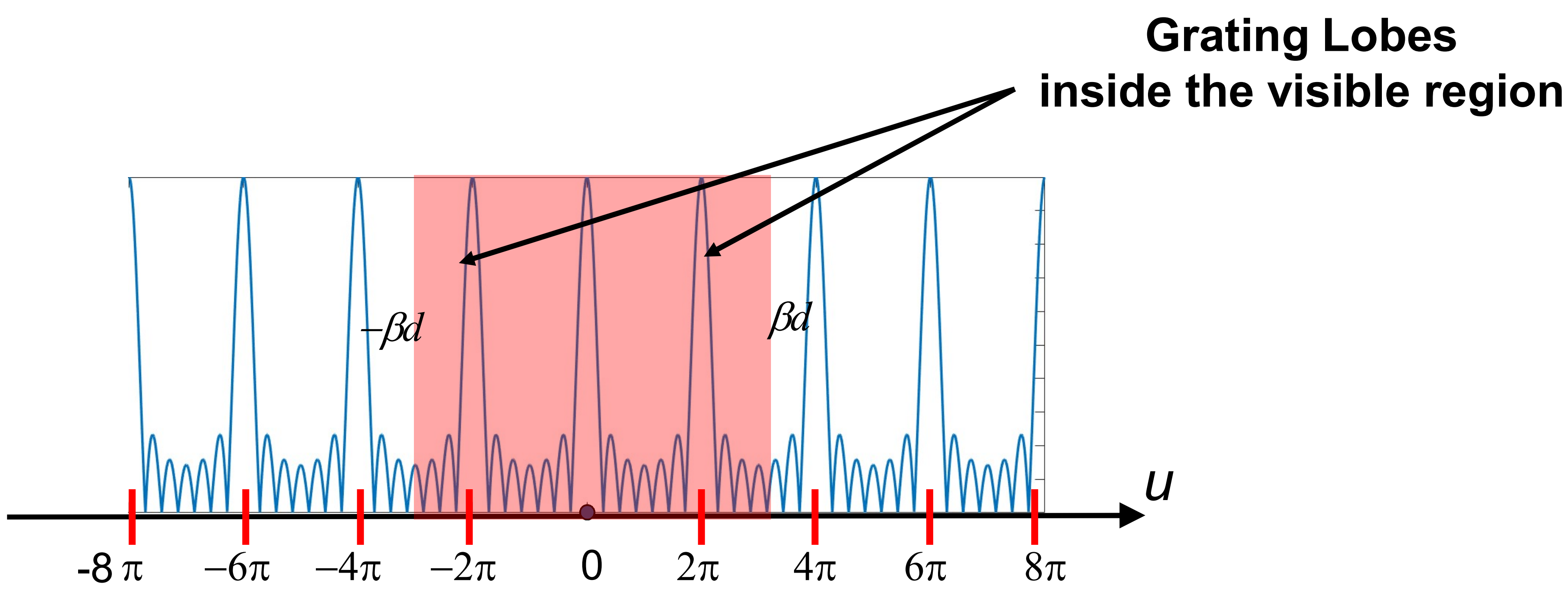


Endfire Arrays



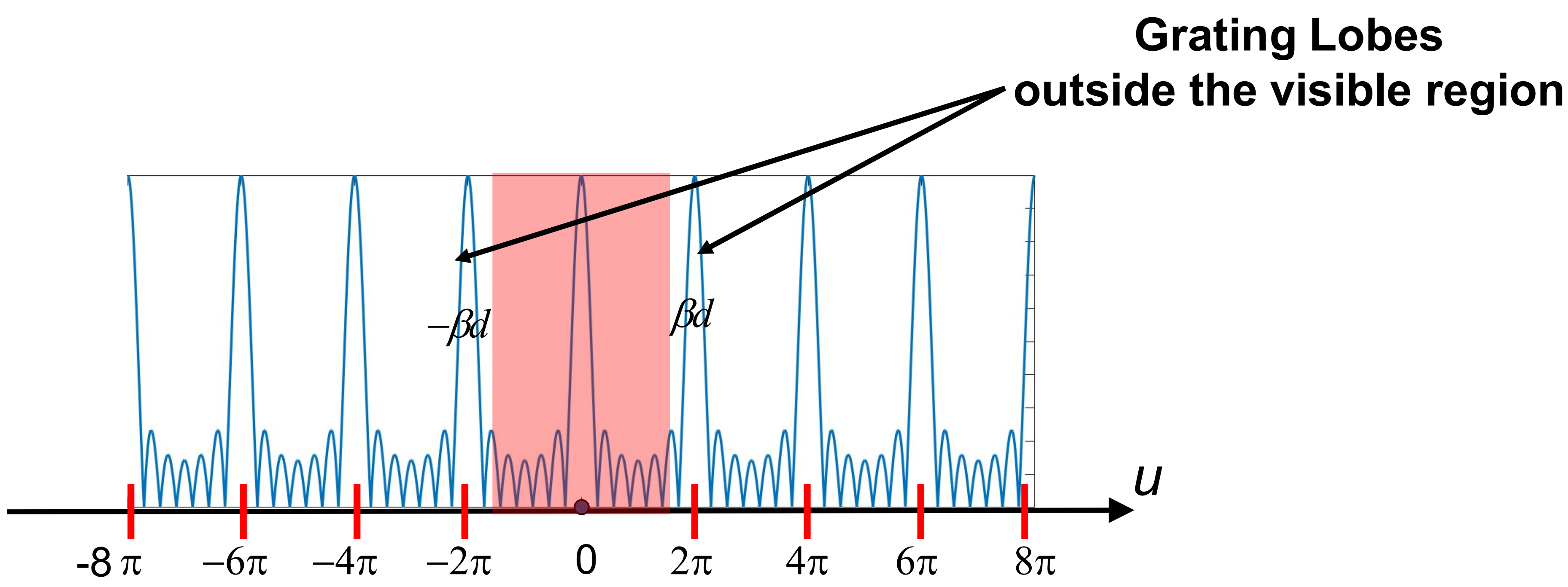
(a) $\theta_0 = 0°$ (b) $\theta_0 = 180°$

Beam scanning and grating lobes



Let's reduce the visible region by acting on the inter-element distance d

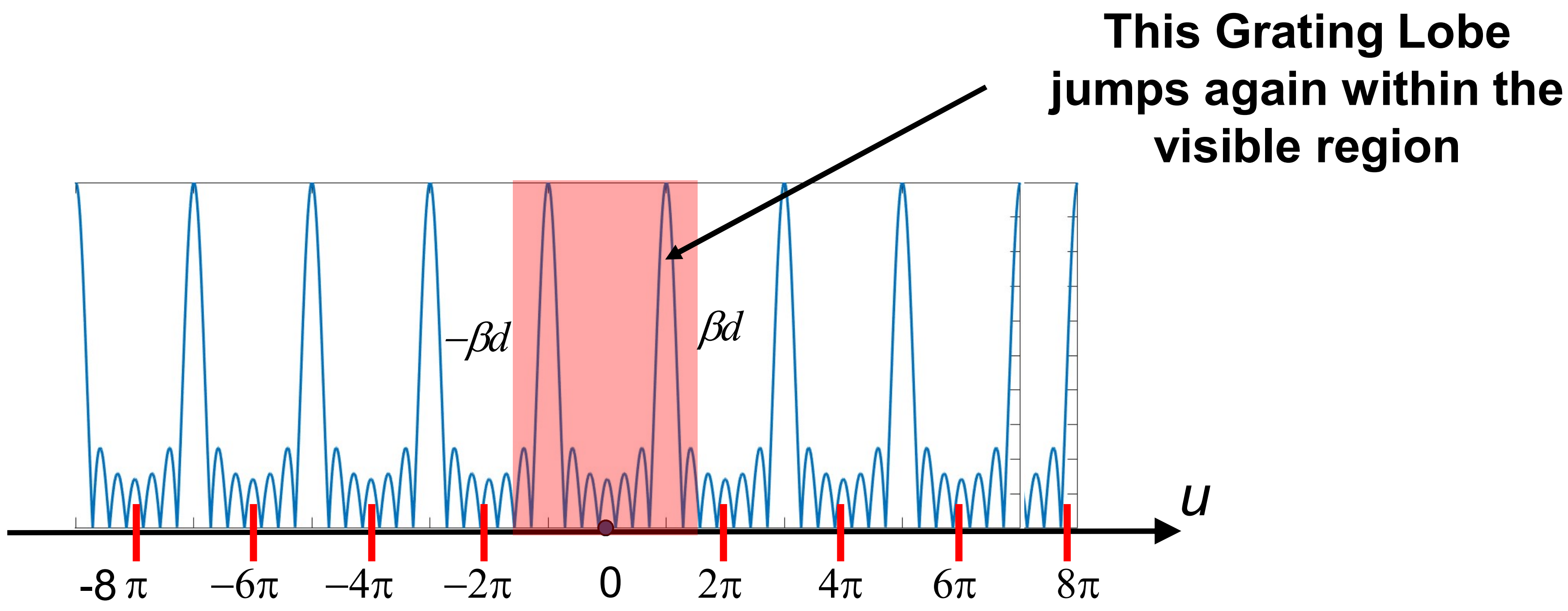
Beam scanning and grating lobes



Let's reduce the visible region by acting on the inter-element distance d

and apply the electronic scanning of the beam

Beam scanning and grating lobes



Let's reduce the visible region by acting on the inter-element distance d

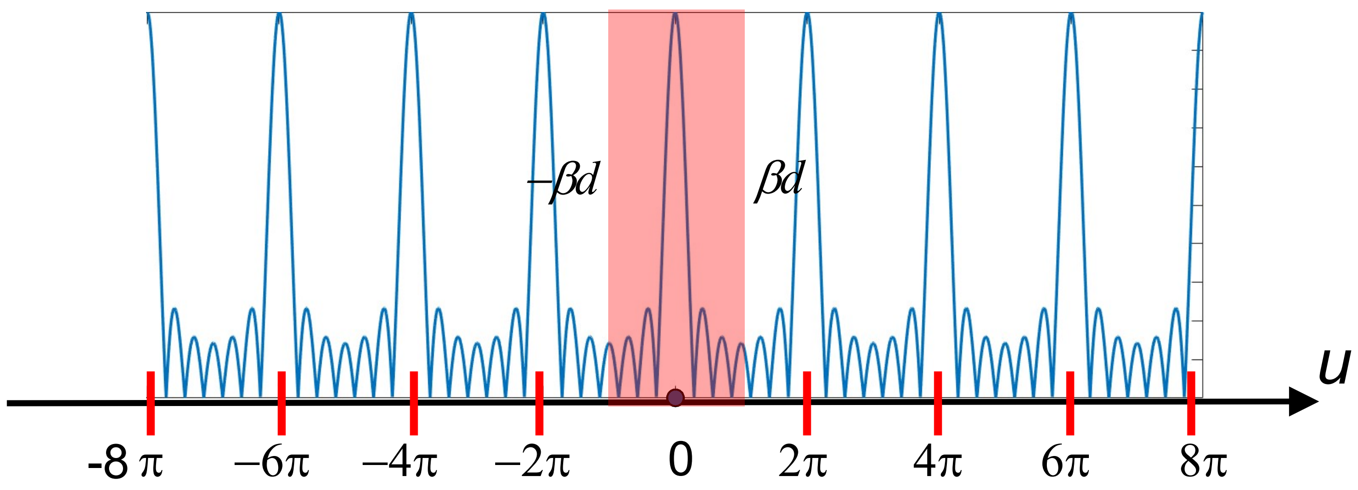
and apply the electronic scanning of the beam

Beam scanning and grating lobes

An example

Let's consider a broadside array and enforce the condition $\mathrm{d} = \frac{\lambda}{2} \quad (\beta d = \pi)$, which puts the grating lobes outside the visible region

Let's then scan the beam to obtain an endfire configuration



Beam scanning and grating lobes

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Let's consider a broadside array and enforce the condition $\mathrm{d} = \frac{\lambda}{2} \quad (\beta d = \pi)$, which puts the grating lobes outside the visible region

Let's then scan the beam to obtain an endfire configuration

Due to the rigid shift of *F*(U), we will have the main beam in two directions: ϑ=0 (corresponding to u=-βd) and ϑ=π (corresponding to u=βd)