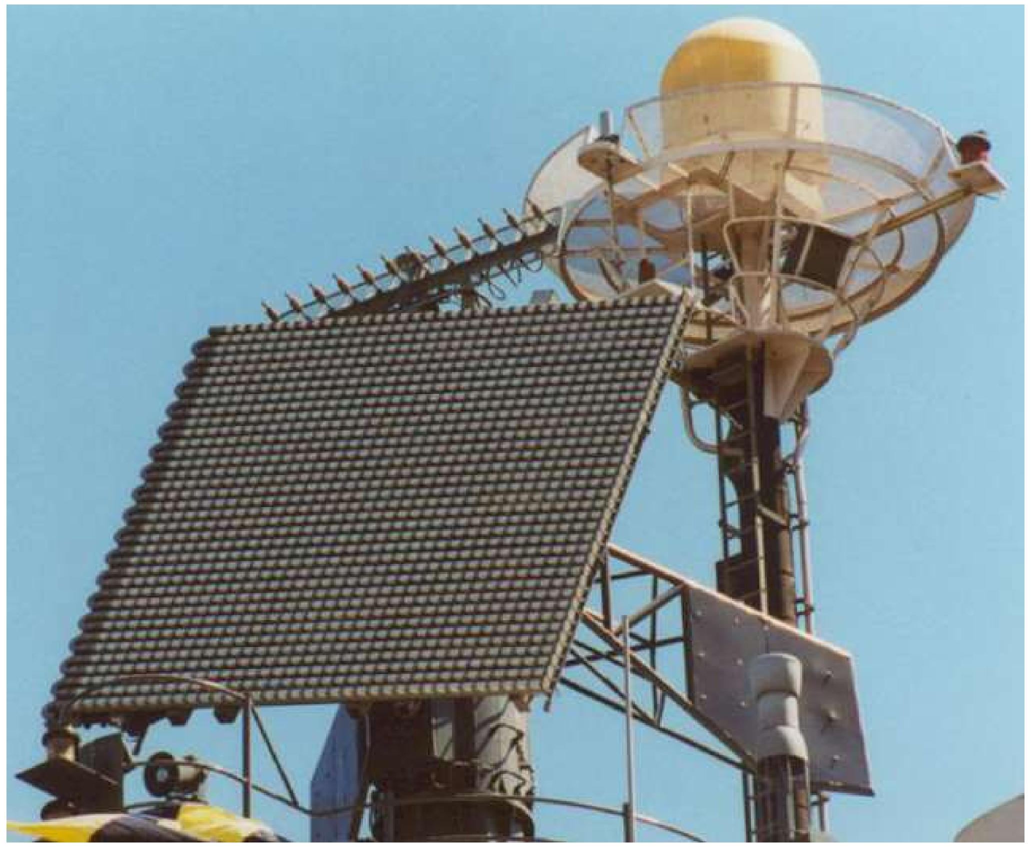
Arrays

Arrays



Color legend

New formulas, important considerations,
important formulas, important concepts

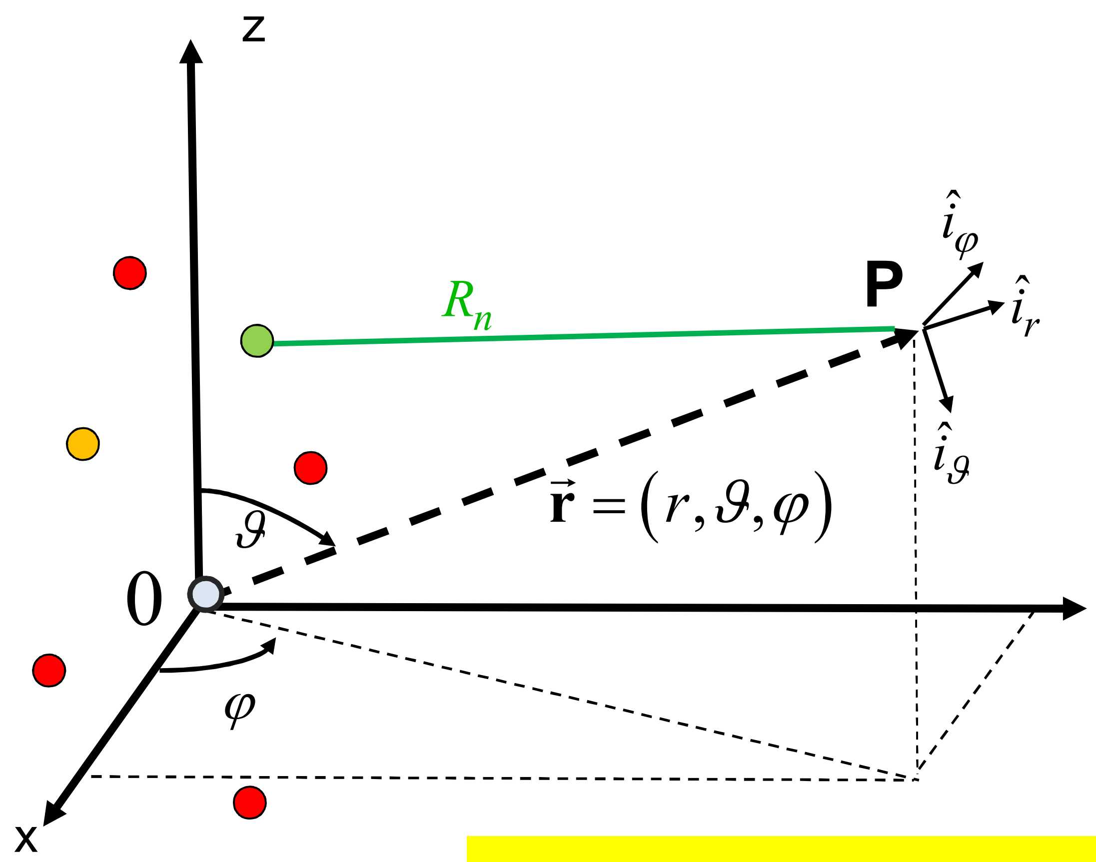
Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Arrays



Let us suppose that P is located in the Fraunhofer Region relevant to the each antenna of the considered array

$$\vec{\mathbf{E}}_0 = j\frac{\zeta I_0}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}_0(\vartheta_0,\varphi_0)$$

$$\vartheta_0 = \vartheta; \quad \varphi_0 = \varphi; \; R_0 = r; \; \hat{i}_{\vartheta_0} = \hat{i}_\vartheta; \; \hat{i}_{\varphi_0} = \hat{i}_\varphi$$

$$\vec{\mathbf{E}}_1 = j\frac{\zeta I_1}{2\lambda}\frac{\exp(-j\beta R_1)}{R_1}\vec{\mathbf{l}}_1(\vartheta_1,\varphi_1)$$

.
.
.
.

$$\vec{\mathbf{E}}_n = j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n,\varphi_n)$$

N **is the number of the array elements**

$$\vec{\mathbf{E}} = \sum_{n=0}^{N-1}\vec{\mathbf{E}}_n = \sum_{n=0}^{N-1} j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n,\varphi_n)$$

Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\,F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta\vec{\mathbf{r}}'_n\cdot\hat{i}_r\right)$$

Principle of pattern multiplication

Array Factor — $F(\vartheta,\varphi)$

Element Factor — $\vec{\mathbf{l}}(\vartheta,\varphi)$

The Array Factor $F(\vartheta,\varphi)$ is a function of the angular coordinates (ϑ,φ) and depends upon:

- **the array geometry (through N and $\vec{\mathbf{r}}'_n$)**
- **the input excitations of the antennas of the array itself (through I_n)**

Very interesting implications relevant to the synthesis of the pattern

Periodic Linear Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

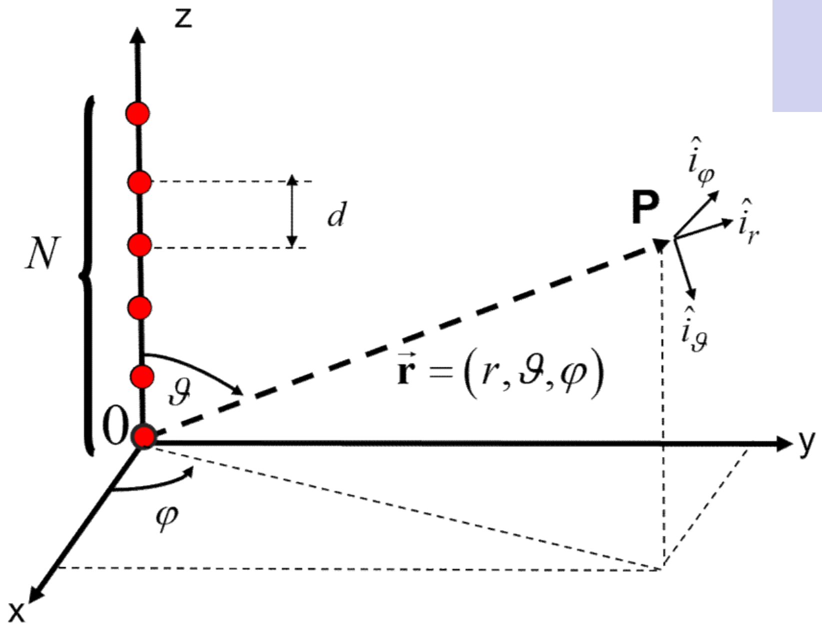
P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

The antennas of the considered array are deployed along one axis (Linear Arrays); f.i., the z-axis

The antennas of the considered array are equispaced (Periodic Arrays)

Periodic Linear Arrays (z-axis)



$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\cos\vartheta)$$

$$u = -\beta d\cos\vartheta$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

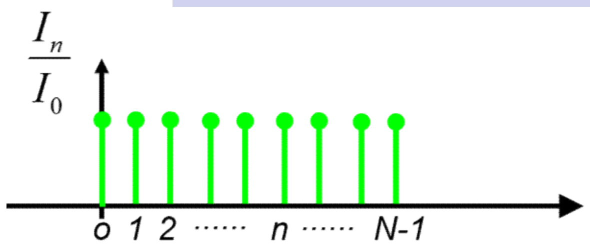
Periodic Linear Arrays (z-axis)

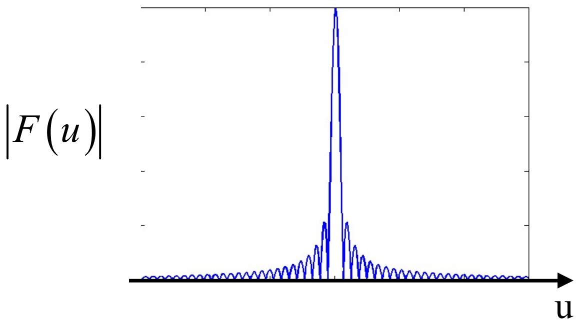
$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$





The properties of the Fourier Transformation suggest some interesting considerations

- **Arrays's size and beamwidth**
- **Scanning of the pattern**
- **Synthesis of the pattern**

Periodic Linear Arrays (z-axis)

Visible region

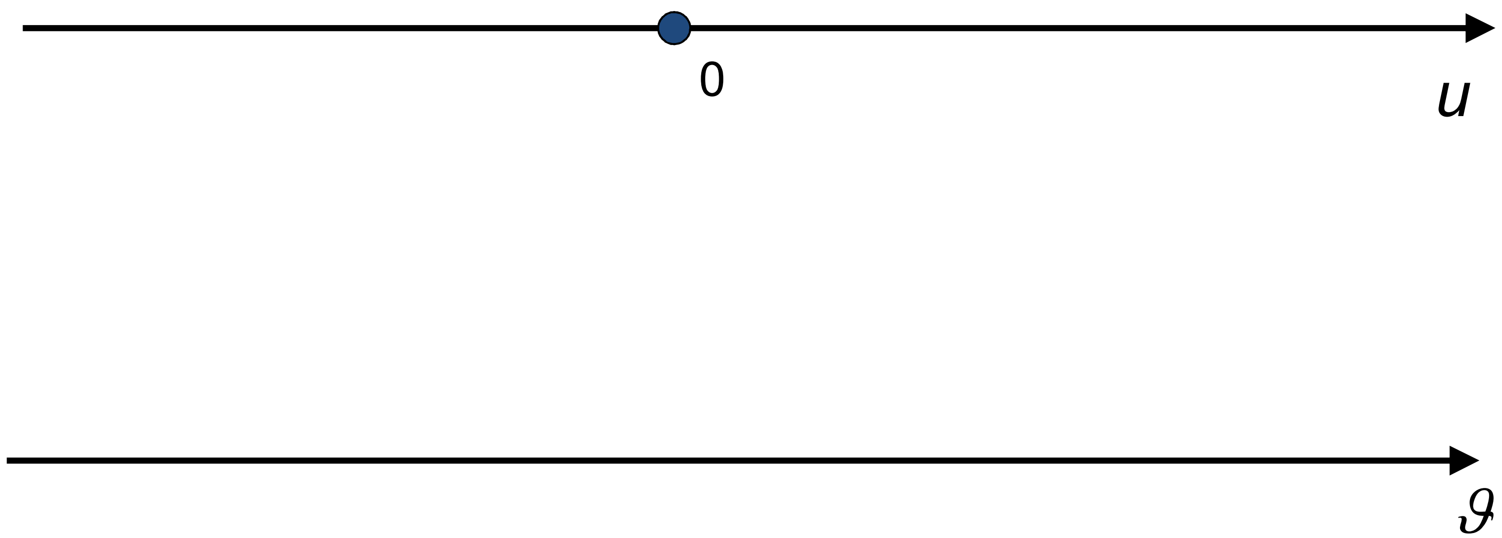
Grating lobes

Periodic Linear Arrays (z-axis): visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$



Periodic Linear Arrays (z-axis): visible region

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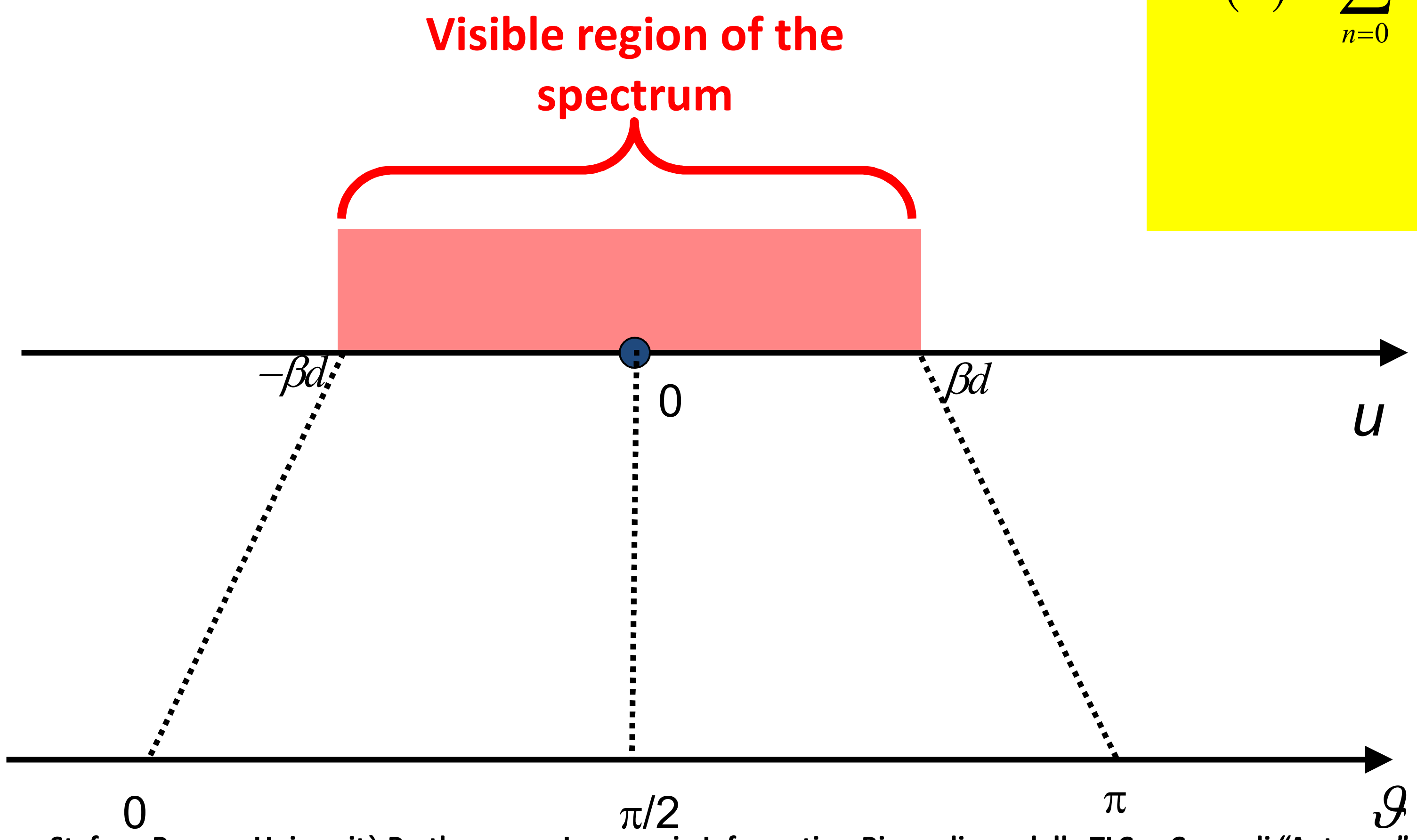

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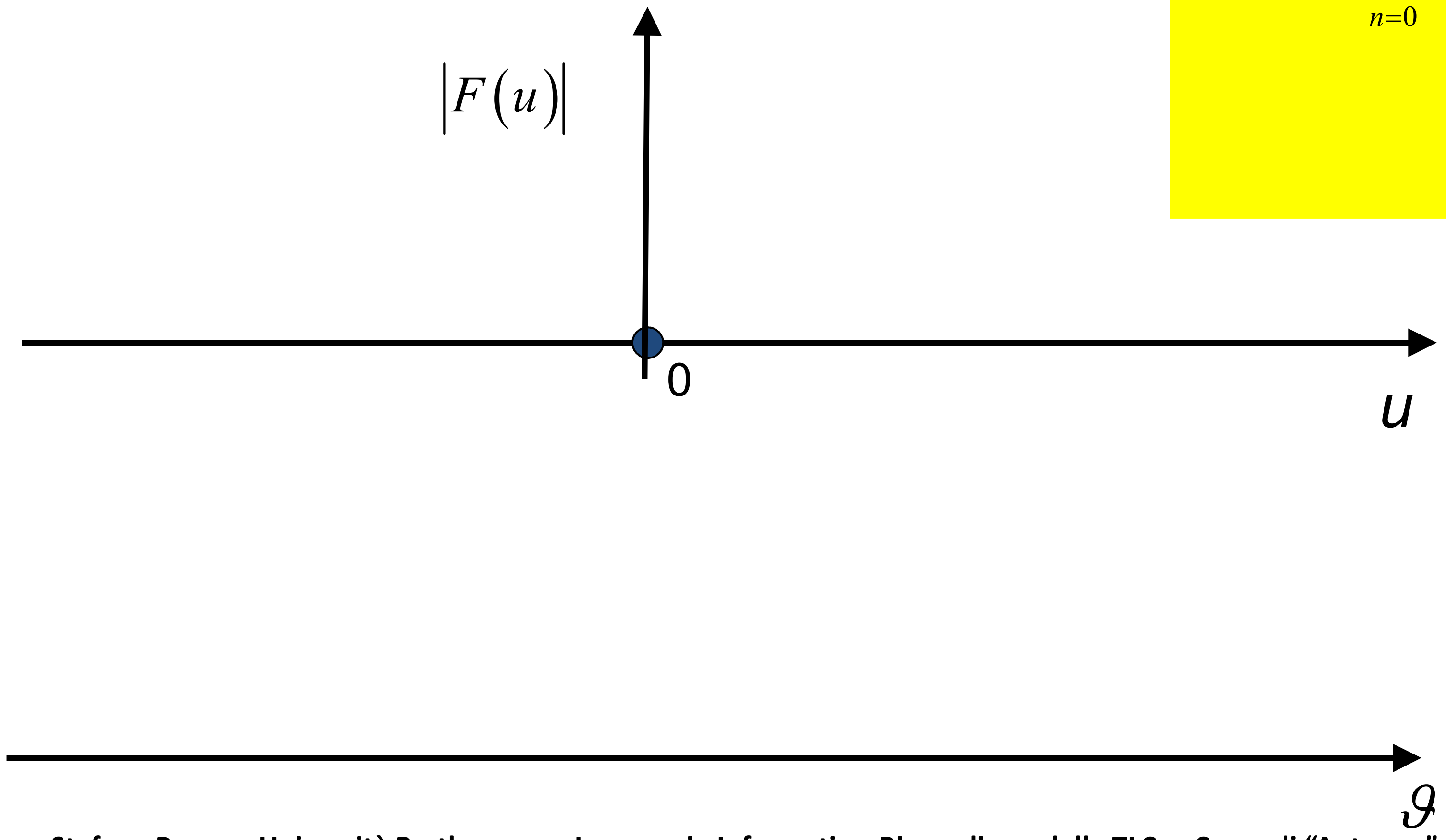

Periodic Linear Arrays (z-axis): visible region

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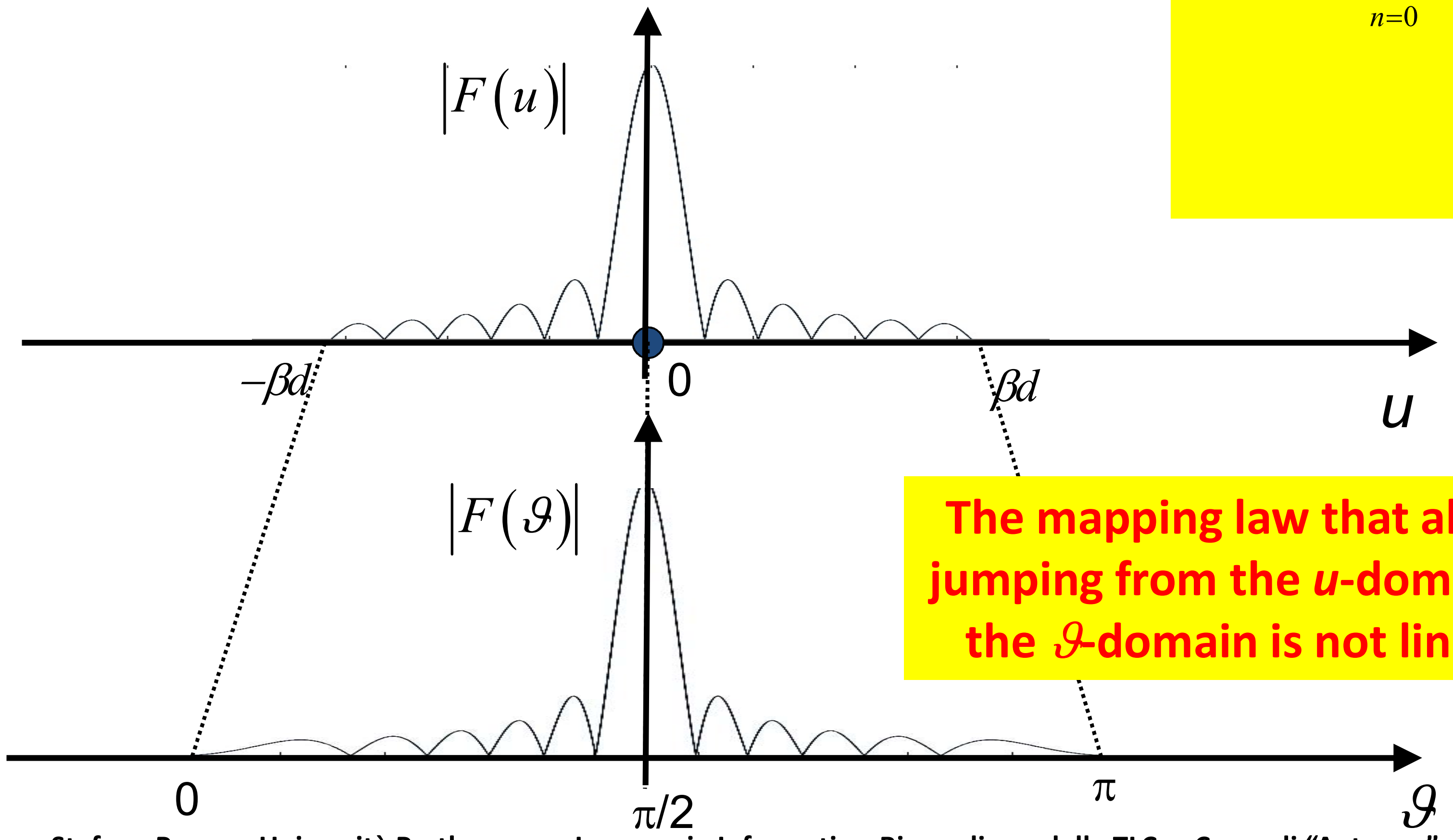

The mapping law that allows jumping from the *u*-domain to the ϑ-domain is not linear!

Periodic Linear Arrays (z-axis): grating lobes

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

F(*u*) is the Fourier Transform of a discrete sequence: it is thus **periodic** with **period equal to 2π**

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$F(u+2k\pi) = \sum_{n=0}^{N-1} I_n \exp\left[-jn(u+2k\pi)\right] = \sum_{n=0}^{N-1} I_n \exp[-jnu]\exp[-jnk2\pi] = \sum_{n=0}^{N-1} I_n \exp[-jnu] = F(u)$$

$\exp[-jnk2\pi] \rightarrow 1$

$k = \pm 1, 2, 3, ...$

$$\Downarrow$$

$$F(u+2k\pi) = F(u)$$

Periodic Linear Arrays (z-axis): grating lobes

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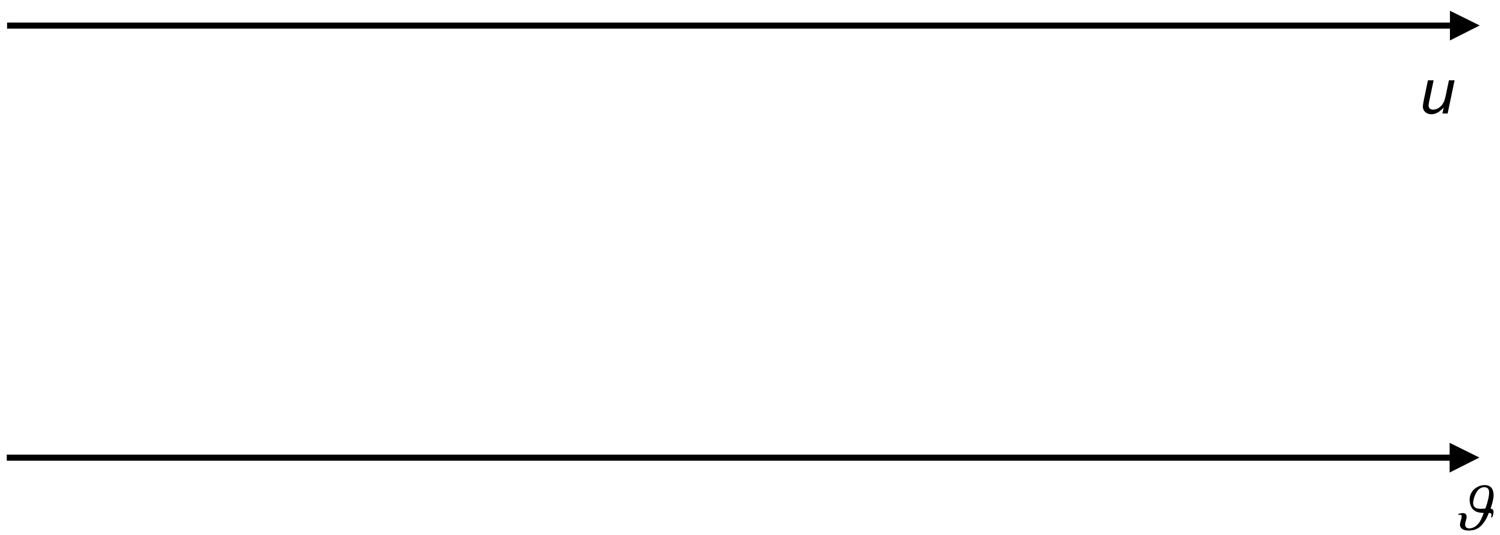

Periodic Linear Arrays (z-axis): grating lobes

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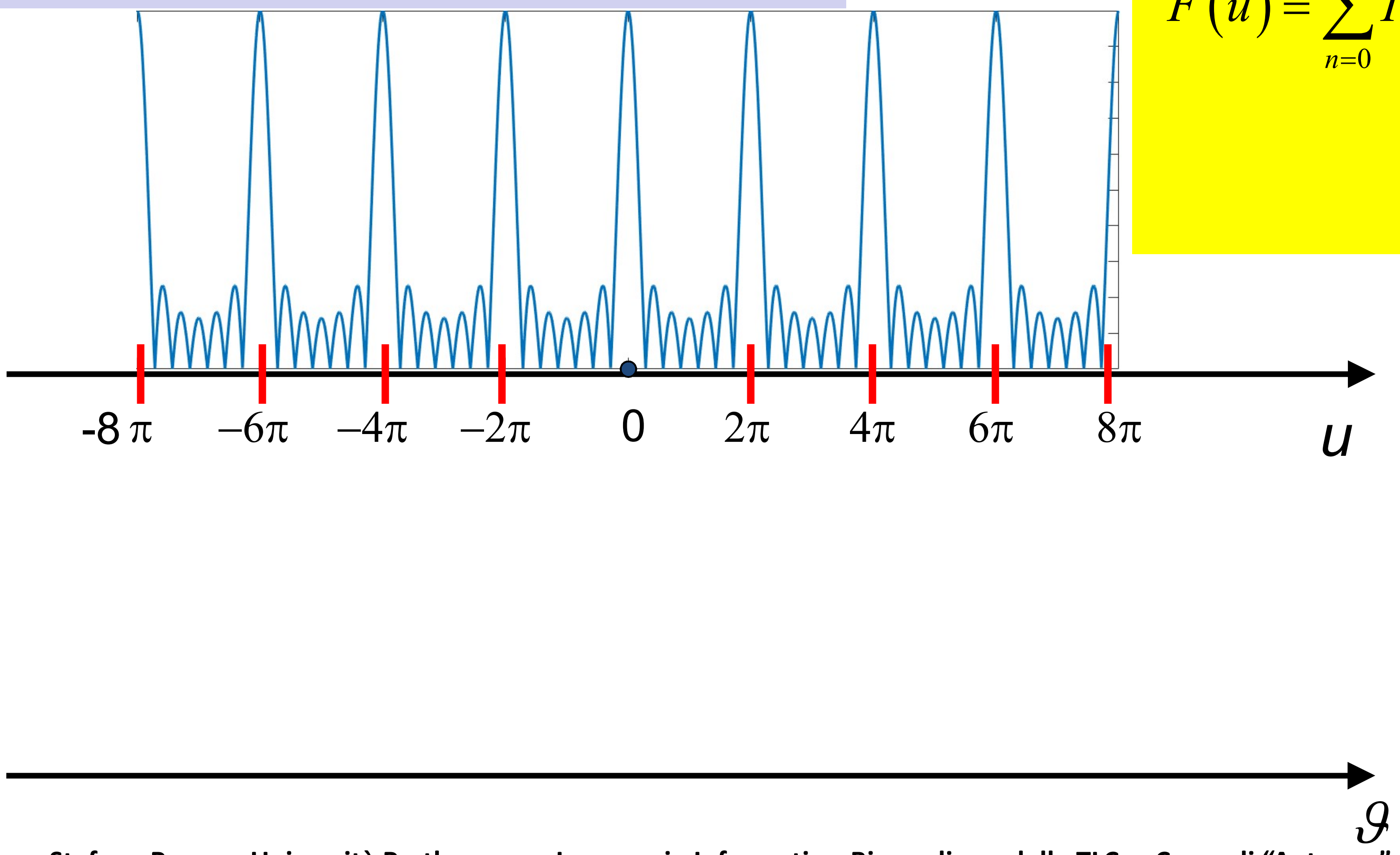

Periodic Linear Arrays (z-axis): grating lobes

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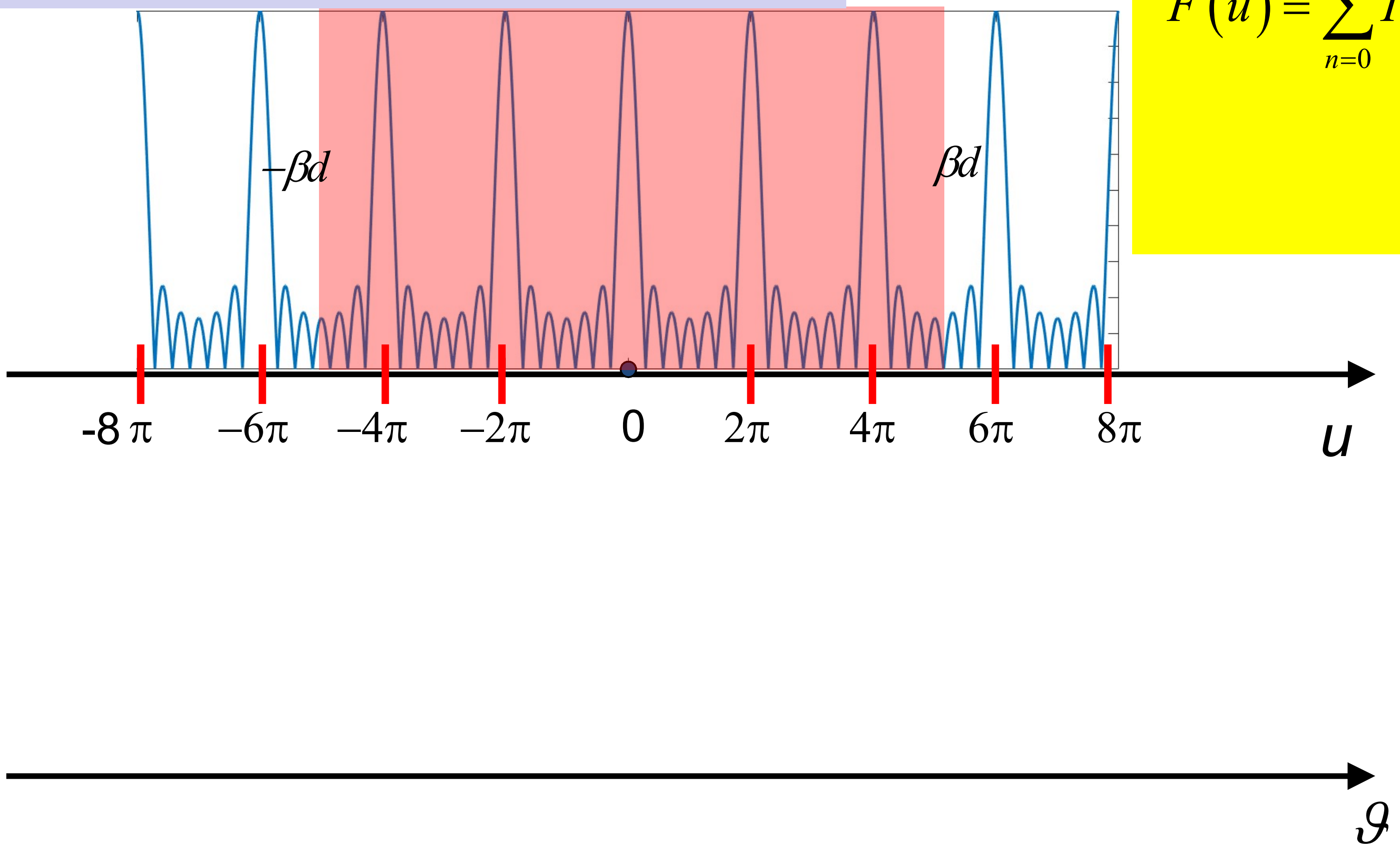

Periodic Linear Arrays (z-axis): grating lobes

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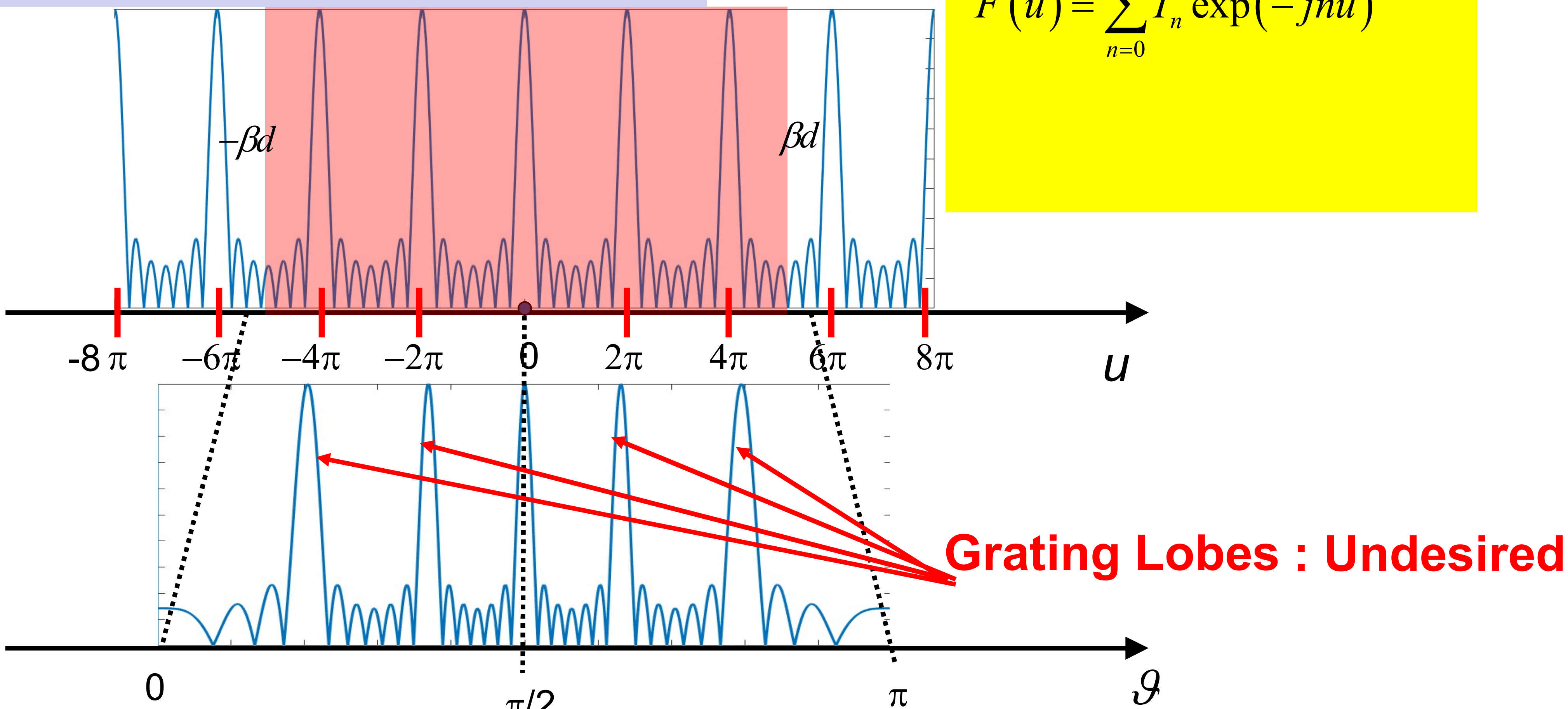
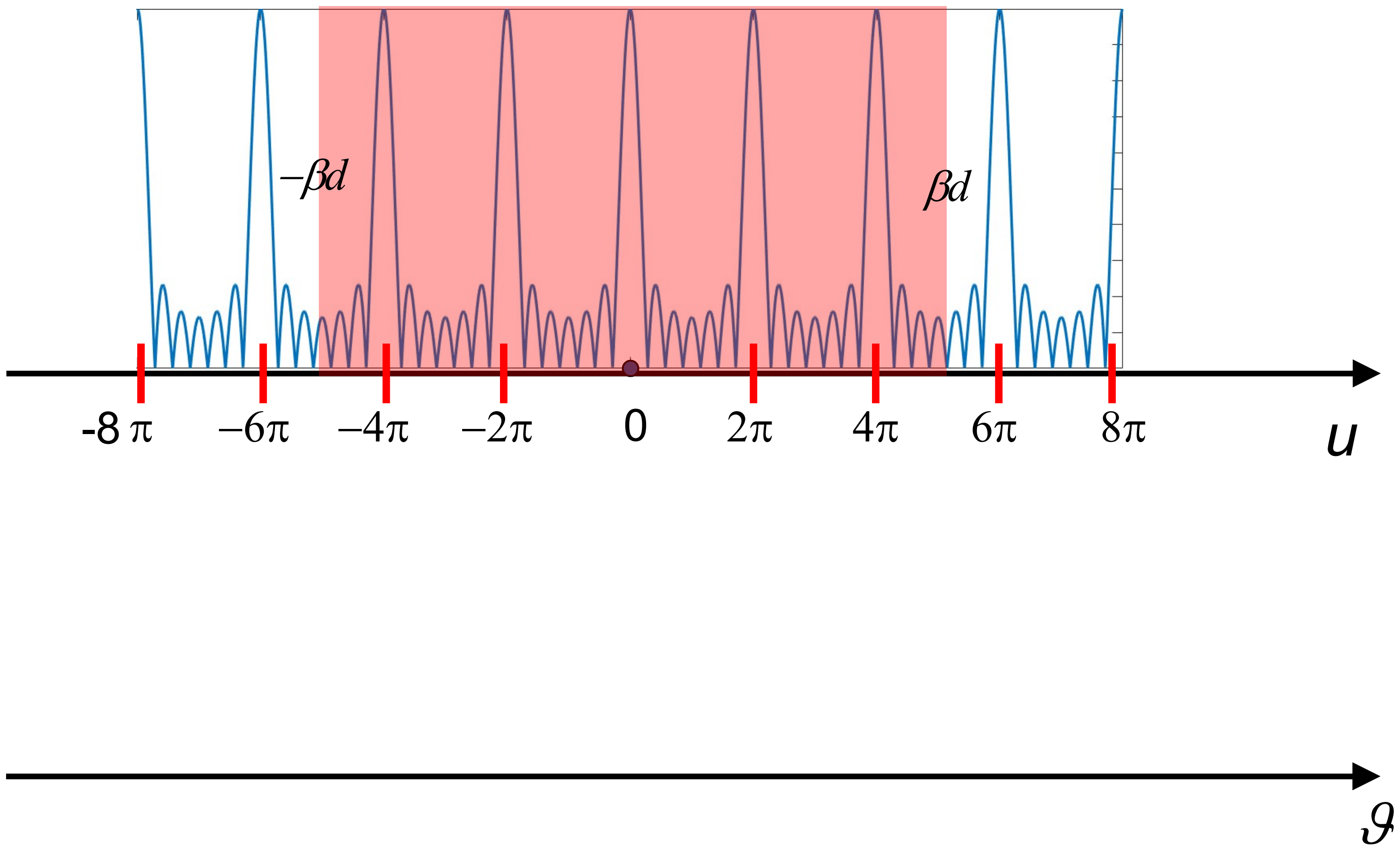


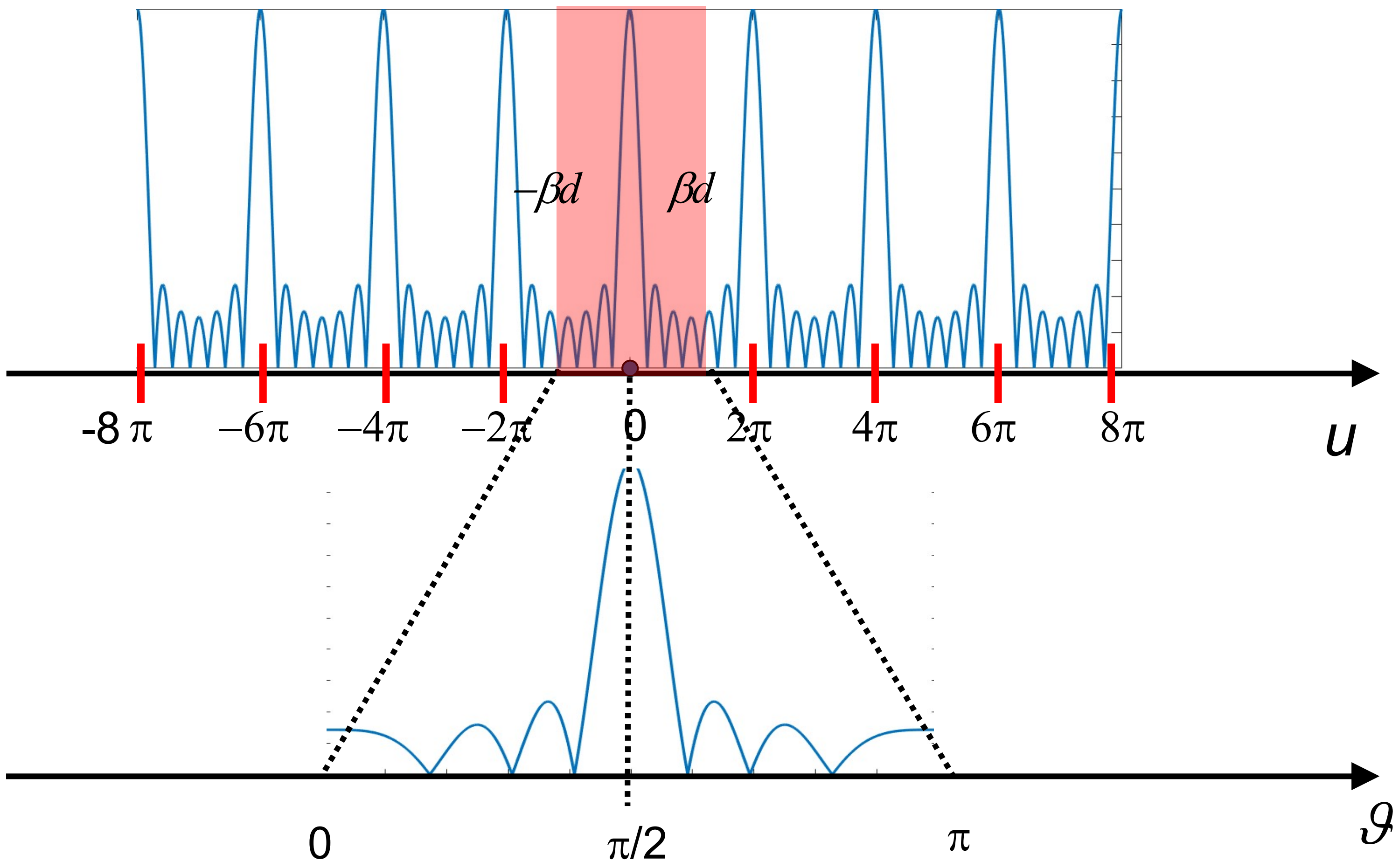
Periodic Linear Arrays (z-axis): grating lobes



Can we circumvent the presence of the grating lobes?

Let’s reduce the width of the visible region!

Periodic Linear Arrays (z-axis): grating lobes



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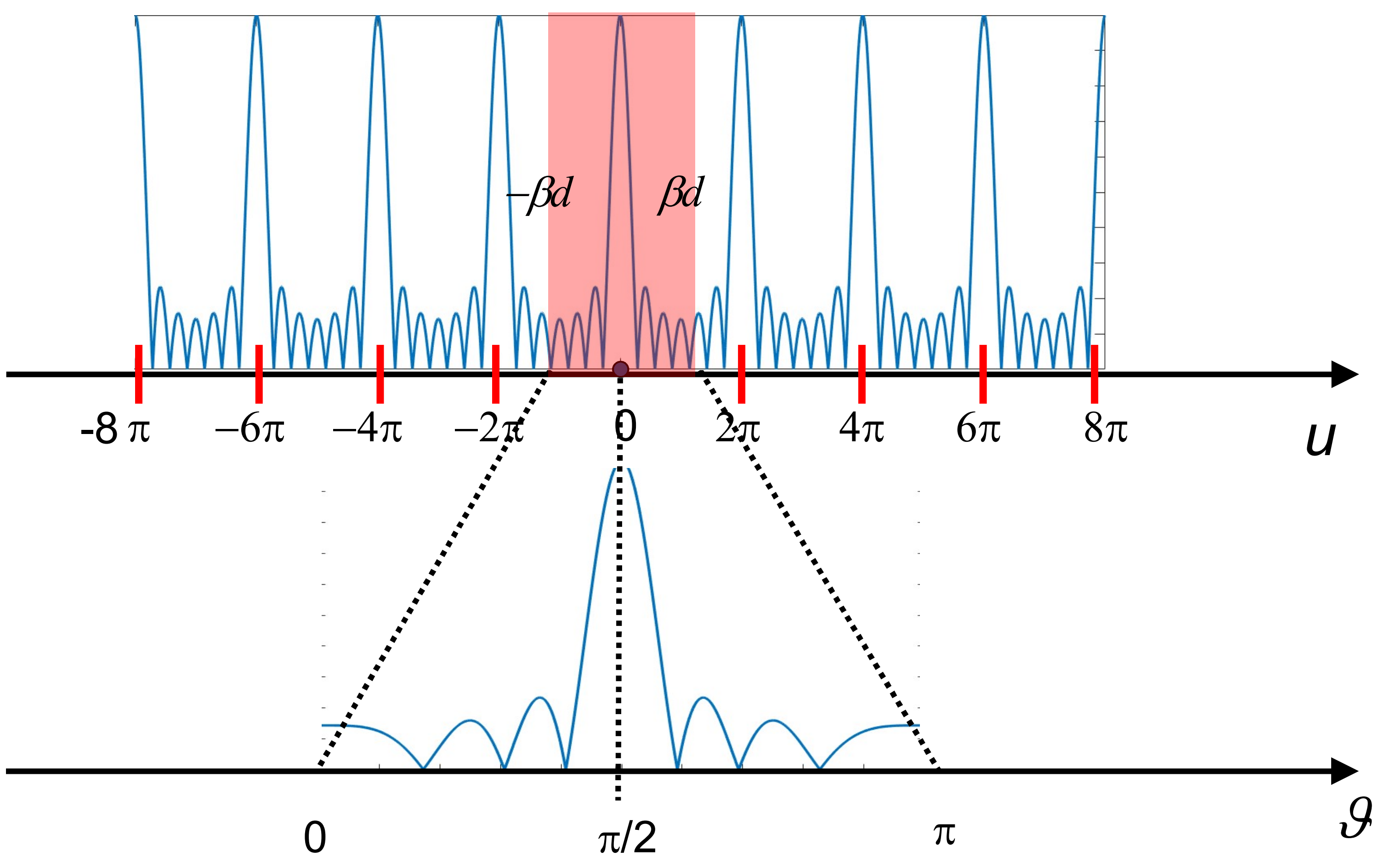
Periodic Linear Arrays (z-axis): grating lobes



The condition

$$\beta d \le \pi$$

guarantees (with a safety margin) absence of grating lobes.

$$\beta d \le \pi \quad \Rightarrow \quad \frac{2\pi}{\lambda} d \le \pi \quad \Rightarrow \quad d \le \frac{\lambda}{2}$$

To avoid the presence of grating lobes the inter-element distance must be thus subject to an upper limit, on the order of half wavelength

Periodic Linear Arrays (z-axis)

Uniform input excitations (Broadside Array)

Beam scanning

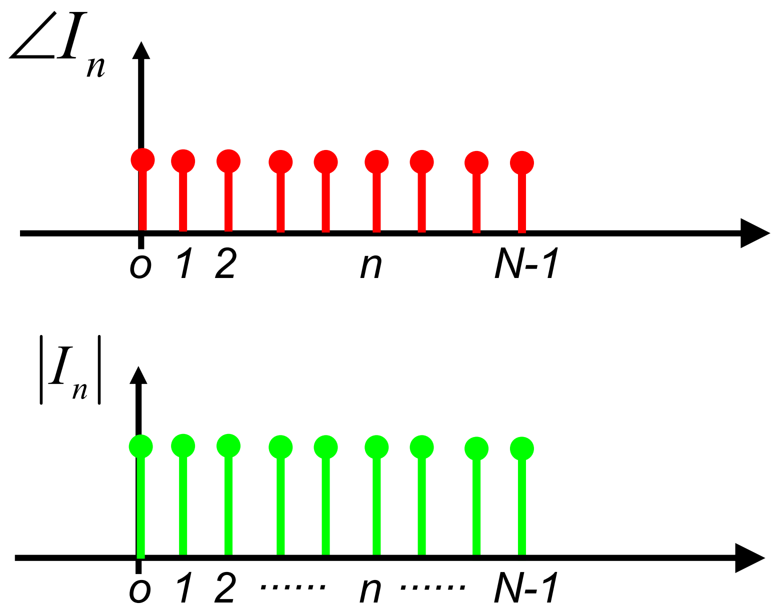
Endfire Array

Beam scanning and grating lobes

Periodic Linear Arrays (z-axis)

Uniform input excitations

$$I_n = I$$



Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$I_n = I$$

$$F(u) = \sum_{n=0}^{N-1} I_n \mathrm{e}^{-jnu} = I\sum_{n=0}^{N-1}\mathrm{e}^{-jnu} = I\frac{1-e^{-jNu}}{1-e^{-ju}} = I\frac{e^{-j\frac{Nu}{2}}}{e^{-j\frac{u}{2}}}\left(\frac{e^{j\frac{Nu}{2}}-e^{-j\frac{Nu}{2}}}{e^{j\frac{u}{2}}-e^{-j\frac{u}{2}}}\right)\frac{2j}{2j} = Ie^{-j\frac{(N-1)u}{2}}\frac{\sin(Nu/2)}{\sin(u/2)}$$

$$|F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$I_n = I \Rightarrow \left|F(u)\right| = \left|I\right|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

Periodic Linear Arrays (z-axis): Uniform Excitations

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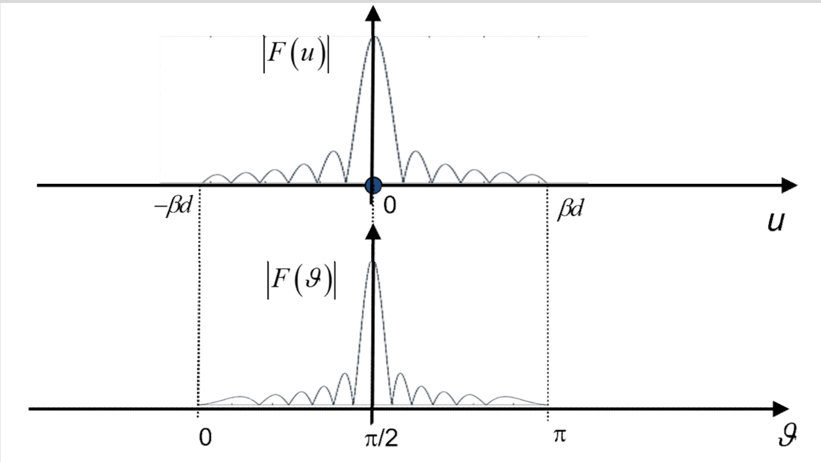
$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

.... Memo



1. Let's depict F(*u*)

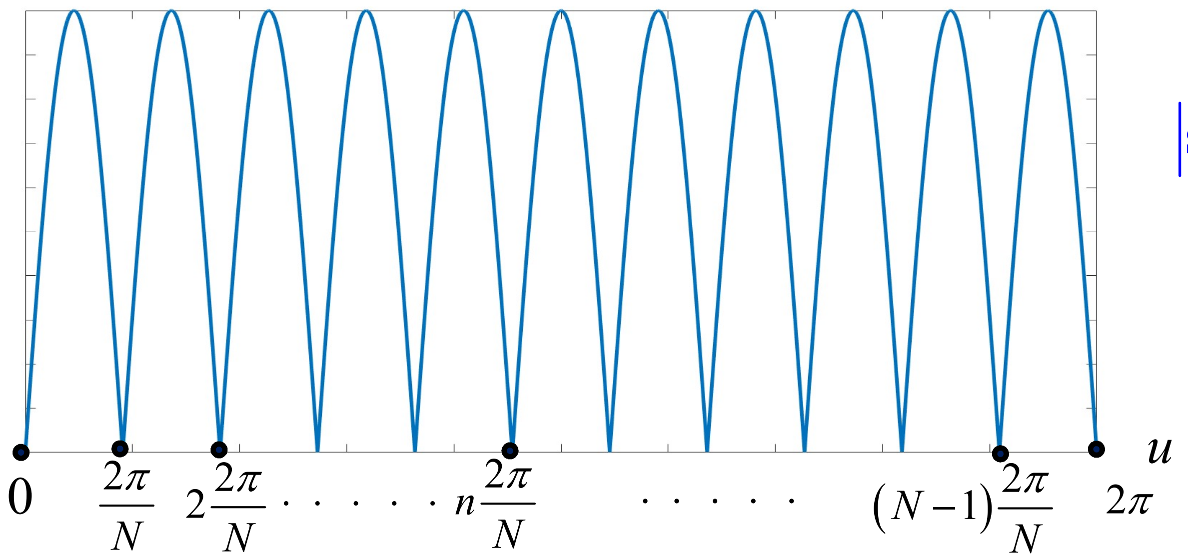
2. Let's jump from *u* to ϑ and calculate:

- **The direction of the Main Lobe**
- **The NNBW / HPBW**
- **The SLL**

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\frac{|F(u)|}{|I|}=\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

Periodic Linear Arrays (z-axis): Uniform Excitations



$|\sin(Nu/2)|$

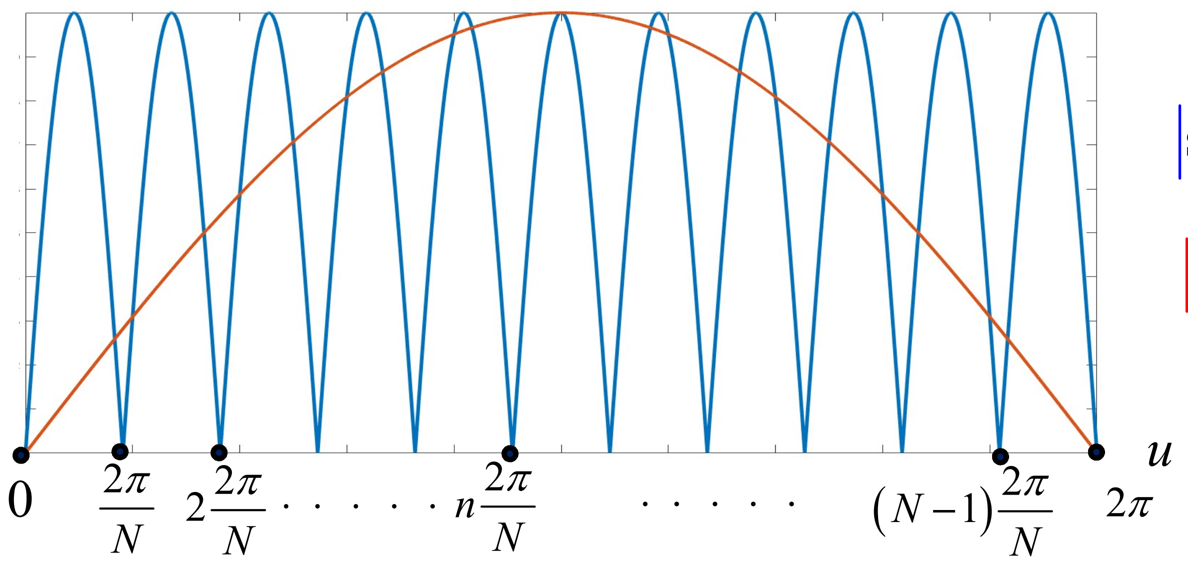
$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$|\sin(Nu/2)|$

Zeroes:

$$\sin(Nu/2) = 0 \Rightarrow \frac{Nu}{2} = k\pi \quad \Rightarrow u = k\frac{2\pi}{N}$$

Periodic Linear Arrays (z-axis): Uniform Excitations



$|\sin(Nu/2)|$

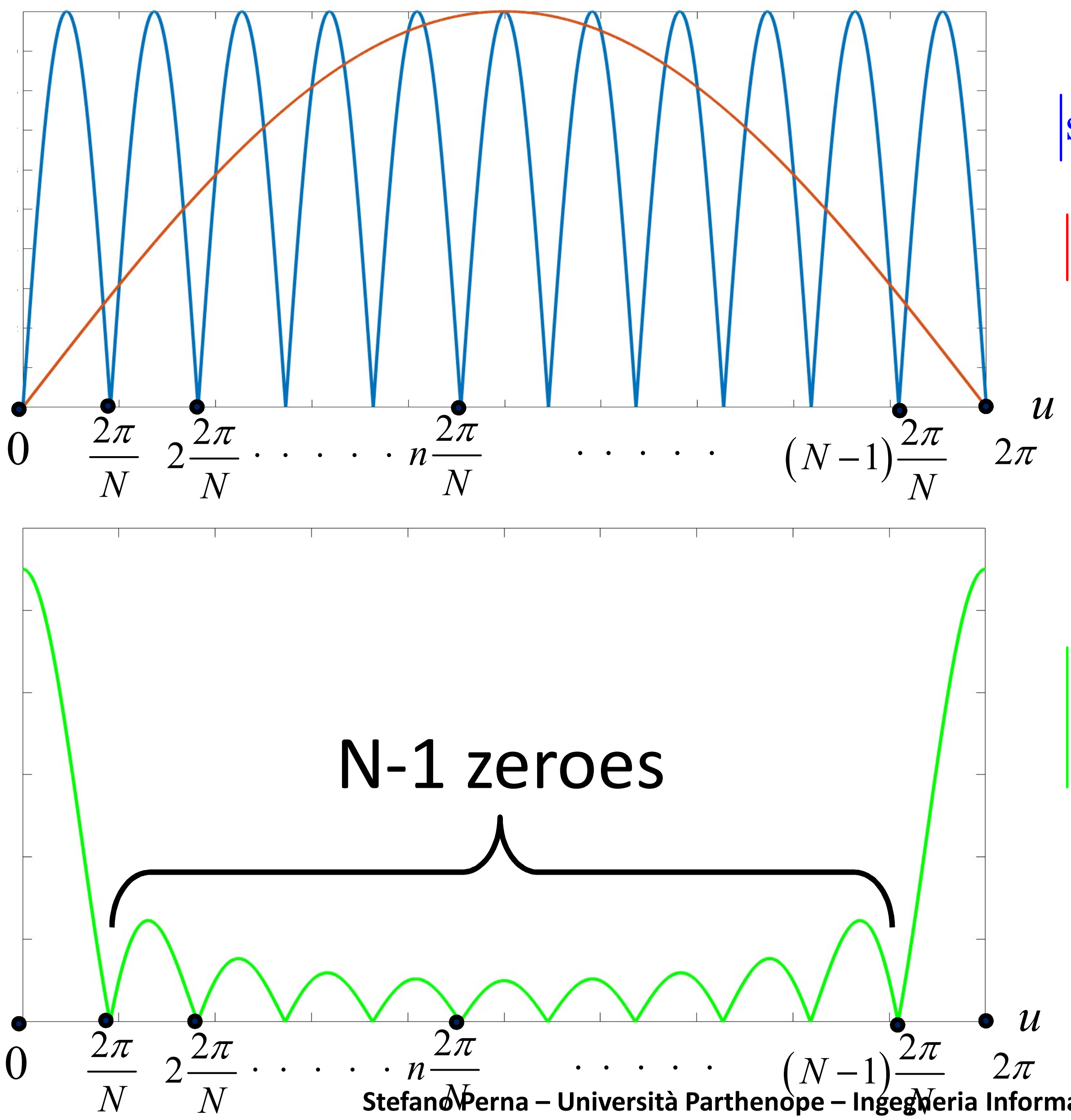
$|\sin(u/2)|$

$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$|\sin(u/2)|$

Period: 4π

Periodic Linear Arrays (z-axis): Uniform Excitations



$|\sin(Nu/2)|$

$|\sin(u/2)|$

$\left|\dfrac{\sin(Nu/2)}{\sin(u/2)}\right|$

$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

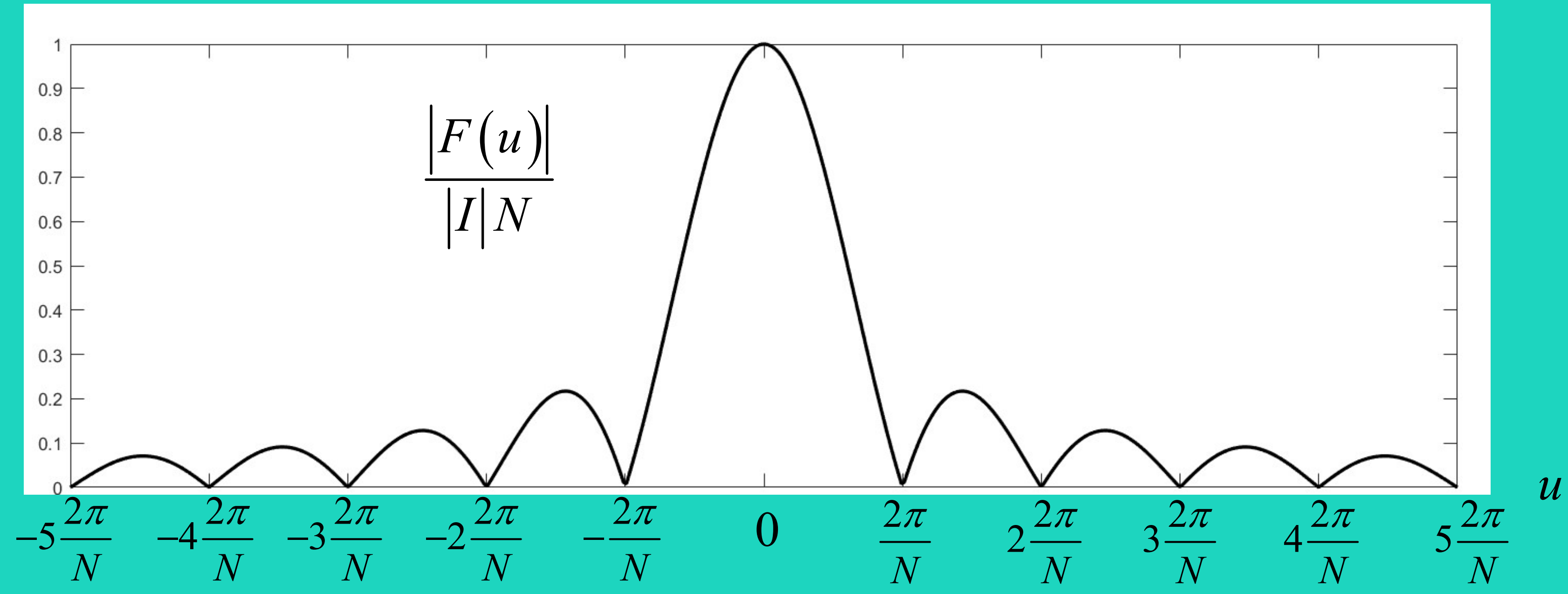
$$u = 0 \quad or \quad u = 2\pi \Rightarrow \quad F(u) = \frac{0}{0}$$

application of the de l'Hopital rule leads to

$$F(0) = F(2\pi) = IN$$

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$



Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

✓ **1. Let's depict F(*u*)**

2. Let's jump from *u* to ϑ and calculate:

The direction of the Main Lobe

The NNBW / HPBW

The SLL

The Directivity

Periodic Linear Arrays (z-axis): Uniform Excitations

u-domain

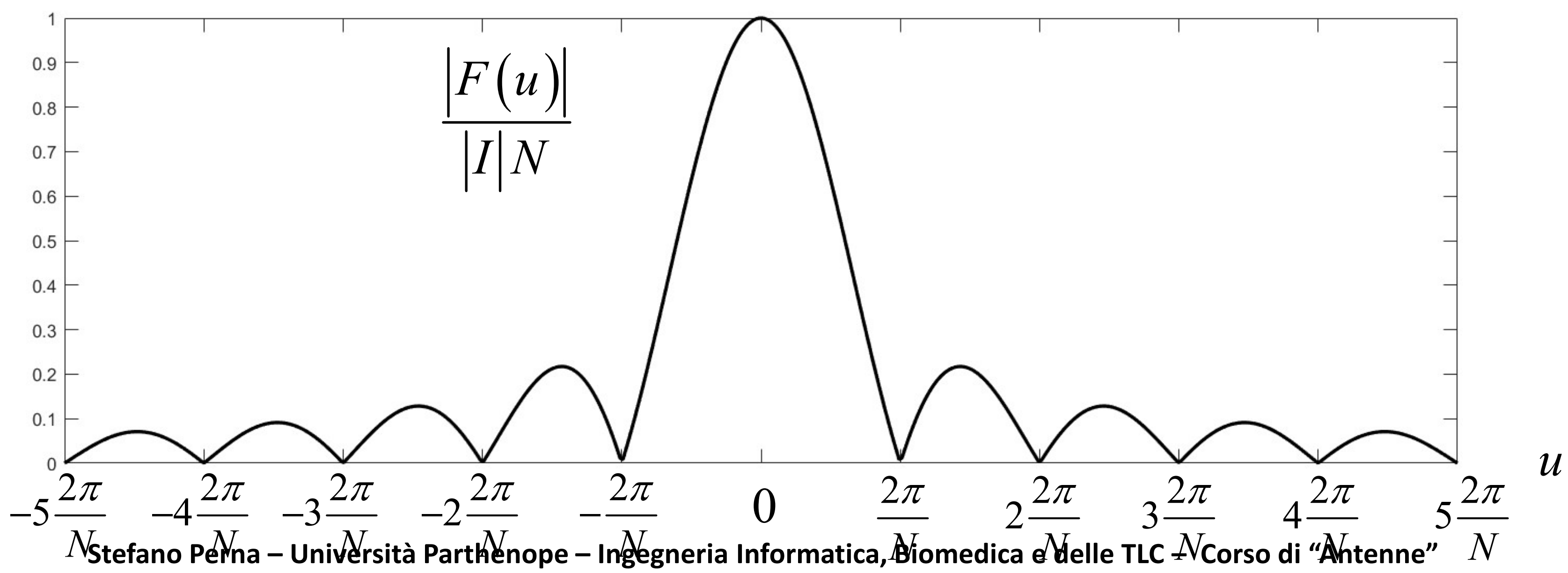
$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$u_{MB} = 0$

ϑ -domain

$$u = -\beta d \cos\vartheta$$

$u_{MB} = -\beta d \cos\vartheta_{MB}$ ➡ $0 = -\beta d \cos\vartheta_{MB}$ ➡ $\vartheta_{MB} = \frac{\pi}{2}$



Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

✓ **1. Let's depict F(*u*)**

2. Let's jump from *u* to ϑ and calculate:

✓ **The direction of the Main Lobe** $\vartheta_{MB} = \frac{\pi}{2}$

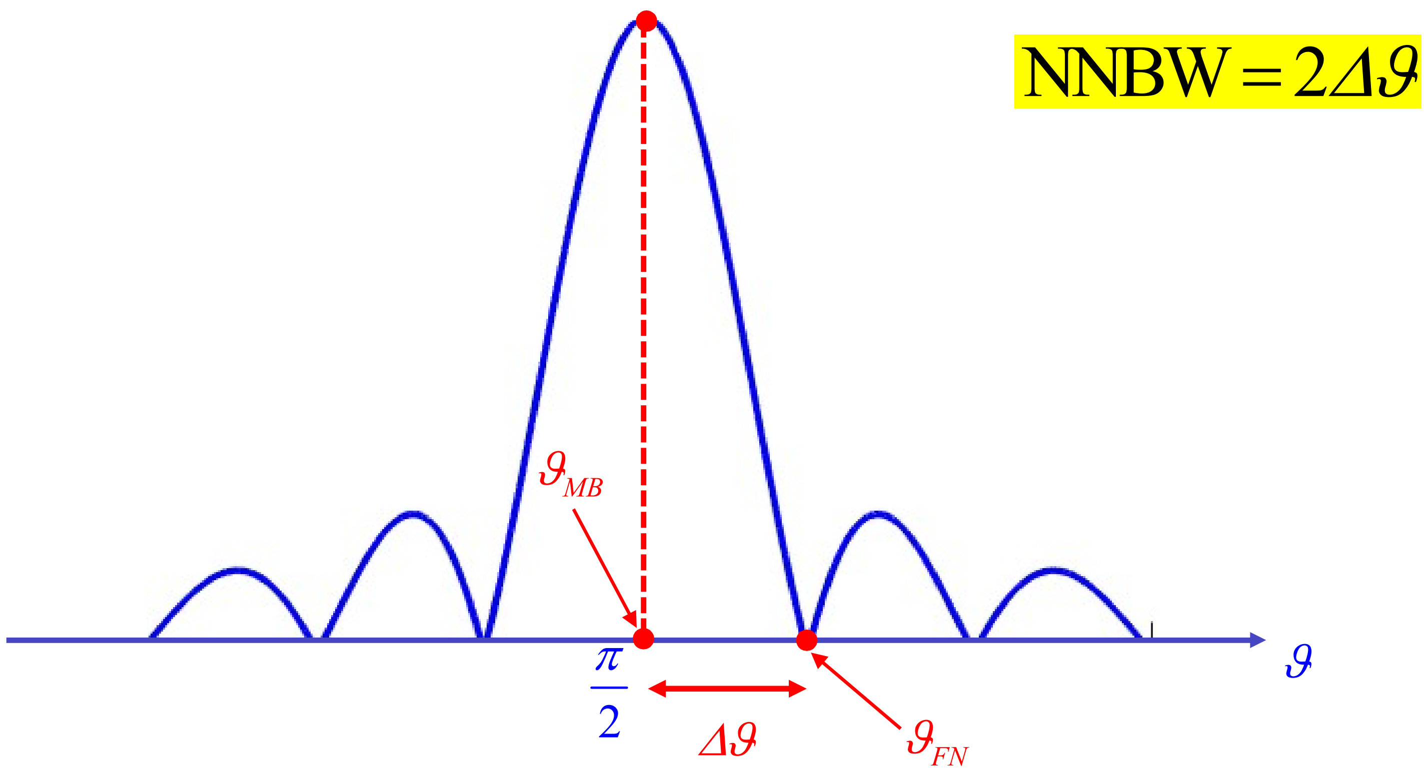
The NNBW / HPBW

The SLL

The Directivity

The main beam is oriented along the direction orthogonal to the array axis. In jargon, they are referred to as **BROADSIDE ARRAYS**

Main lobe and Beamwidth (NNBW)



Periodic Linear Arrays (z-axis): Uniform Excitations

u-domain

$$\frac{|F(u)|}{|I|} = \left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$u_{FN} = \frac{2\pi}{N}$$

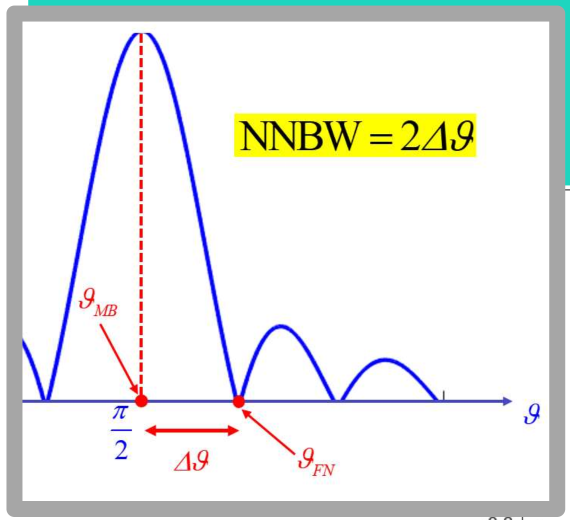
ϑ -domain

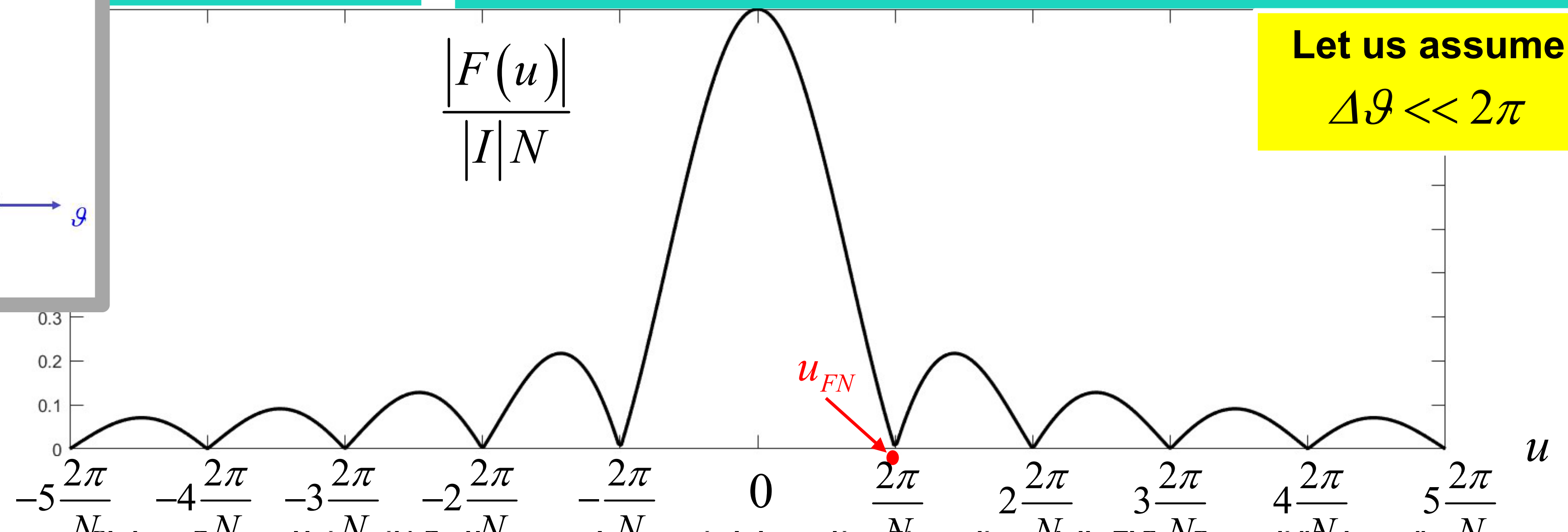
$$u = -\beta d \cos\vartheta$$

$$u_{FN} = -\beta d \cos\vartheta_{FN} = -\beta d \cos(\vartheta_{MB} + \Delta\vartheta)$$

$$\frac{2\pi}{N} = -\frac{2\pi}{\lambda} d \cos\left(\frac{\pi}{2} + \Delta\vartheta\right) = \frac{2\pi}{\lambda} d \sin(\Delta\vartheta) \approx \frac{2\pi}{\lambda} d\Delta\vartheta \Rightarrow \frac{2\pi}{N} \approx \frac{2\pi}{\lambda} d\Delta\vartheta$$

$$\Rightarrow \Delta\vartheta \approx \frac{\lambda}{Nd} \Rightarrow \mathrm{NNBW} = 2\Delta\vartheta \approx 2\frac{\lambda}{Nd}$$





Let us assume

$$\Delta\vartheta << 2\pi$$

$$\vartheta_{MB} = \frac{\pi}{2}$$

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

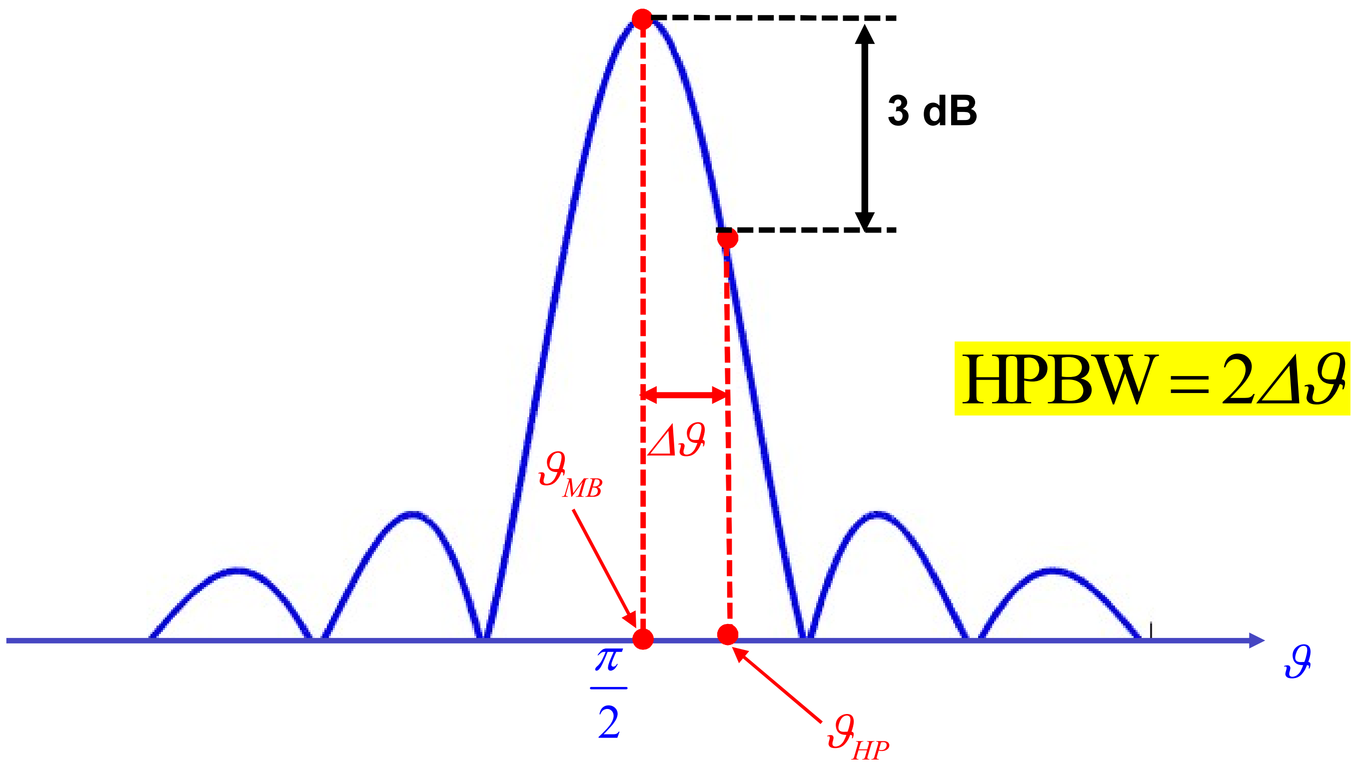
$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

- ✓ **1. Let's depict F(*u*)**
- **2. Let's jump from *u* to ϑ and calculate:**
  - ✓ **The direction of the Main Lobe** $\vartheta_{MB} = \frac{\pi}{2}$
  - ✓ **The NNBW / HPBW** $\mathrm{NNBW} \approx 2\frac{\lambda}{Nd}$
  - **The SLL**

Main lobe and Beamwidth (HPBW)



$$\mathrm{HPBW} = 2\Delta\vartheta$$

Periodic Linear Arrays (z-axis): Uniform Excitations

u-domain

$$\frac{|F(u)|}{|I|} = \left| \frac{\sin(Nu/2)}{\sin(u/2)} \right|$$

$$u_{HP} = 0.88\frac{\pi}{N}$$

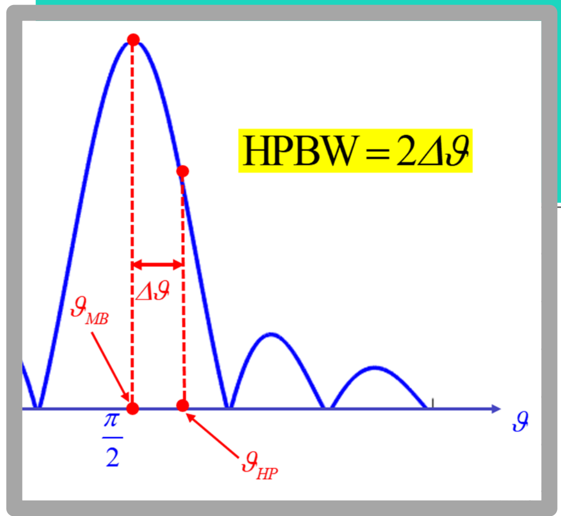
ϑ -domain

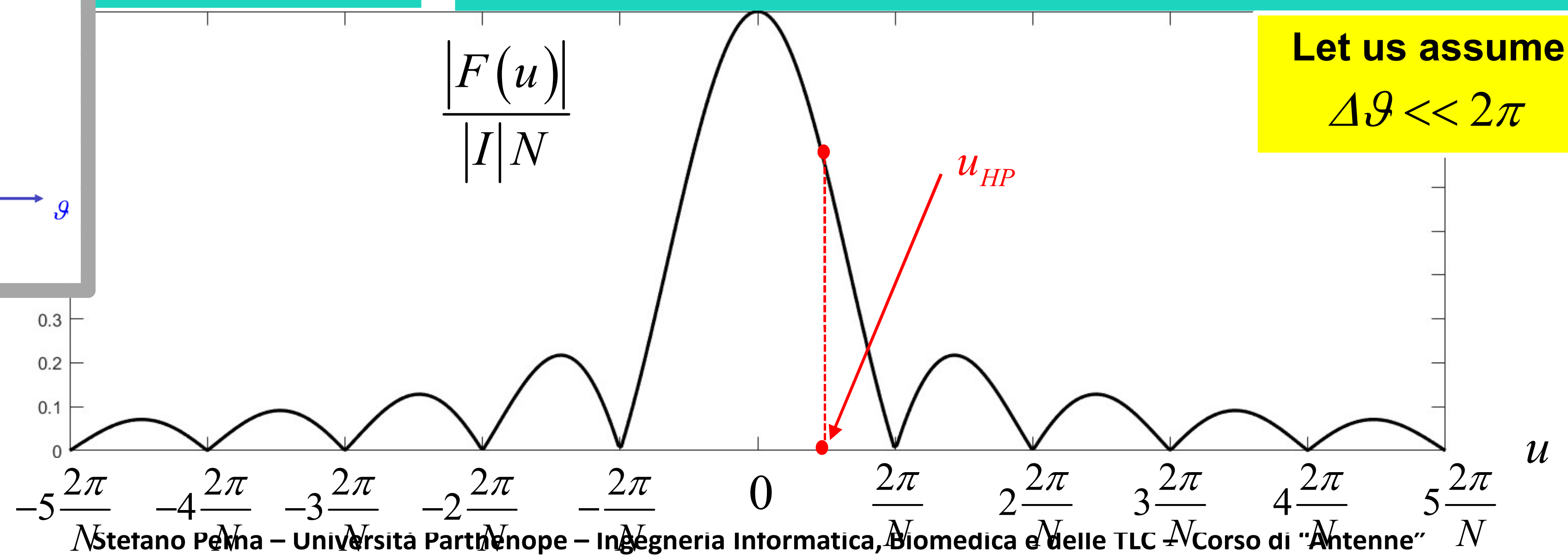
$$u = -\beta d \cos\vartheta$$

$$u_{HP} = -\beta d \cos\vartheta_{HP} = -\beta d \cos(\vartheta_{MB} + \Delta\vartheta)$$

$$0.88\frac{\pi}{N} = -\frac{2\pi}{\lambda} d \cos\left(\frac{\pi}{2} + \Delta\vartheta\right) = \frac{2\pi}{\lambda} d \sin(\Delta\vartheta) \approx \frac{2\pi}{\lambda} d \Delta\vartheta \Rightarrow 0.88\frac{\pi}{N} \approx \frac{2\pi}{\lambda} d \Delta\vartheta$$

$$\Rightarrow \Delta\vartheta \approx 0.88\frac{\lambda}{2Nd} \Rightarrow \text{HPBW} = 2\Delta\vartheta \approx 0.88\frac{\lambda}{Nd}$$





Let us assume
$\Delta\vartheta << 2\pi$

$$\vartheta_{MB} = \frac{\pi}{2}$$

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

✓ **1. Let's depict F(*u*)**

2. Let's jump from *u* to ϑ and calculate:

✓ **The direction of the Main Lobe** $\vartheta_{MB} = \frac{\pi}{2}$

✓ **The NNBW / HPBW** $\text{NNBW} \approx 2\frac{\lambda}{Nd}$ $\text{HPBW} \approx 0.88\frac{\lambda}{Nd}$

The SLL

……. Memo: Wire Antennas - an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right] \Rightarrow |F(u)| = 2L\left|\frac{\sin(uL)}{uL}\right|$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

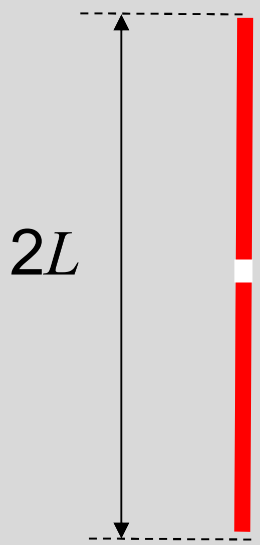
- ✓ **1. Let's depict F(*u*)**

2. Let's jump from *u* to ϑ and calculate:

- ✓ **The direction of the Main Lobe** $\vartheta_{MB} = \frac{\pi}{2}$
- ✓ **The NNBW / HPBW** $\mathrm{NNBW} \approx \frac{\lambda}{L}$ $\mathrm{HPBW} \approx 0.88\frac{\lambda}{2L}$
- ✓ **The SLL** $\mathrm{SLL} = -13.46\, dB$

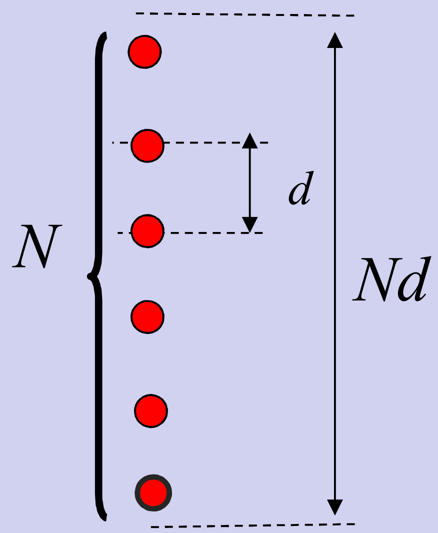
Periodic Linear Arrays (z-axis): Uniform Excitations vs. Wire Antennas with Uniform Current Distribution

Wire Antennas



$$\mathrm{HPBW} \approx 0.88 \frac{\lambda}{2L}$$

Linear Arrays



$$\mathrm{HPBW} \approx 0.88 \frac{\lambda}{Nd}$$

Periodic Linear Arrays (z-axis): Uniform Excitations

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$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

$$I_n = I \Rightarrow |F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

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   The SLL

The case at hand is **real**!

Periodic Linear Arrays (z-axis): Uniform Excitations

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

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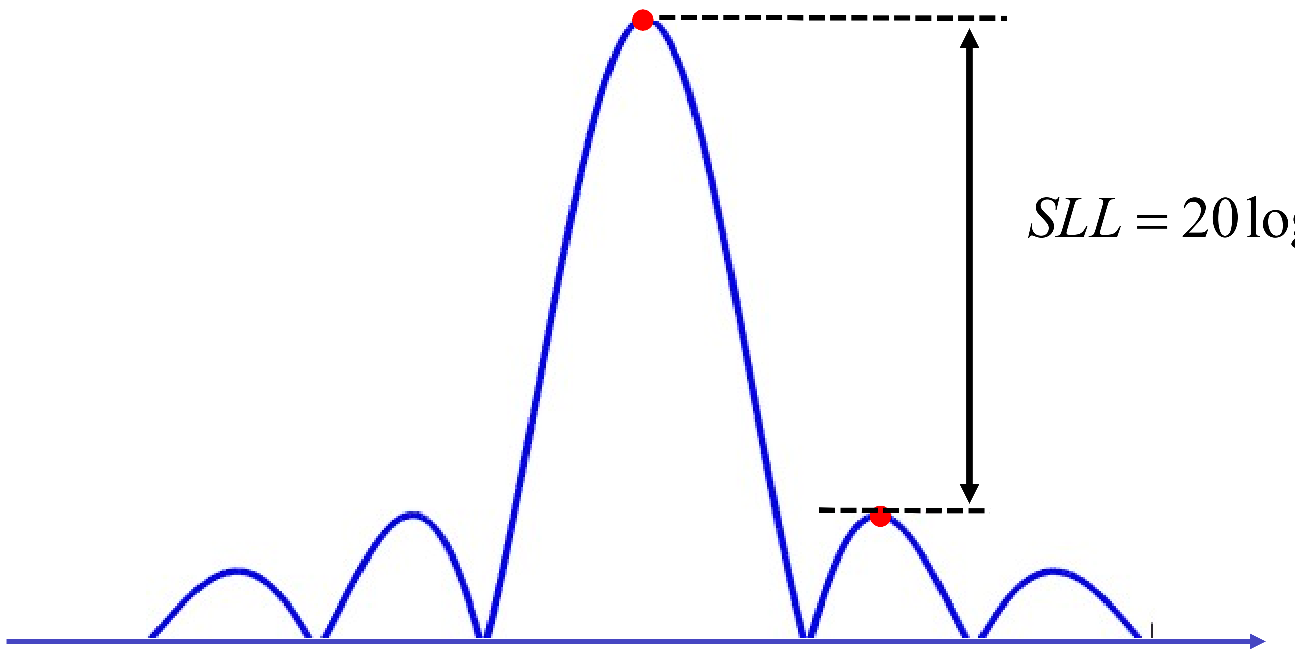
The SLL

For a fixed wavelength, to reduce the beamwidth, one can increase

- **N** … but take care of the overall cost of the array

- **d** … but take care of the grating lobes

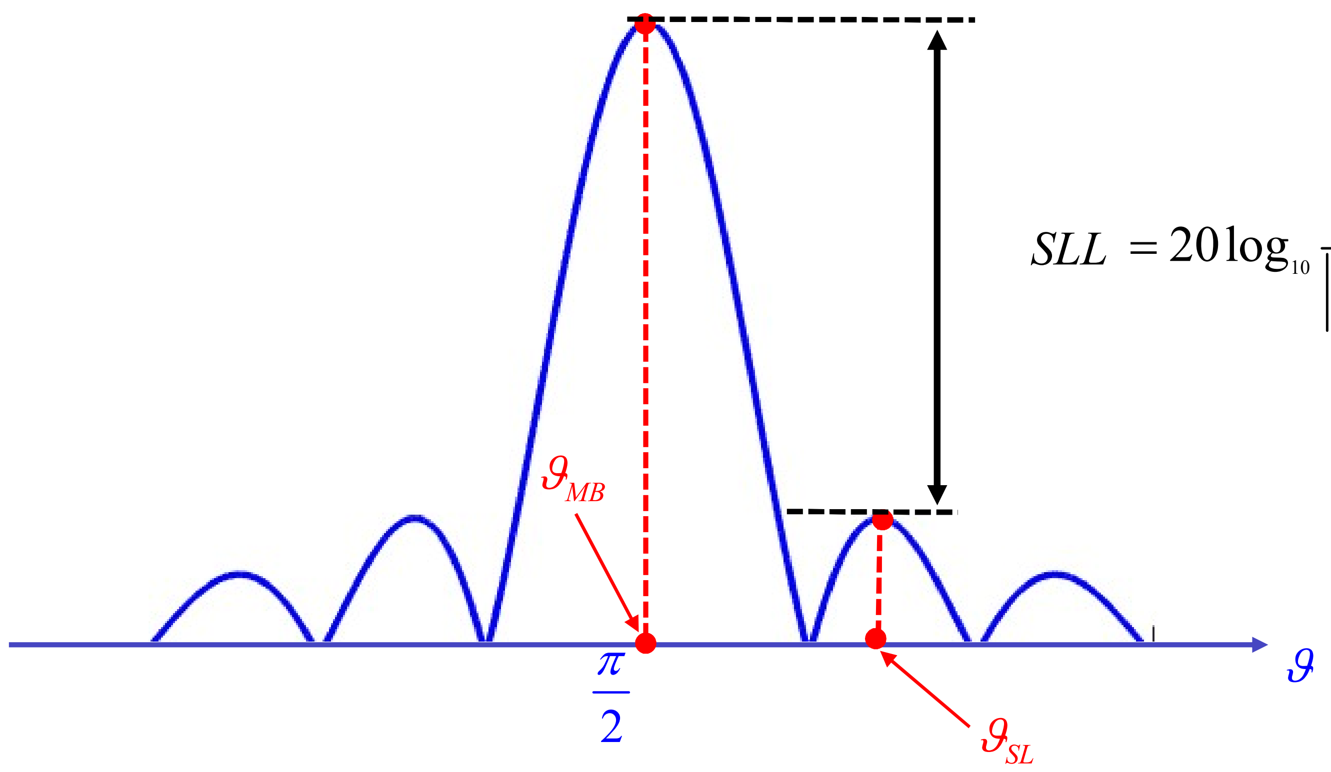
Side Lobe Level (SLL)



$$SLL = 20\log_{10}\frac{\left|\vec{\mathbf{E}}_{SL}\right|}{\left|\vec{\mathbf{E}}_{ML}\right|}$$

Side Lobe Level (SLL)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$



$$SLL = 20\log_{10}\frac{\left|\vec{\mathbf{I}}(\vartheta_{SL},\varphi_{SL})F(\vartheta_{SL})\right|}{\left|\vec{\mathbf{I}}(\vartheta_{MB},\varphi_{MB})F(\vartheta_{MB})\right|} = 20\log_{10}\frac{\left|F(\vartheta_{SL})\right|}{\left|F(\vartheta_{MB})\right|}$$

Let us assume that the array element is isotropic

$$\left|\vec{\mathbf{I}}(\vartheta_{SL},\varphi_{SL})\right| = \left|\vec{\mathbf{I}}(\vartheta_{MB},\varphi_{MB})\right|$$

Periodic Linear Arrays (z-axis): Uniform Excitations

u-domain

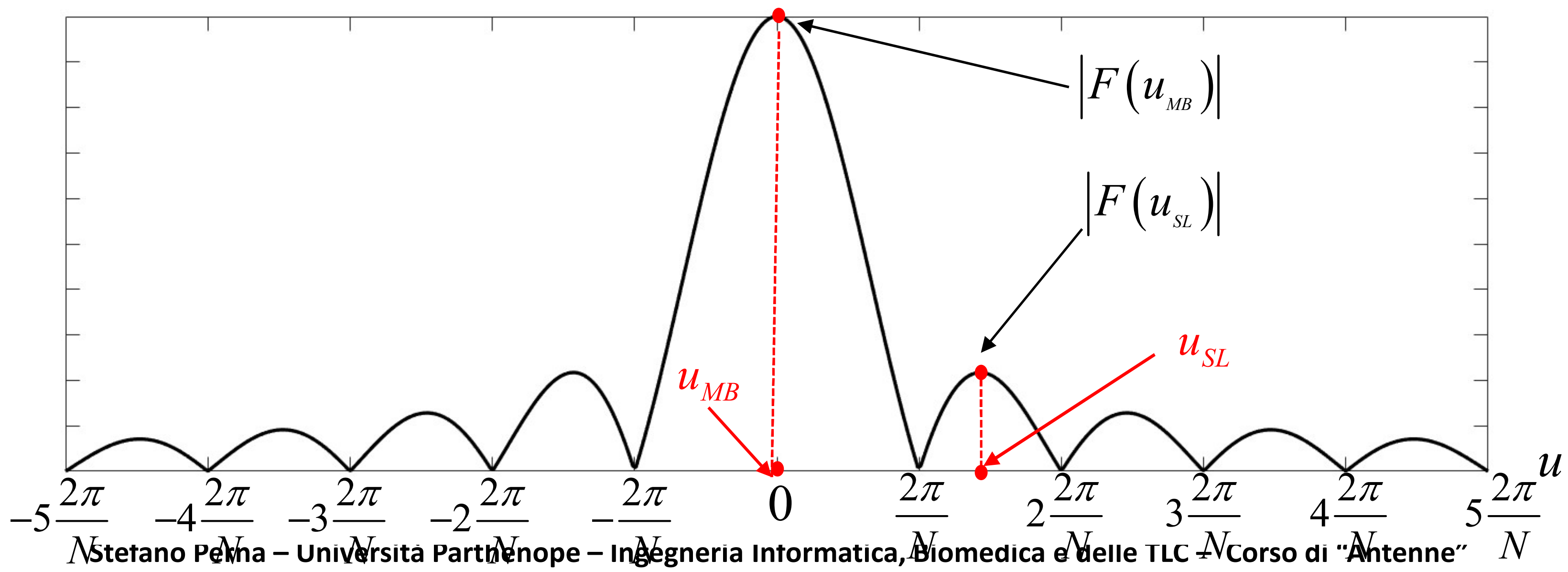
$$|F(u)| = |I| \left| \frac{\sin(Nu/2)}{\sin(u/2)} \right|$$

$$SLL \approx 20 \log_{10} \frac{|F(u_{SL})|}{|F(u_{MB})|}$$

ϑ -domain

$$u = -\beta d \cos \vartheta$$

$$SLL \approx 20 \log_{10} \frac{|F(\vartheta_{SL})|}{|F(\vartheta_{MB})|}$$



Periodic Linear Arrays (z-axis): Uniform Excitations

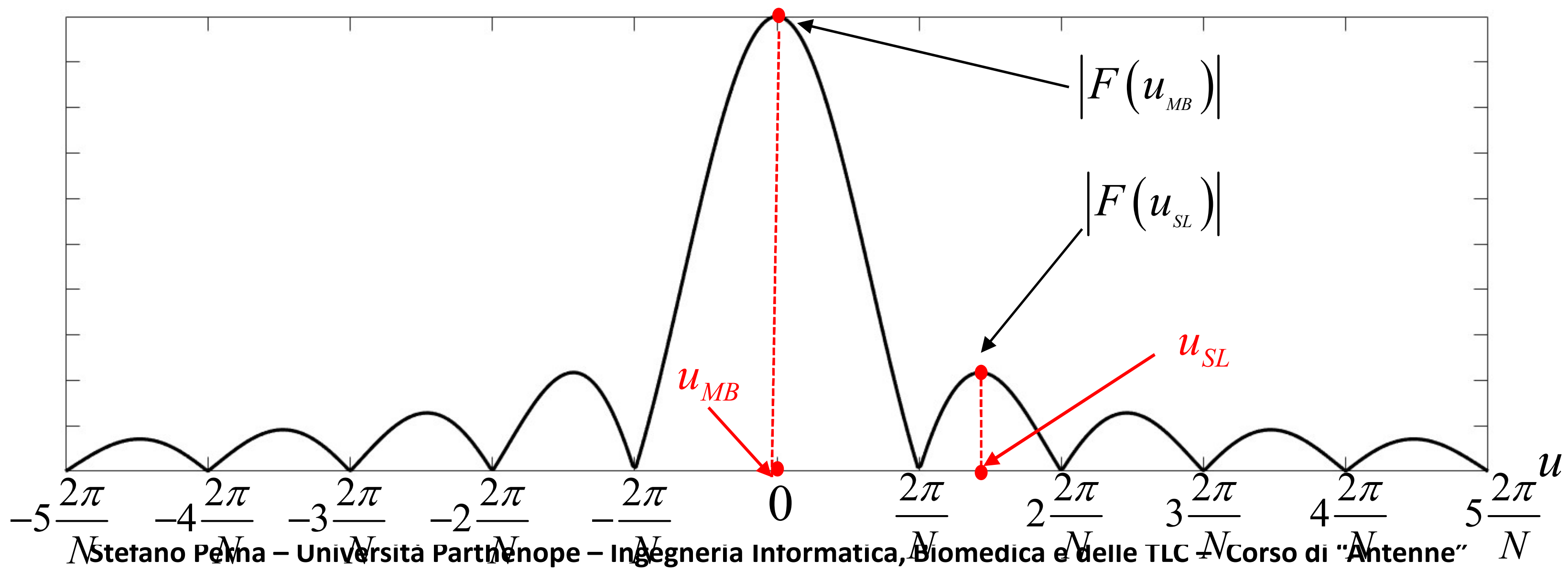
u-domain

$$|F(u)| = |I|\left|\frac{\sin(Nu/2)}{\sin(u/2)}\right|$$

$$SLL = 20\log_{10}\frac{|F(u_{SL})|}{|F(u_{MB})|}$$

$$u_{SL} \approx 3\frac{\pi}{N} \Rightarrow F(u_{SL}) = I\frac{\sin\left(\frac{N}{2}\frac{3\pi}{N}\right)}{\sin\left(\frac{1}{2}\frac{3\pi}{N}\right)} = -I\frac{1}{\sin\left(\frac{1}{2}\frac{3\pi}{N}\right)} \approx -I\frac{2N}{3\pi} \Rightarrow SLL \approx 20\log_{10}\frac{|2|}{|3\pi|} = -13.46\,dB$$

$$u_{MB} = 0 \Rightarrow F(u_{MB}) = I\,N$$



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BROADSIDE ARRAYS