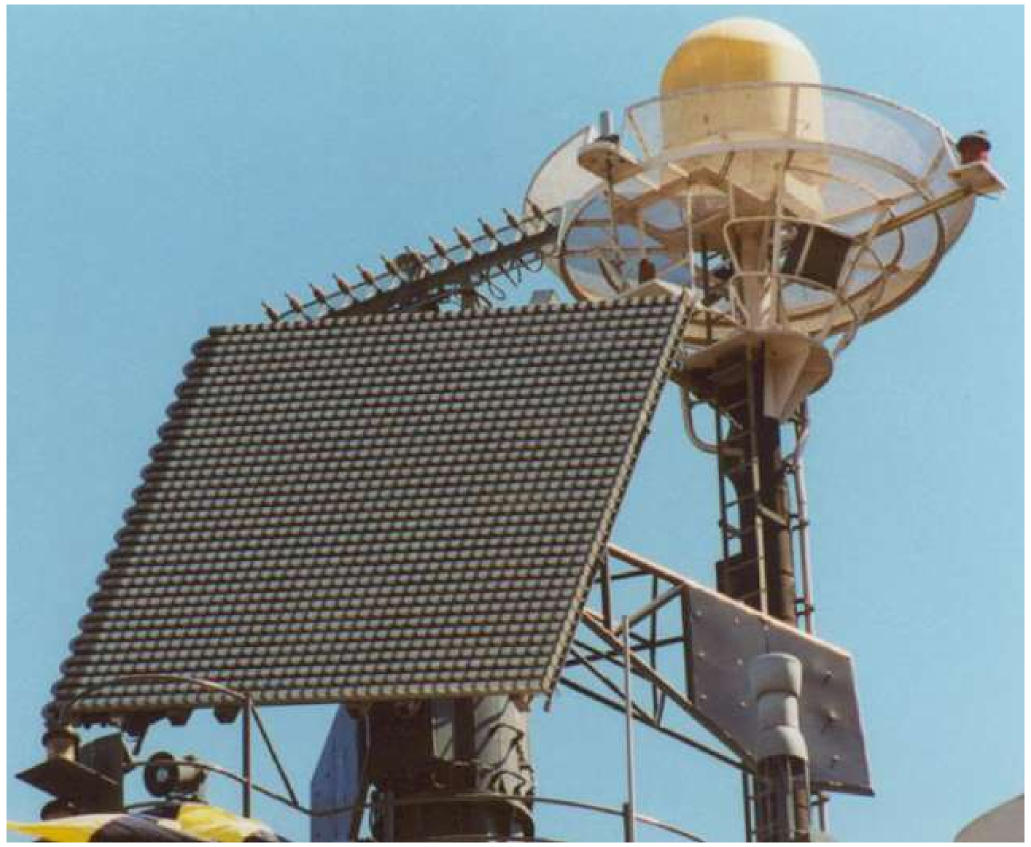
Arrays

Arrays



Color legend

New formulas, important considerations,
important formulas, important concepts

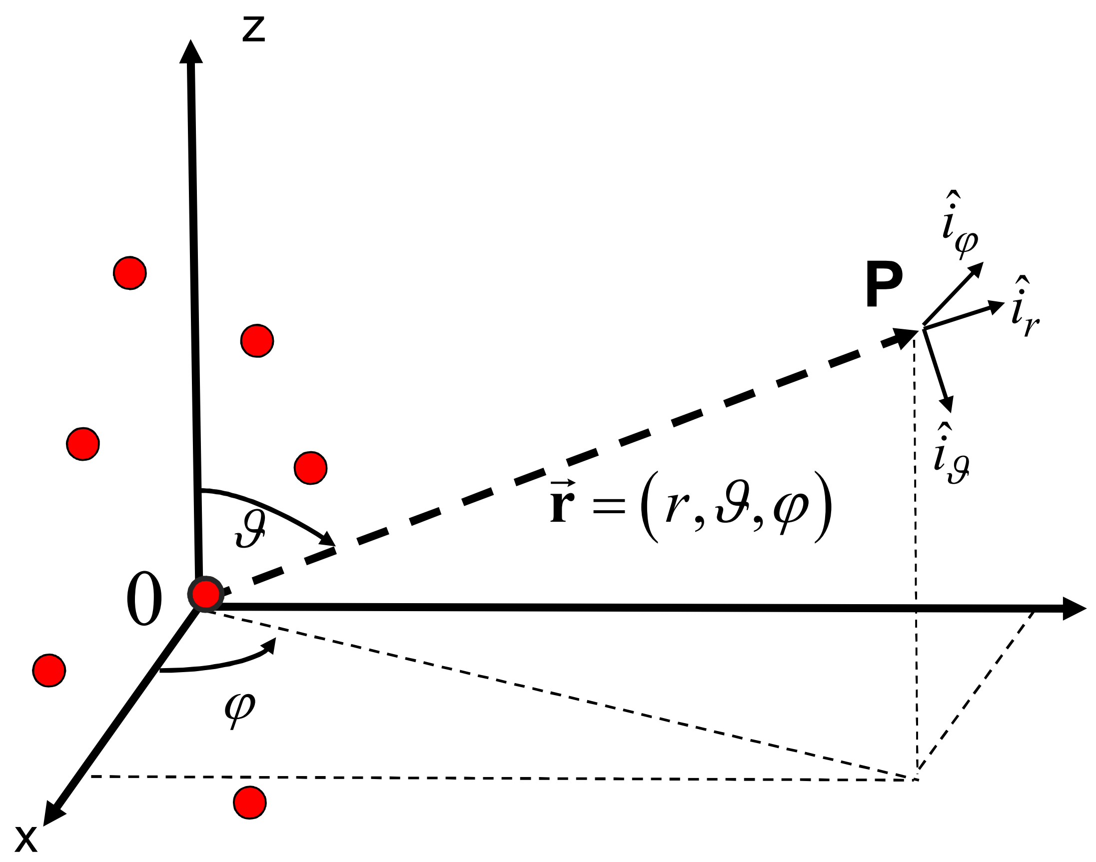
Very important for the discussion

Memo

Mathematical tools to be exploited

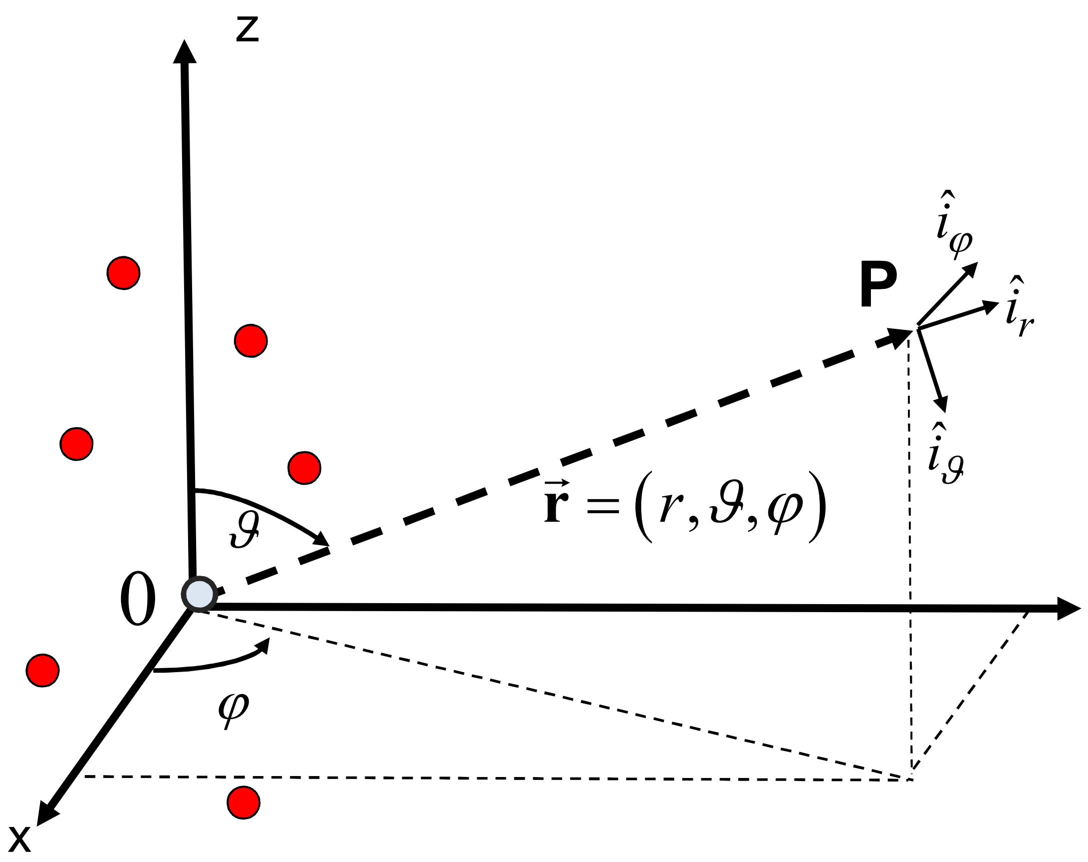
Mathematics

Arrays



Let us suppose that P is located in the Fraunhofer Region relevant to the each antenna of the considered array

Arrays

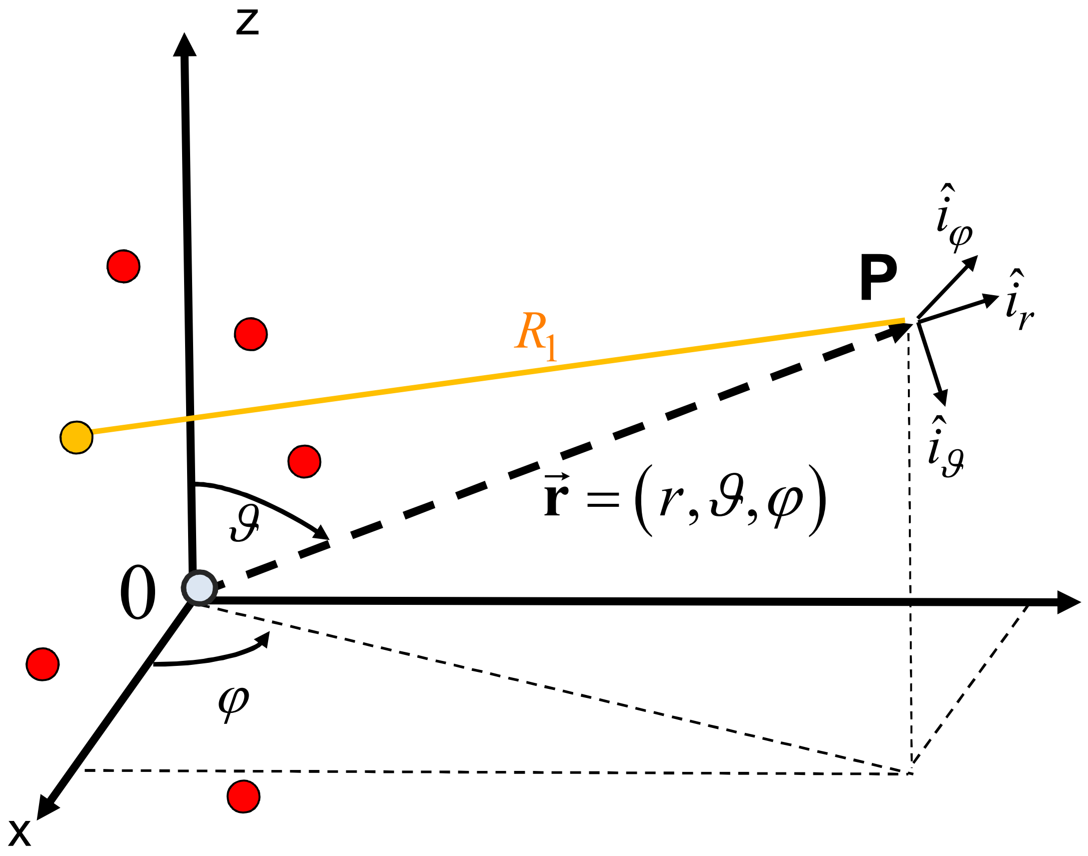


Let us suppose that P is located in the Fraunhofer Region relevant to the each antenna of the considered array

$$\vec{\mathbf{E}}_0 = j\frac{\zeta I_0}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}_0(\vartheta_0, \varphi_0)$$

$$\vartheta_0 = \vartheta; \quad \varphi_0 = \varphi; \; R_0 = r; \; \hat{i}_{\vartheta_0} = \hat{i}_\vartheta; \; \hat{i}_{\varphi_0} = \hat{i}_\varphi$$

Arrays



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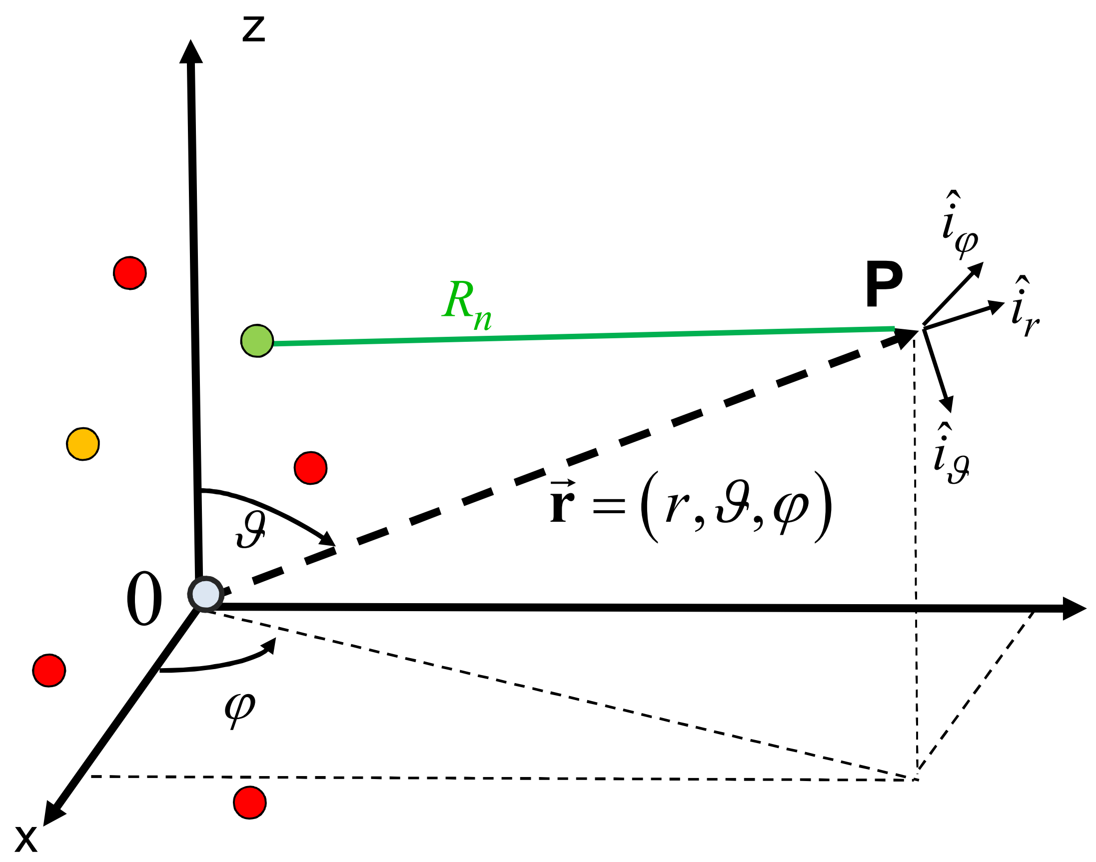
$$\vec{\mathbf{E}}_0 = j\frac{\zeta I_0}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}_0(\vartheta_0,\varphi_0)$$

$$\vartheta_0 = \vartheta; \quad \varphi_0 = \varphi; \; R_0 = r; \; \hat{i}_{\vartheta_0} = \hat{i}_\vartheta; \; \hat{i}_{\varphi_0} = \hat{i}_\varphi$$

$$\vec{\mathbf{E}}_1 = j\frac{\zeta I_1}{2\lambda}\frac{\exp(-j\beta R_1)}{R_1}\vec{\mathbf{l}}_1(\vartheta_1,\varphi_1)$$

Arrays



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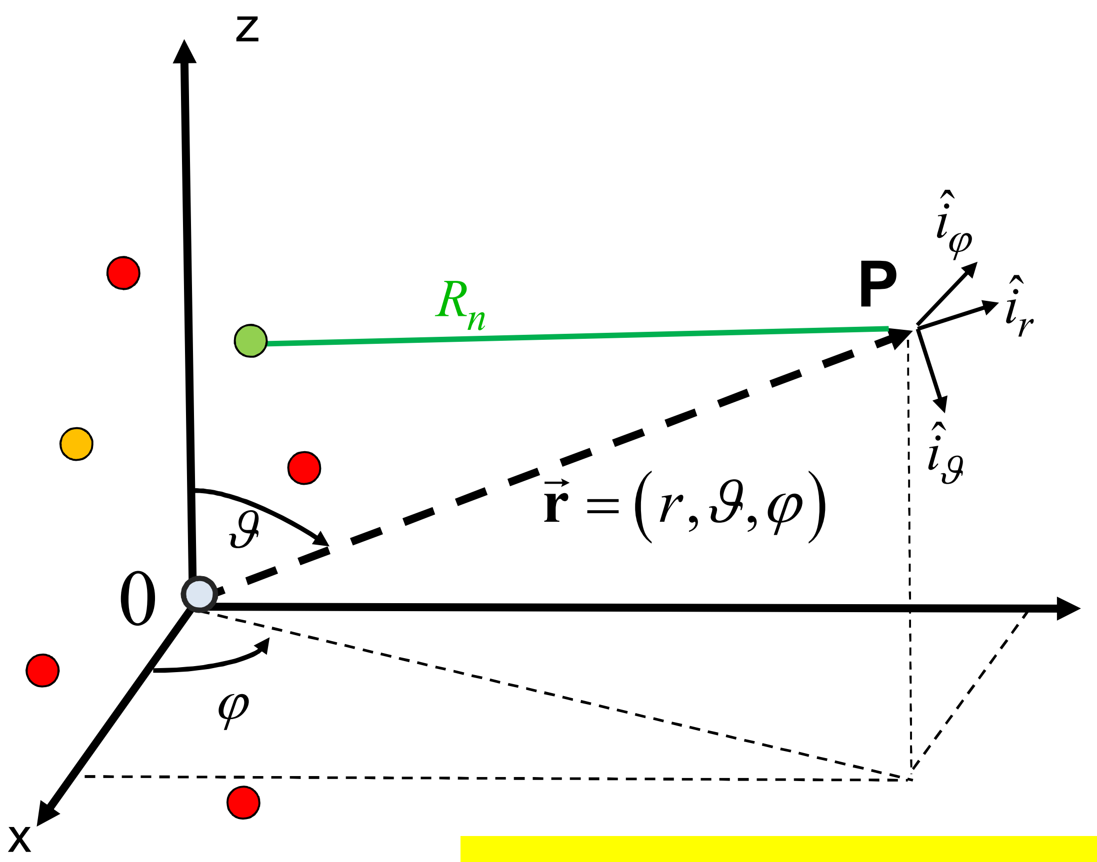
$$\vec{\mathbf{E}}_n = j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n, \varphi_n)$$

Arrays



Let us suppose that P is located in the Fraunhofer Region relevant to the each antenna of the considered array

$$\vec{\mathbf{E}}_0 = j\frac{\zeta I_0}{2\lambda}\frac{\exp(-j\beta R_0)}{R_0}\vec{\mathbf{l}}_0(\vartheta_0, \varphi_0)$$

$$\vartheta_0 = \vartheta; \quad \varphi_0 = \varphi; \; R_0 = r; \; \hat{i}_{\vartheta_0} = \hat{i}_\vartheta; \; \hat{i}_{\varphi_0} = \hat{i}_\varphi$$

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$$\vec{\mathbf{E}}_n = j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n, \varphi_n)$$

N **is the number of the array elements**

$$\vec{\mathbf{E}} = \sum_{n=0}^{N-1}\vec{\mathbf{E}}_n = \sum_{n=0}^{N-1} j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n, \varphi_n)$$

Arrays

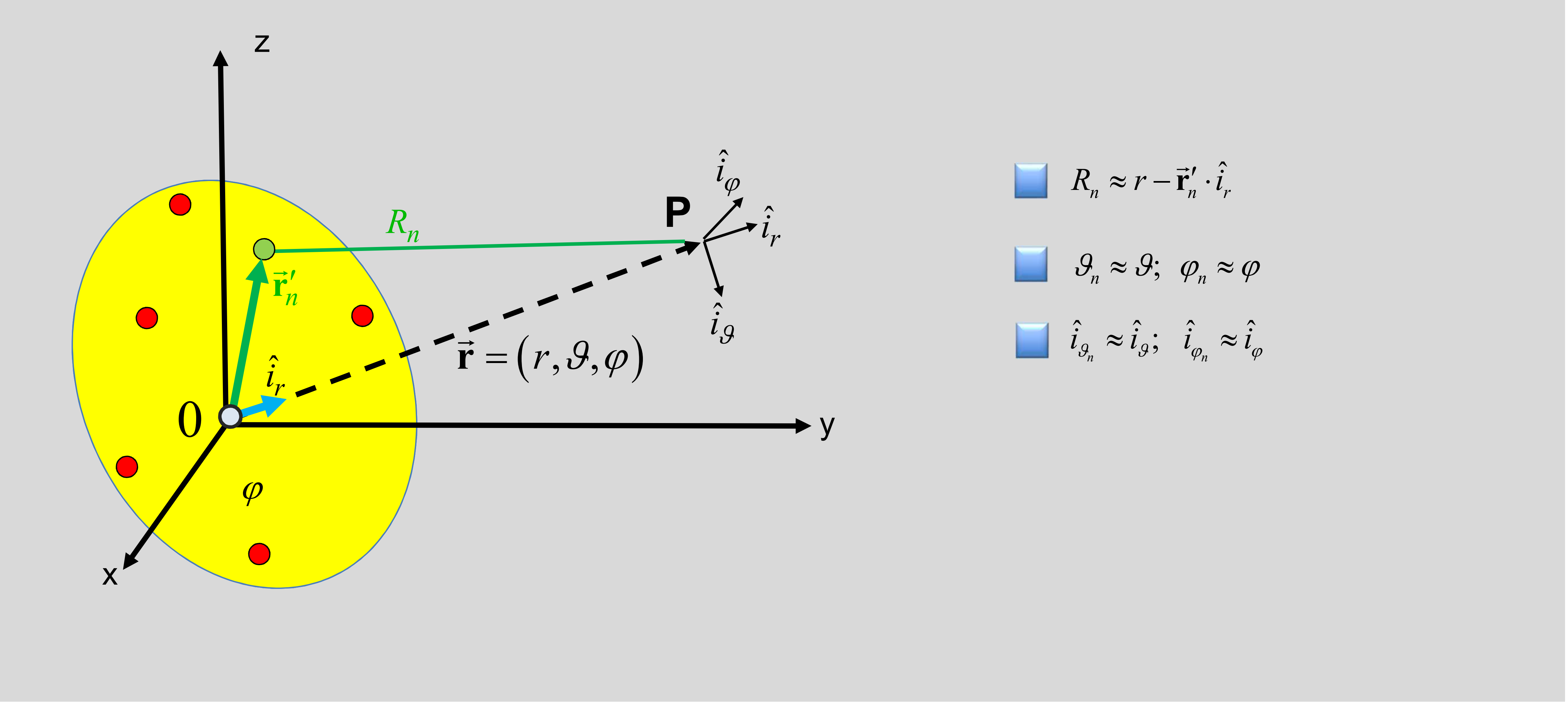
P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

$$\vec{\mathbf{E}} = \sum_{n=0}^{N-1} j \frac{\zeta I_n}{2\lambda} \frac{\exp(-j\beta R_n)}{R_n} \vec{\mathbf{l}}_n \left(\vartheta_n, \varphi_n\right) = j \frac{\zeta}{2\lambda} \sum_{n=0}^{N-1} I_n \frac{\exp(-j\beta R_n)}{R_n} \vec{\mathbf{l}}_n \left(\vartheta_n, \varphi_n\right)$$

Memo: Fraunhofer Region

P is located in the Fraunhofer Region relevant to the overall array antenna



Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

- $R_n \approx r - \vec{\mathbf{r}}'_n \cdot \hat{i}_r \qquad \Rightarrow \frac{\exp\left(-j\beta R_n\right)}{R_n} \approx \frac{\exp\left(-j\beta r\right)\exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)}{r}$
- $\vartheta_n \approx \vartheta; \quad \varphi_n \approx \varphi$
- $\hat{i}_{\vartheta_n} \approx \hat{i}_\vartheta; \quad \hat{i}_{\varphi_n} \approx \hat{i}_\varphi$

$$\vec{\mathbf{E}} = \sum_{n=0}^{N-1} j \frac{\zeta I_n}{2\lambda} \frac{\exp(-j\beta R_n)}{R_n} \vec{\mathbf{l}}_n\left(\vartheta_n, \varphi_n\right) = j\frac{\zeta}{2\lambda}\sum_{n=0}^{N-1} I_n \frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n\left(\vartheta_n, \varphi_n\right) = j\frac{\zeta}{2\lambda}\sum_{n=0}^{N-1} I_n \frac{\exp\left(-j\beta r\right)\exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)}{r}\vec{\mathbf{l}}_n\left(\vartheta, \varphi\right)$$

$$= j\frac{\zeta}{2\lambda}\frac{\exp\left(-j\beta r\right)}{r}\sum_{n=0}^{N-1} I_n \exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)\vec{\mathbf{l}}_n\left(\vartheta, \varphi\right)$$

Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

$$\vec{\mathbf{l}}_n(\vartheta,\varphi)=\vec{\mathbf{l}}(\vartheta,\varphi)$$

$$\vec{\mathbf{E}}=\sum_{n=0}^{N-1} j\frac{\zeta I_n}{2\lambda}\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n,\varphi_n)=j\frac{\zeta}{2\lambda}\sum_{n=0}^{N-1} I_n\frac{\exp(-j\beta R_n)}{R_n}\vec{\mathbf{l}}_n(\vartheta_n,\varphi_n)=j\frac{\zeta}{2\lambda}\sum_{n=0}^{N-1} I_n\frac{\exp(-j\beta r)\exp\left(j\beta\vec{\mathbf{r}}'_n\cdot\hat{i}_r\right)}{r}\vec{\mathbf{l}}_n(\vartheta,\varphi)$$

$$=j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\sum_{n=0}^{N-1} I_n\exp\left(j\beta\vec{\mathbf{r}}'_n\cdot\hat{i}_r\right)\vec{\mathbf{l}}_n(\vartheta,\varphi)=j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{n=0}^{N-1} I_n\exp\left(j\beta\vec{\mathbf{r}}'_n\cdot\hat{i}_r\right)$$

$$=j\frac{\zeta}{2\lambda}I_0\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\sum_{n=0}^{N-1}\frac{I_n}{I_0}\exp\left(j\beta\vec{\mathbf{r}}'_n\cdot\hat{i}_r\right)$$

Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp(-j\beta r)}{r} \vec{\mathbf{l}}(\vartheta,\varphi)\; F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \frac{1}{I_0}\sum_{n=0}^{N-1} I_n \exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)$$

Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)\,F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta\vec{\mathbf{r}}'_n \cdot \hat{i}_r\right)$$

Principle of pattern multiplication

Array Factor — $F(\vartheta,\varphi)$

Element Factor — $\vec{\mathbf{l}}(\vartheta,\varphi)$

The Array Factor $F(\vartheta,\varphi)$ is a function of the angular coordinates (ϑ,φ) and depends upon:

- **the array geometry (through N and $\vec{\mathbf{r}}'_n$)**
- **the input excitations of the antennas of the array itself (through I_n)**

Very interesting implications relevant to the synthesis of the pattern

Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

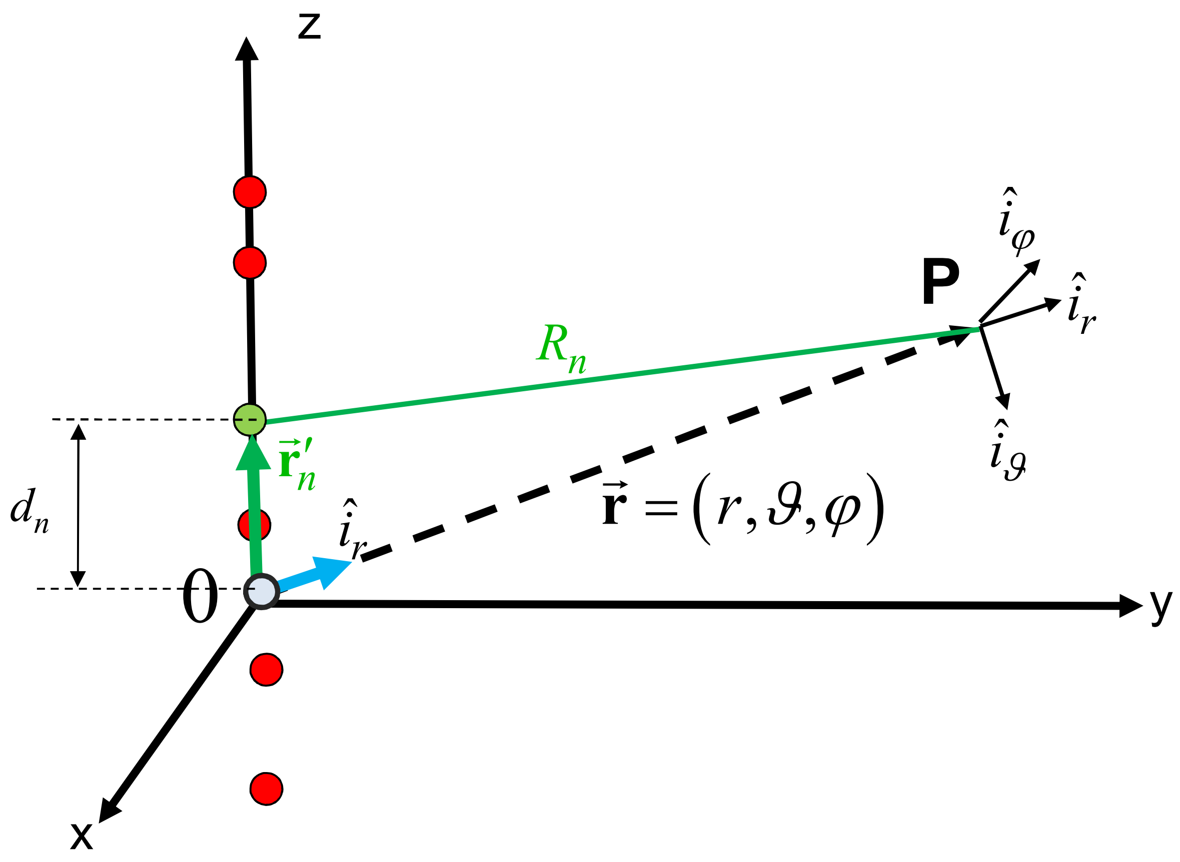
The antennas of the considered array are deployed along one axis (Linear Arrays)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta\vec{\mathbf{r}}_n' \cdot \hat{i}_r\right)$$

Linear Arrays

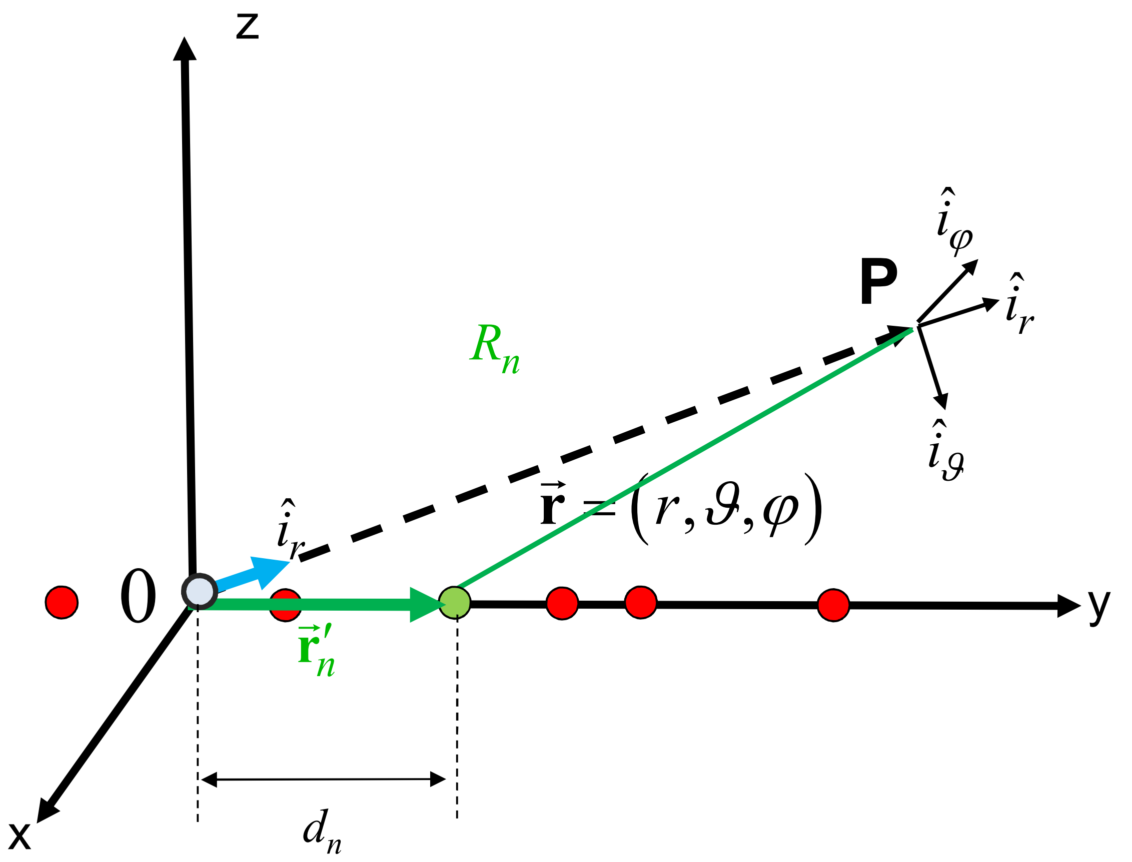
The antennas of the considered array are deployed along the z-axis



$$\vec{\mathbf{r}}'_n = d_n \hat{i}_z$$

Linear Arrays

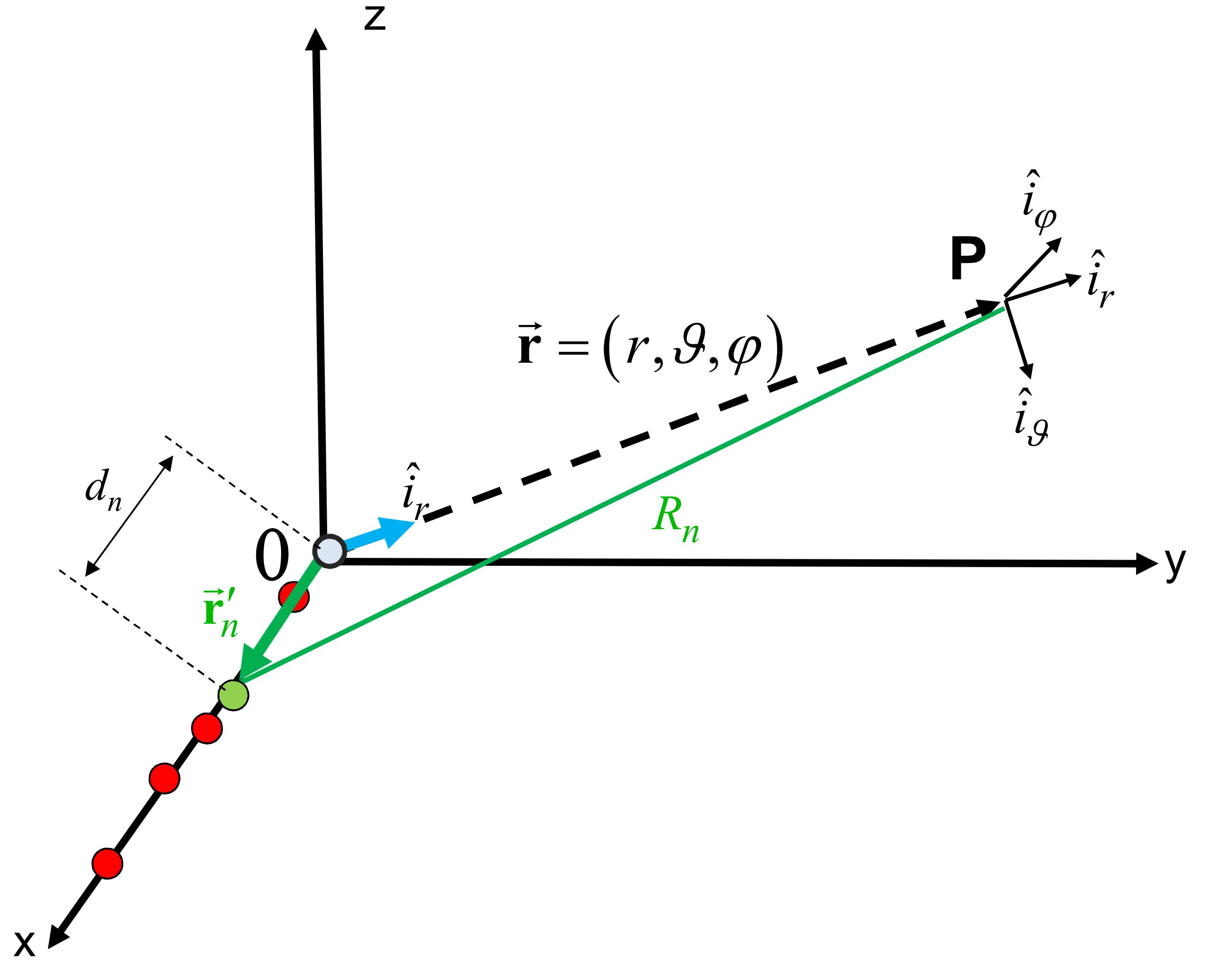
The antennas of the considered array are deployed along the y-axis



$$\vec{\mathbf{r}}'_n = d_n \hat{i}_y$$

Linear Arrays

The antennas of the considered array are deployed along the x-axis



$$\vec{\mathbf{r}}'_n = d_n \hat{i}_x$$

Linear Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

The antennas of the considered array are deployed along one axis (Linear Arrays); f.i., the z-axis

$$\vec{\mathbf{r}}'_n = d_n \hat{i}_z \quad \Rightarrow \vec{\mathbf{r}}'_n \cdot \hat{i}_r = d_n \hat{i}_z \cdot \hat{i}_r = d_n \cos\vartheta$$

$$\vec{\mathbf{r}}'_n = d_n \hat{i}_y \quad \Rightarrow \vec{\mathbf{r}}'_n \cdot \hat{i}_r = d_n \hat{i}_y \cdot \hat{i}_r = d_n \sin\vartheta \sin\varphi$$

$$\vec{\mathbf{r}}'_n = d_n \hat{i}_x \quad \Rightarrow \vec{\mathbf{r}}'_n \cdot \hat{i}_r = d_n \hat{i}_x \cdot \hat{i}_r = d_n \sin\vartheta \cos\varphi$$

$$\hat{i}_r = \sin\vartheta\cos\varphi\,\hat{i}_x + \sin\vartheta\sin\varphi\,\hat{i}_y + \cos\vartheta\,\hat{i}_z$$

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta\vec{\mathbf{r}}'_n \cdot \hat{i}_r\right) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta d_n \cos\vartheta\right)$$

Periodic Linear Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

The antennas of the considered array are deployed along one axis (Linear Arrays); f.i., the z-axis

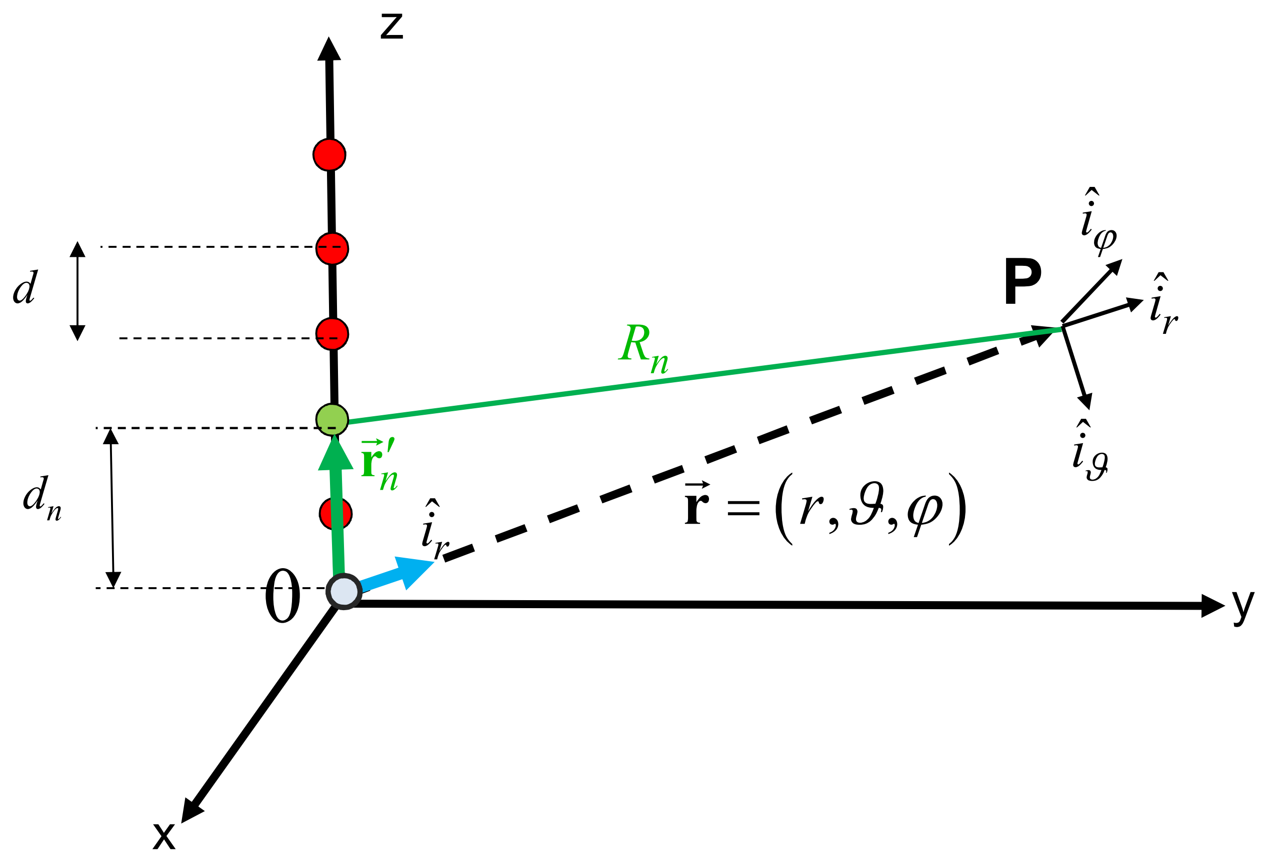
The antennas of the considered array are equispaced (Periodic Arrays)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta d_n \cos\vartheta\right)$$

Periodic Linear Arrays

The antennas of the considered array are equispaced (Periodic Arrays)



$d_n = n\,d$

Periodic Linear Arrays

P is located in the Fraunhofer Region relevant to the each antenna of the considered array

P is located in the Fraunhofer Region relevant to the overall array antenna

The antennas of the considered array are equal

The antennas of the considered array are deployed along one axis (Linear Arrays); f.i., the z-axis

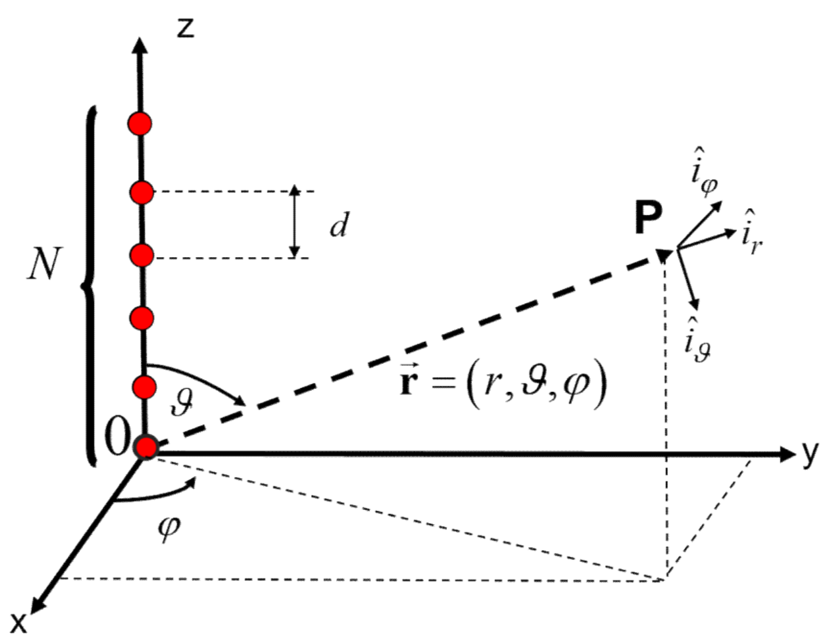
The antennas of the considered array are equispaced (Periodic Arrays)

$$d_n = n\,d$$

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta,\varphi)$$

$$F(\vartheta,\varphi) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta \vec{\mathbf{r}}'_n \cdot \hat{i}_r\right) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta d_n \cos\vartheta\right) = \sum_{n=0}^{N-1} I_n \exp\left(j\beta n d \cos\vartheta\right)$$

Periodic Linear Arrays (z-axis)



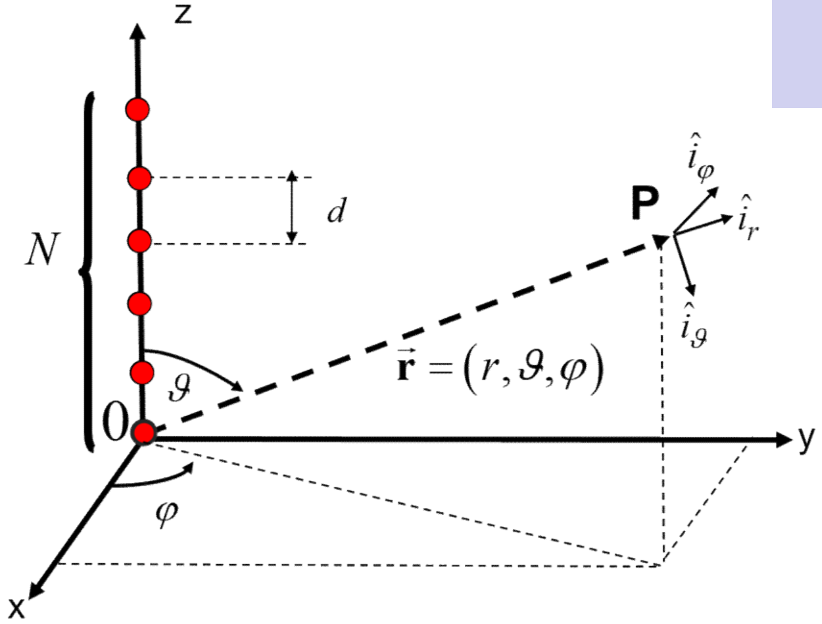
The expression of the array factor *F*(·) simplifies as

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\cos\vartheta)$$

The array Factor *F*(·) is independent of φ … absolutely not surprising

Periodic Linear Arrays (z-axis)



$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

$$F(\vartheta) = \sum_{n=0}^{N-1} I_n \exp(j\beta nd\cos\vartheta)$$

$$u = -\beta d\cos\vartheta$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

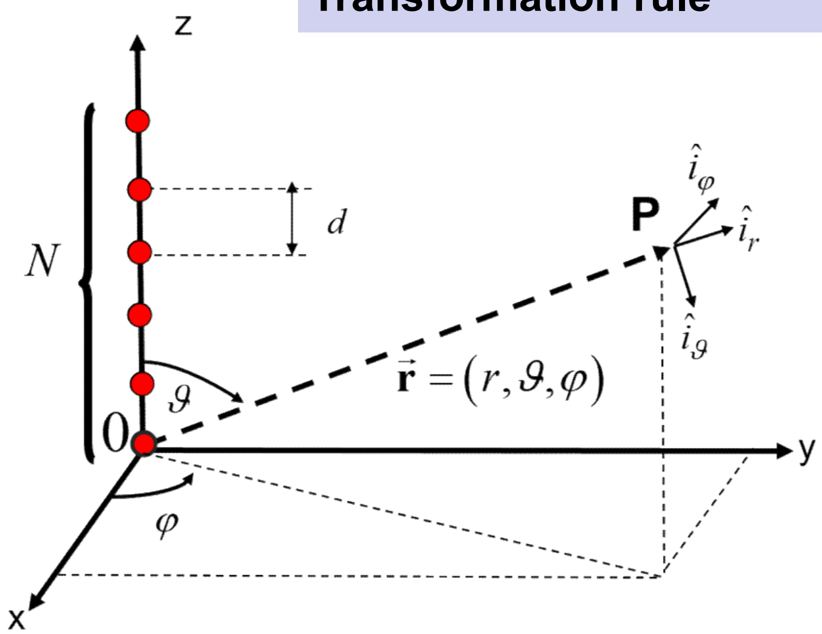
Periodic Linear Arrays (z-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

$$F(u) = \sum_{n=0}^{N-1} I_n \exp(-jnu)$$



The properties of the Fourier Transformation suggest some interesting considerations

Periodic Linear Arrays (z-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

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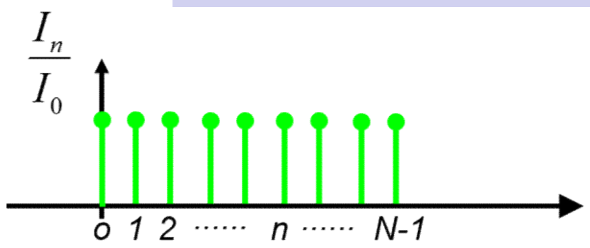

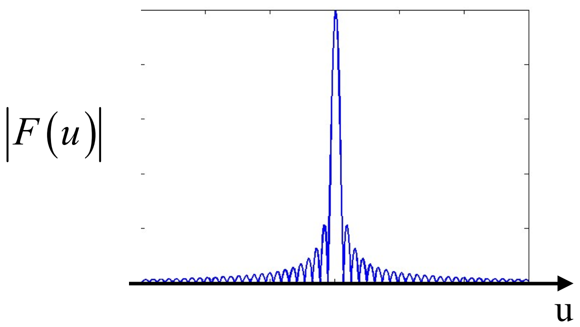

The properties of the Fourier Transformation suggest some interesting considerations

- **Arrays's size and beamwidth**

Periodic Linear Arrays (z-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{l}}(\vartheta,\varphi)F(\vartheta)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

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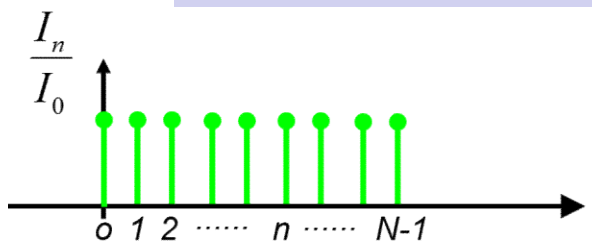

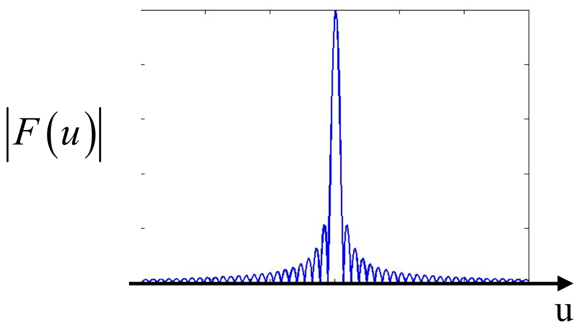

The properties of the Fourier Transformation suggest some interesting considerations

- **Arrays's size and beamwidth**
- **Scanning of the pattern**

Periodic Linear Arrays (z-axis)

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda}\frac{\exp(-j\beta r)}{r}\vec{\mathbf{I}}(\vartheta,\varphi)F(\vartheta)$$

For the periodic linear arrays the input excitations of the antennas of the array are related to the array factor through the Fourier Transformation rule

$$F(\vartheta) = F(u)\Big|_{u=-\beta d\cos\vartheta}$$

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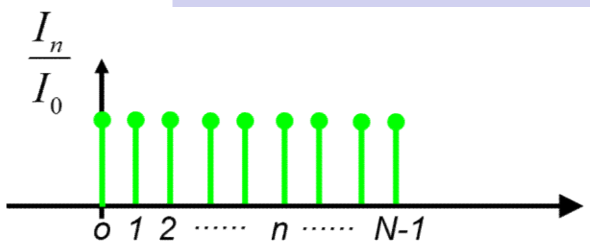

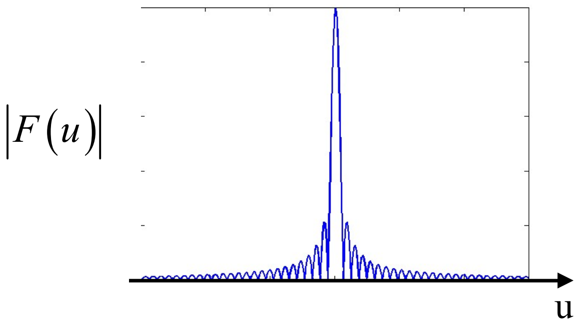

The properties of the Fourier Transformation suggest some interesting considerations

- **Arrays's size and beamwidth**
- **Scanning of the pattern**
- **Synthesis of the pattern**