

# Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle  
Telecomunicazioni

Università degli Studi di Napoli “Parthenope”

a.a. 2023–2024 – Laurea “Triennale” – Secondo semestre – Terzo anno

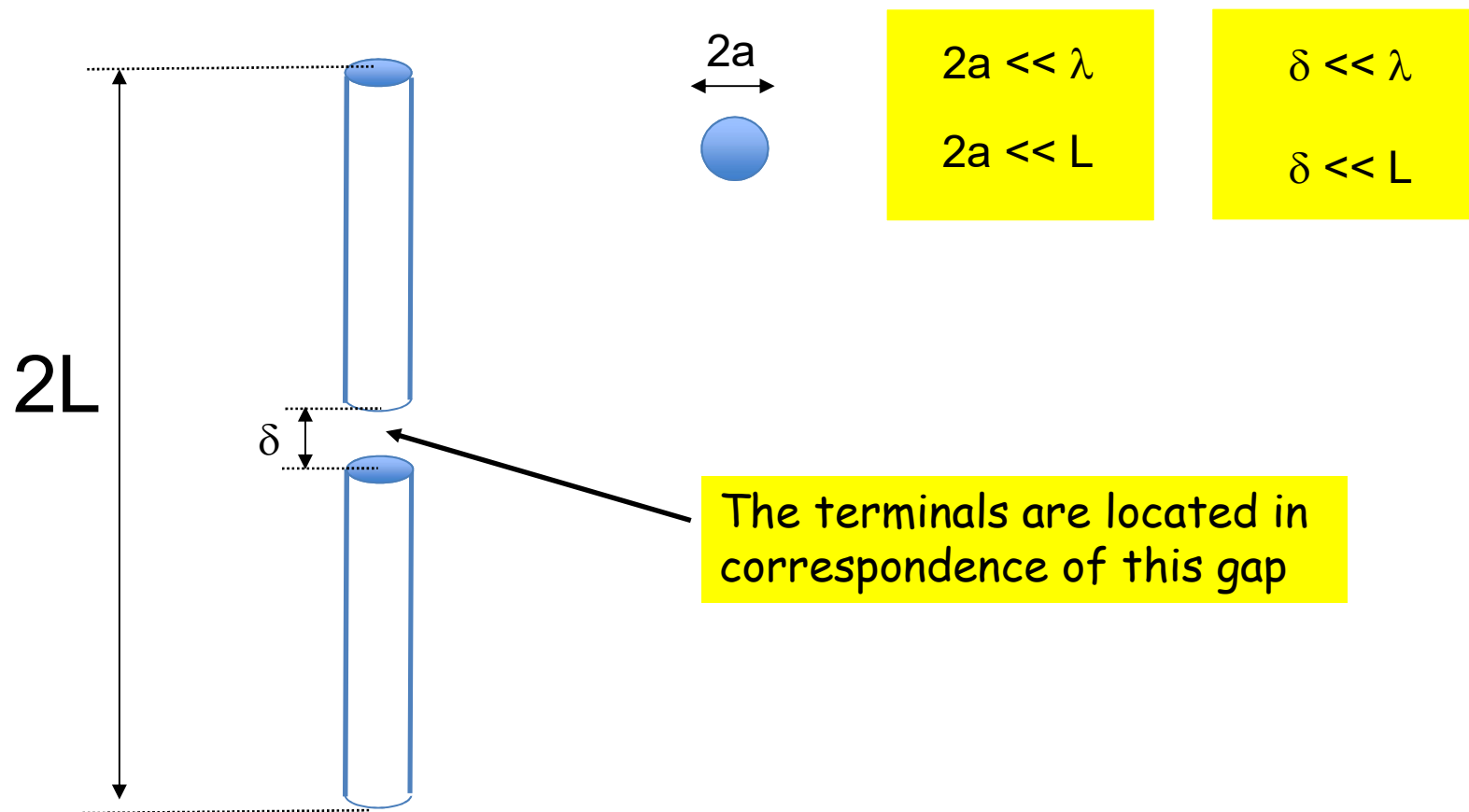
Ing. Stefano Perna

# Wire antennas

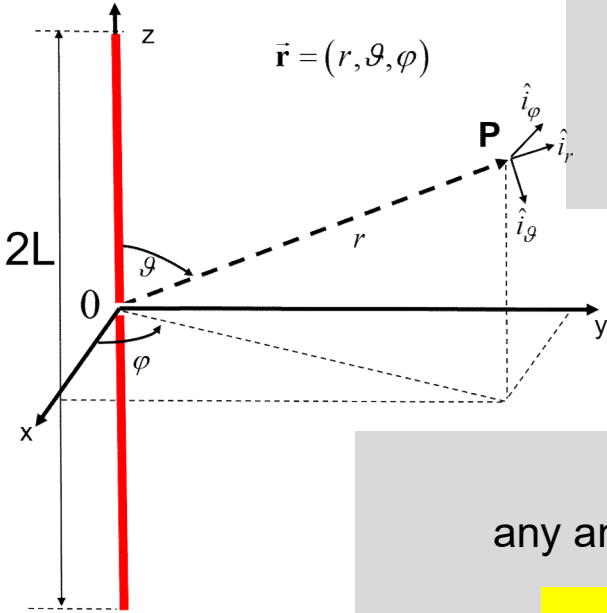
# Wire antennas



# Wire antennas



# Wire antennas



In the Fraunhofer Region the expression of the radiated field simplifies as

$$\vec{E} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \left[ \int_{-L}^L dz \frac{I(z)}{I_0} \exp(j\beta z \cos \vartheta) \right] \hat{i}_\vartheta$$

Effective length of the wire antenna

.... Memo

any antenna, in the Fraunhofer region, behaves as follows

$$\begin{aligned} r &\gg D \\ r &> \frac{2D^2}{\lambda} \\ r &\gg \lambda \end{aligned}$$

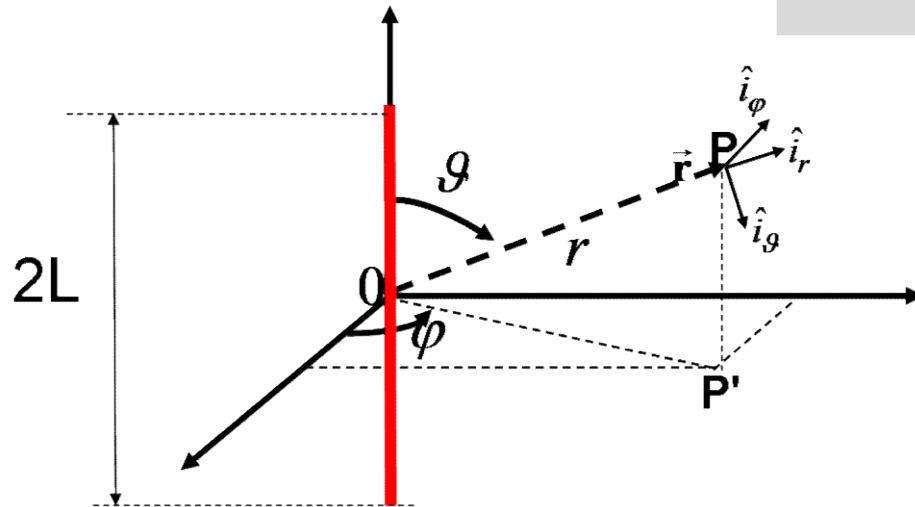
$$\begin{cases} \mathbf{E}(\vec{r}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I e^{-j\beta r}}{2\lambda r} \mathbf{I}(\vartheta, \varphi) \\ \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E} \end{cases}$$

$$\mathbf{I}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi) \hat{i}_\vartheta + l_\varphi(\vartheta, \varphi) \hat{i}_\varphi \quad \text{Effective length}$$

# Current distribution

In wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rules.

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \left[ \sin \vartheta F(\vartheta) \hat{i}_\vartheta \right]$$



$$F(\vartheta) = F(u) \Big|_{u = -\beta \cos \vartheta}$$

$$F(u) = \int_{-L}^L dz \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

# Wire antennas: visible region

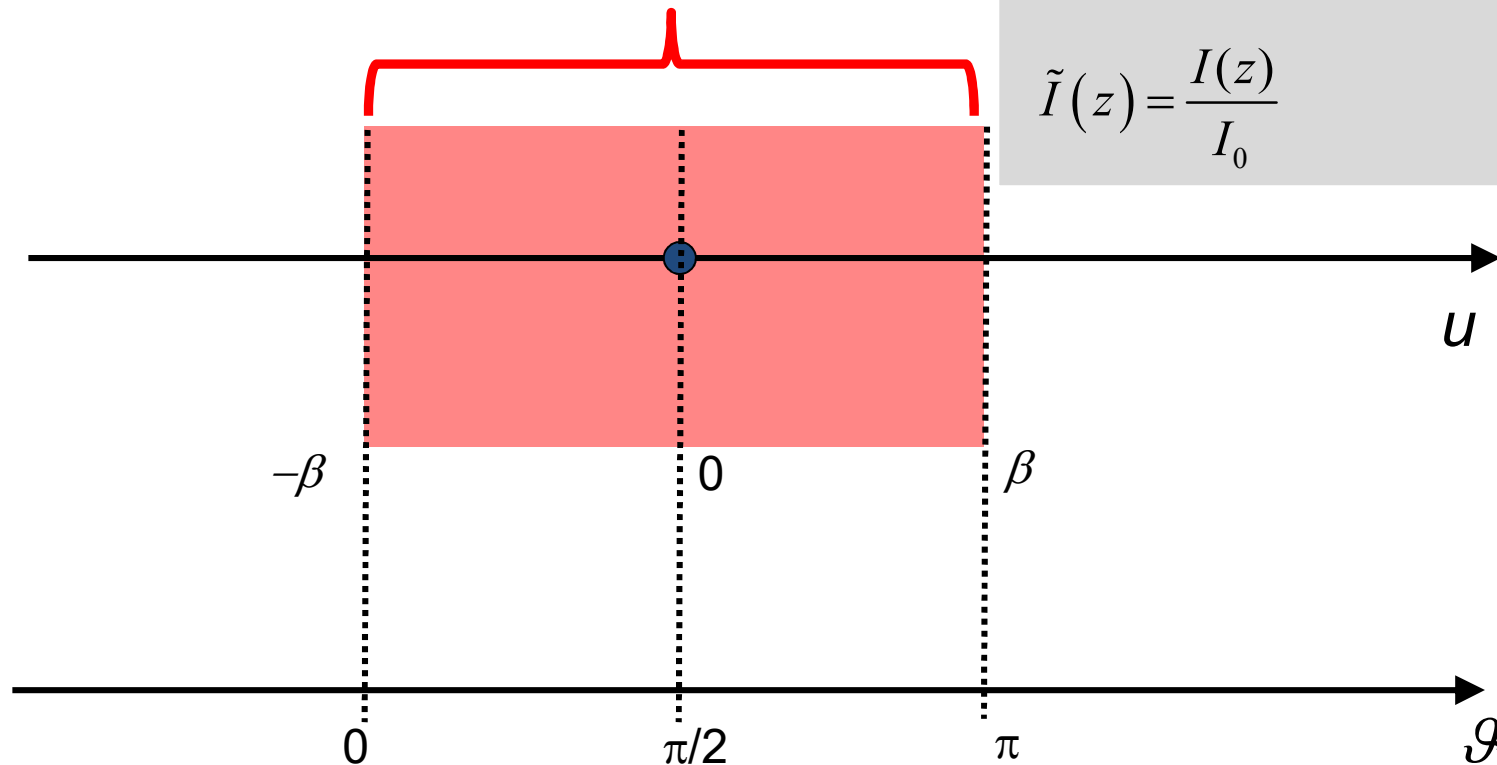
$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \left[ \sin \vartheta F(\vartheta) \hat{i}_\vartheta \right]$$

$$F(\vartheta) = F(u) \Big|_{u = -\beta \cos \vartheta}$$

$$F(u) = \int_{-L}^L dz \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

Visible region of the spectrum



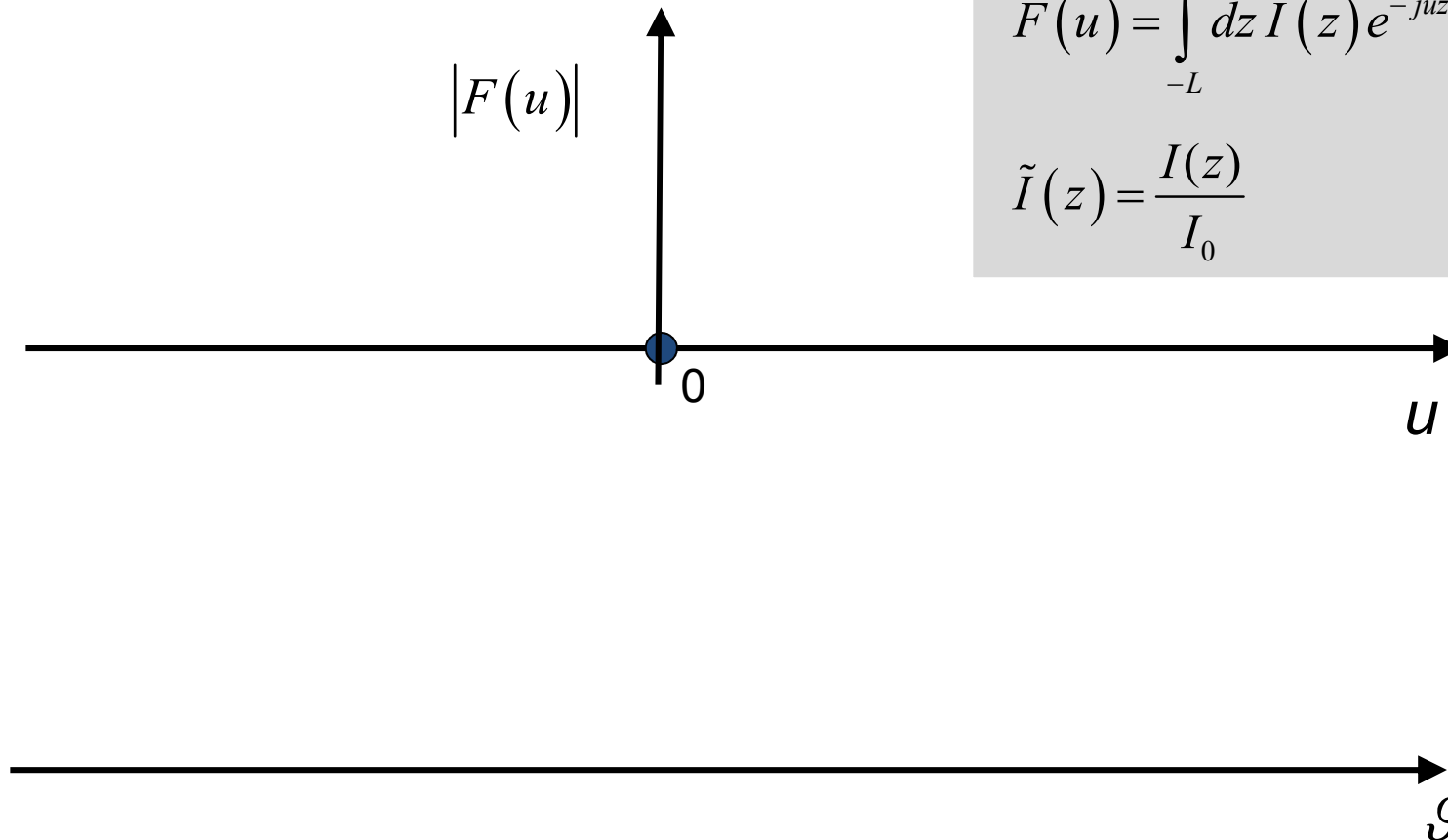
# Wire antennas: visible region

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \left[ \sin \vartheta F(\vartheta) \hat{i}_\vartheta \right]$$

$$F(\vartheta) = F(u) \Big|_{u = -\beta \cos \vartheta}$$

$$F(u) = \int_{-L}^L dz \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$





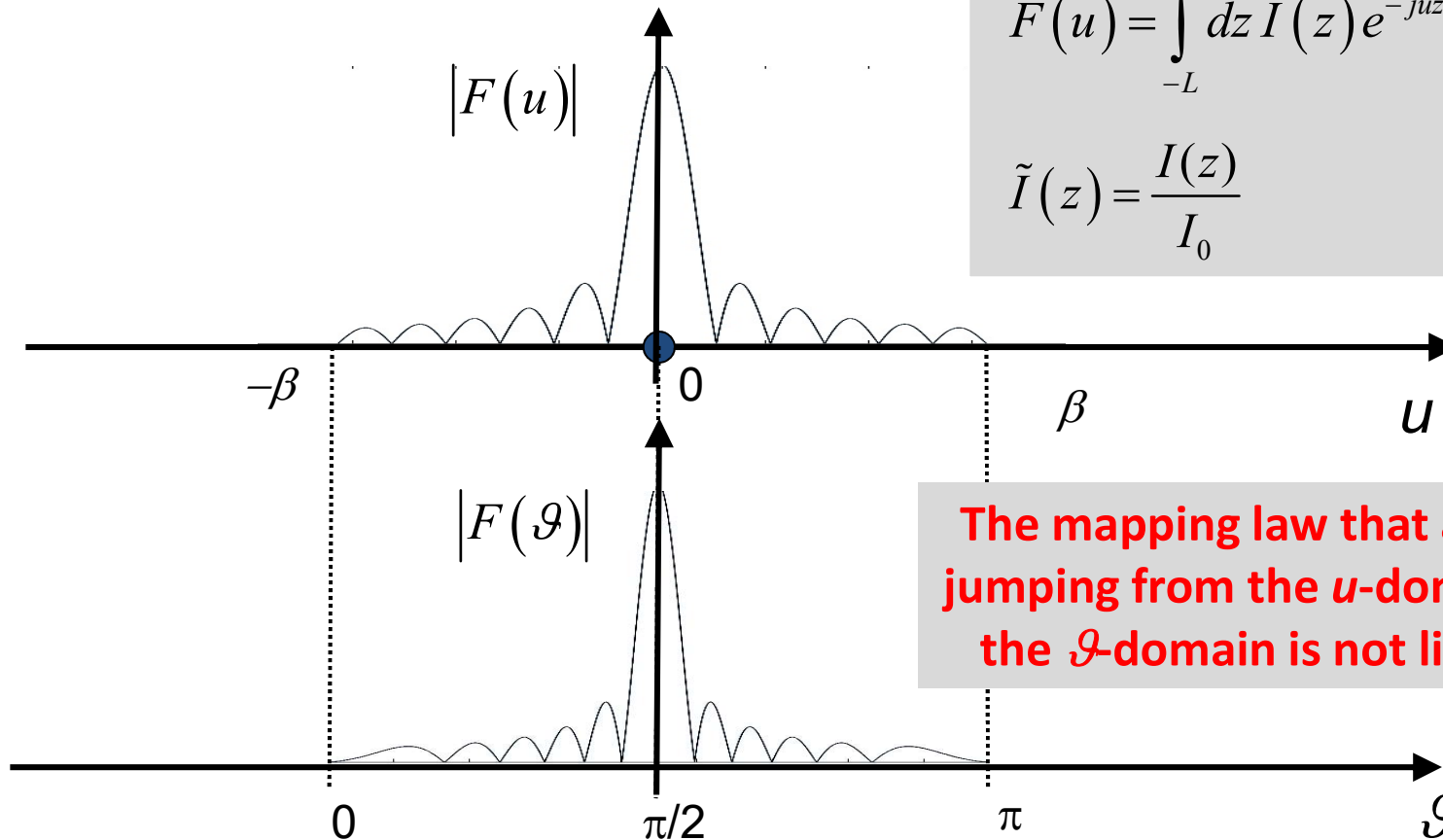
# Wire antennas: visible region

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} [\sin \vartheta F(\vartheta) \hat{i}_\vartheta]$$

$$F(\vartheta) = F(u) \Big|_{u = -\beta \cos \vartheta}$$

$$F(u) = \int_{-L}^L dz \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



**The mapping law that allows jumping from the  $u$ -domain to the  $\vartheta$ -domain is not linear!**

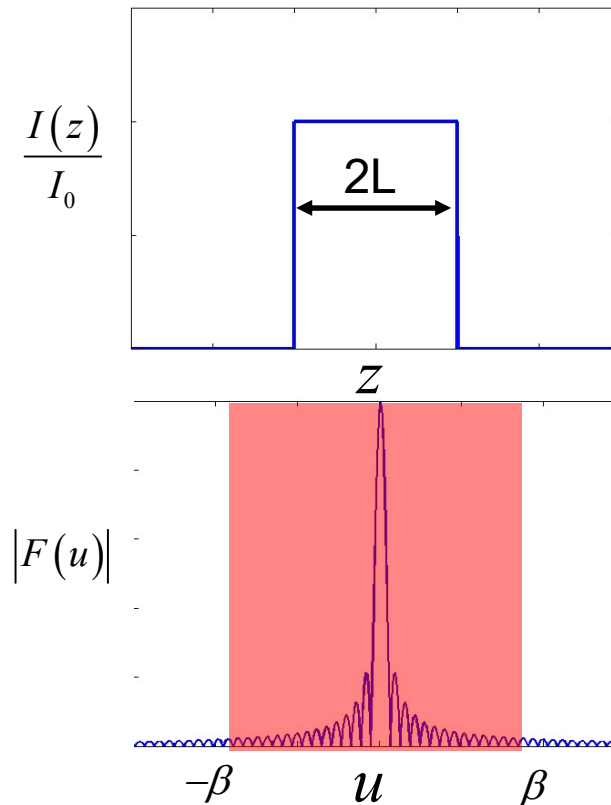
# Current distribution

## An ideal case

$$\frac{I(z)}{I_0} = \text{rect}\left[\frac{z}{2L}\right]$$

$$F(u) = \int \frac{I(z)}{I_0} e^{-juz} dz = 2L \frac{\sin(uL)}{uL}$$

$$u = -\beta \cos \vartheta$$

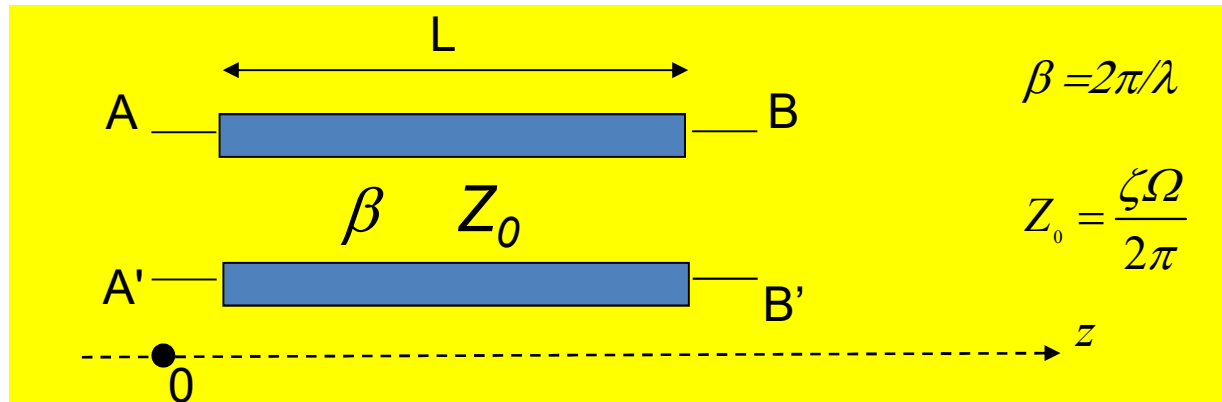
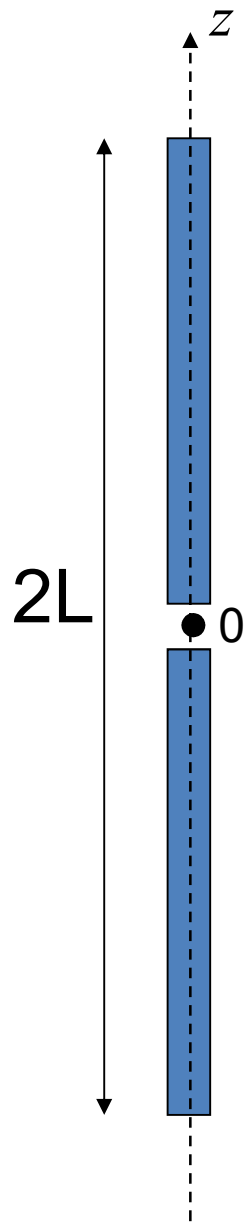


**Direction of the Main Lobe**  $\vartheta_{MB} = \frac{\pi}{2}$

**NNBW / HPBW**  $\text{NNBW} \approx \frac{\lambda}{L}$   $\text{HPBW} \approx 0.88 \frac{\lambda}{2L}$

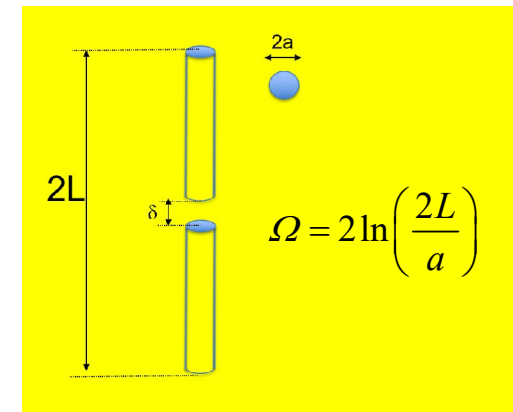
**SLL**  $\text{SLL} = -13.46 \text{ dB}$

# Hallen Formulation



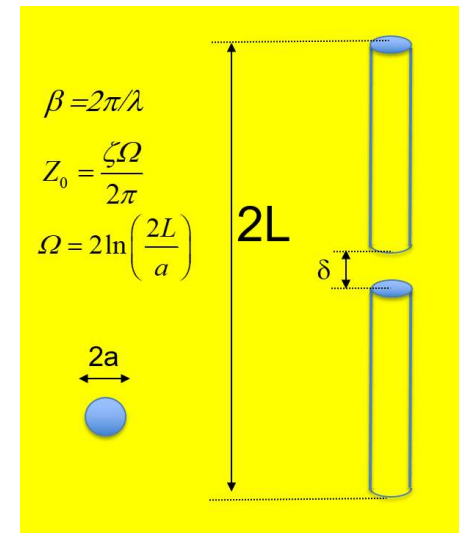
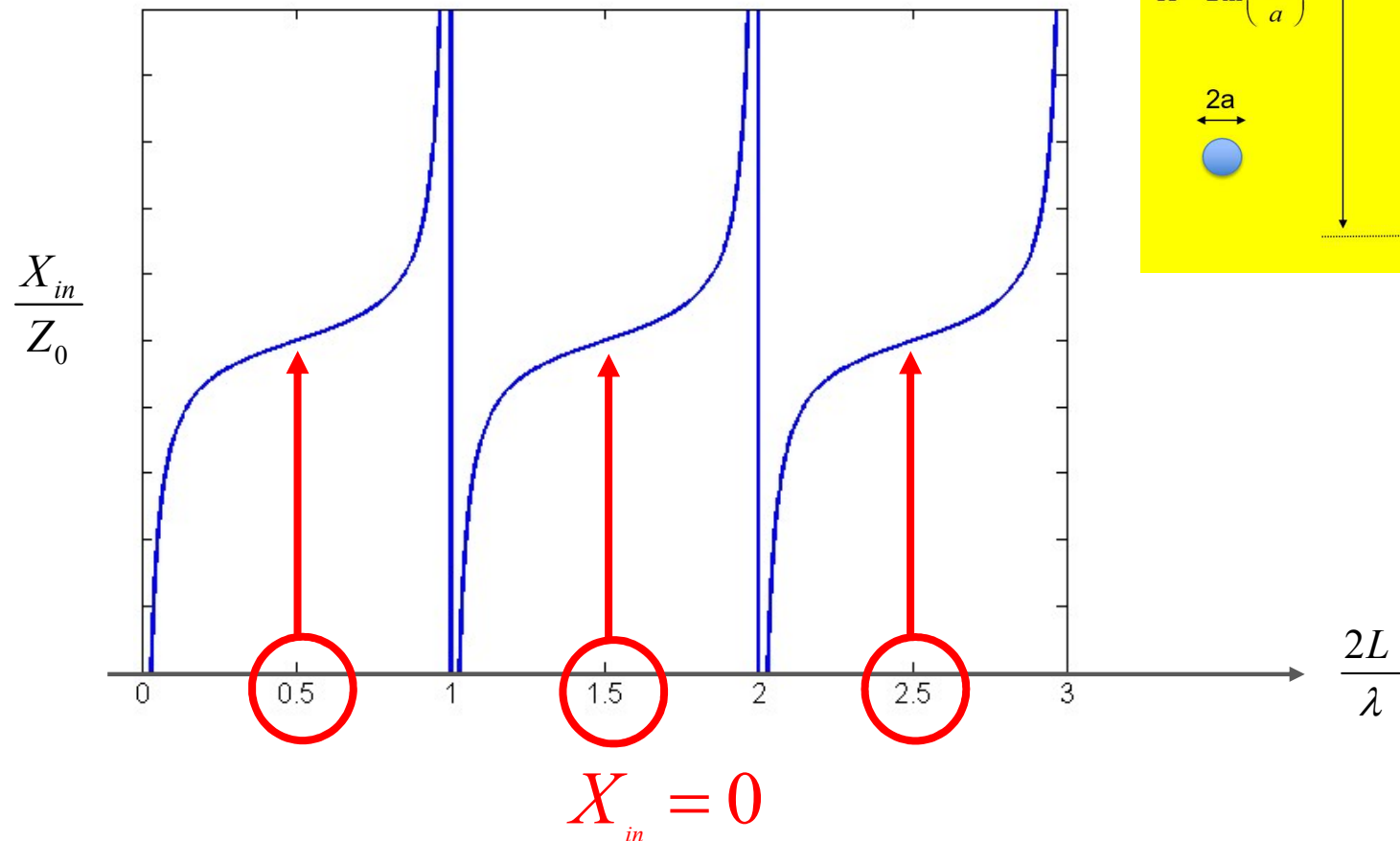
$$Z_{in} = -jZ_0 \operatorname{ctg}(\beta L)$$

$$I(z) = I_0 \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$



# Hallen Formulation

$$Z_{in} = -jZ_o \operatorname{ctg}(\beta L) = -jZ_o \operatorname{ctg}\left(\frac{2\pi}{\lambda} L\right) = jX_{in}$$



# Wire antennas

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \left[ \sin \vartheta F(\vartheta) \hat{i}_\vartheta \right]$$

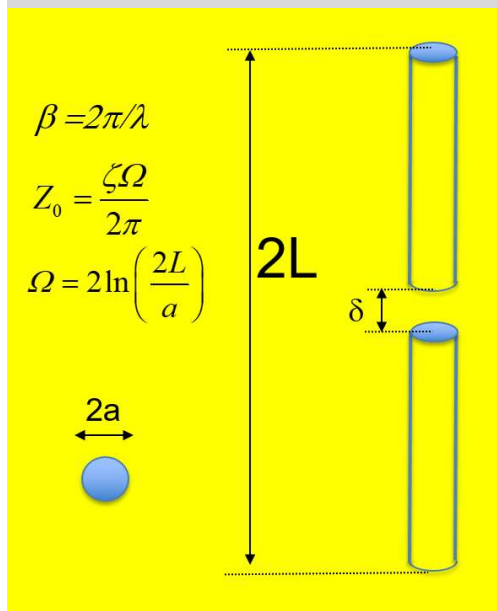
$$\vec{\mathbf{I}}(\vartheta) = \sin \vartheta F(\vartheta) \hat{i}_\vartheta$$

$$F(\vartheta) = F(u) \Big|_{u = -\beta \cos \vartheta}$$

$$F(u) = \int_{-L}^L dz \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0} = \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$

$$X_{\text{in}} = -Z_0 \text{ctg}(\beta L)$$



# Short dipole

## Short dipole

$$2L \ll \lambda$$

$$\vec{I}(\vartheta) = L \sin \vartheta \hat{i}_\vartheta$$

$$D(\vartheta, \varphi) = \frac{3}{2} \sin^2 \vartheta \quad D_{\max} = 1.76 \text{ dB}$$

$$Z_{in} = R_{in} + jX_{in}$$

$$R_{rad} = \frac{2\pi}{3} \zeta \left( \frac{L}{\lambda} \right)^2$$

$$X_{in} = -Z_o \operatorname{ctg}(\beta L)$$

$$\beta = 2\pi/\lambda$$

$$Z_o = \frac{\zeta \Omega}{2\pi}$$

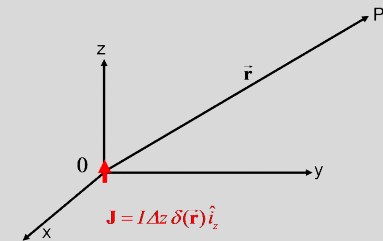
$$\Omega = 2 \ln \left( \frac{2L}{a} \right)$$

## Elementary electrical dipole

$$\vec{I}(\vartheta) = \Delta z \sin \vartheta \hat{i}_\vartheta$$

$$D(\vartheta, \varphi) = \frac{3}{2} \sin^2 \vartheta \quad D_{\max} = 1.76 \text{ dB}$$

$$R_{rad} = \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2$$

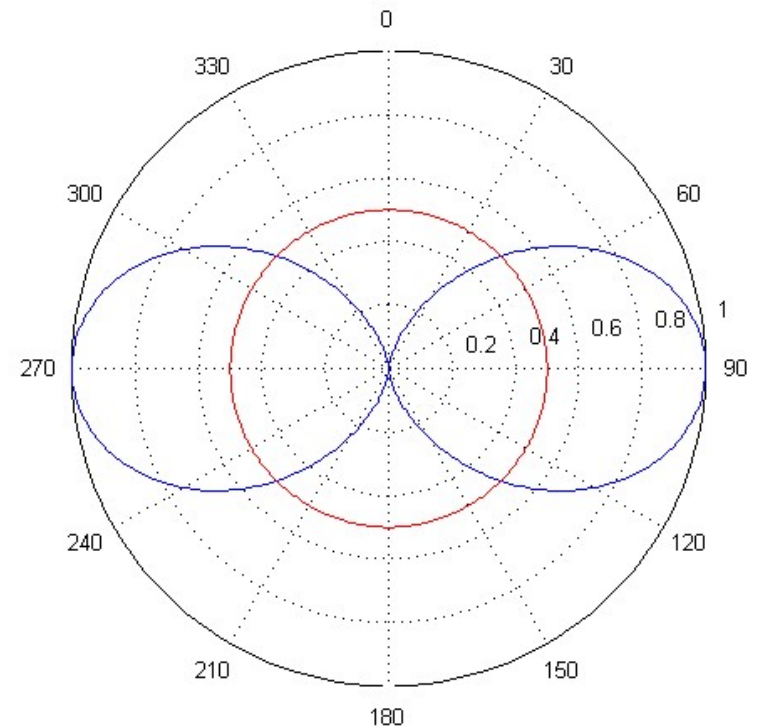
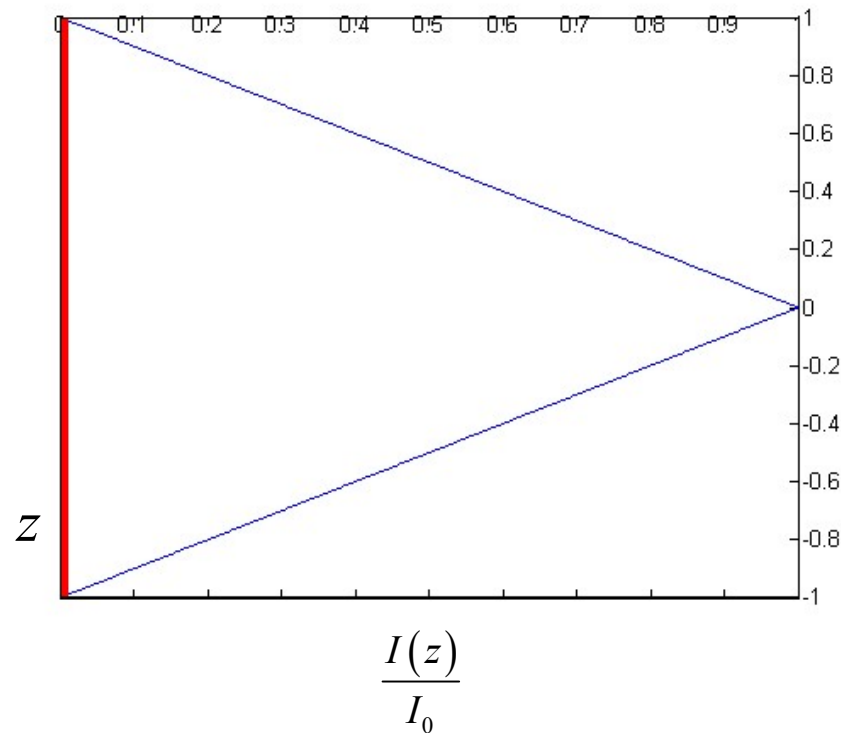


# Short dipole

Power pattern  
(vertical plane)

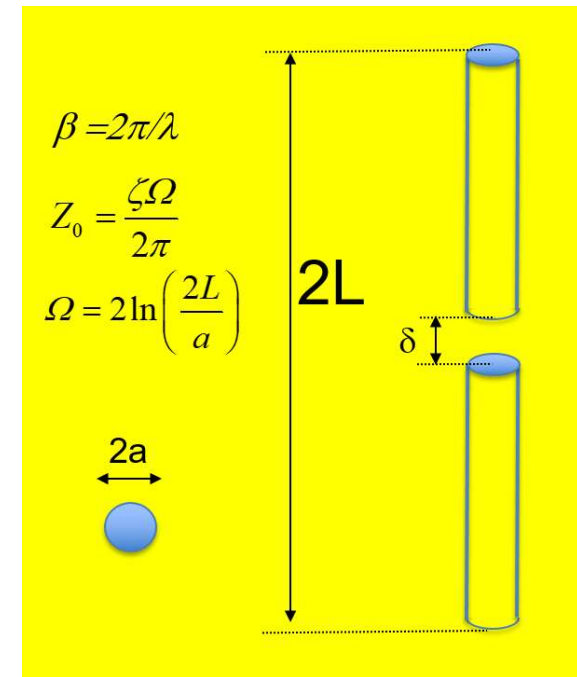
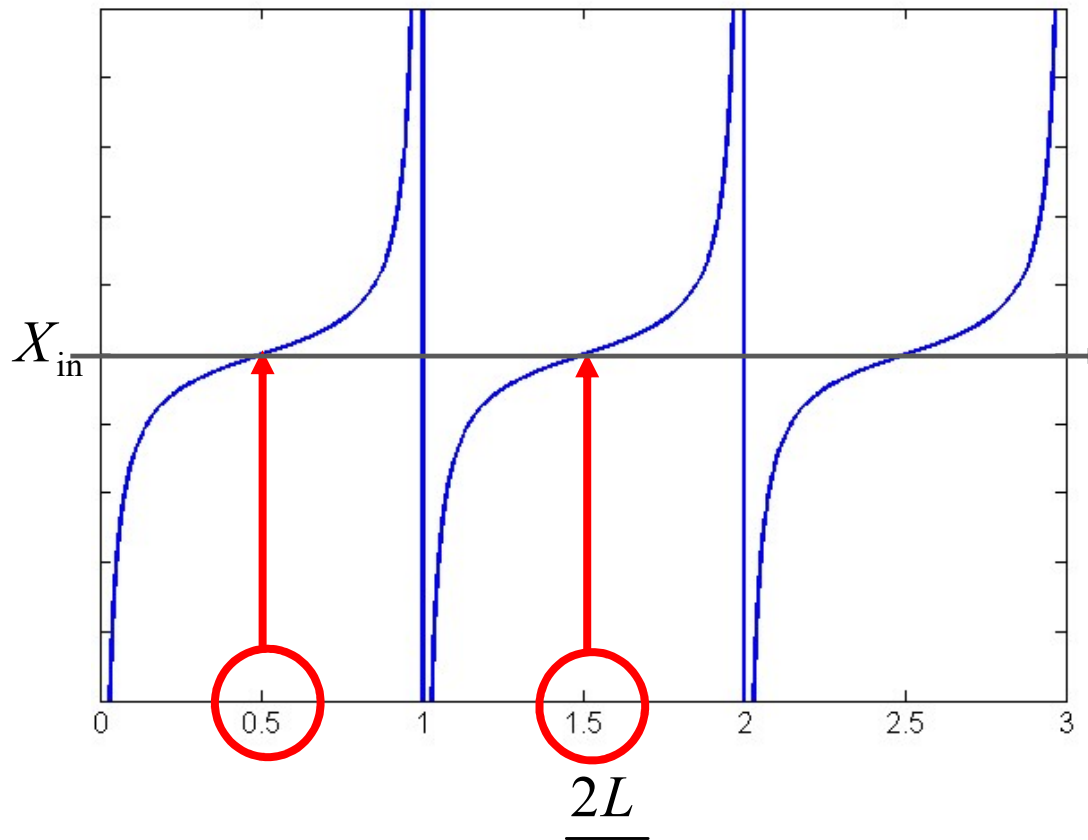
$$2L=0.01\lambda$$

Current distribution



# Wire antennas

$\lambda/2$  antenna &  $3\lambda/2$  antennas



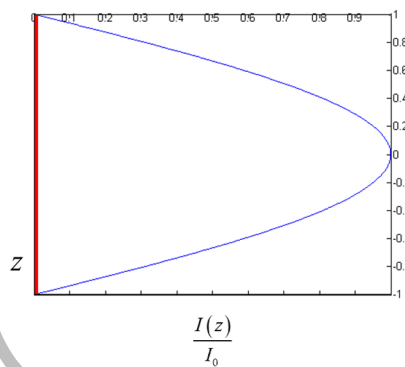


# $\lambda/2$ vs. $3\lambda/2$

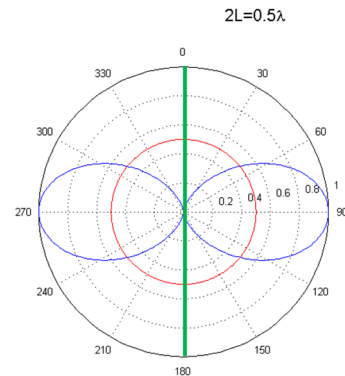
$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0} \exp(j\beta z \cos \vartheta)$$

## Half wavelength antenna

Current distribution

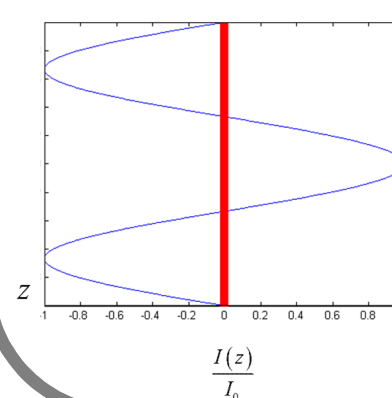


Power pattern  
(vertical plane)

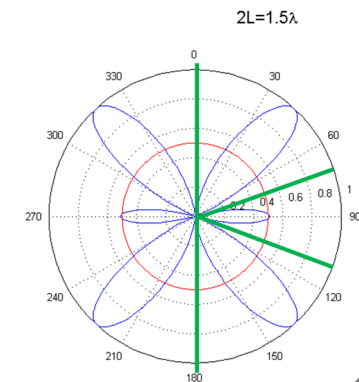


## 3/2 wavelength antenna

Current distribution

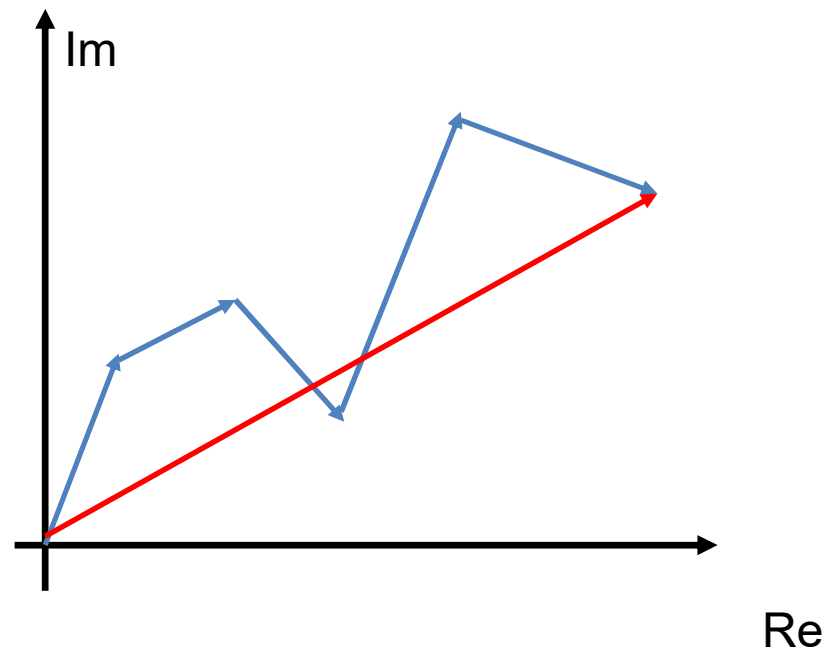


Power pattern  
(vertical plane)



# $\lambda/2$ vs. $3\lambda/2$

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0} \exp(j\beta z \cos \vartheta)$$



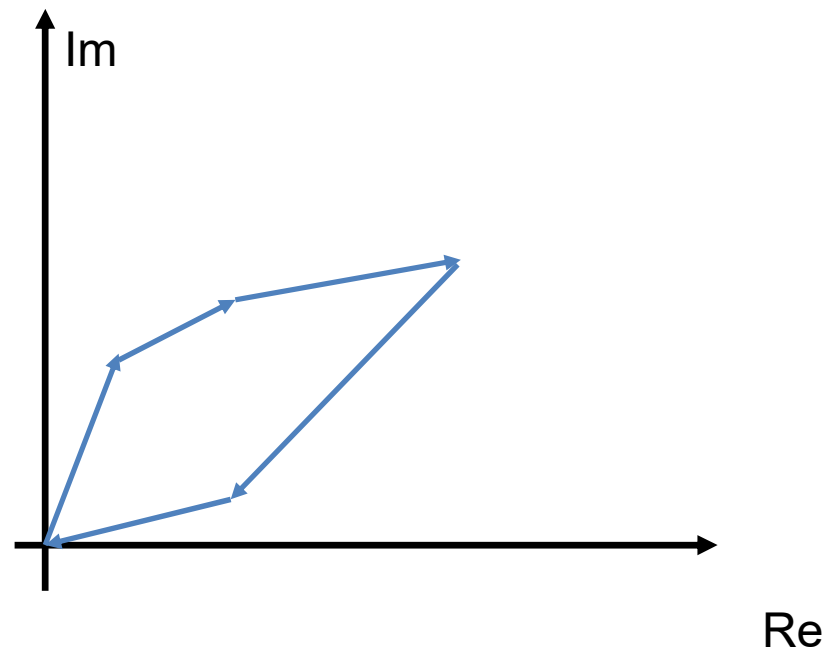
# $\lambda/2$ vs. $3\lambda/2$

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0} \exp(j\beta z \cos \vartheta)$$



# $\lambda/2$ vs. $3\lambda/2$

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0} \exp(j\beta z \cos \vartheta)$$



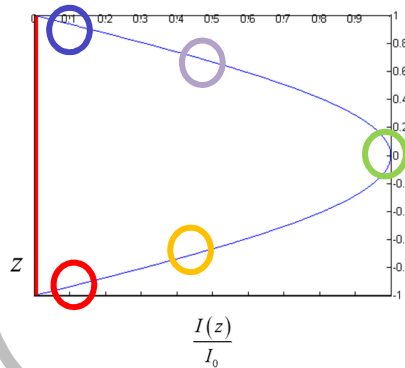
# $\lambda/2$ vs. $3\lambda/2$

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0}$$

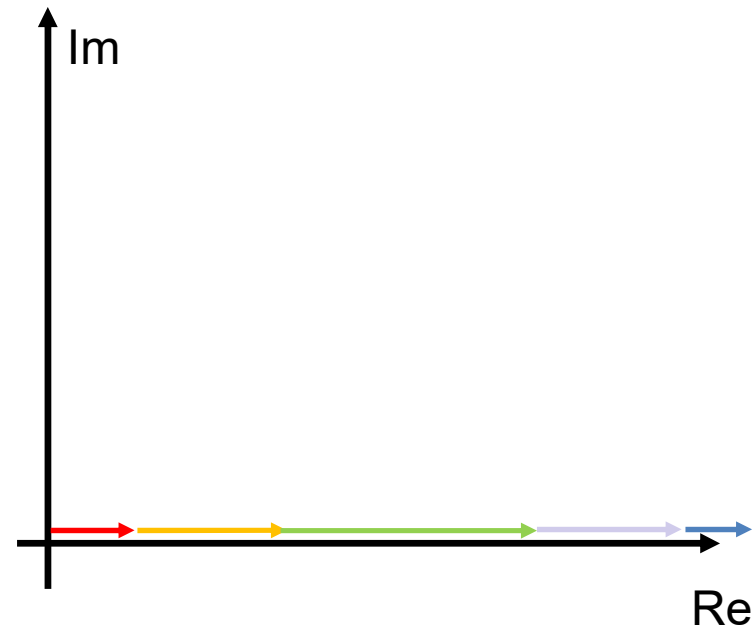
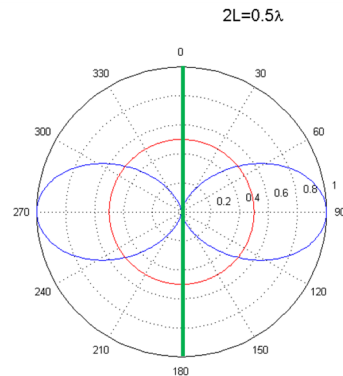
$$\vartheta = \pi/2$$

## Half wavelength antenna

Current distribution



Power pattern  
(vertical plane)



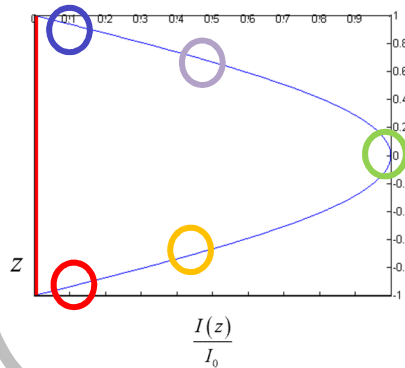
# $\lambda/2$ vs. $3\lambda/2$

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0} \exp(j\beta z \cos \vartheta)$$

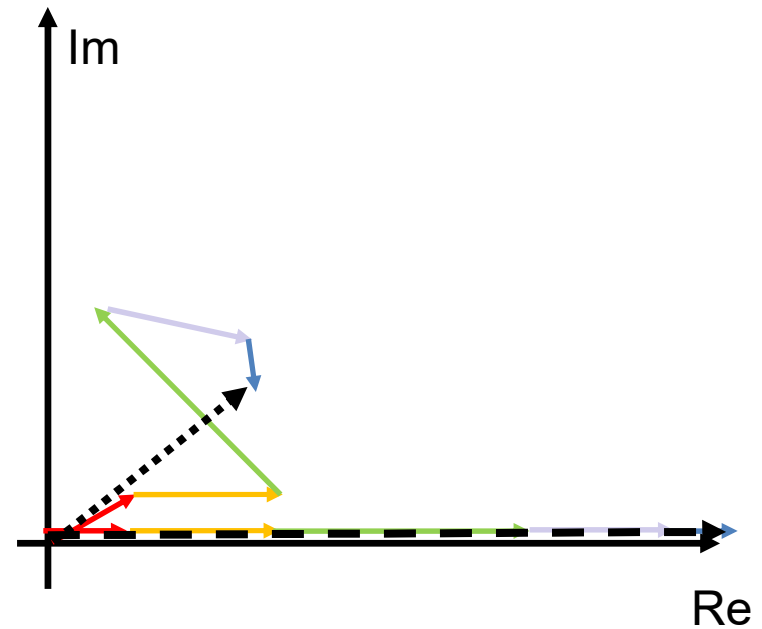
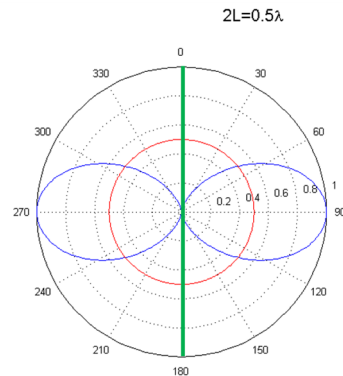
$$\vartheta \neq \pi/2$$

## Half wavelength antenna

Current distribution



Power pattern  
(vertical plane)



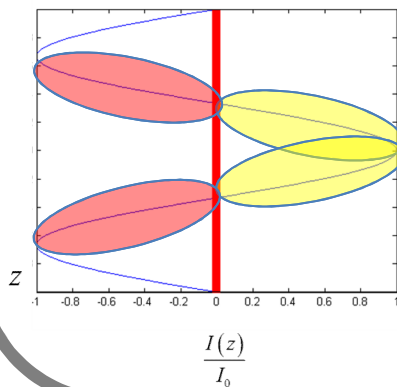
# $\lambda/2$ vs. $3\lambda/2$

$$\vec{\mathbf{E}} = j \frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin \vartheta \hat{i}_\vartheta \int_{-l}^l dz \frac{I(z)}{I_0}$$

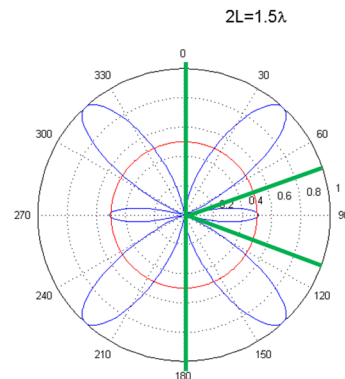
$$\vartheta = \pi/2$$

## 3/2 wavelength antenna

Current distribution



Power pattern  
(vertical plane)

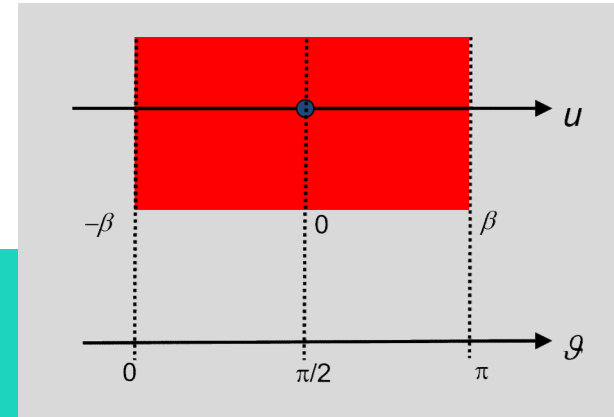


# Numerical examples



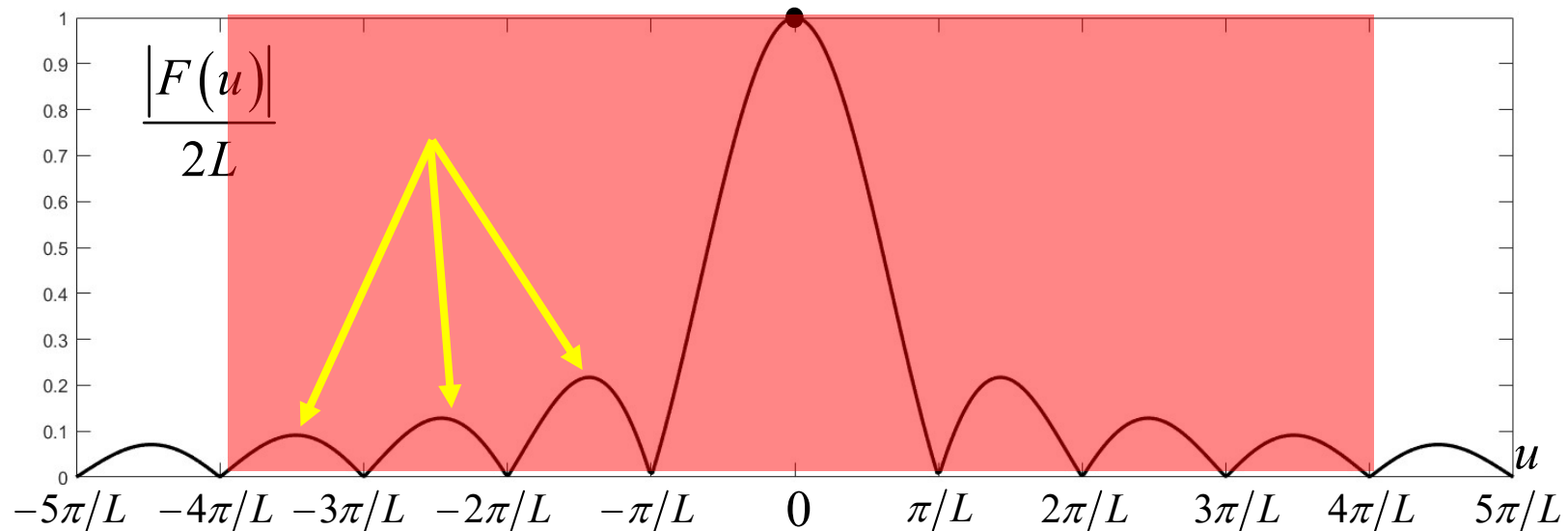
# Wire antennas: an ideal case

$$F(u) = 2L \frac{\sin(uL)}{uL}$$



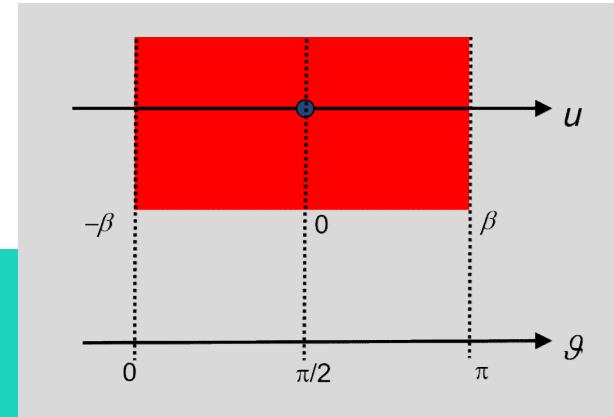
## Exercise n.1

$$2L = 4\lambda \quad \Rightarrow \quad \lambda = \frac{L}{2} \quad \Rightarrow \quad \beta = \frac{2\pi}{\lambda} = \frac{4\pi}{L}$$



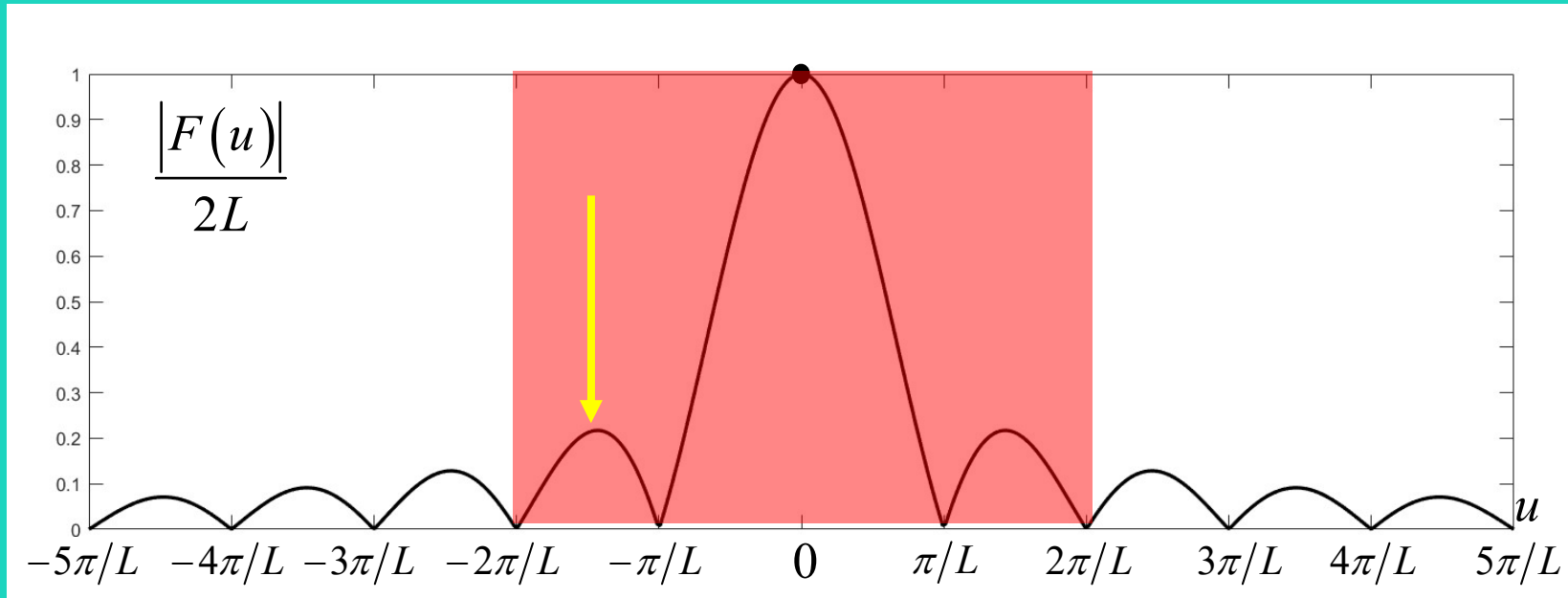
# Wire antennas: an ideal case

$$F(u) = 2L \frac{\sin(uL)}{uL}$$



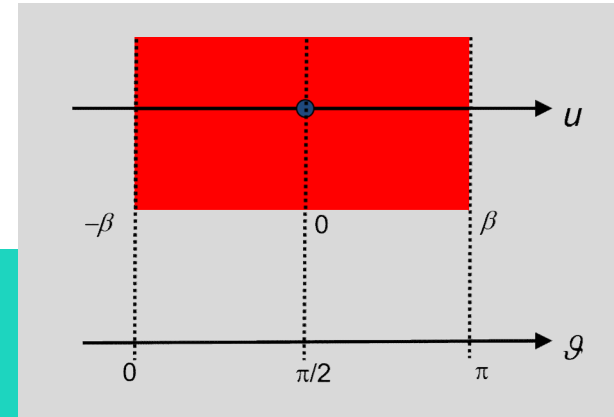
## Exercise n.2

$$2L = 2\lambda \quad \longrightarrow \quad \lambda = L \quad \longrightarrow \quad \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{L}$$



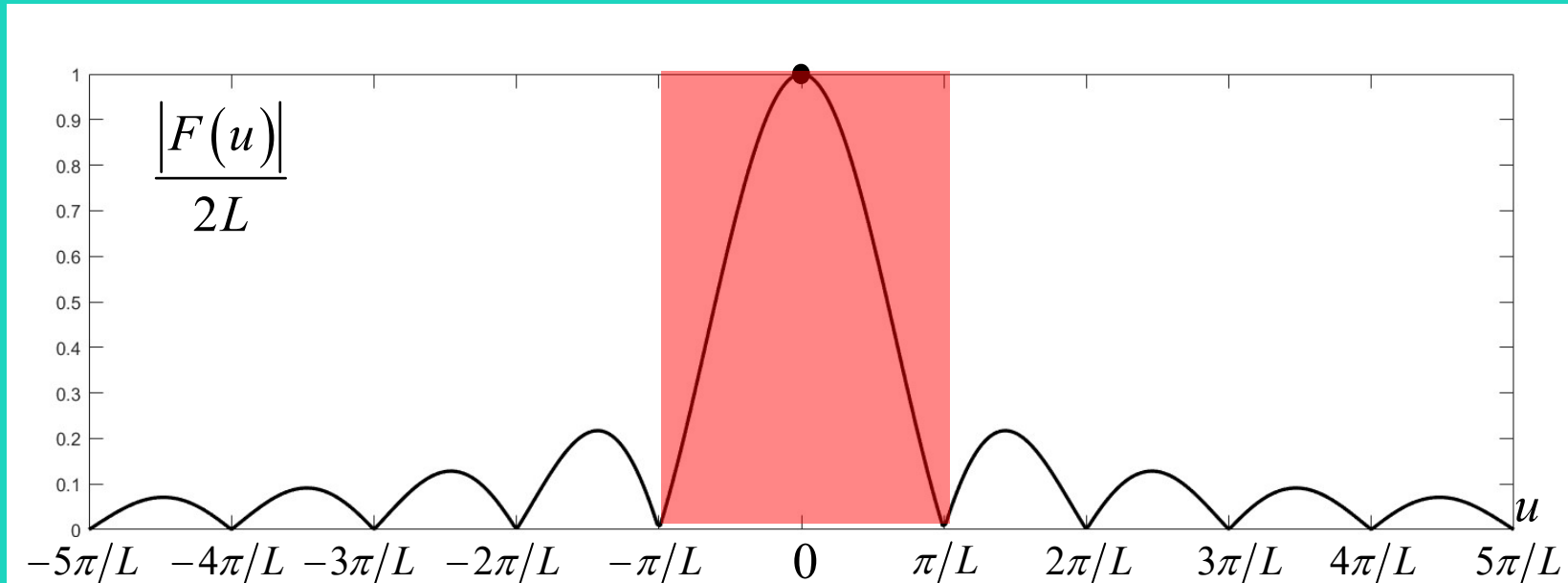
# Wire antennas: an ideal case

$$F(u) = 2L \frac{\sin(uL)}{uL}$$



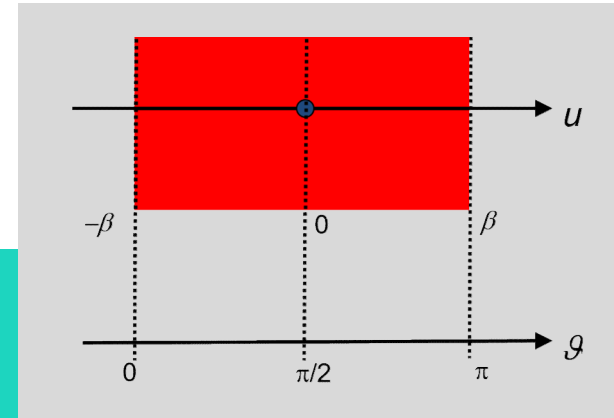
## Exercise n.3

$$2L = \lambda \quad \Rightarrow \quad \beta = \frac{2\pi}{\lambda} = \frac{\pi}{L}$$



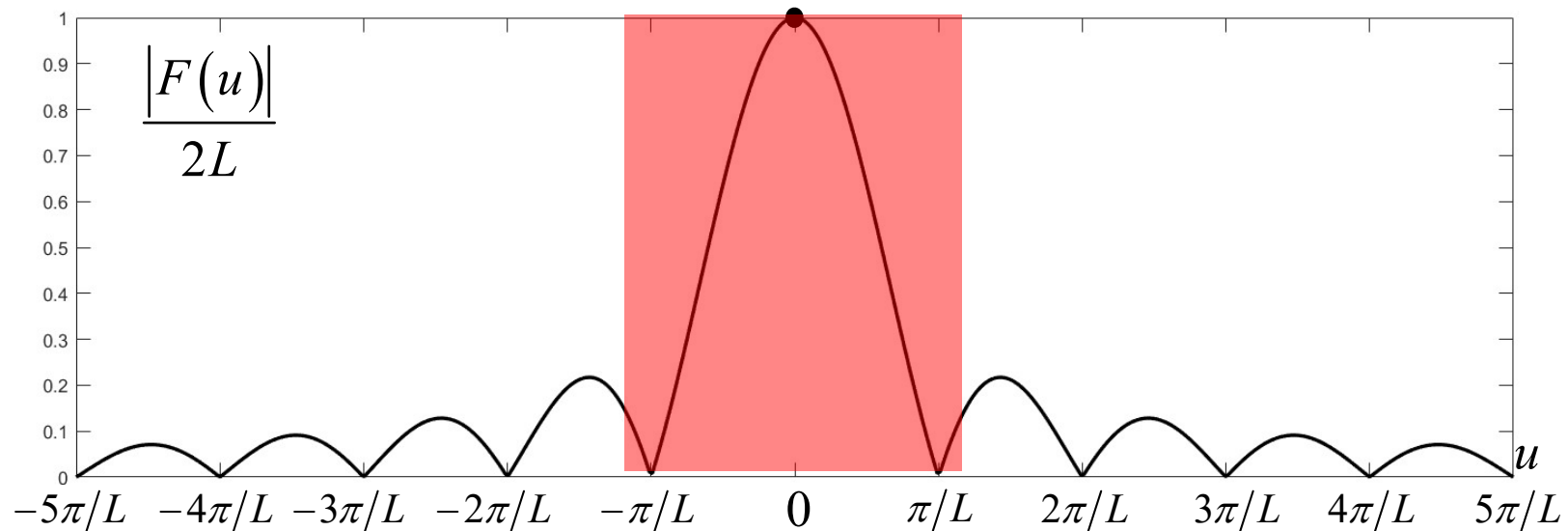
# Wire antennas: an ideal case

$$F(u) = 2L \frac{\sin(uL)}{uL}$$



## Exercise n.4

$$2L = 1.2 \lambda \quad \Rightarrow \quad \lambda = \frac{2}{1.2} L \quad \Rightarrow \quad \beta = \frac{2\pi}{\lambda} = 1.2 \frac{\pi}{L}$$



# Wire antennas: the **real** case

## Directivity

