Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Università degli Studi di Napoli “Parthenope”

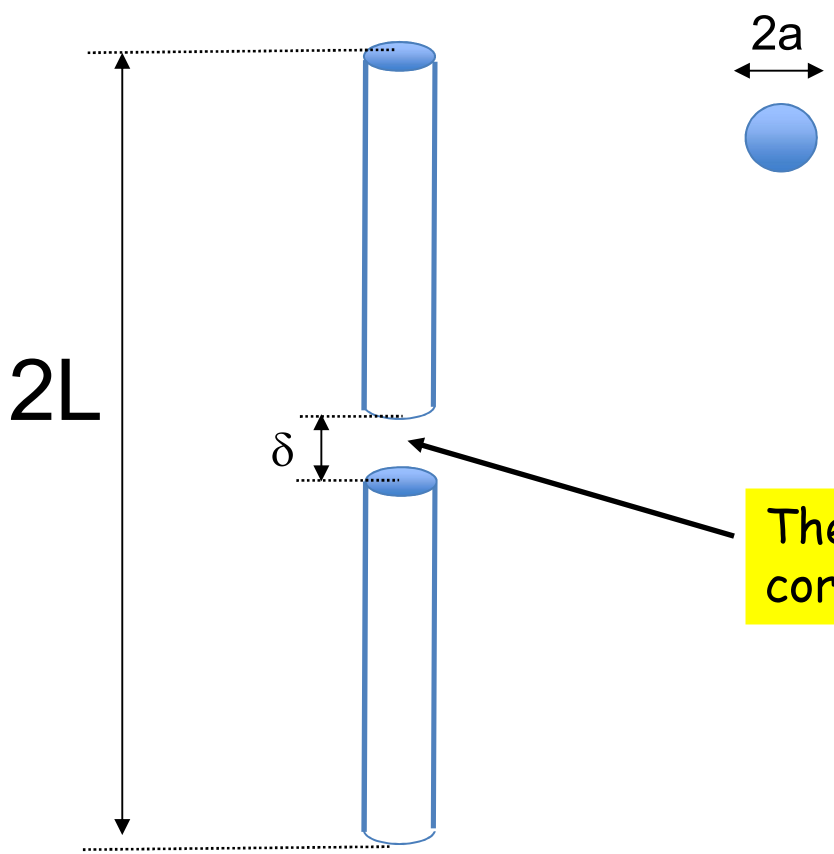
a.a. 2023–2024 – Laurea “Triennale” – Secondo semestre – Terzo anno

Ing. Stefano Perna

Wire antennas

Wire antennas

Wire antennas



$2a << \lambda$

$2a << L$

$\delta << \lambda$

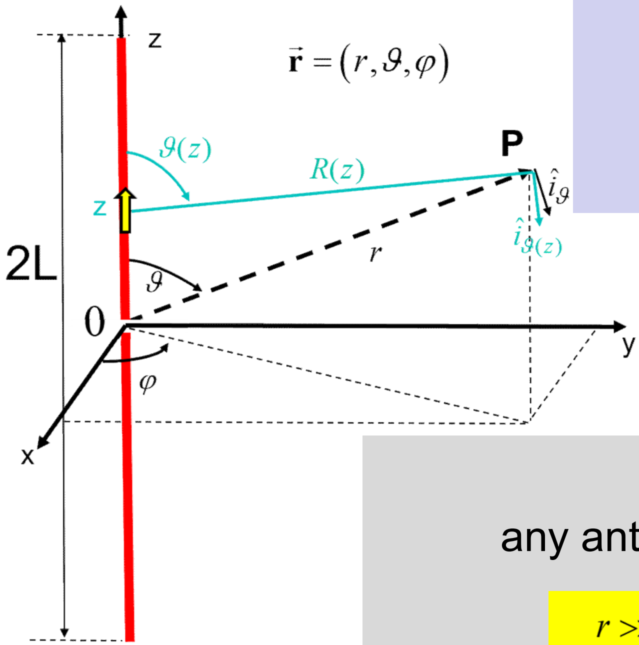
$\delta << L$

The terminals are located in correspondence of this gap

Wire antennas

In the Fraunhofer Region the expression of the radiated field simplifies as

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta \left[\int_{-L}^{L} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta) \right] \hat{i}_\vartheta$$



Effective length of the wire antenna

.... Memo

any antenna, in the Fraunhofer region, behaves as follows

$r >> D$

$r > \frac{2D^2}{\lambda}$

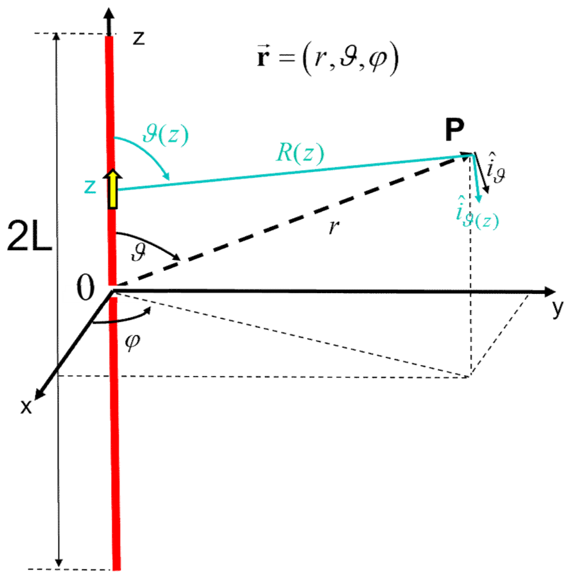
$r >> \lambda$

$$\begin{cases} \mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi) \\ \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E} \end{cases}$$

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$ **Effective length**

Wire antennas: effective length

$$\vec{\mathbf{l}}(\vartheta) = l_\vartheta(\vartheta)\hat{i}_\vartheta = \sin\vartheta\left[\int_{-L}^{L} dz \frac{I(z)}{I_0}\exp(j\beta z\cos\vartheta)\right]\hat{i}_\vartheta$$



$$u = -\beta\cos\vartheta \qquad \tilde{I}(z) = \frac{I(z)}{I_0}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z)\, e^{-juz}$$

For the wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rule

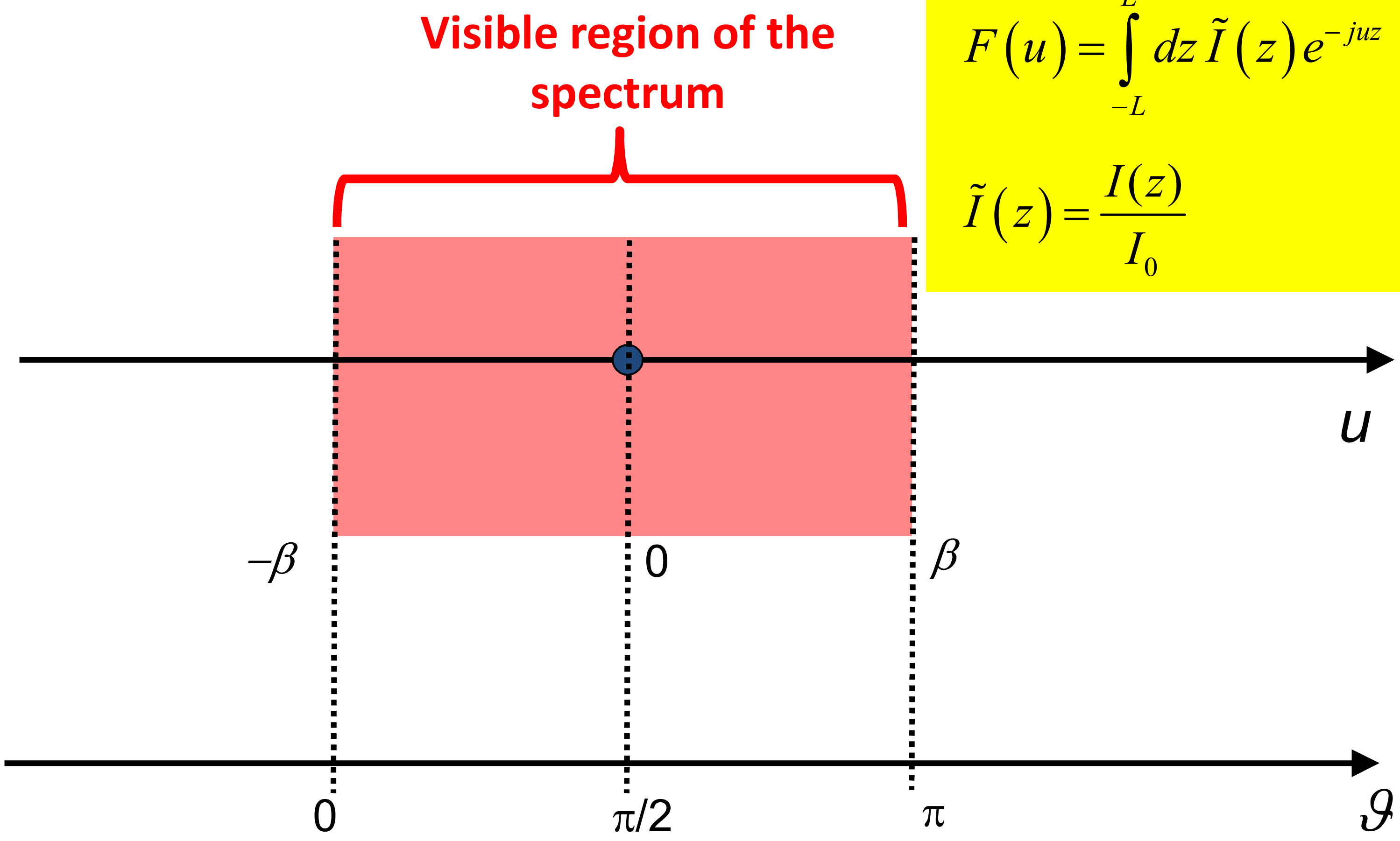
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$



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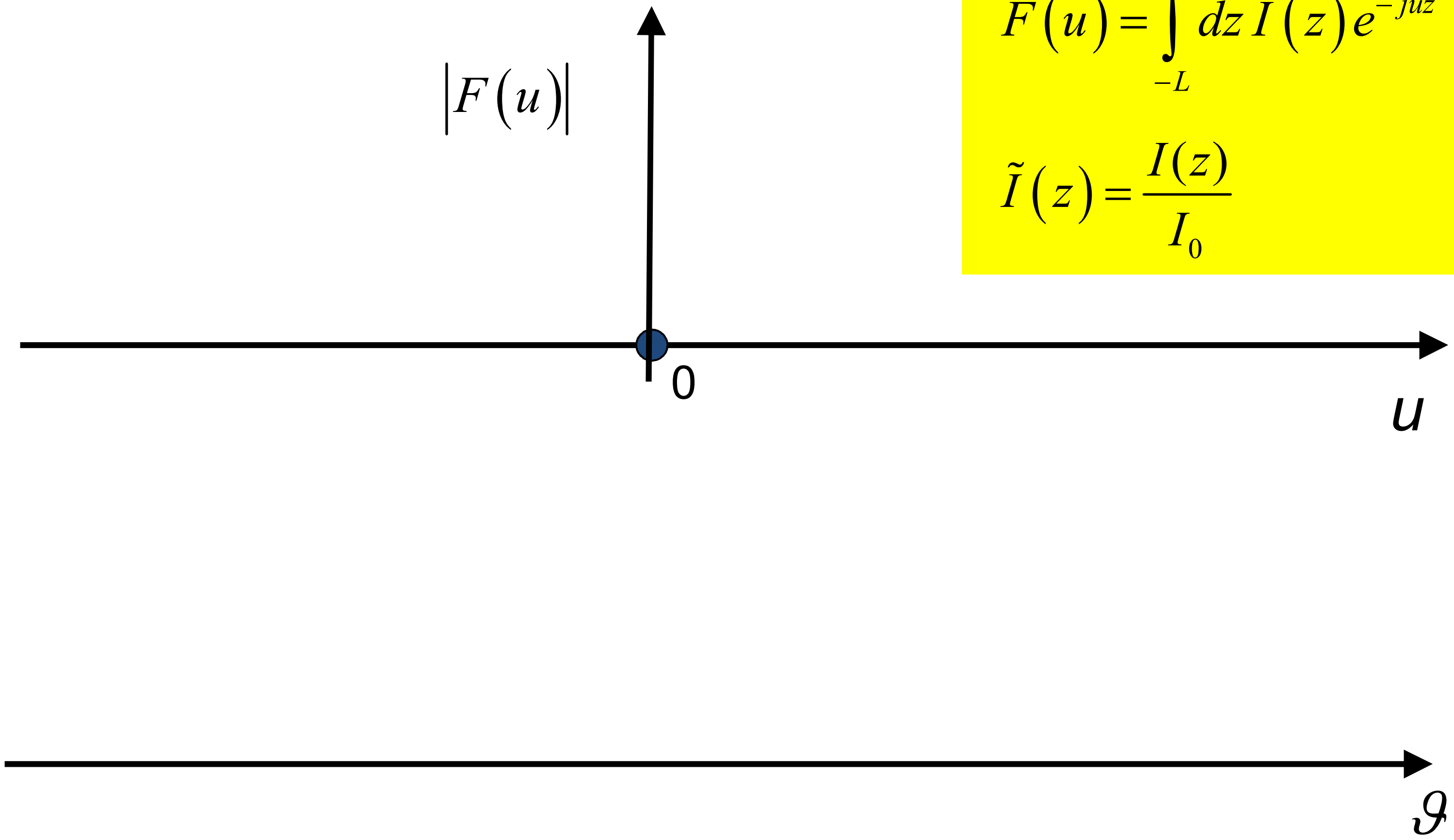

Wire antennas: visible region

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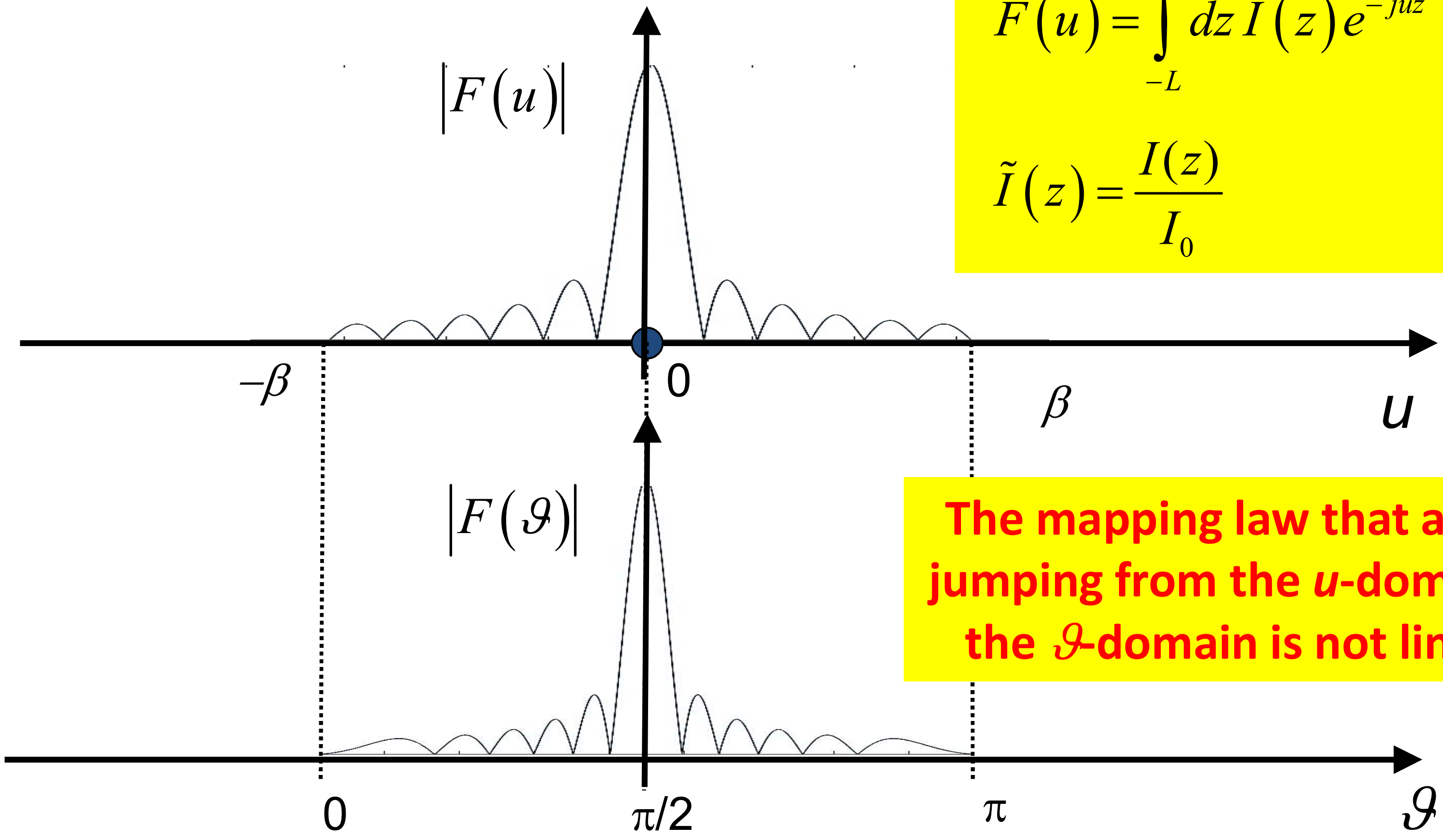


The mapping law that allows jumping from the *u*-domain to the ϑ-domain is not linear!

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

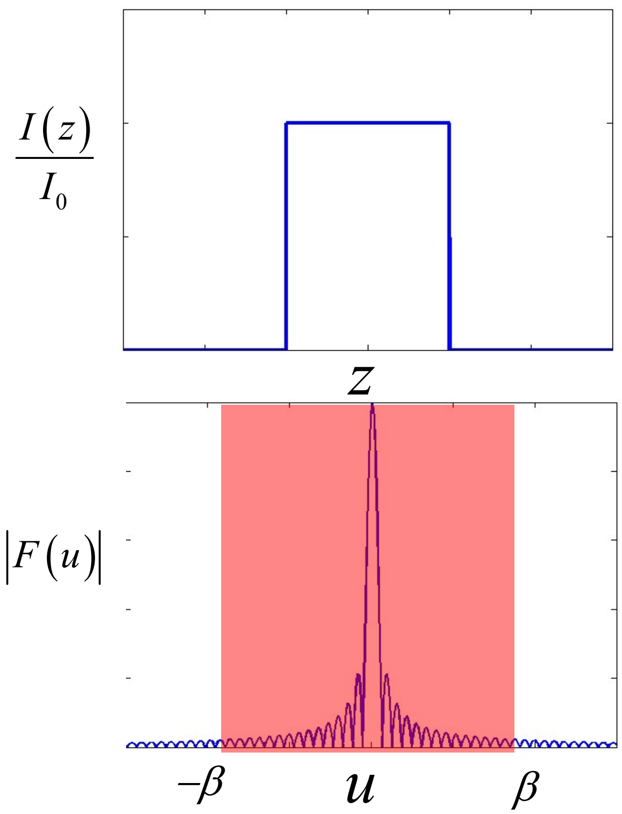
Current distribution

An ideal case

$$\frac{I(z)}{I_0} = \mathrm{rect}\left[\frac{z}{2L}\right]$$

$$F(u) = \int \frac{I(z)}{I_0} e^{-juz}\, dz \; = 2L\frac{\sin(uL)}{uL}$$

$$u = -\beta \cos\vartheta$$

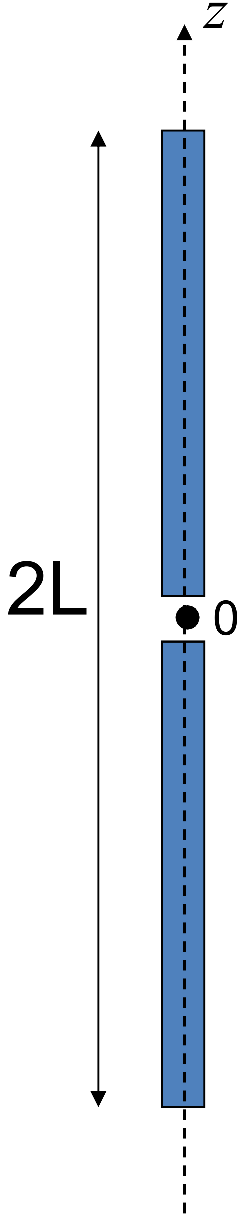


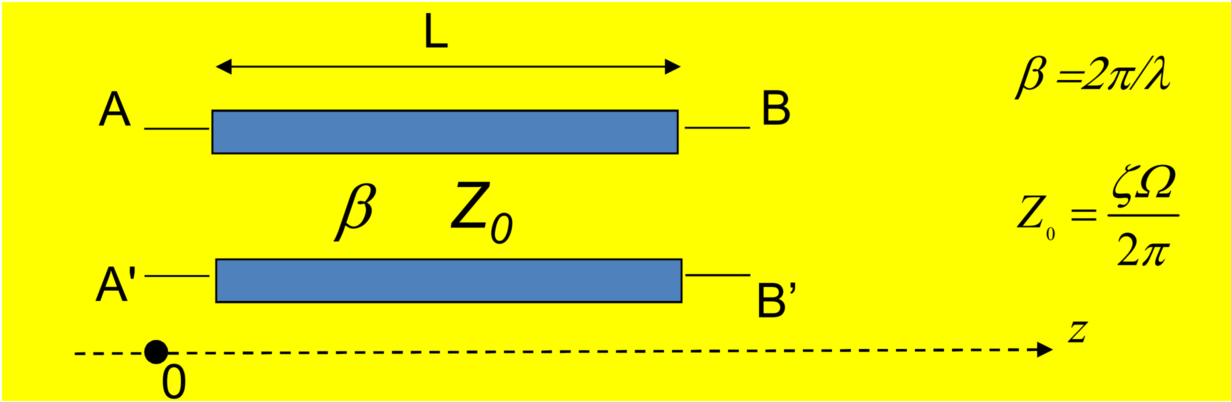
Direction of the Main Lobe $\vartheta_{MB} = \frac{\pi}{2}$

NNBW / HPBW $\mathrm{NNBW} \approx \frac{\lambda}{L} \quad \mathrm{HPBW} \approx 0.88\frac{\lambda}{2L}$

SLL $\mathrm{SLL} = -13.46\, dB$

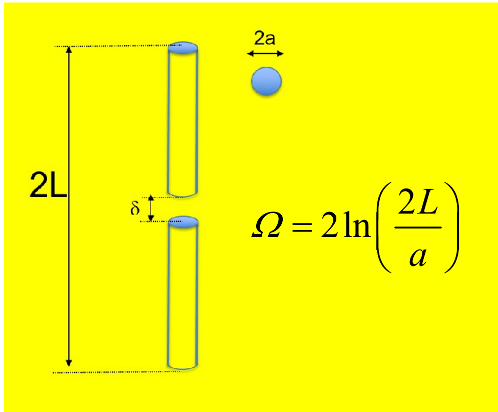
Hallen Formulation



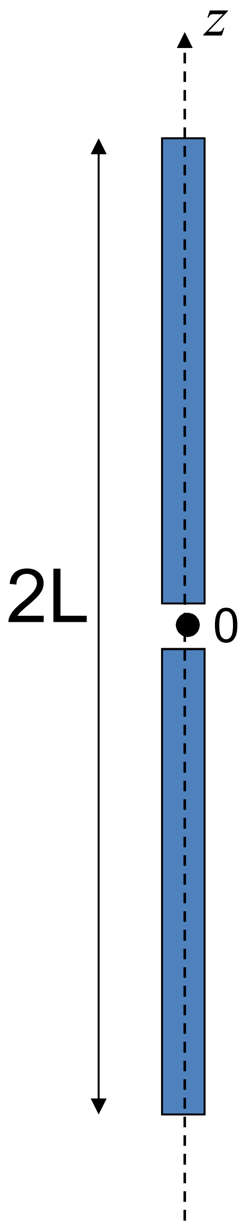


$$Z_{\text{in}} = -jZ_o ctg\left(\beta L\right)$$

$$I\left(z\right) = I_0 \frac{\sin\left(\beta L - \beta\left|z\right|\right)}{\sin\left(\beta L\right)}$$

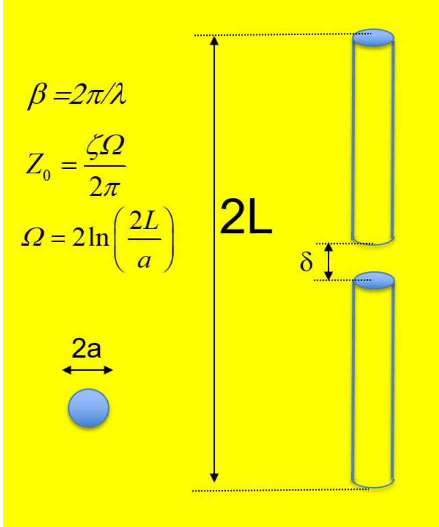


Hallen Formulation



$$Z_{in} = -jZ_o ctg(\beta L) = jX_{in}$$

$$I(z) = I_0 \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$



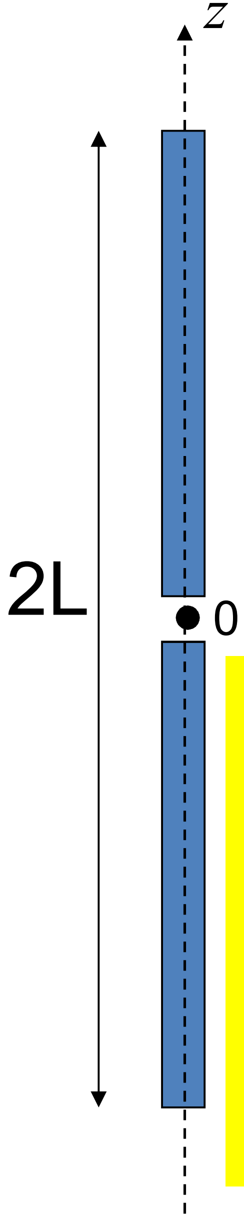
The Hallen model provides a wrong value of the input resistance R_{in}, due to the employed approximations

Notwithstanding, measurements carried out in laboratory show that the input reactance X_{in} provided by the Hallen model is quite accurate.

Measurements carried out in laboratory show also that the far field obtained by employing the expression I(z) provided by the Hallen model is very accurate.

From the analytical expression of the far field obtained by employing the expression I(z) provided by the Hallen model, we can obtain a quite accurate value of the radiation resistance R_{rad} (and thus a sound estimate of the input resistance R_{in}).

Hallen Formulation



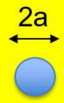
$$Z_{\text{in}} = -jZ_o ctg(\beta L) = jX_{\text{in}}$$

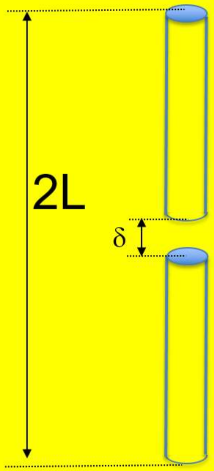
$$I(z) = I_0 \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$

$$\beta = 2\pi/\lambda$$

$$Z_0 = \frac{\zeta\Omega}{2\pi}$$

$$\Omega = 2\ln\left(\frac{2L}{a}\right)$$





$$P_{\text{in}} = \frac{1}{2} R_{\text{in}} |I_0|^2$$

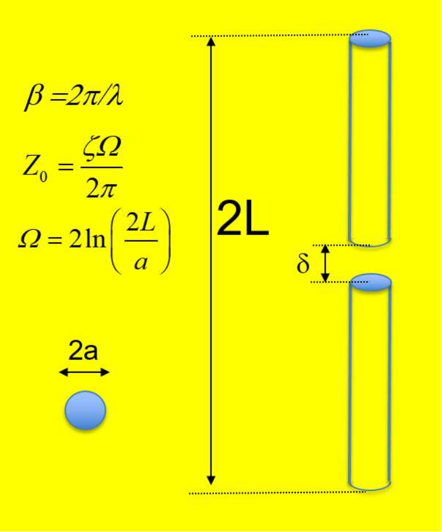
$$P_{rad} = \frac{1}{2} R_{rad} |I_0|^2$$

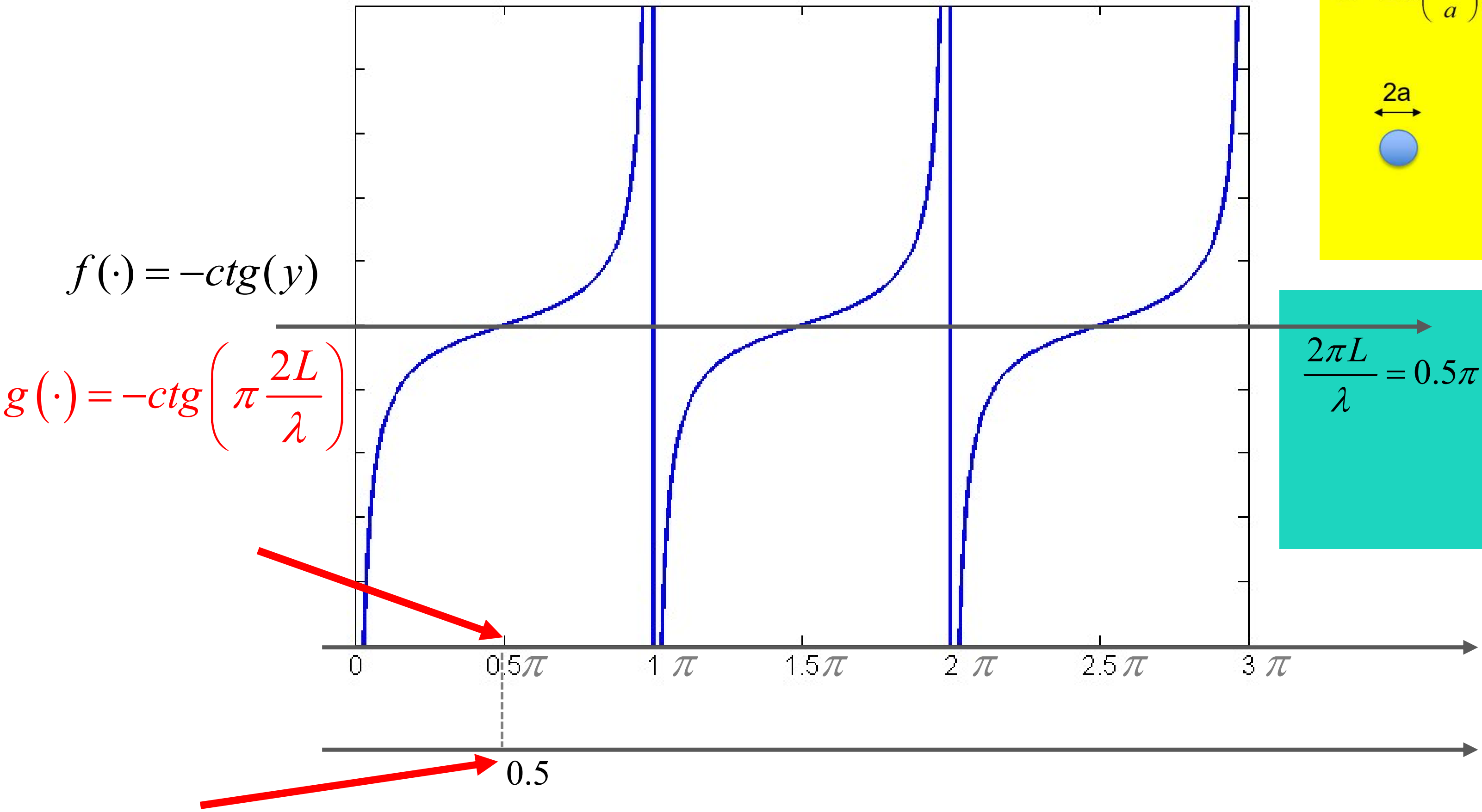
$$P_{rad} = \oiint_S \frac{1}{2\zeta} |\vec{\mathbf{E}}|^2$$

This antenna radiates!

Hallen Formulation

$$Z_{in} = -jZ_o ctg(\beta L) = -jZ_o ctg\left(\frac{2\pi}{\lambda}L\right) = jX_{in}$$

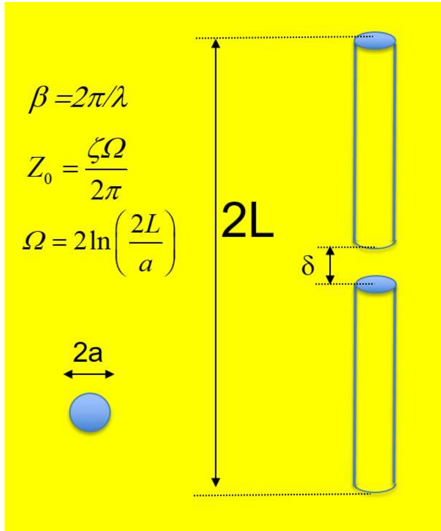




$$\frac{2\pi L}{\lambda} = 0.5\pi \Rightarrow \frac{2L}{\lambda} = 0.5$$

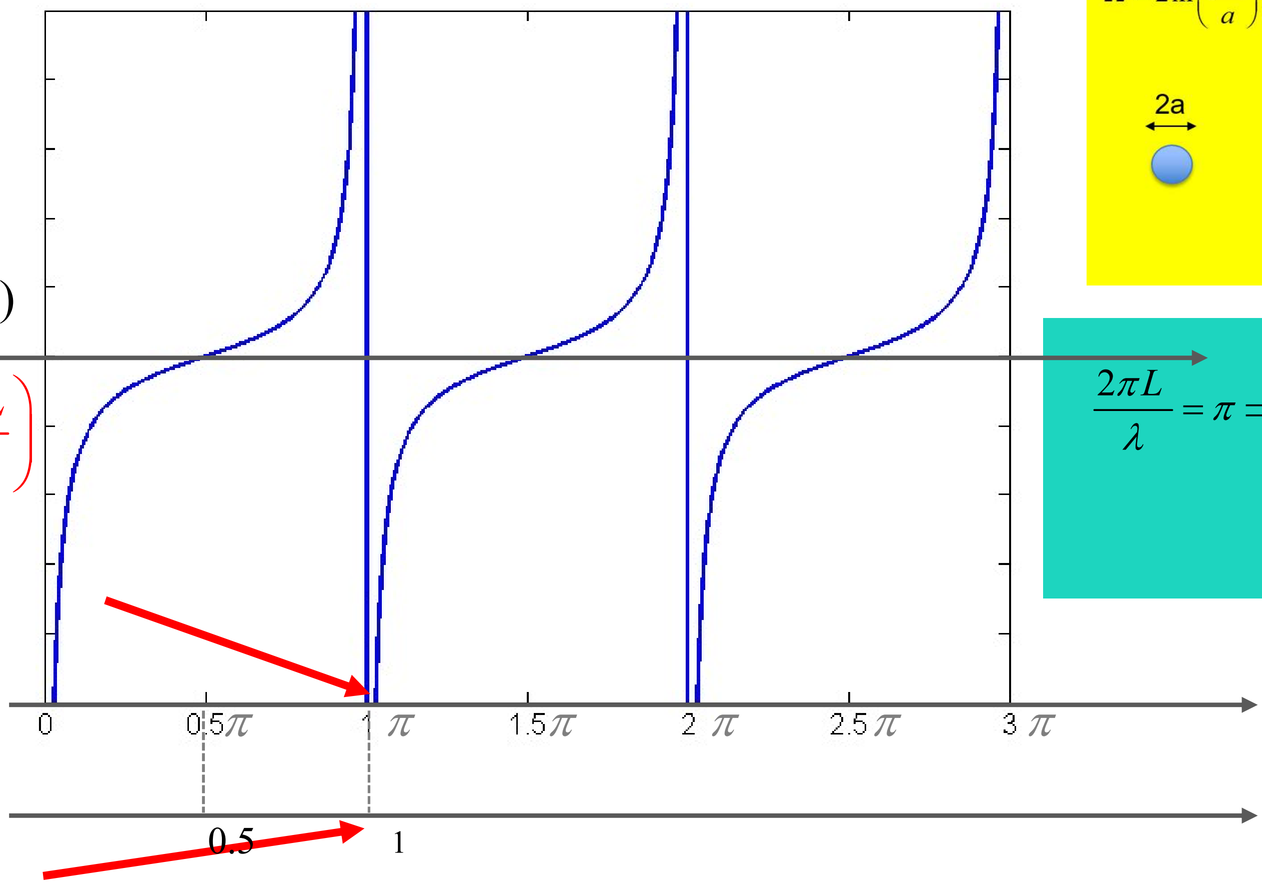
Hallen Formulation

$$Z_{\text{in}} = -jZ_o ctg(\beta L) = -jZ_o ctg\left(\frac{2\pi}{\lambda} L\right) = jX_{\text{in}}$$



$$f(y) = -ctg(y)$$

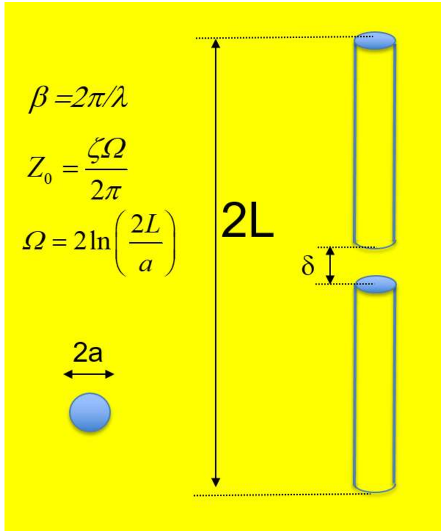
$$g(\cdot) = -ctg\left(\pi \frac{2L}{\lambda}\right)$$



$$\frac{2\pi L}{\lambda} = \pi \Rightarrow \frac{2L}{\lambda} = 1$$

Hallen Formulation

$$Z_{in} = -jZ_o ctg(\beta L) = -jZ_o ctg\left(\frac{2\pi}{\lambda}L\right) = jX_{in}$$



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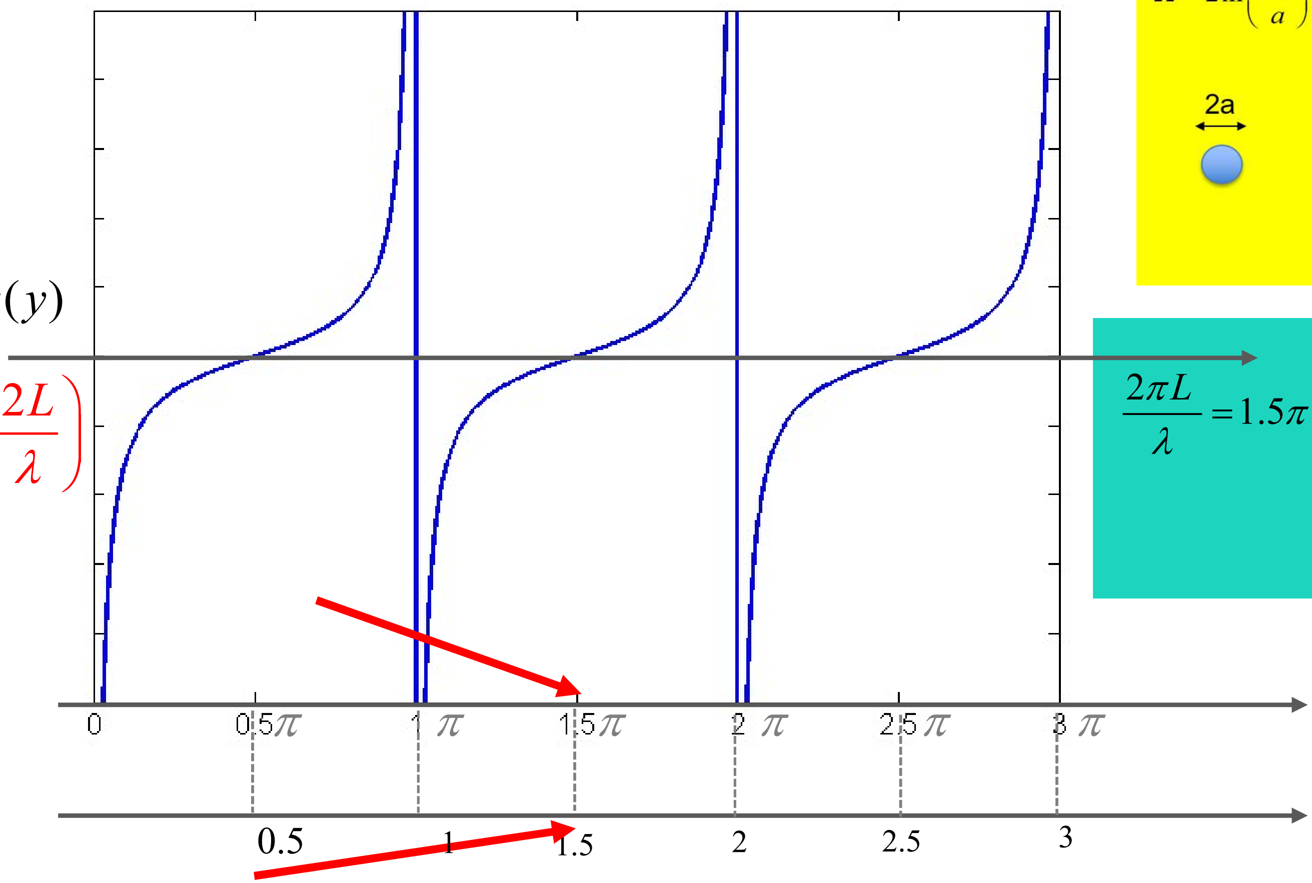


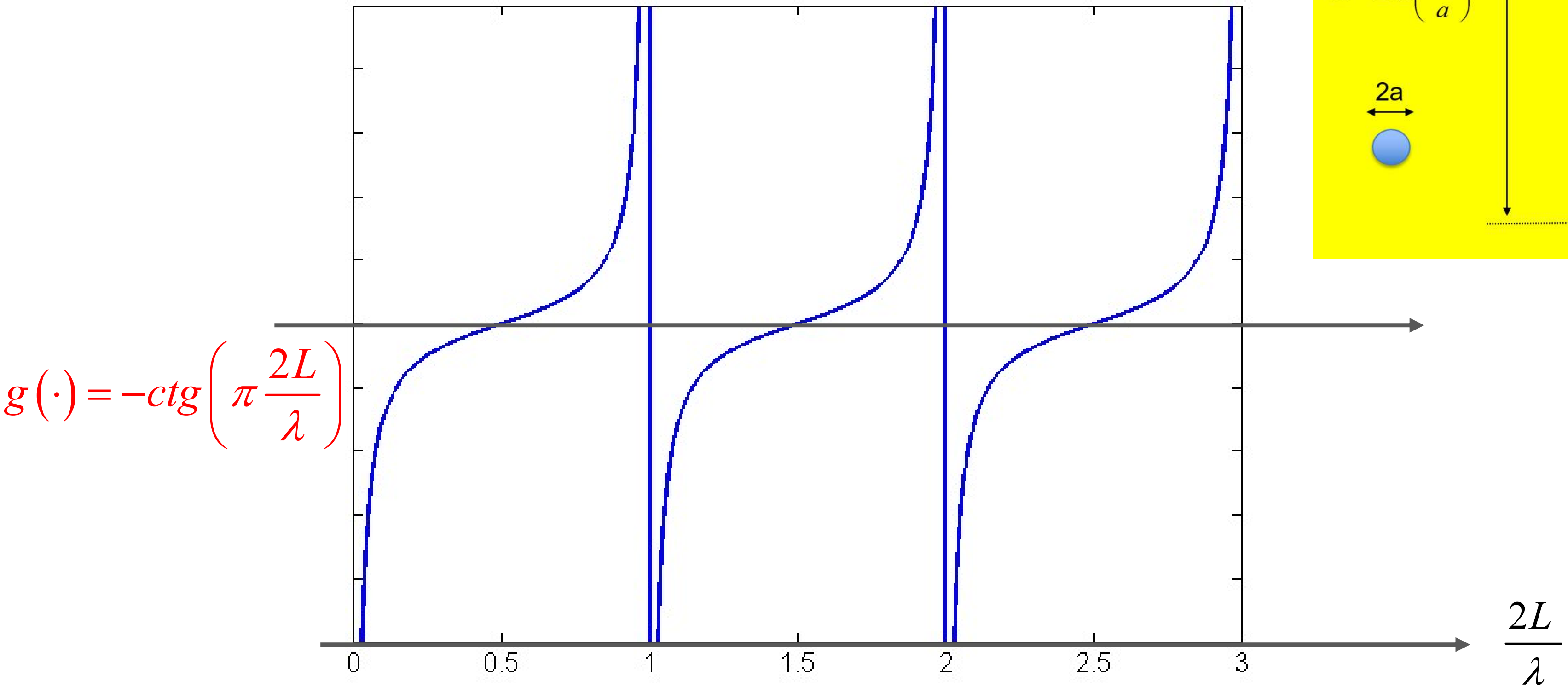
$y = \beta L = \frac{2\pi}{\lambda}L$

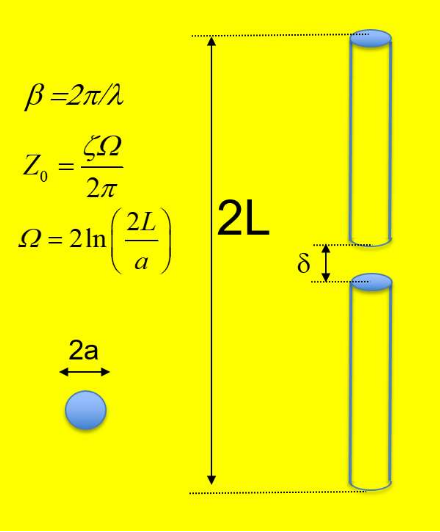
$\frac{2L}{\lambda}$

$$\frac{2\pi L}{\lambda} = 1.5\pi \Rightarrow \frac{2L}{\lambda} = 1.5$$

Hallen Formulation

$$Z_{\text{in}} = -jZ_o ctg(\beta L) = -jZ_o ctg\left(\frac{2\pi}{\lambda}L\right) = jX_{\text{in}}$$





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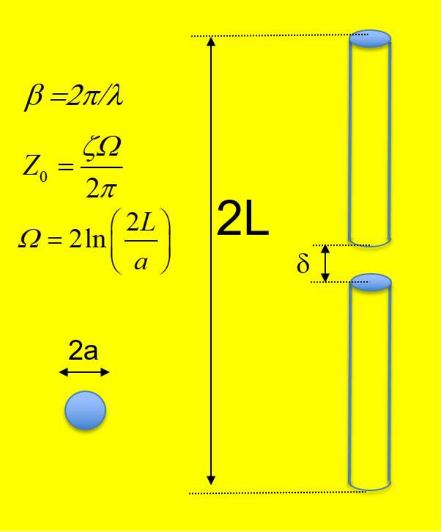

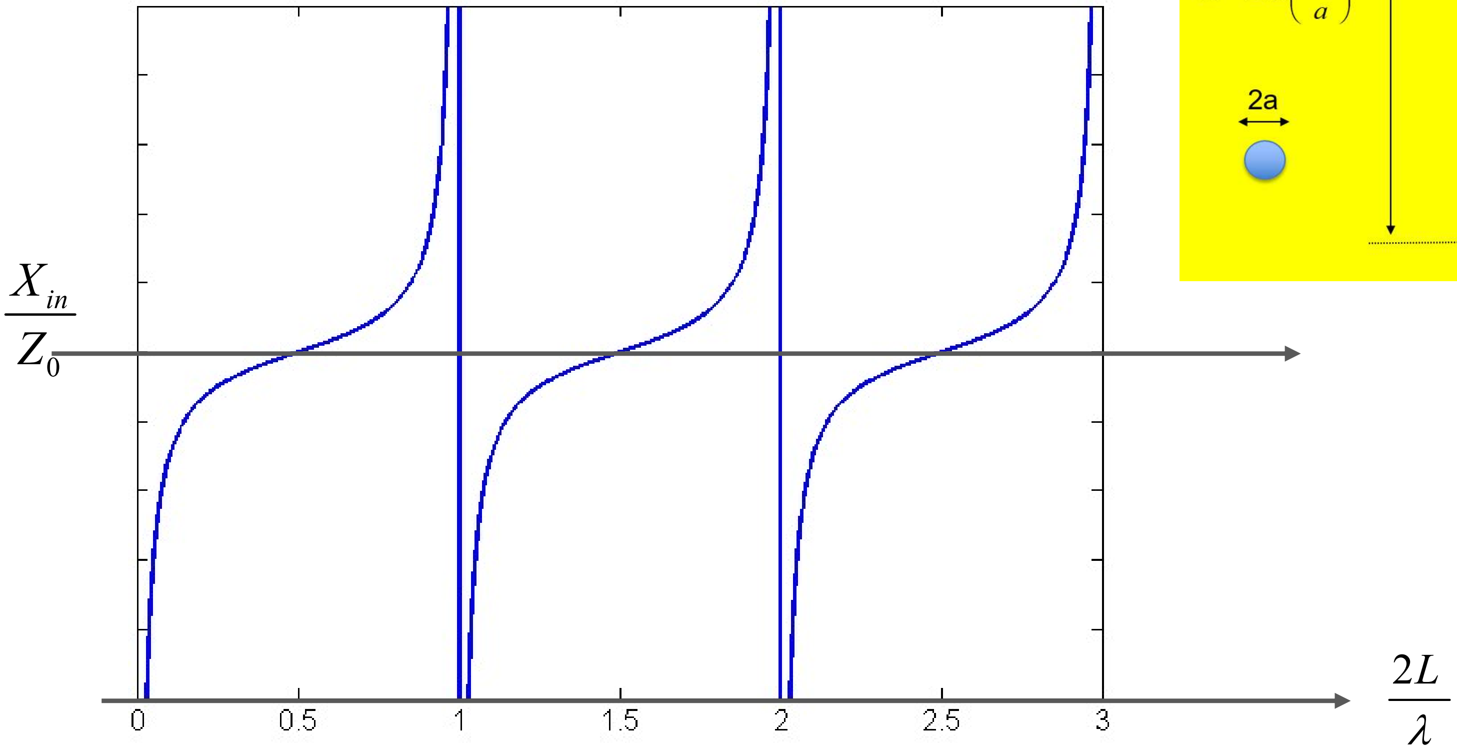

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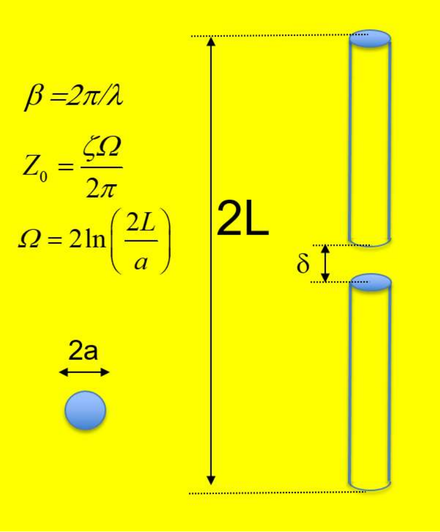
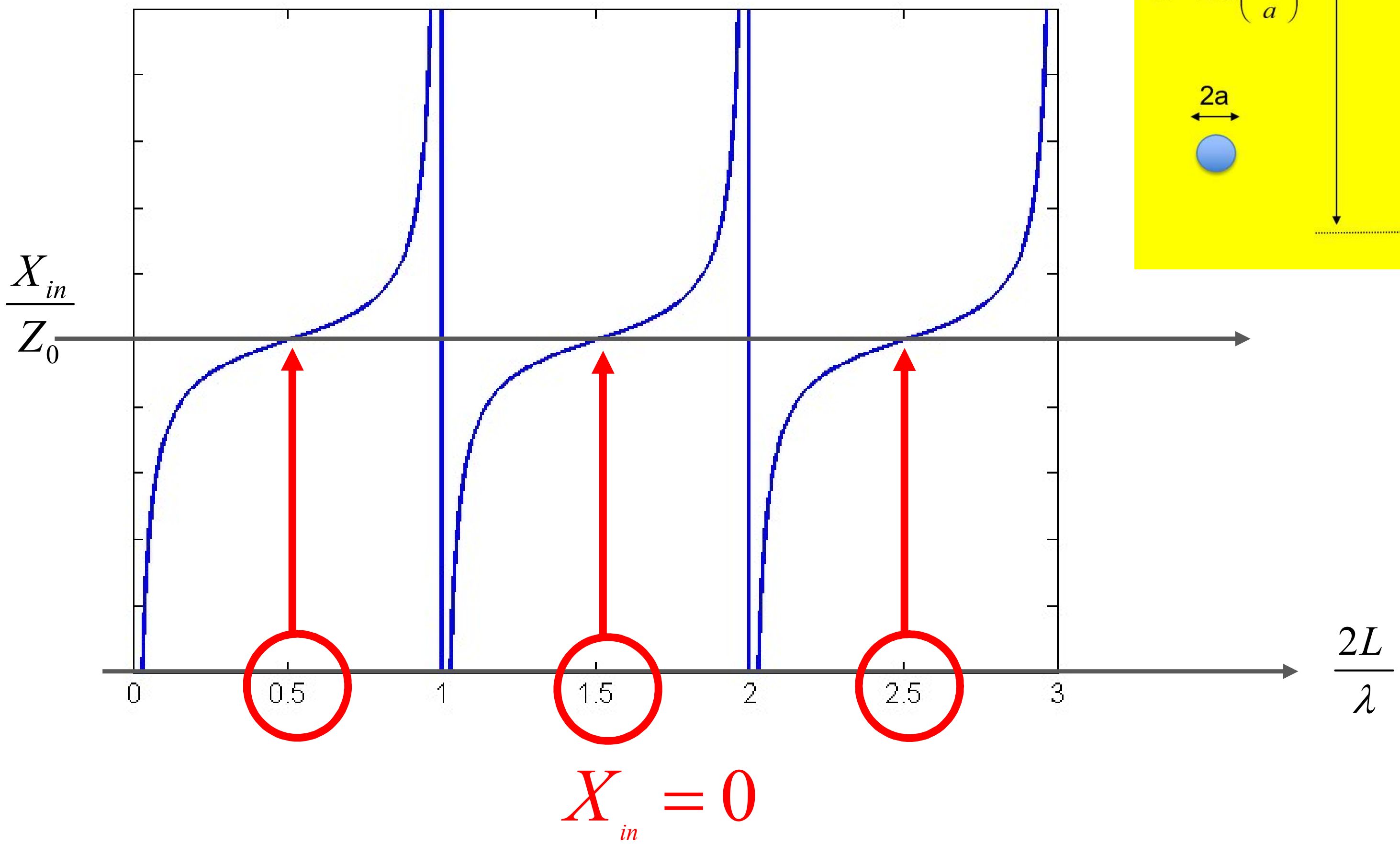
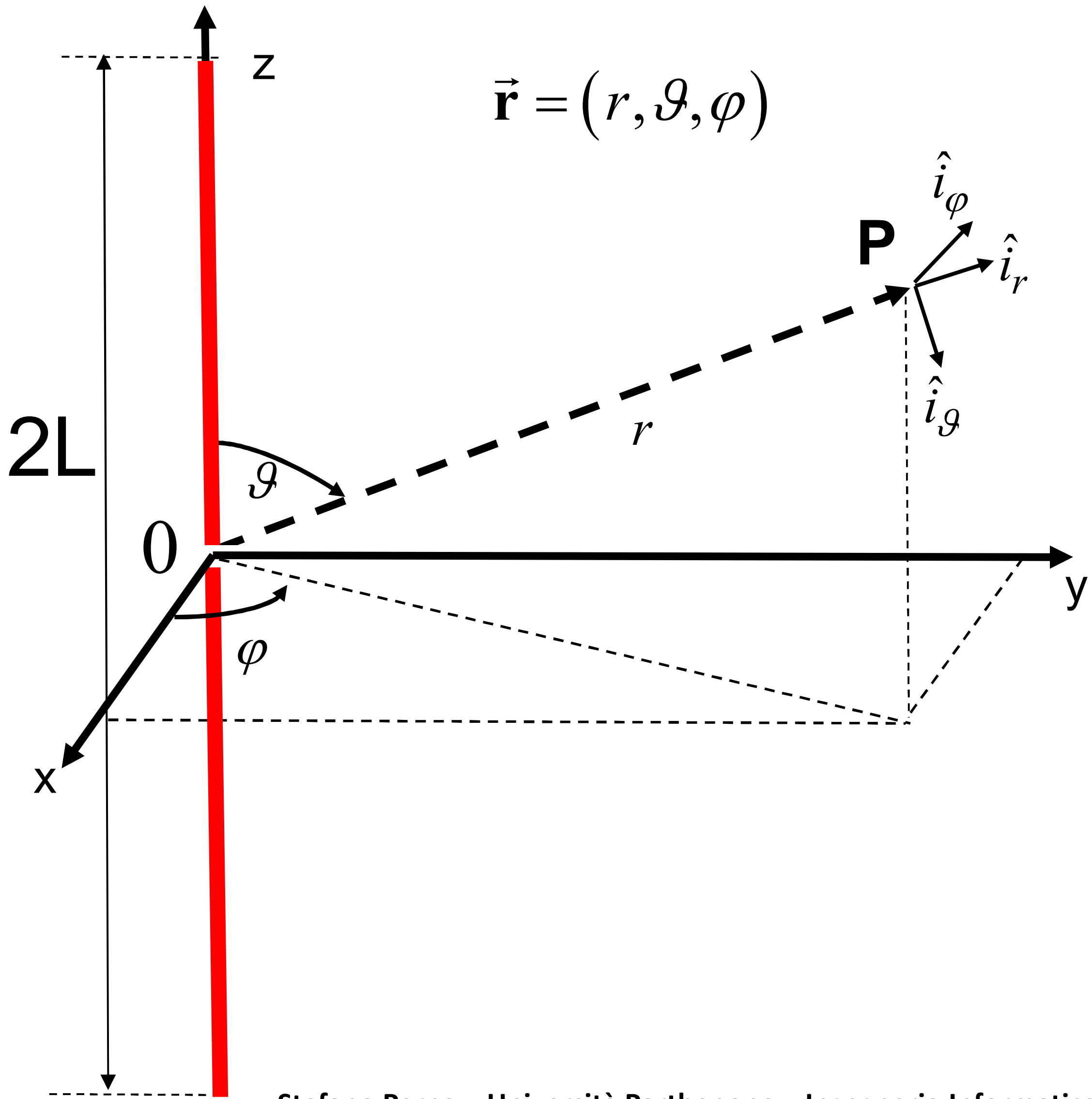




Wire antennas



Wire antennas

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$\vec{\mathbf{I}}(\vartheta) = \sin\vartheta F(\vartheta)\hat{i}_\vartheta$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0} = \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$

$$X_{\text{in}} = -Z_o ctg(\beta L)$$

$\beta = 2\pi/\lambda$

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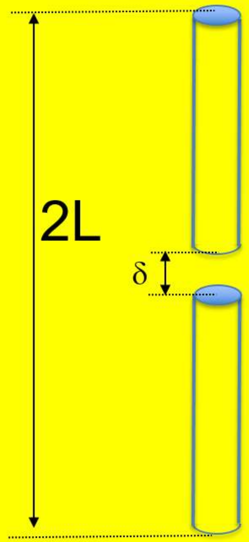

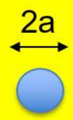

Short dipole

Short dipole

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0} = \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$

Short dipole

$$2L \ll \lambda \Rightarrow \beta L = \frac{2\pi}{\lambda} L \ll \pi$$

$$F(\vartheta) = \int_{-L}^{L} dz \frac{I(z)}{I_0}\exp(j\beta z\cos\vartheta) = \int_{-L}^{L} dz \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}\exp(j\beta z\cos\vartheta) \approx \int_{-L}^{L} dz\left(1 - \frac{|z|}{L}\right)$$

$$\beta L \ll \pi \Rightarrow \begin{cases} \sin(\beta L) \approx \beta L \\ \sin[\beta(L-|z|)] \approx \beta(L-|z|) \\ \exp(j\beta z\cos\vartheta) \approx 1 \end{cases} \Rightarrow \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}\exp(j\beta z\cos\vartheta) \approx \frac{\beta(L-|z|)}{\beta L}$$

Short dipole

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$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

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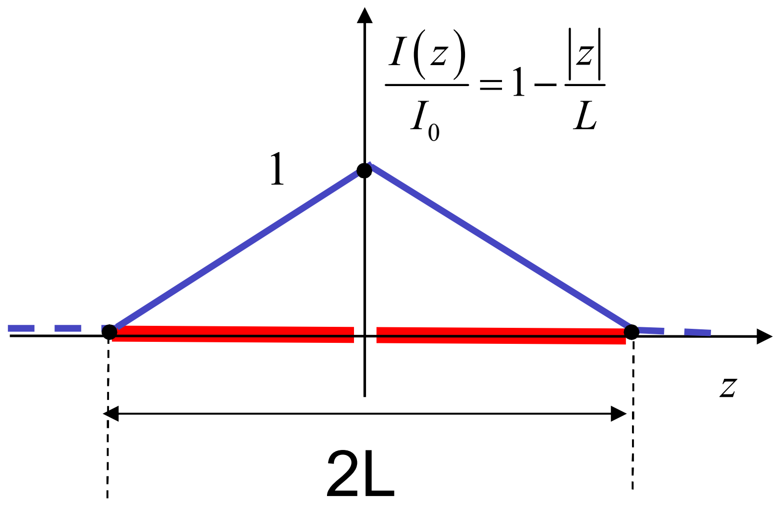
Short dipole

$$2L \ll \lambda \Rightarrow \beta L = \frac{2\pi}{\lambda} L \ll \pi$$

$$F(\vartheta) \approx \int_{-L}^{L} dz\left(1 - \frac{|z|}{L}\right) = L$$

$$z = 0 \Rightarrow 1 - \frac{|z|}{L} = 1$$

$$z = \pm L \Rightarrow 1 - \frac{|z|}{L} = 0$$



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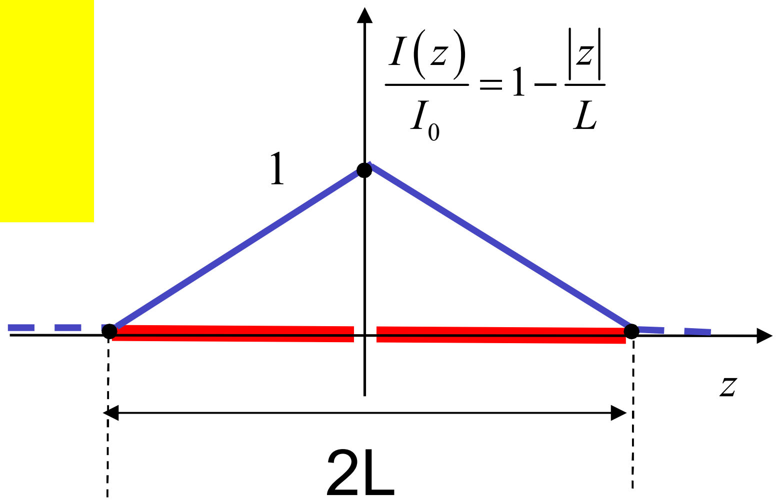
Short dipole

$$2L \ll \lambda \;\Rightarrow\; \beta L = \frac{2\pi}{\lambda} L \ll \pi$$

$$F(\vartheta) \approx L \;\Rightarrow\; \vec{\mathbf{l}}(\vartheta) = L\sin\vartheta\, \hat{i}_\vartheta$$

$$\vec{\mathbf{l}}(\vartheta) = \sin\vartheta F(\vartheta)\hat{i}_\vartheta$$

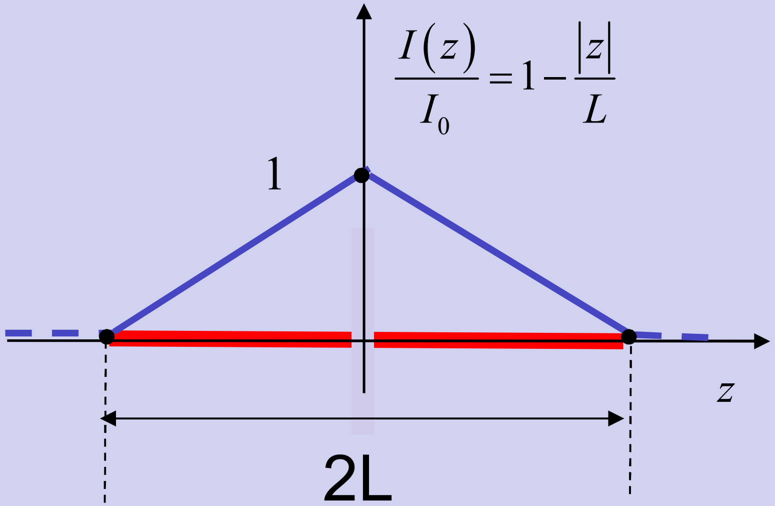


Short dipole

Short dipole

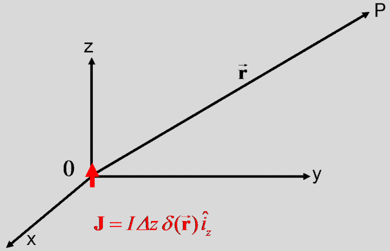
$2L << \lambda$

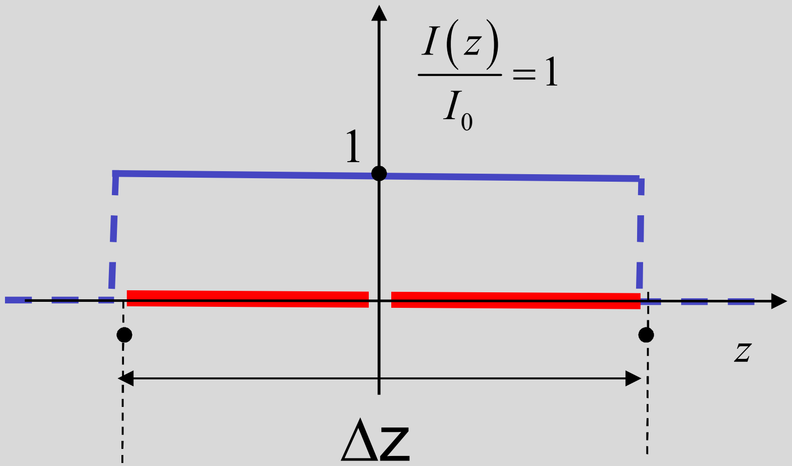
$\vec{\mathbf{l}}(\vartheta) = L \sin\vartheta \, \hat{i}_\vartheta$



Elementary electrical dipole

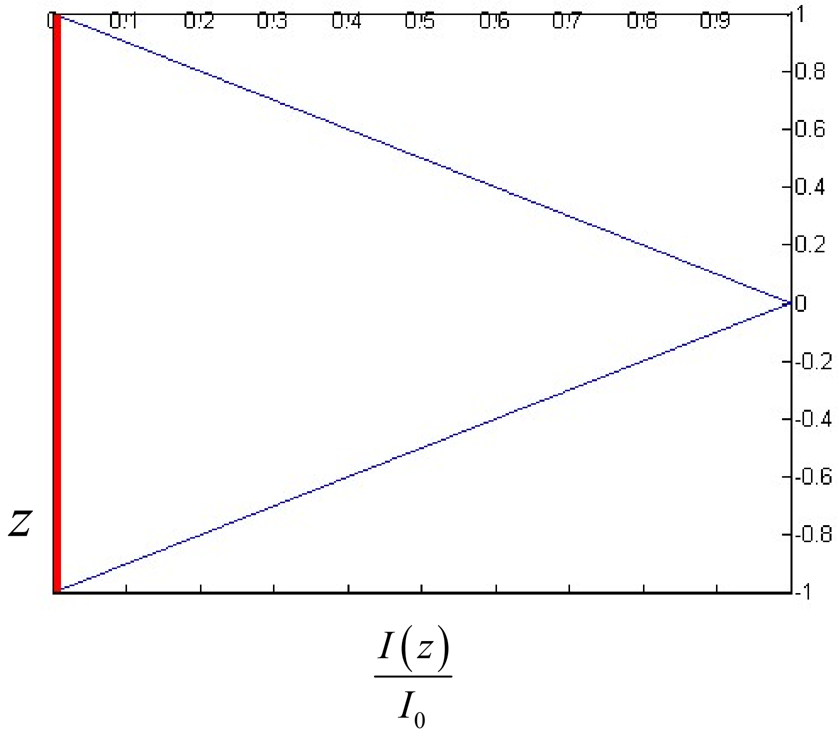
$\vec{\mathbf{l}}(\vartheta) = \Delta z \sin\vartheta \, \hat{i}_\vartheta$



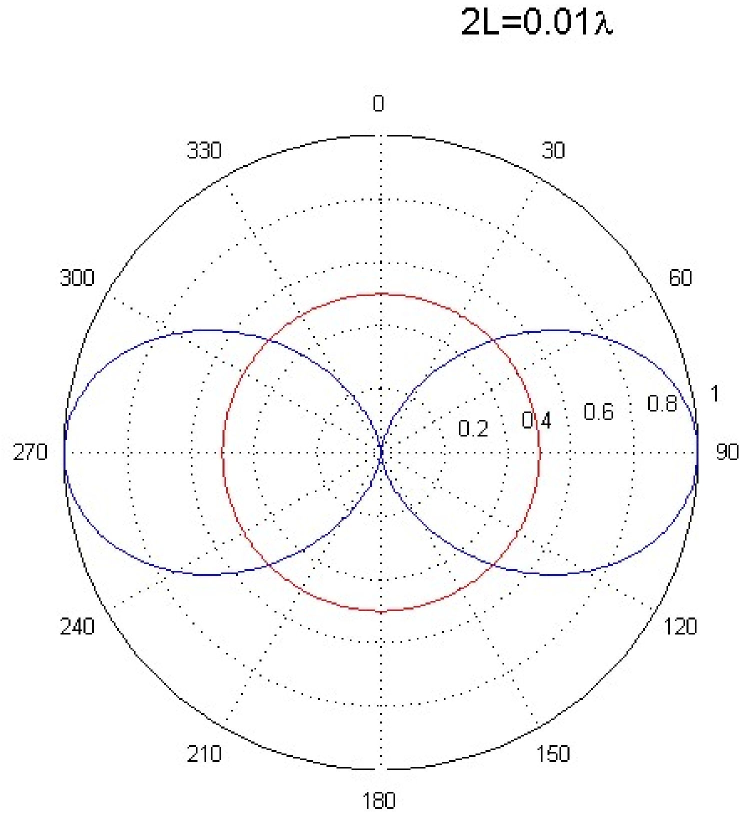


Short dipole

Current distribution



Power pattern
(vertical plane)



Short dipole

Short dipole

$2L << \lambda$

$\vec{\mathbf{l}}(\vartheta) = L \sin\vartheta \, \hat{i}_\vartheta$

$D(\vartheta,\varphi) = \frac{3}{2}\sin^2\vartheta$ $\qquad D_{\max} = 1.76 \text{ dB}$

$Z_{in} = R_{in} + jX_{\text{in}}$

$R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{L}{\lambda}\right)^2$

$X_{\text{in}} = -Z_o ctg(\beta L)$

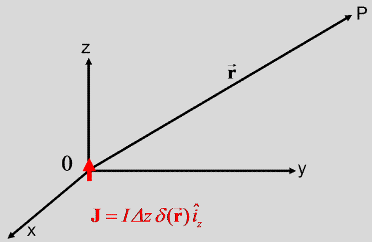
$\beta = 2\pi/\lambda$

$Z_0 = \frac{\zeta\Omega}{2\pi}$

$\Omega = 2\ln\left(\frac{2L}{a}\right)$

Elementary electrical dipole

$\vec{\mathbf{l}}(\vartheta) = \Delta z \sin\vartheta \, \hat{i}_\vartheta$

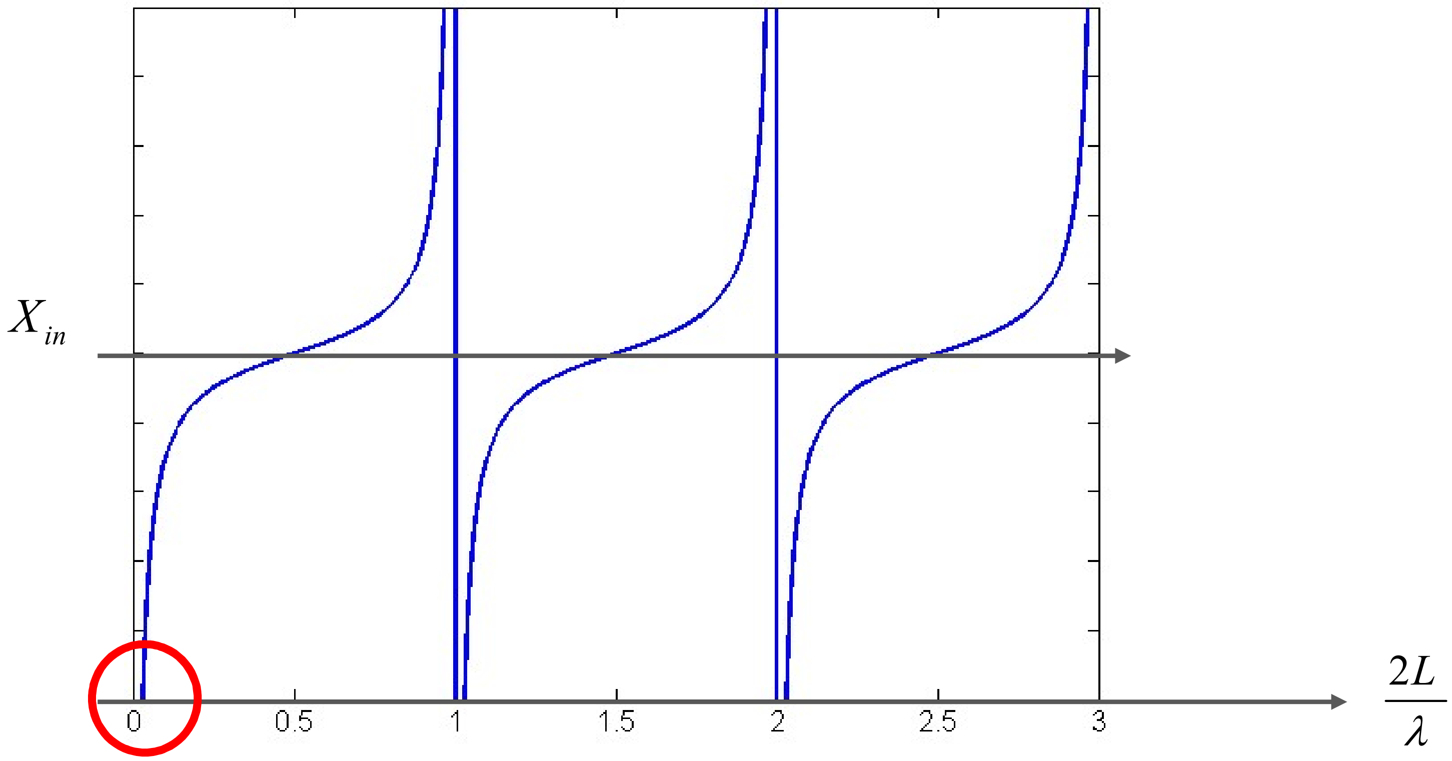


$D(\vartheta,\varphi) = \frac{3}{2}\sin^2\vartheta$ $\qquad D_{\max} = 1.76 \text{ dB}$

$R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2$

Short dipole

$2L << \lambda$



Wire antennas

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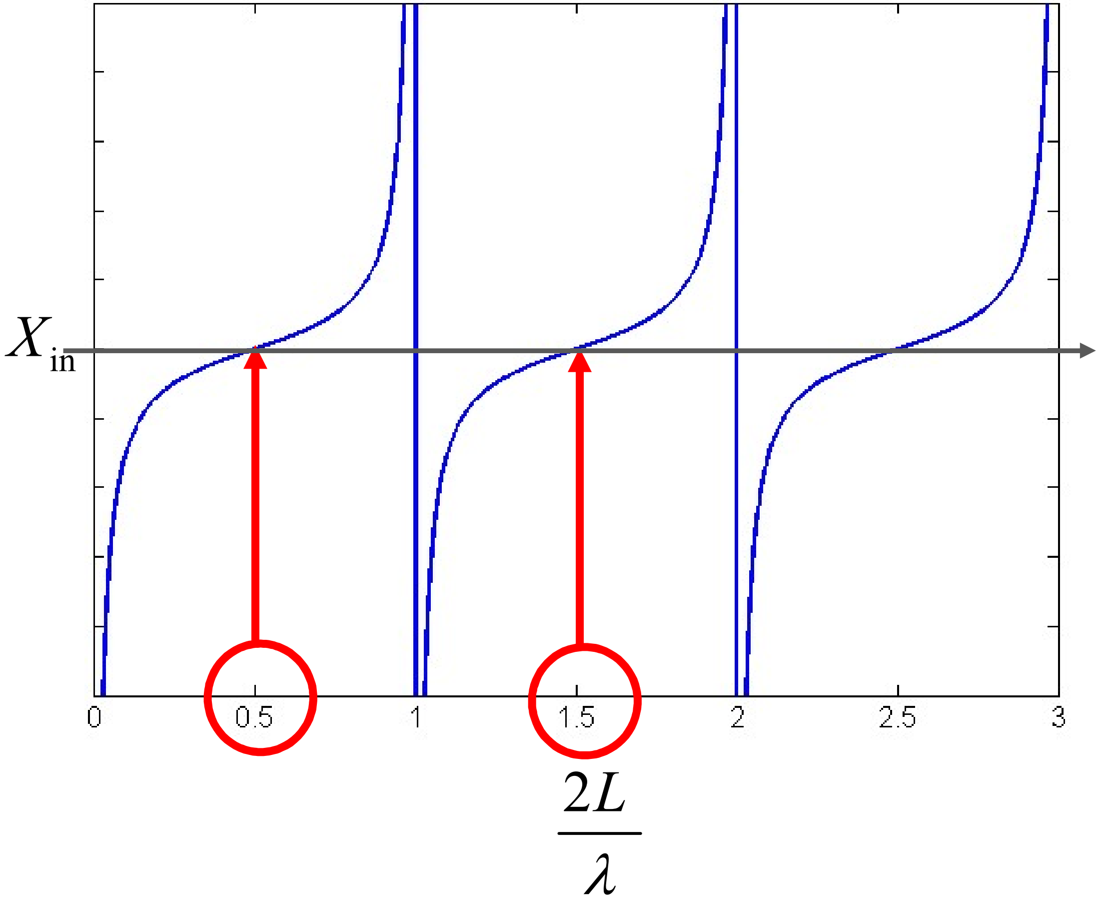
$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

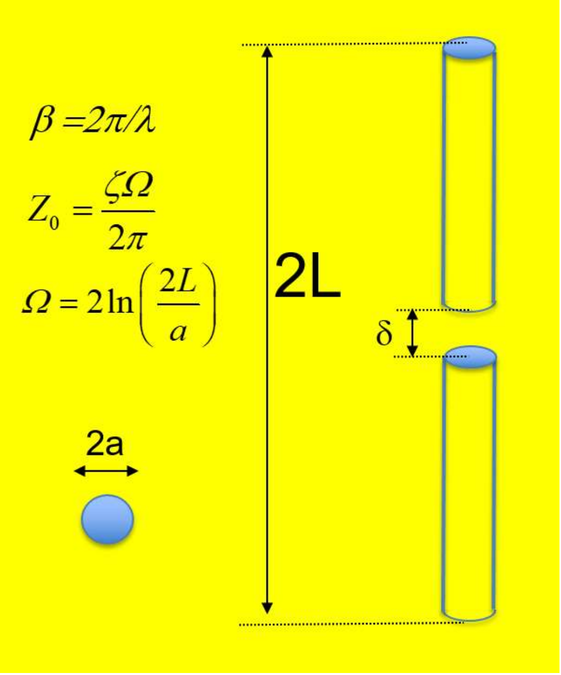
$$\tilde{I}(z) = \frac{I(z)}{I_0} = \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$

$$\vec{\mathbf{I}} = \frac{\lambda}{\pi} \frac{\left[\cos(\beta L\cos\vartheta) - \cos(\beta L)\right]}{\sin(\beta L)\sin\vartheta} \hat{i}_\vartheta$$

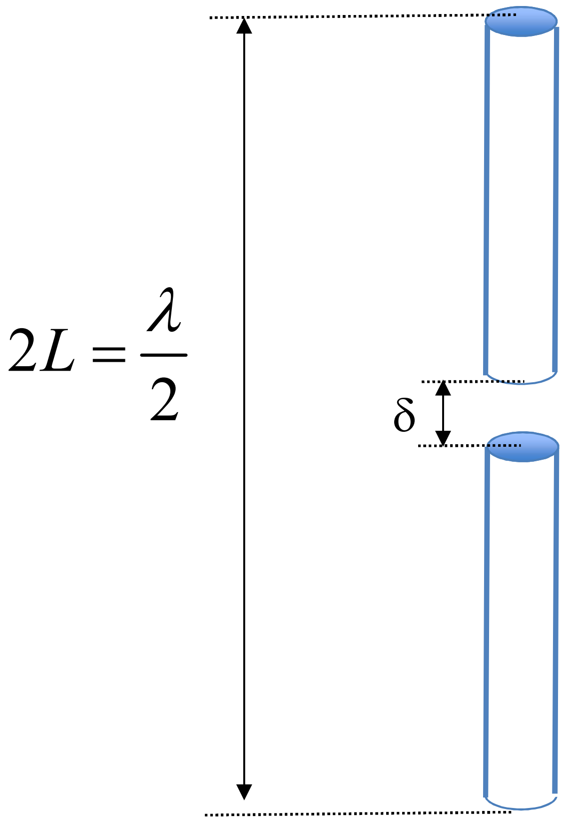
Wire antennas

λ/2 antenna & 3λ/2 antennas





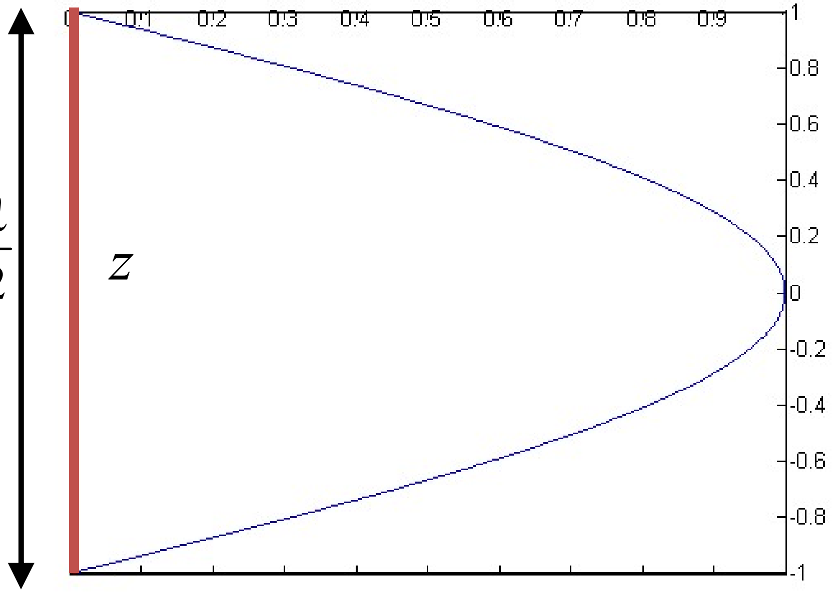
Half wavelength antenna



Half wavelength antenna

Current distribution

$$\tilde{I}(z)=\frac{I(z)}{I_0}=\frac{\sin\left(\beta L-\beta|z|\right)}{\sin\left(\beta L\right)}=\frac{\sin\left(\frac{2\pi}{\lambda}L-\frac{2\pi}{\lambda}|z|\right)}{\sin\left(\frac{2\pi}{\lambda}L\right)}$$



$$2L=\frac{\lambda}{2}$$

z

$$\frac{I(z)}{I_0}$$

$$z=0 \quad\Rightarrow\quad \tilde{I}(z)=\frac{I(z)}{I_0}=\frac{I_0}{I_0}=1$$

Half wavelength antenna

Pattern

$$2L = \frac{\lambda}{2} \quad \Rightarrow \quad \beta L = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\vec{\mathbf{l}} = \frac{\lambda}{\pi}\frac{\left[\cos\left(\beta L\cos\vartheta\right) - \cos\left(\beta L\right)\right]}{\sin\left(\beta L\right)\sin\vartheta}\hat{i}_\vartheta = \frac{\lambda}{\pi}\frac{\left[\cos\left(\frac{\pi}{2}\cos\vartheta\right) - \cos\left(\frac{\pi}{2}\right)\right]}{\sin\left(\frac{\pi}{2}\right)\sin\vartheta}\hat{i}_\vartheta = \frac{\lambda}{\pi}\frac{\left[\cos\left(\frac{\pi}{2}\cos\vartheta\right)\right]}{\sin\vartheta}\hat{i}_\vartheta$$

Half wavelength antenna

Pattern

$$2L = \frac{\lambda}{2} \qquad \vec{\mathbf{l}} = \frac{\lambda}{\pi} \frac{\left[\cos\left(\frac{\pi}{2} \cos\vartheta \right) \right]}{\sin\vartheta} \hat{i}_\vartheta$$

Zeroes

$$\cos\left(\frac{\pi}{2} \cos\vartheta \right) = 0 \Rightarrow \frac{\pi}{2}\cos\vartheta = \frac{\pi}{2} + n\pi \Rightarrow \cos\vartheta = 1 + 2n$$

$$\begin{cases} n = -2 \Rightarrow \cos\vartheta = -3 \\ n = -1 \Rightarrow \cos\vartheta = -1 \Rightarrow \vartheta = \pi \\ n = 0 \Rightarrow \cos\vartheta = 1 \Rightarrow \vartheta = 0 \\ n = 1 \Rightarrow \cos\vartheta = 3 \end{cases}$$

… application of the de l'Hopital rule leads to

$$\lim_{\vartheta \to 0} \frac{\left[\cos\left(\frac{\pi}{2} \cos\vartheta \right) \right]}{\sin\vartheta} = \frac{0}{0} \lim_{\vartheta \to 0} \frac{\pi}{2} \frac{\sin\vartheta \cdot \sin\left(\frac{\pi}{2} \cos\vartheta \right)}{\cos\vartheta} = 0$$

Half wavelength antenna

Pattern

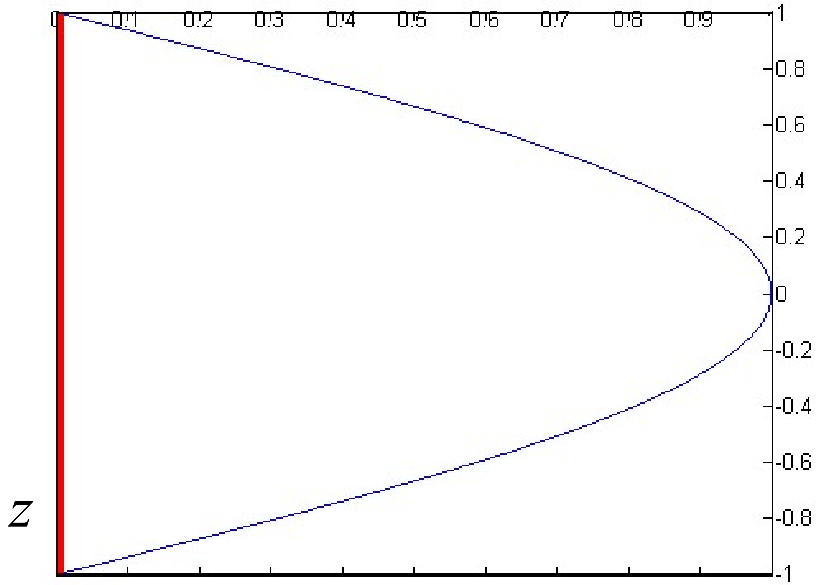
$$2L = \frac{\lambda}{2}$$

$$\vec{\mathbf{l}} = \frac{\lambda}{\pi} \frac{\left[\cos\left(\frac{\pi}{2}\cos\vartheta\right)\right]}{\sin\vartheta} \hat{i}_{\vartheta}$$

Zeroes in the region $\vartheta \in [0, \pi]$ **:** $\vartheta = 0$ $\vartheta = \pi$

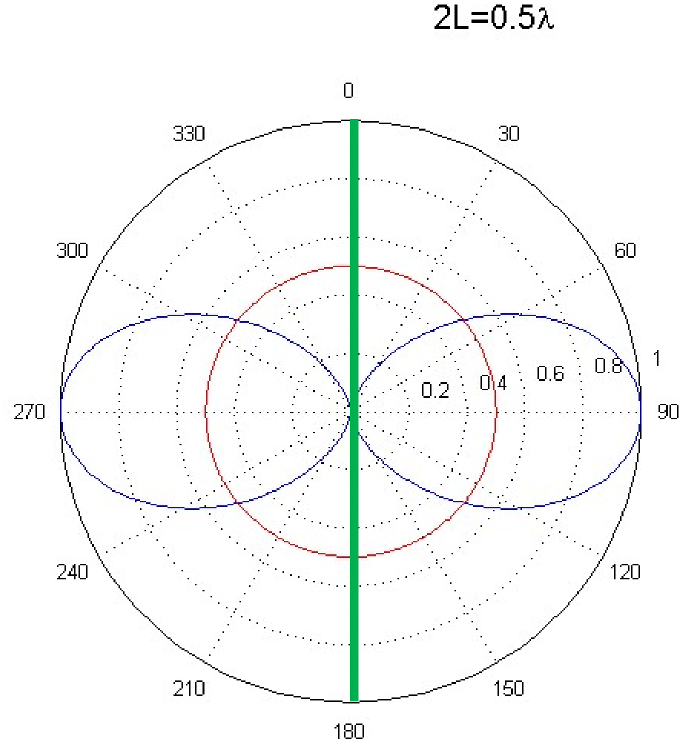
Half wavelength antenna

Current distribution



$$\frac{I(z)}{I_0}$$

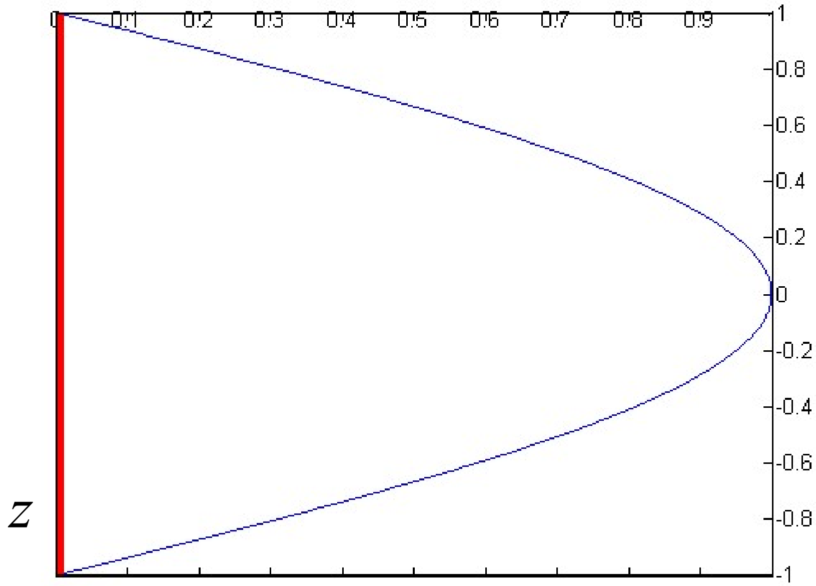
Power pattern
(vertical plane)



Half wavelength antenna

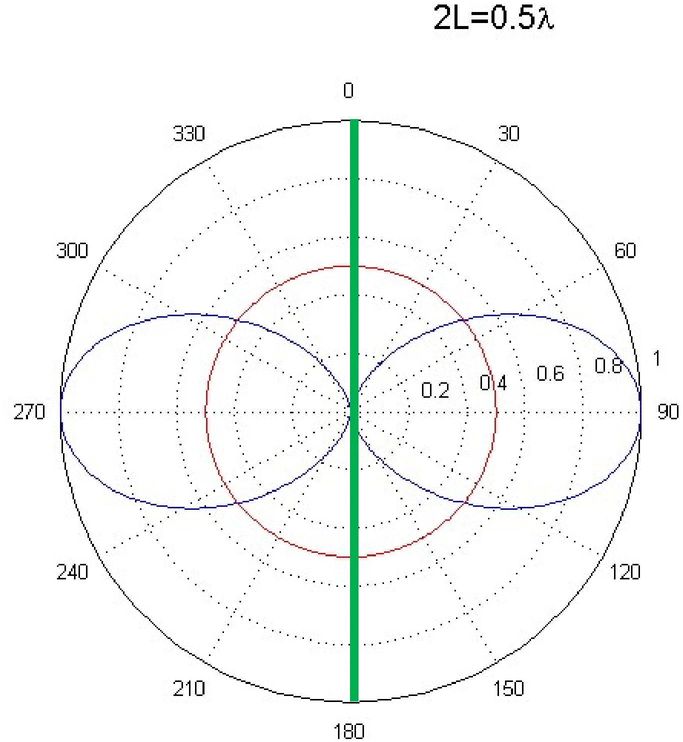
$$R_{in} \approx 75\Omega$$

Current distribution



$$\frac{I(z)}{I_0}$$

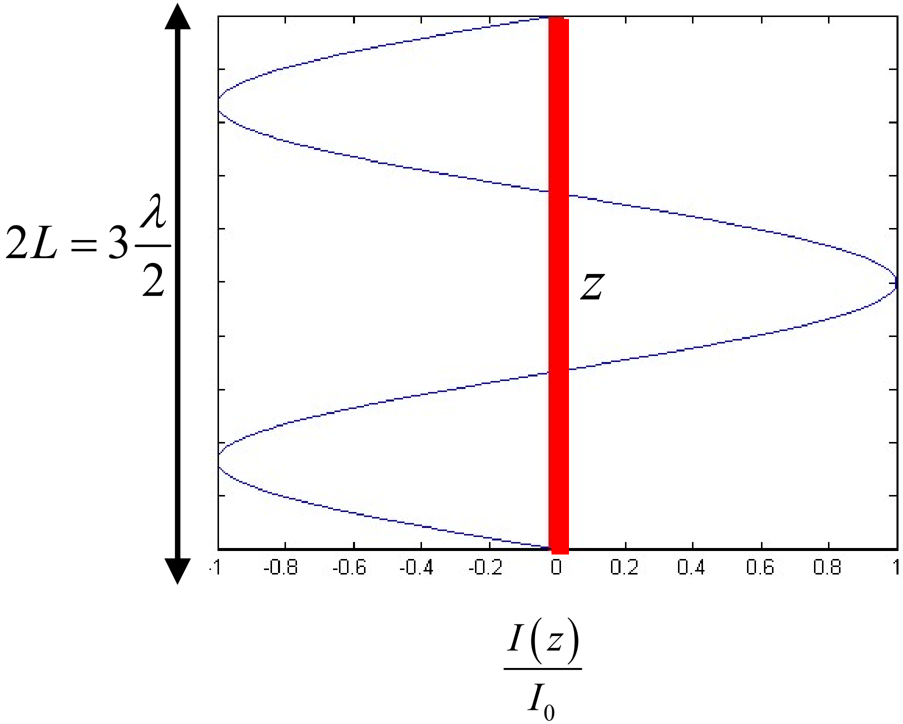
Power pattern (vertical plane)



3/2 wavelength antenna

Current distribution

$$\tilde{I}(z) = \frac{I(z)}{I_0} = \frac{\sin\left(\beta L - \beta|z|\right)}{\sin\left(\beta L\right)} = \frac{\sin\left(\frac{2\pi}{\lambda}L - \frac{2\pi}{\lambda}|z|\right)}{\sin\left(\frac{2\pi}{\lambda}L\right)}$$



$$z = 0 \quad \Rightarrow \quad \tilde{I}(z) = \frac{I(z)}{I_0} = \frac{I_0}{I_0} = 1$$

3/2 wavelength antenna

Half wavelength antenna

$$2L = \frac{3}{2}\lambda \quad \Rightarrow \quad \beta L = \frac{2\pi}{\lambda}\frac{3}{4}\lambda = \frac{3}{2}\pi$$

$$\vec{\mathbf{I}} = \frac{\lambda}{\pi}\frac{\left[\cos\left(\beta L\cos\vartheta\right) - \cos\left(\beta L\right)\right]}{\sin\left(\beta L\right)\sin\vartheta}\hat{i}_\vartheta = \frac{\lambda}{\pi}\frac{\left[\cos\left(\frac{3}{2}\pi\cos\vartheta\right) - \cos\left(\frac{3}{2}\pi\right)\right]}{\sin\left(\frac{3}{2}\pi\right)\sin\vartheta}\hat{i}_\vartheta = -\frac{\lambda}{\pi}\frac{\left[\cos\left(\frac{3}{2}\pi\cos\vartheta\right)\right]}{\sin\vartheta}\hat{i}_\vartheta$$

3/2 wavelength antenna

$$2L = \frac{3}{2}\lambda \qquad \vec{\mathbf{I}} = -\frac{\lambda}{\pi}\frac{\left[\cos\left(\frac{3}{2}\pi\cos\vartheta\right)\right]}{\sin\vartheta}\hat{i}_\vartheta$$

Zeroes

$$\cos\left(\frac{3}{2}\pi\cos\vartheta\right) = 0 \Rightarrow \frac{3}{2}\pi\cos\vartheta = \frac{\pi}{2} + n\pi \Rightarrow \cos\vartheta = \frac{1}{3} + \frac{2}{3}n$$

$$\begin{cases} n=-3 \Rightarrow \cos\vartheta = -\frac{5}{3} \\ n=-2 \Rightarrow \cos\vartheta = -1 \Rightarrow \vartheta = \pi \\ n=-1 \Rightarrow \cos\vartheta = -\frac{1}{3} \Rightarrow \vartheta = 0.6\pi \\ n=0 \Rightarrow \cos\vartheta = \frac{1}{3} \Rightarrow \vartheta = 0.4\pi \\ n=1 \Rightarrow \cos\vartheta = 1 \Rightarrow \vartheta = 0 \\ n=2 \Rightarrow \cos\vartheta = \frac{5}{3} \end{cases}$$

… application of the de l'Hopital rule leads to

$$\lim_{\vartheta\to 0}\frac{\left[\cos\left(\frac{3}{2}\pi\cos\vartheta\right)\right]}{\sin\vartheta} = \frac{0}{0}\lim_{\vartheta\to 0}\frac{3}{2}\pi\frac{\sin\vartheta\cdot\sin\left(\frac{3\pi}{2}\cos\vartheta\right)}{\cos\vartheta} = 0$$

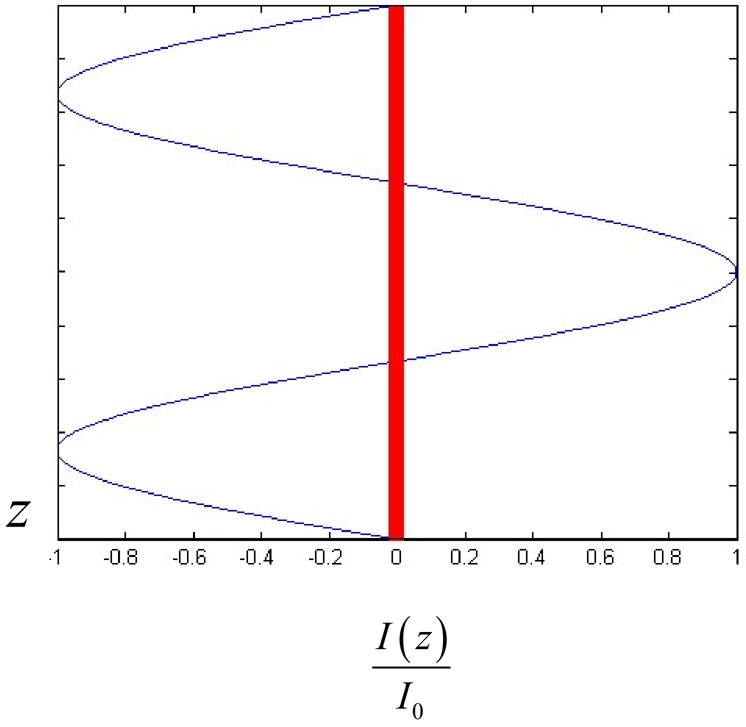
3/2 wavelength antenna

$$2L = \frac{3}{2}\lambda \qquad \vec{\mathbf{l}} = -\frac{\lambda}{\pi}\frac{\left[\cos\left(\frac{3}{2}\pi\cos\vartheta\right)\right]}{\sin\vartheta}\hat{i}_\vartheta$$

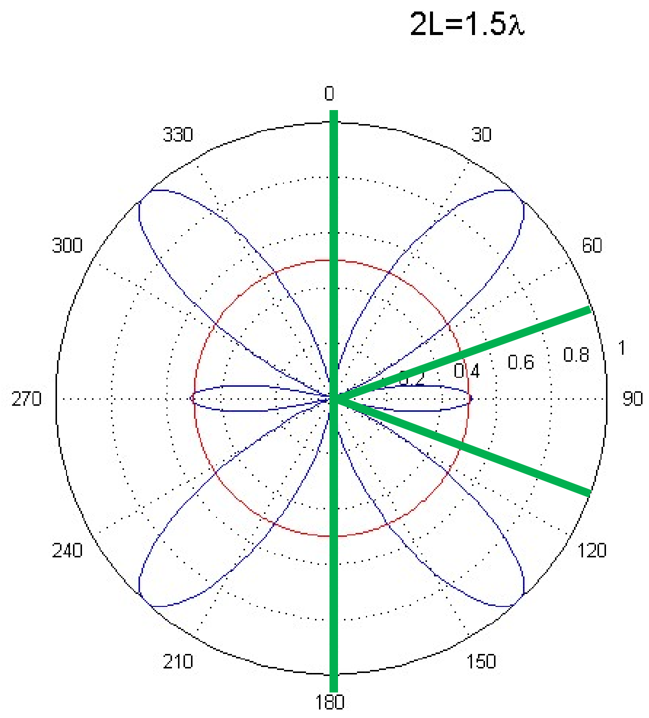
Zeroes in the region $\vartheta \in [0, \pi]$ **:** $\vartheta = 0$ $\vartheta = 0.4\pi$ $\vartheta = 0.7\pi$ $\vartheta = \pi$

3/2 wavelength antenna

Current distribution



Power pattern
(vertical plane)

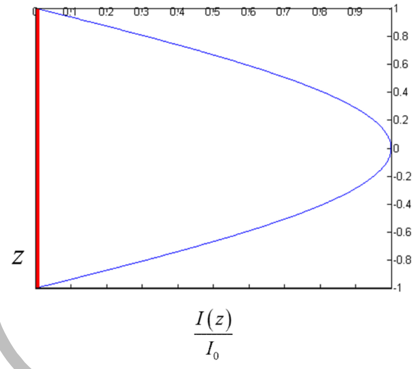


λ/2 vs. 3λ/2

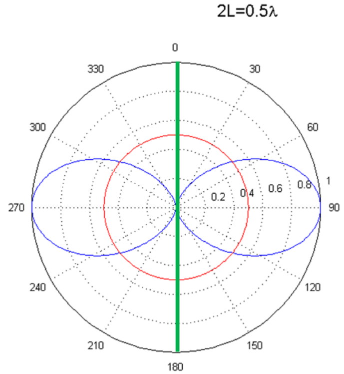
$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta\, \hat{i}_\vartheta \int_{-l}^{l} dz \frac{I(z)}{I_0} \exp(j\beta z \cos\vartheta)$$

Half wavelength antenna

Current distribution

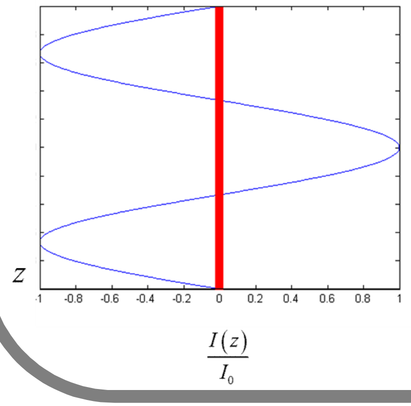


Power pattern (vertical plane)



3/2 wavelength antenna

Current distribution



Power pattern (vertical plane)