Corso di “Antenne”

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Università degli Studi di Napoli “Parthenope”

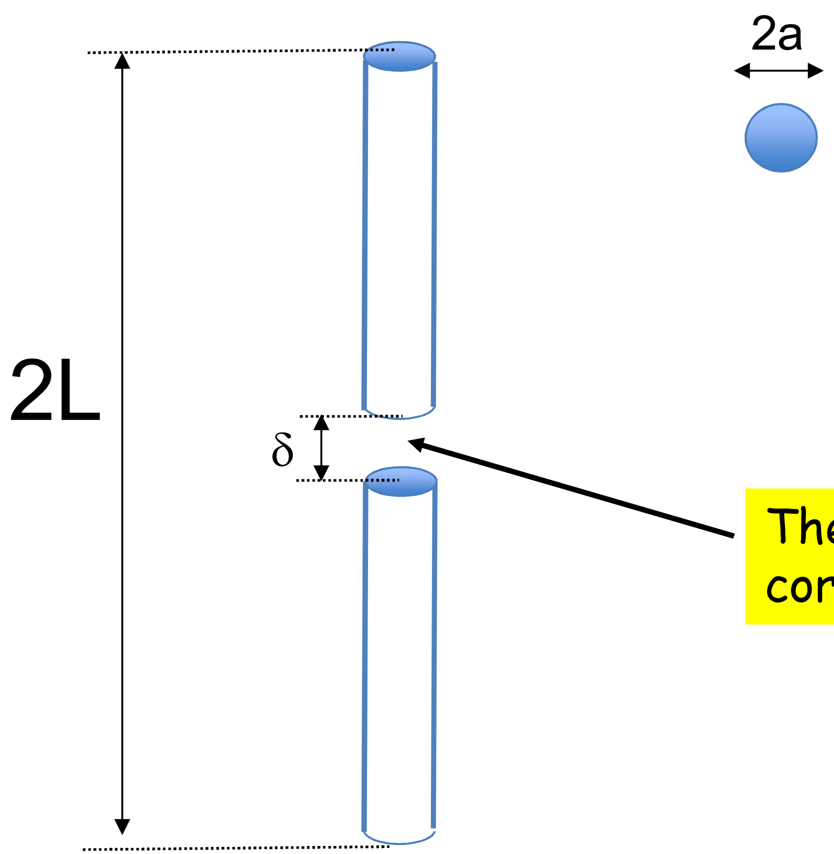
a.a. 2023–2024 – Laurea “Triennale” – Secondo semestre – Terzo anno

Ing. Stefano Perna

Wire antennas

Wire antennas

Wire antennas



$2a \ll \lambda$

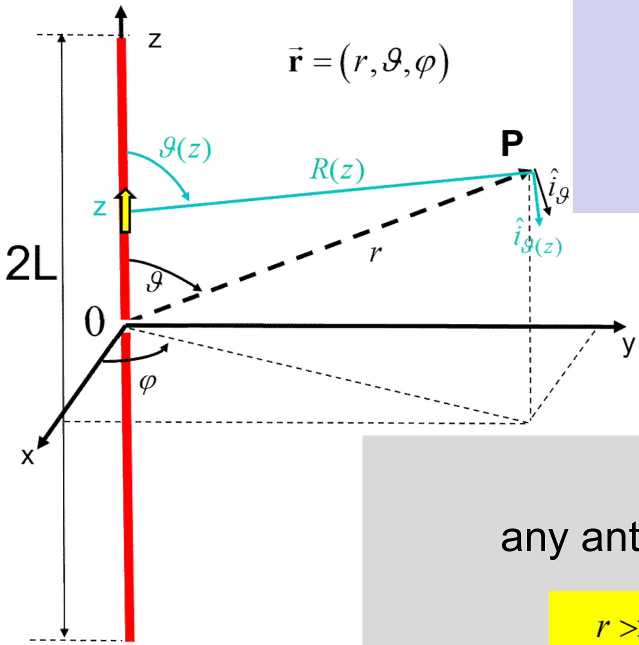
$2a \ll L$

$\delta \ll \lambda$

$\delta \ll L$

The terminals are located in correspondence of this gap

Wire antennas



In the Fraunhofer Region the expression of the radiated field simplifies as

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r} \sin\vartheta \left[\int_{-L}^{L} dz \frac{I(z)}{I_0} \exp(j\beta z\cos\vartheta) \right] \hat{i}_\vartheta$$

Effective length of the wire antenna

.... Memo

any antenna, in the Fraunhofer region, behaves as follows

$r >> D$

$r > \dfrac{2D^2}{\lambda}$

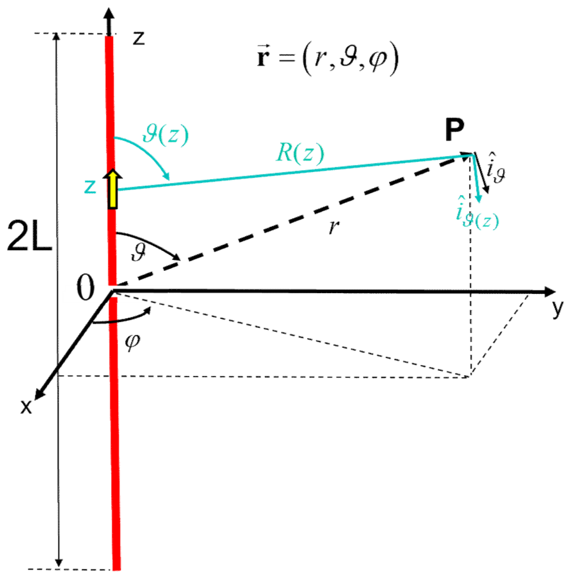
$r >> \lambda$

$$\begin{cases} \mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \dfrac{j\zeta I}{2\lambda} \dfrac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi) \\ \zeta \mathbf{H} = \hat{i}_r \times \mathbf{E} \end{cases}$$

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$ **Effective length**

Wire antennas: effective length

$$\vec{\mathbf{l}}(\vartheta)=l_\vartheta(\vartheta)\hat{i}_\vartheta=\sin\vartheta\left[\int_{-L}^{L}dz\frac{I(z)}{I_0}\exp(j\beta z\cos\vartheta)\right]\hat{i}_\vartheta$$



$$u=-\beta\cos\vartheta \qquad \tilde{I}(z)=\frac{I(z)}{I_0}$$

$$F(u)=\int_{-L}^{L}dz\,\tilde{I}(z)e^{-juz}$$

For the wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rule

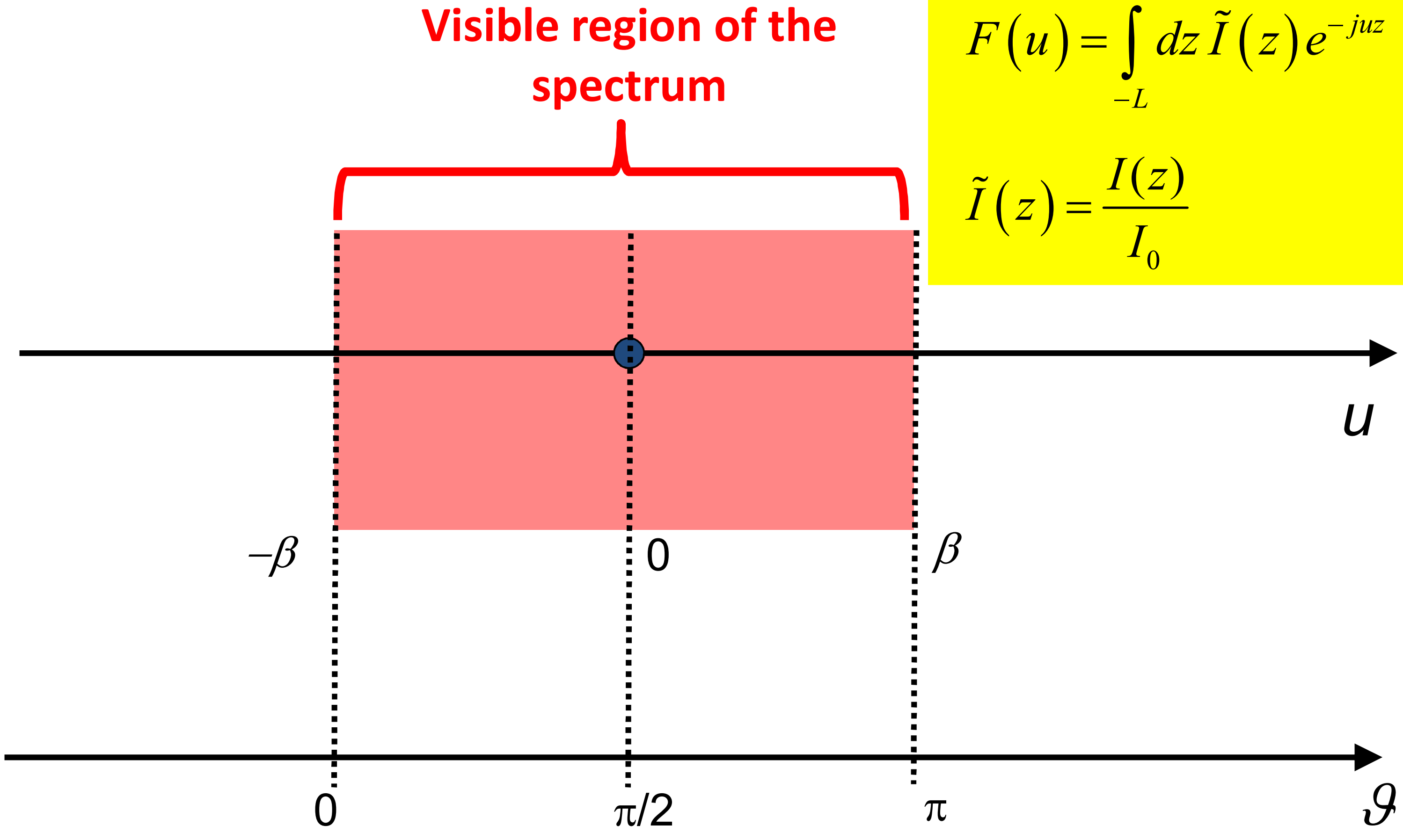
Wire antennas: visible region

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta)=F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u)=\int_{-L}^{L} dz\,\tilde{I}(z)e^{-juz}$$

$$\tilde{I}(z)=\frac{I(z)}{I_0}$$



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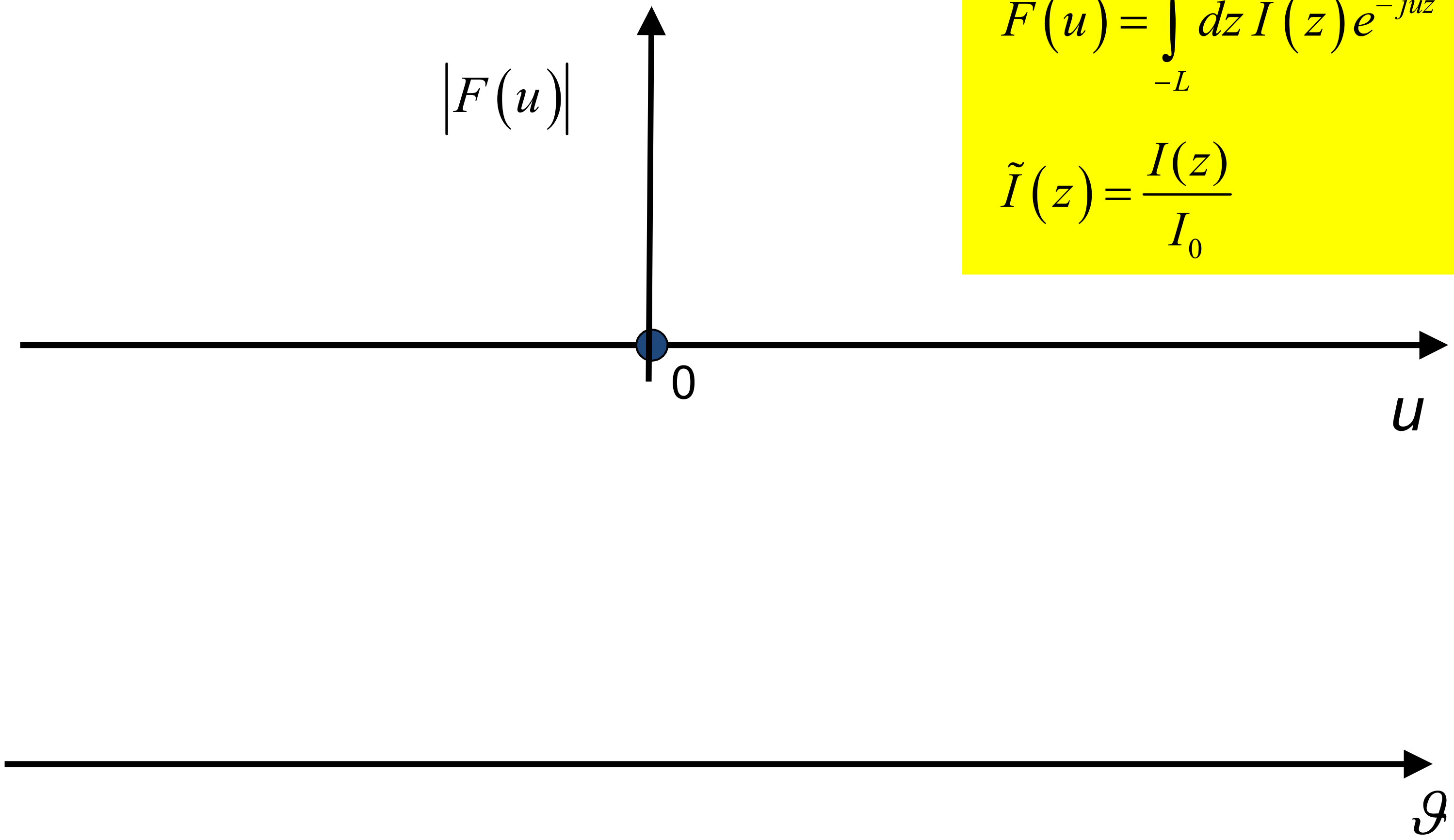

Wire antennas: visible region

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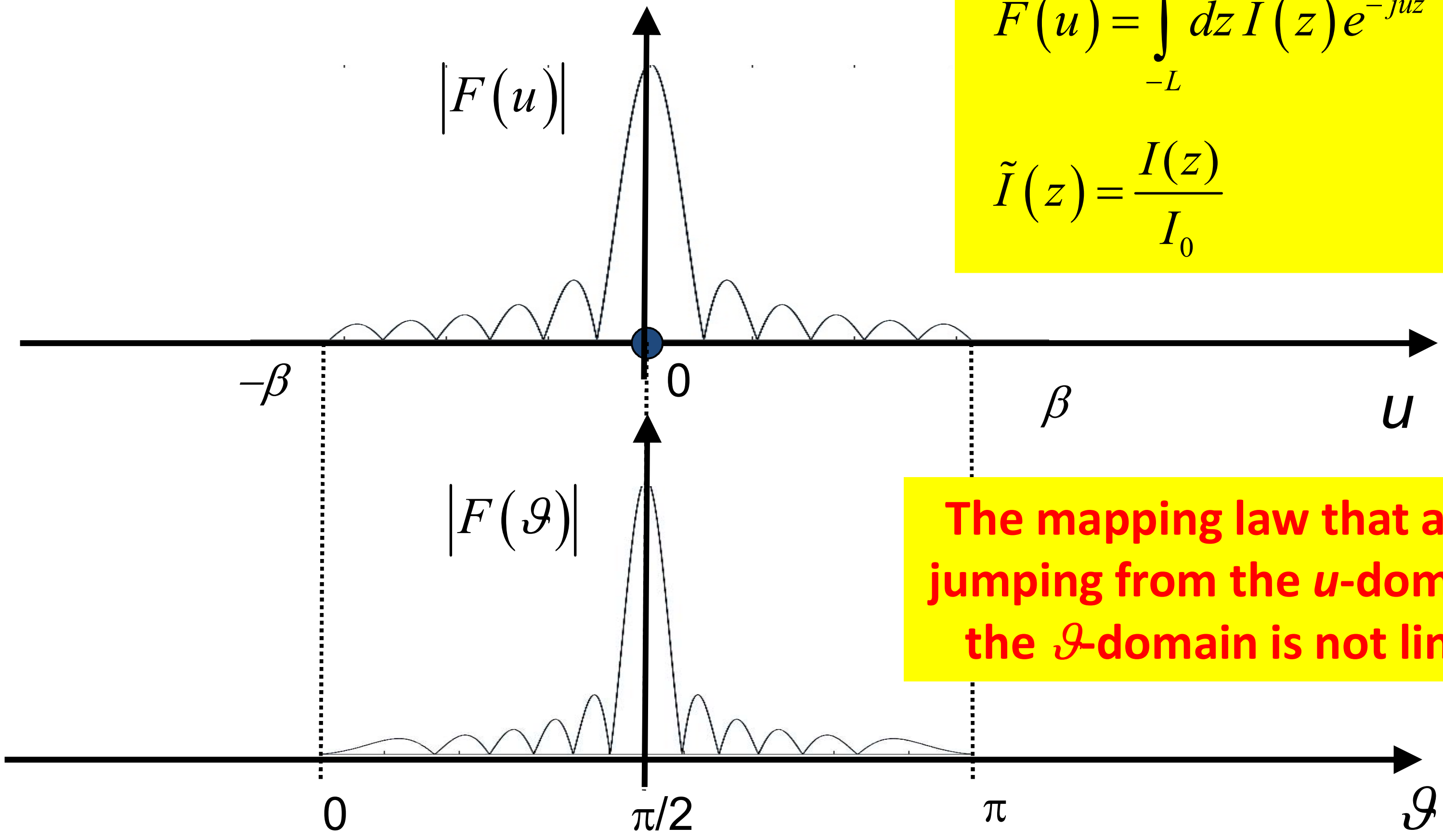


The mapping law that allows jumping from the *u*-domain to the ϑ-domain is not linear!

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

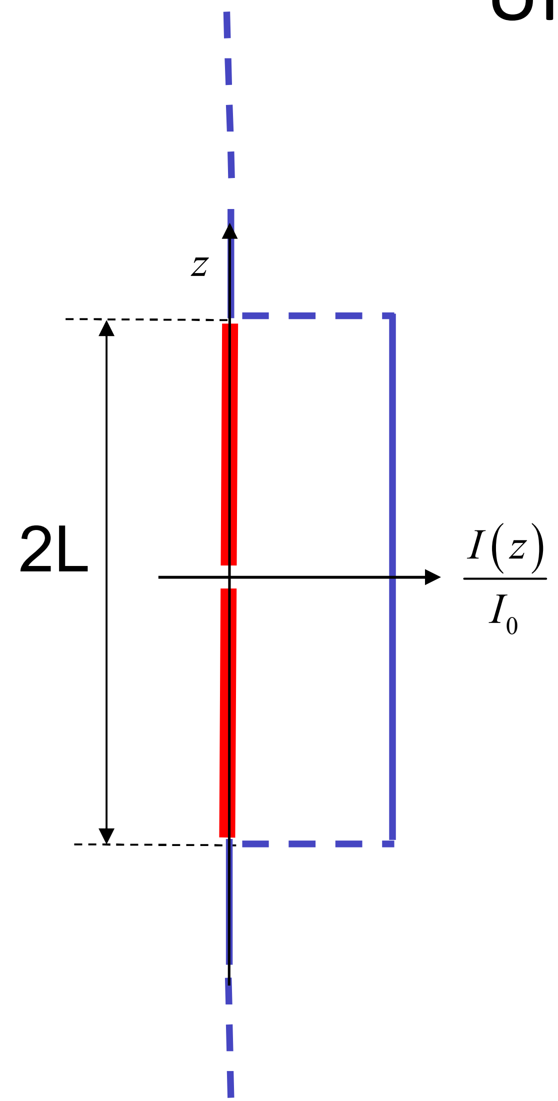
Memo

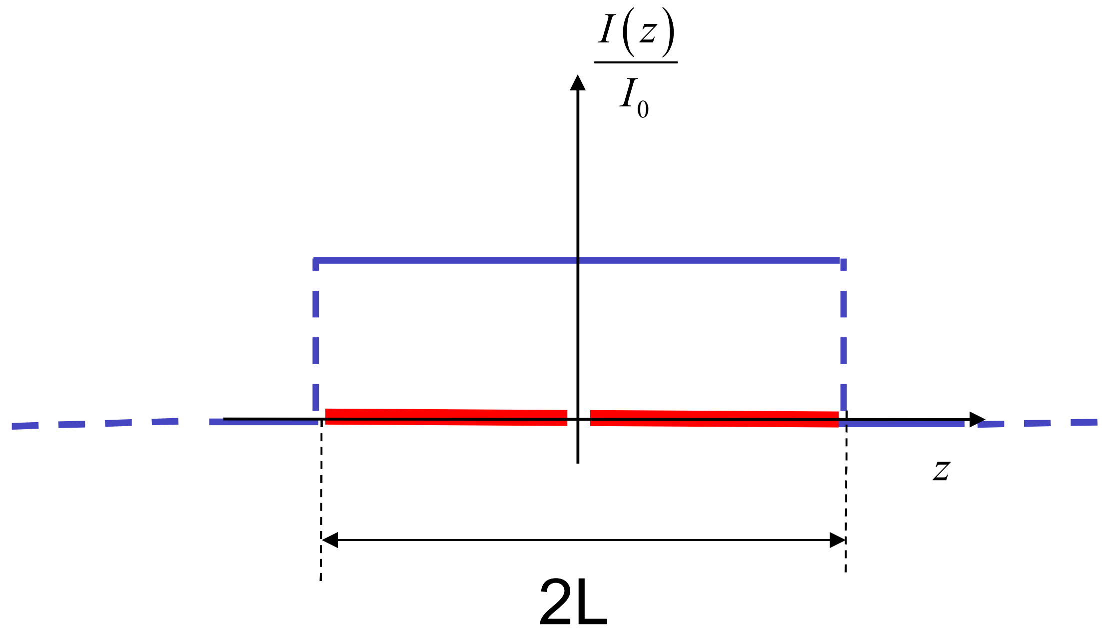
Mathematical tools to be exploited

Mathematics

Wire antennas: an ideal case

Uniform current distribution





$$\frac{I(z)}{I_0} = \tilde{I}(z) = \mathrm{rect}\left[\frac{z}{2L}\right]$$

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

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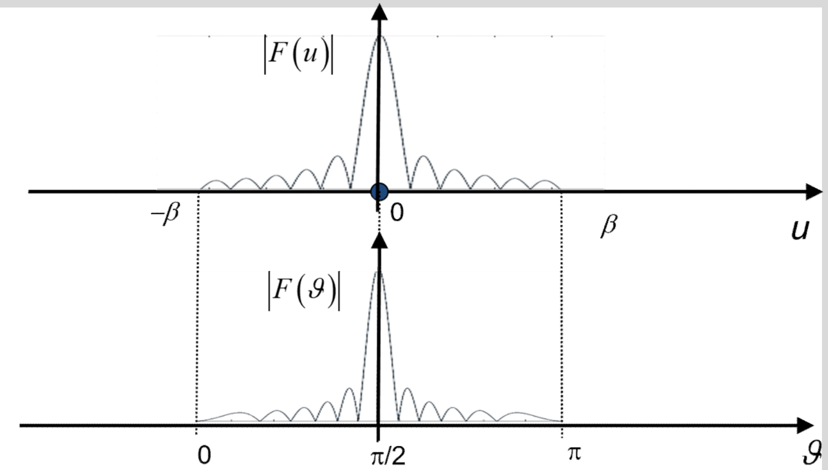
$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \text{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

.... Memo



1. Let's depict F(*u*)
2. Let's jump from *u* to ϑ and calculate:
   - The direction of the Main Lobe
   - The NNBW / HPBW
   - The SLL
   - The Directivity

Wire antennas: an ideal case

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\frac{I(z)}{I_0} = \tilde{I}(z) = \text{rect}\left[\frac{z}{2L}\right] \Rightarrow F(u) = 2L\frac{\sin(uL)}{uL}$$

1. **Let's depict F(*u*)** ✓

2. **Let's jump from *u* to ϑ and calculate:**

- **The direction of the Main Lobe** ✓
- **The NNBW / HPBW** ✓
- **The SLL** ✓
- **The Directivity**

$$\vartheta_{MB} = \frac{\pi}{2}$$

$$\text{NNBW} \approx \frac{\lambda}{L} \quad \text{HPBW} \approx 0.88\frac{\lambda}{2L}$$

$$\text{SLL} = -13.46\, dB$$

Color legend

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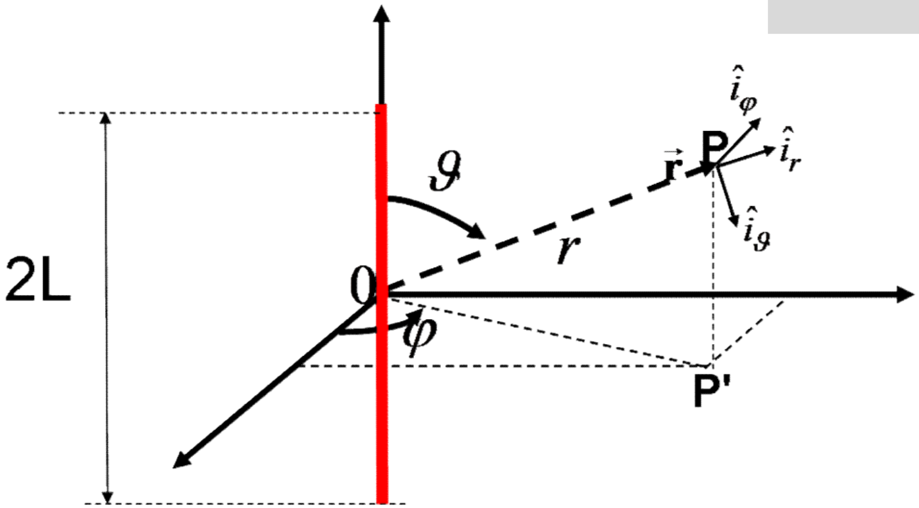
Current distribution

In wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rules.

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

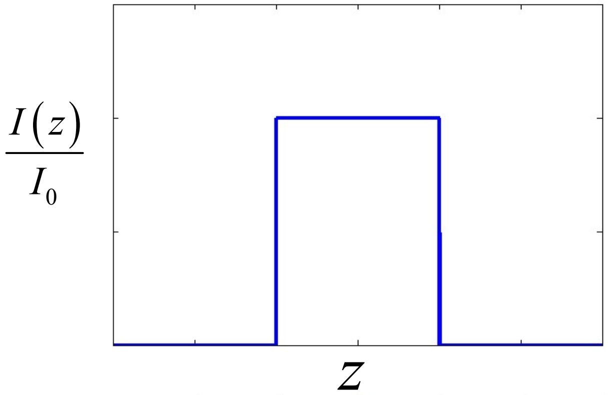


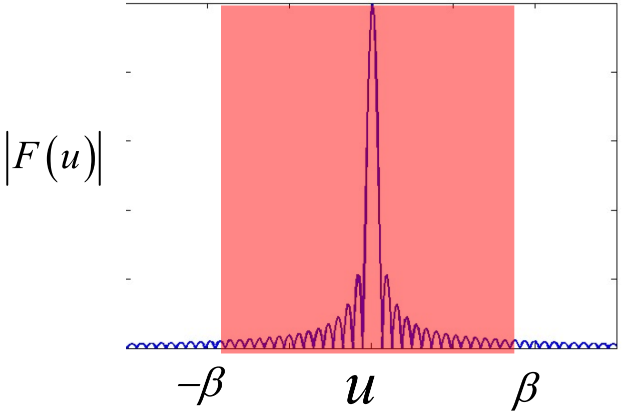
$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

$$F(u) = \int_{-L}^{L} dz\, \tilde{I}(z) e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

Current distribution





An ideal case

$$\frac{I(z)}{I_0} = \mathrm{rect}\left[\frac{z}{2L}\right]$$

$$F(u) = \int \frac{I(z)}{I_0} e^{-juz}\, dz \; = 2L\frac{\sin(uL)}{uL}$$

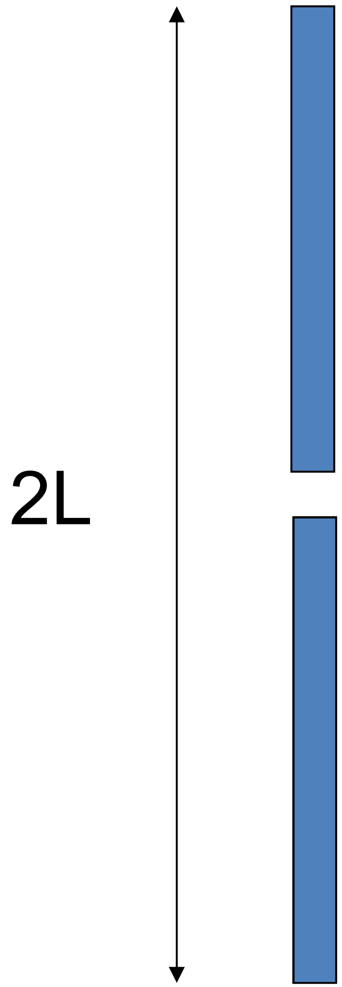
$$u = -\beta \cos\vartheta$$

Direction of the Main Lobe $\quad \vartheta_{MB} = \frac{\pi}{2}$

NNBW / HPBW $\quad \mathrm{NNBW} \approx \frac{\lambda}{L} \quad \mathrm{HPBW} \approx 0.88\frac{\lambda}{2L}$

SLL $\quad \mathrm{SLL} = -13.46\, dB$

Hallen Formulation



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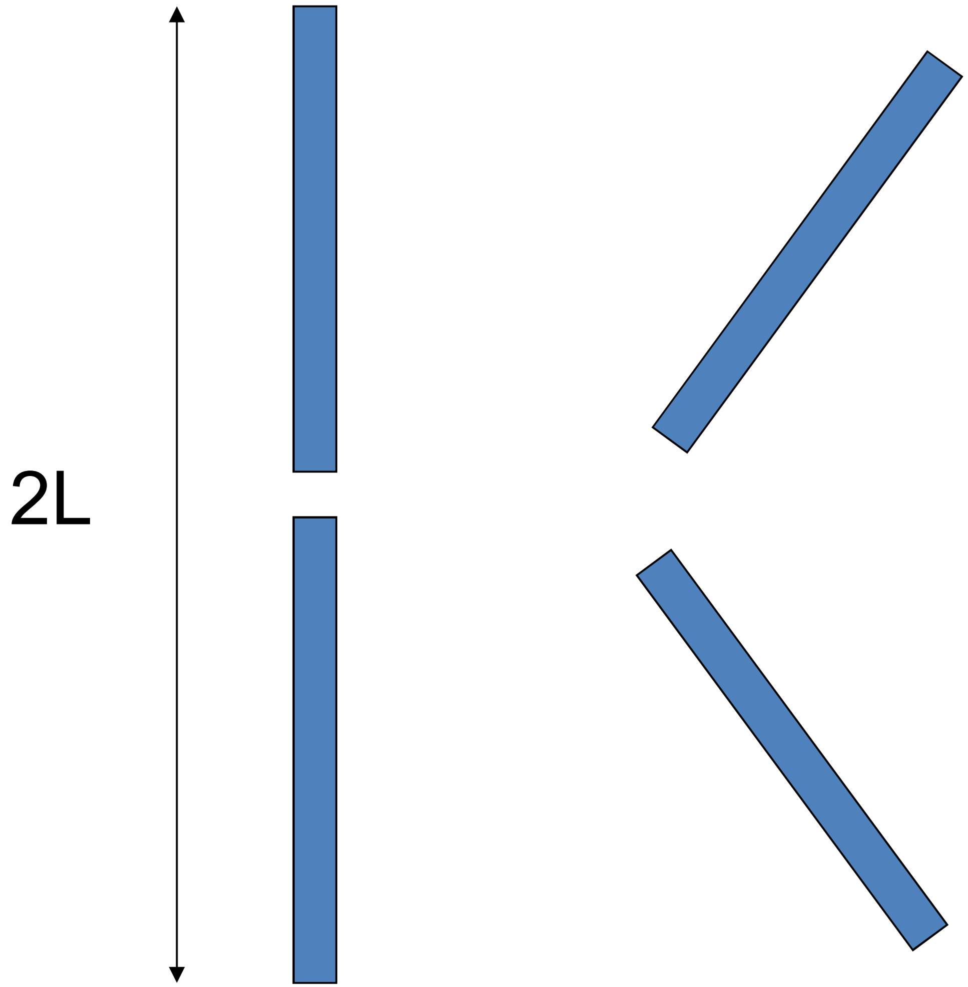

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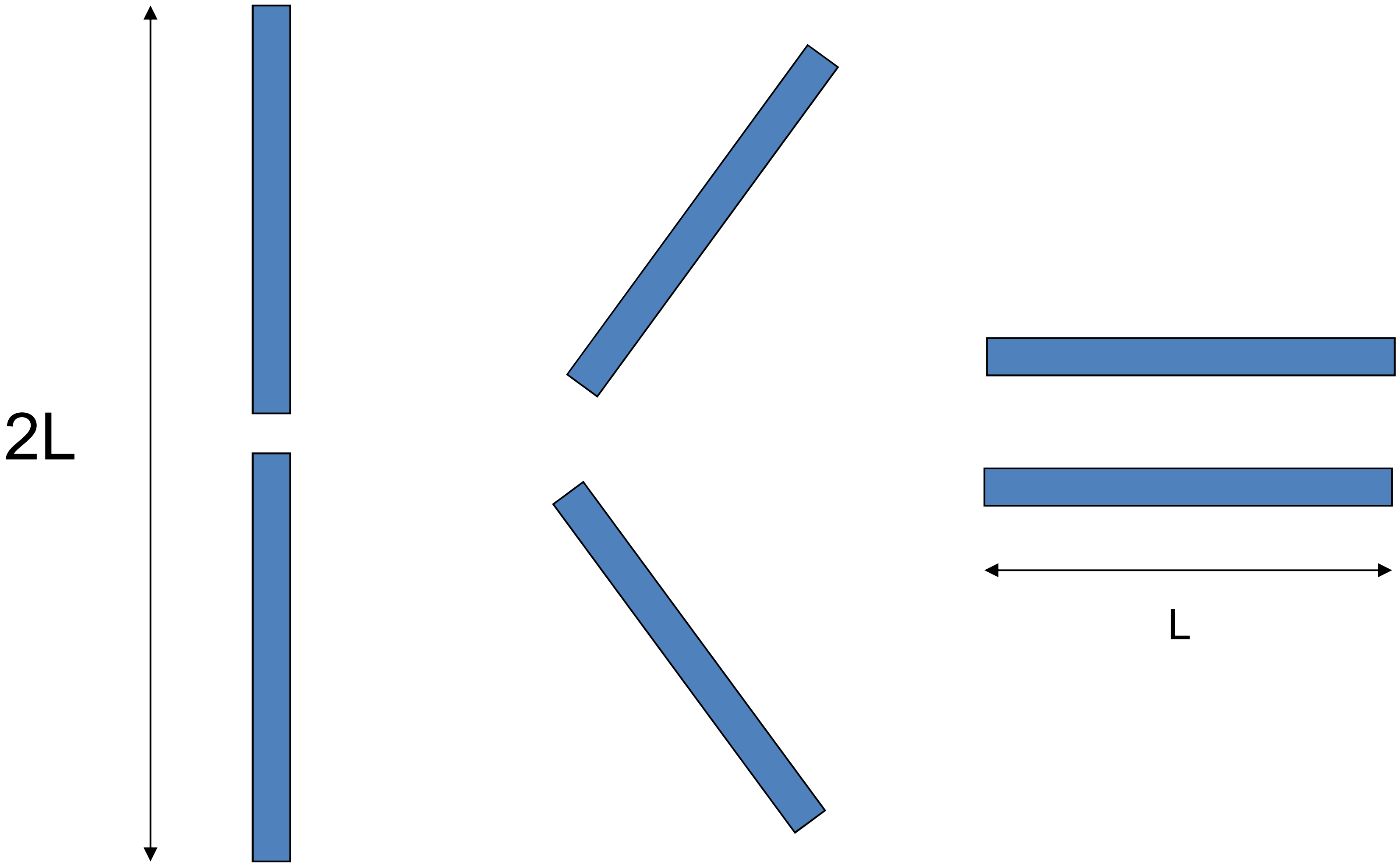

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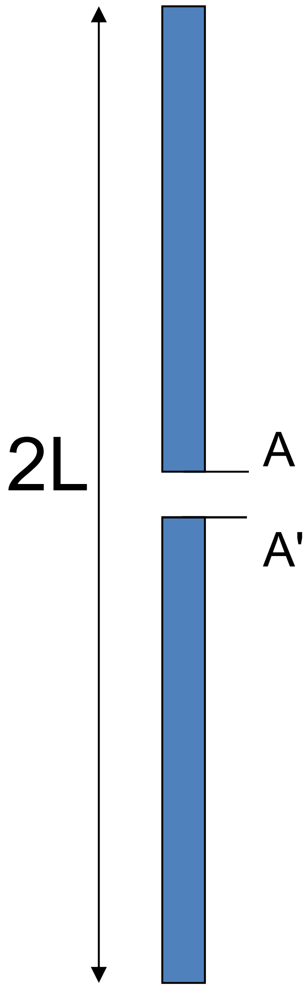
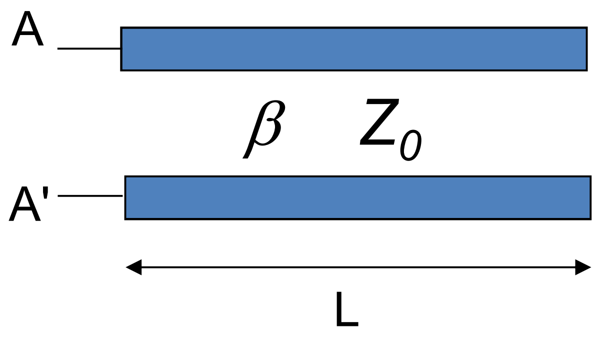
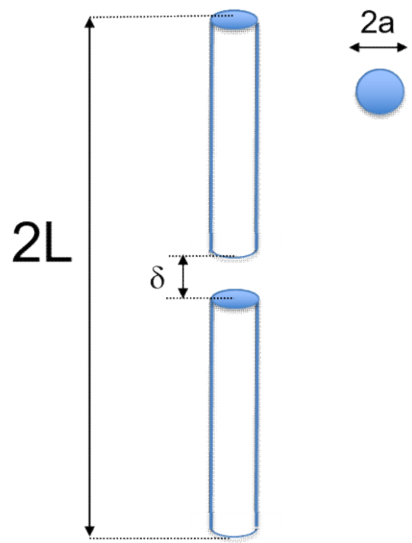




$$\beta = 2\pi/\lambda$$

$$Z_0 = \frac{\zeta \Omega}{2\pi}$$

$$\Omega = 2 \ln\left(\frac{2L}{a}\right)$$



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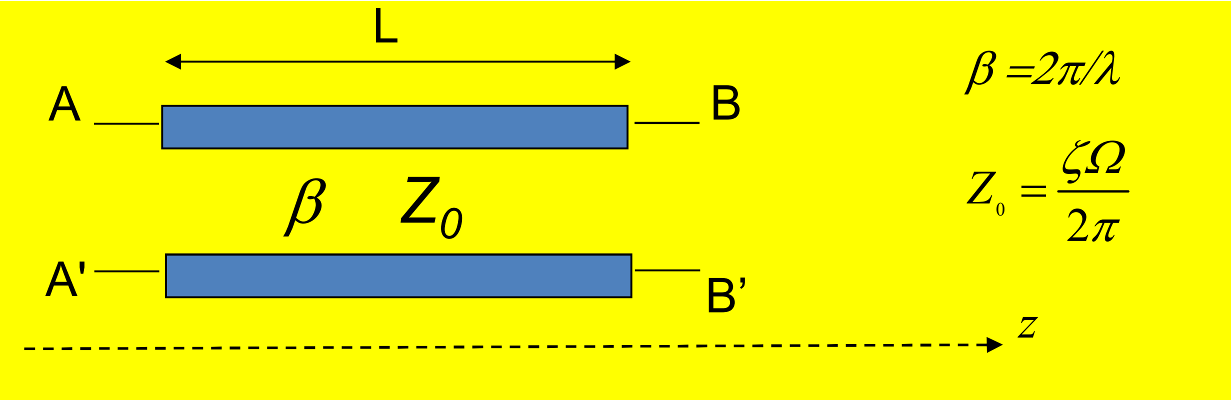

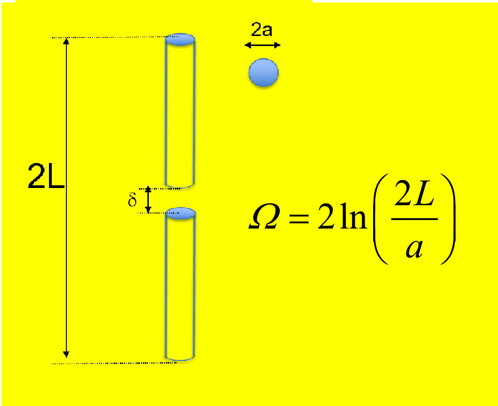

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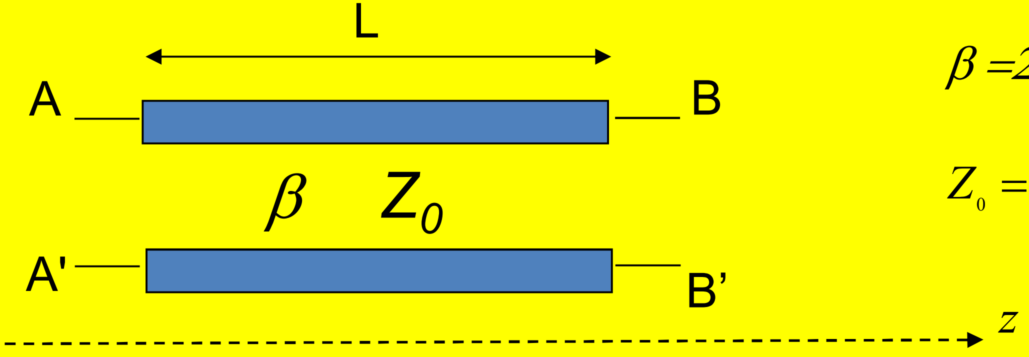

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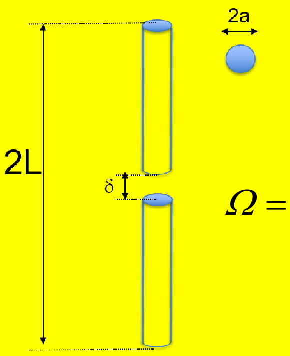
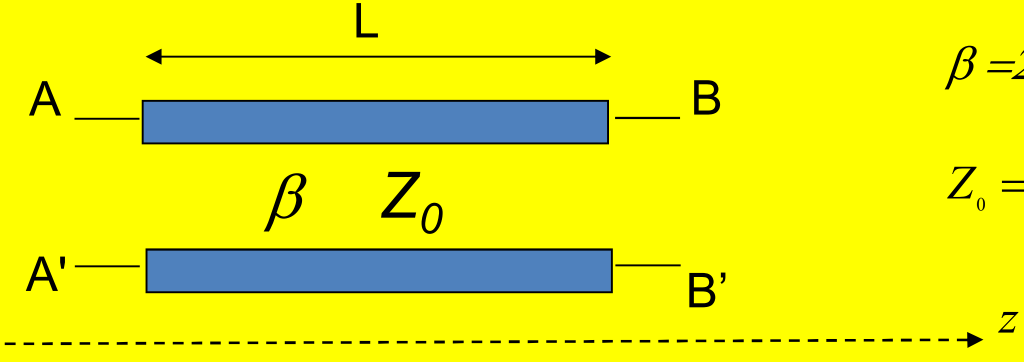


$$\Omega = 2\ln\left(\frac{2L}{a}\right)$$

$$\begin{cases} V(z) = V_{AA'}\cos(\beta z) - jZ_o I_{AA'}\sin(\beta z) \\ I(z) = I_{AA'}\cos(\beta z) - j\dfrac{V_{AA'}}{Z_o}\sin(\beta z) \end{cases}$$

$$Z_{AA'} = Z_o \frac{Z_{BB'} + jZ_o tg(\beta L)}{Z_o + jZ_{BB'} tg(\beta L)}$$

Hallen Formulation



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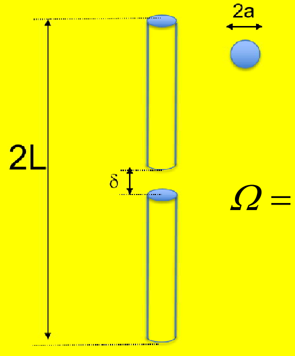

$$\Omega = 2\ln\left(\frac{2L}{a}\right)$$

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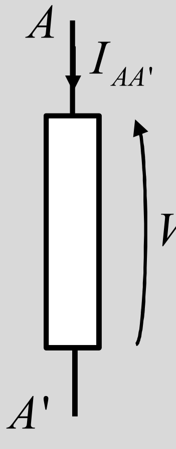
$$Z_{AA'} = Z_o \frac{Z_{BB'} + jZ_o tg(\beta L)}{Z_o + jZ_{BB'} tg(\beta L)}$$

$$Z_{BB'} \to \infty \quad \Rightarrow \quad Z_{AA'} = \frac{Z_o}{j\,tg(\beta L)} = -jZ_o ctg(\beta L)$$

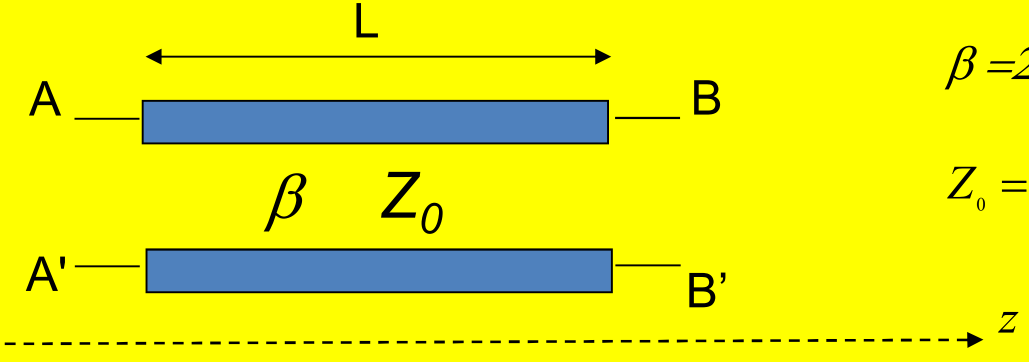
$$I(z) = I_{AA'}\cos(\beta z) - j\frac{V_{AA'}}{Z_o}\sin(\beta z) = I_{AA'}\cos(\beta z) - j\frac{Z_{AA'}I_{AA'}}{Z_o}\sin(\beta z)$$

$$= I_{AA'}\cos(\beta z) - j\frac{(-jZ_o ctg(\beta L))I_{AA'}}{Z_o}\sin(\beta z) = I_{AA'}\cos(\beta z) - I_{AA'}\frac{\cos(\beta \ldots}{\sin(\beta \ldots}$$



$$V_{AA'} = Z_{AA'} I_{AA'}$$

Hallen Formulation



$$\beta = 2\pi/\lambda$$

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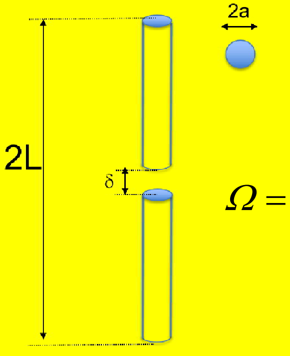


$$\Omega = 2\ln\left(\frac{2L}{a}\right)$$

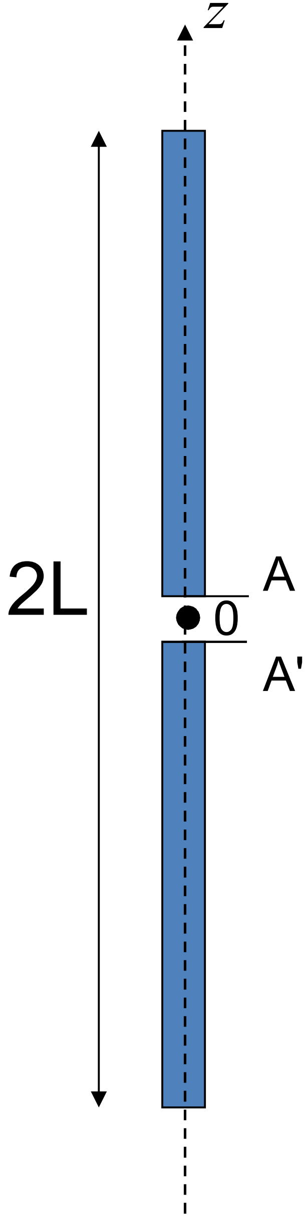
$$z_{AA'} = \frac{Z_o}{j\,tg(\beta L)} = -jZ_o ctg(\beta L)$$

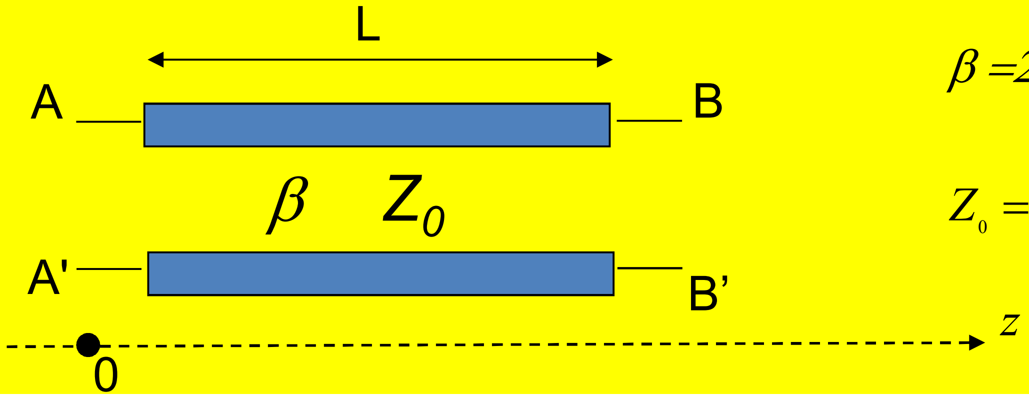
$$I(z) = I_{AA'}\cos(\beta z) - I_{AA'}\frac{\cos(\beta L)}{\sin(\beta L)}\sin(\beta z) = I_{AA'}\cos(\beta z)\frac{\sin(\beta L)}{\sin(\beta L)} - I_{AA'}\frac{\cos(\beta L)}{\sin(\beta L)}\sin(\beta z)$$

$$= \frac{I_{AA'}}{\sin(\beta L)}\left[\sin(\beta L)\cos(\beta z) - \cos(\beta L)\sin(\beta z)\right] = \frac{I_{AA'}}{\sin(\beta L)}\sin(\beta L - \beta z)$$

$$\sin(\alpha - \beta) = \sin\alpha\,\cos\beta - \cos\alpha\sin\beta$$

Hallen Formulation



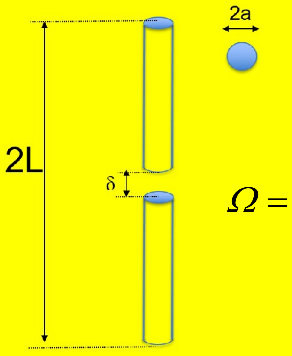


$$\beta = 2\pi/\lambda$$

$$Z_0 = \frac{\zeta\Omega}{2\pi}$$

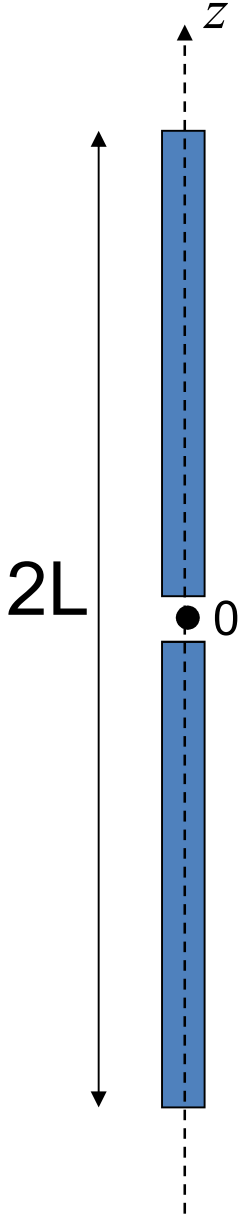
$$Z_{AA'} = -jZ_o ctg(\beta L)$$

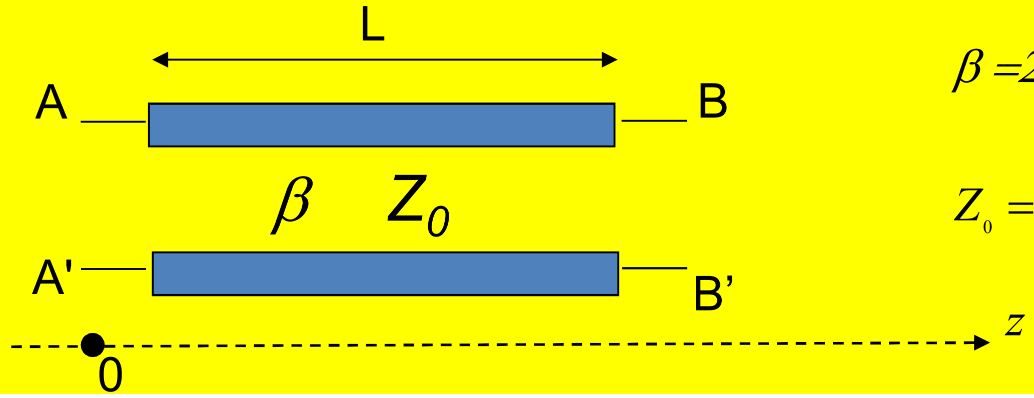
$$I(z) = I_{AA'}\frac{\sin(\beta L - \beta z)}{\sin(\beta L)}$$



$$\Omega = 2\ln\left(\frac{2L}{a}\right)$$

Hallen Formulation



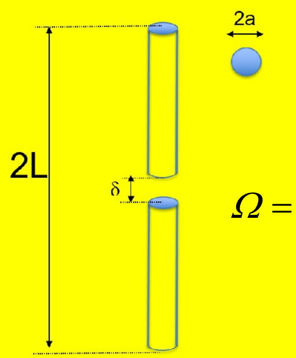


$$\beta = 2\pi/\lambda$$

$$Z_0 = \frac{\zeta \Omega}{2\pi}$$

$$Z_{in} = -jZ_o ctg(\beta L)$$

$$I(z) = I_0 \frac{\sin(\beta L - \beta |z|)}{\sin(\beta L)}$$



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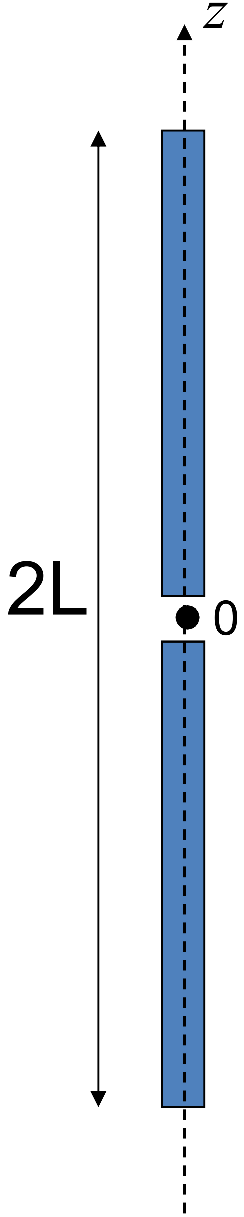

$$Z_{\text{in}} = -jZ_o ctg\left(\beta L\right) = jX_{\text{in}}$$

$$I\left(z\right) = I_0 \frac{\sin\left(\beta L - \beta\left|z\right|\right)}{\sin\left(\beta L\right)}$$

$\beta = 2\pi/\lambda$

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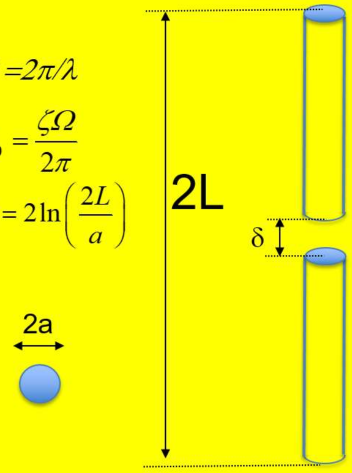
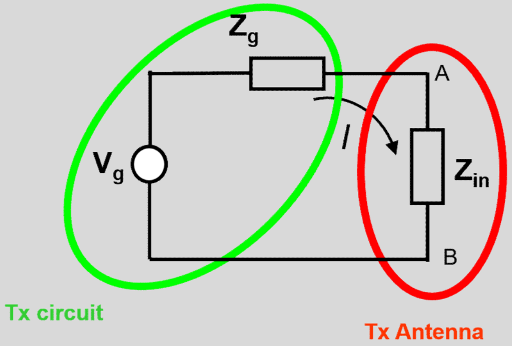


....MEMO..

Equivalent circuit of the Tx antenna

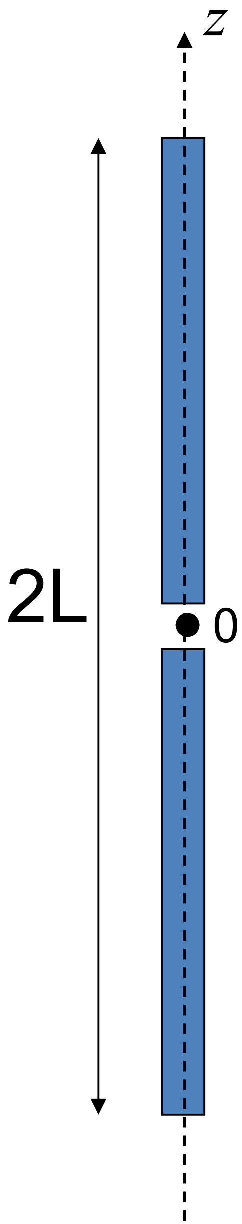


$Z_{in} = R_{in} + jX_{in}$

$P_{in} = \frac{1}{2} R_{in} \left|I\right|^2$

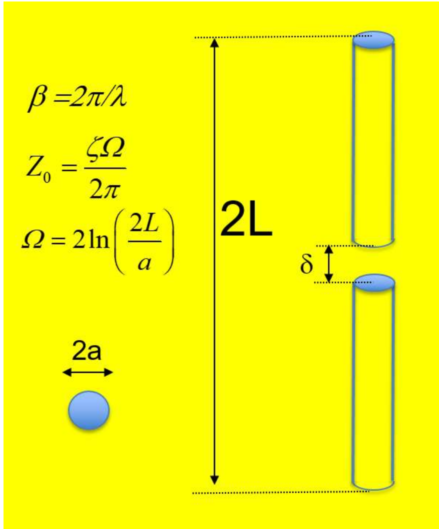
This antenna does NOT radiate!

Hallen Formulation



$$Z_{\text{in}} = -jZ_o ctg(\beta L) = jX_{\text{in}}$$

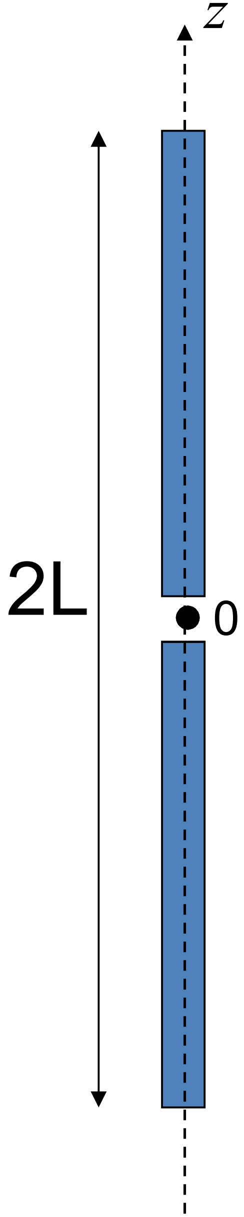
$$I(z) = I_0 \frac{\sin(\beta L - \beta |z|)}{\sin(\beta L)}$$



The Hallen model provides a wrong value of the input resistance R_{in}, due to the employed approximations. Actually, measurements carried out in laboratory show that, in contrast to the Hallen formulation, $R_{in} \neq 0$.

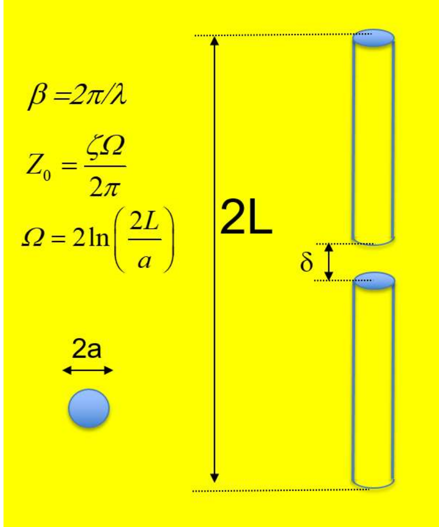
This antenna does NOT radiate!

Hallen Formulation



$$Z_{in} = -jZ_o ctg(\beta L) = jX_{in}$$

$$I(z) = I_0 \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$



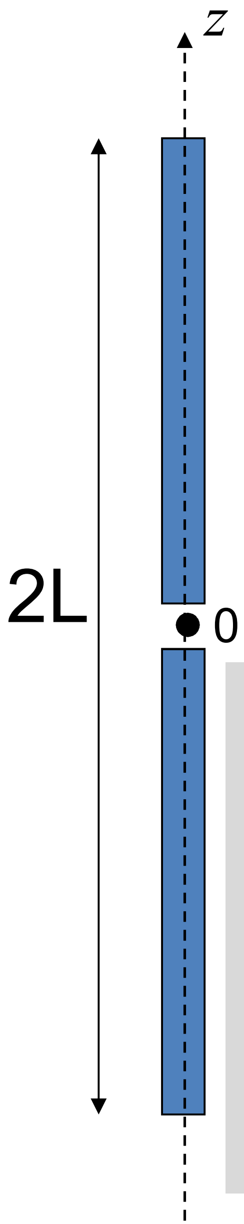
The Hallen model provides a wrong value of the input resistance R_{in}, due to the employed approximations. Actually, measurements carried out in laboratory show that, in contrast to the Hallen formulation, $R_{in} \neq 0$.

On the other side, measurements carried out in laboratory show that the input reactance X_{in} provided by the Hallen model is quite accurate.

Measurements carried out in laboratory show also that the far field obtained by employing the expression I(z) provided by the Hallen model is very accurate.

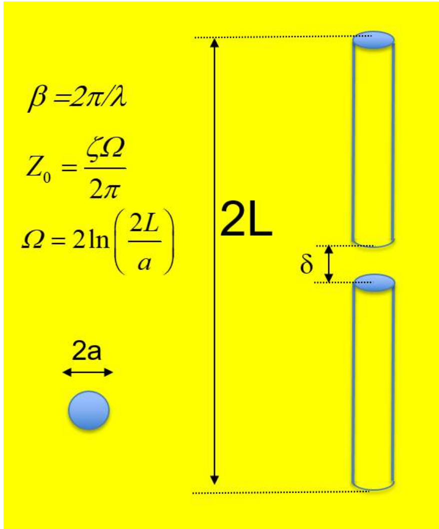
This antenna radiates!

Hallen Formulation



$$Z_{\text{in}} = -jZ_o ctg(\beta L) = jX_{\text{in}}$$

$$I(z) = I_0 \frac{\sin(\beta L - \beta |z|)}{\sin(\beta L)}$$



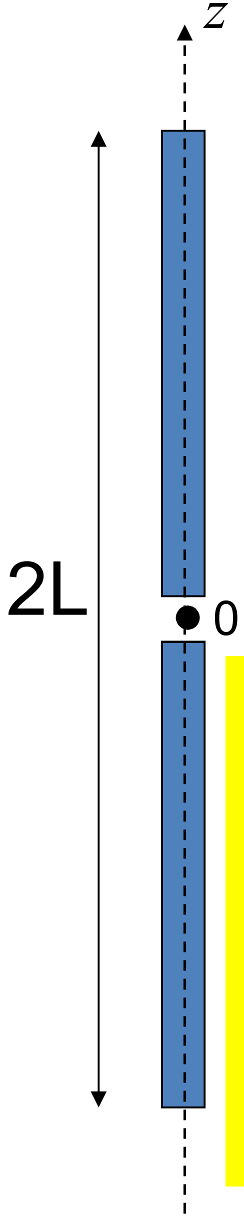
$$P_{\text{in}} = \frac{1}{2} R_{\text{in}} |I_0|^2 \qquad P_{rad} = \frac{1}{2} R_{rad} |I_0|^2$$

MEMO

$$P_{rad} = \oiint_S \frac{1}{2\zeta} |\vec{\mathbf{E}}|^2$$

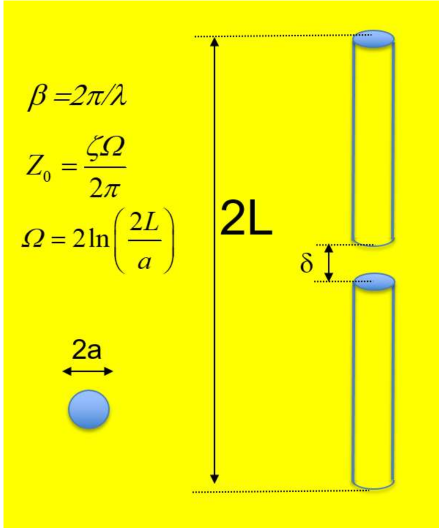
This antenna radiates!

Hallen Formulation



$$Z_{\text{in}} = -jZ_o ctg\left(\beta L\right) = jX_{\text{in}}$$

$$I\left(z\right) = I_0 \frac{\sin\left(\beta L - \beta\left|z\right|\right)}{\sin\left(\beta L\right)}$$

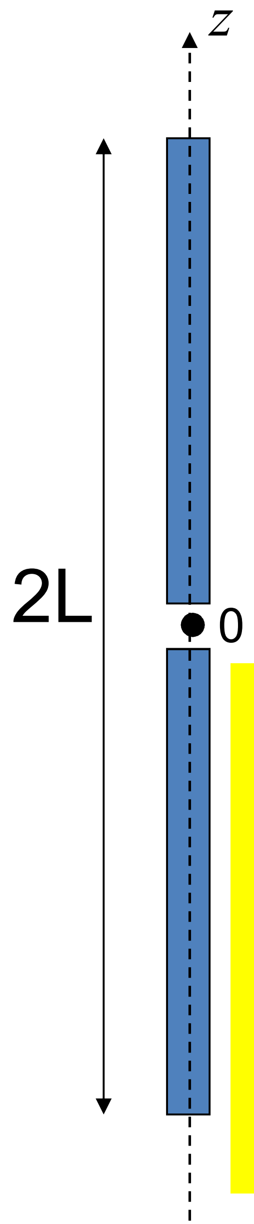


$$P_{\text{in}} = \frac{1}{2} R_{\text{in}} \left|I_0\right|^2$$

$$P_{rad} = \frac{1}{2} R_{rad} \left|I_0\right|^2$$

$$P_{rad} = \oiint_S \frac{1}{2\zeta} \left|\vec{\mathbf{E}}\right|^2$$

This antenna radiates!



Hallen Formulation

Memo

In wire antennas the source impressed on the antenna is related to the radiated field through the Fourier Transformation rules.

$$\vec{\mathbf{E}} = j\frac{\zeta}{2\lambda} I_0 \frac{\exp[-j\beta r]}{r}\left[\sin\vartheta F(\vartheta)\hat{i}_\vartheta\right]$$

$$F(\vartheta) = F(u)\Big|_{u=-\beta\cos\vartheta}$$

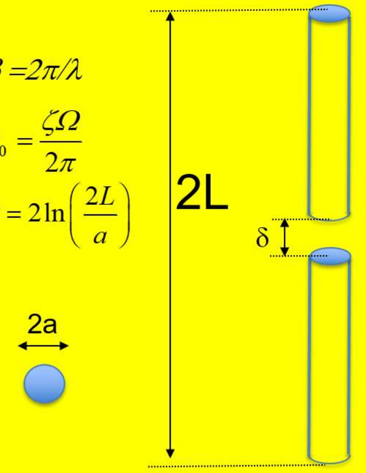
$$F(u) = \int_{-L}^{L} dz\,\tilde{I}(z)e^{-juz}$$

$$\tilde{I}(z) = \frac{I(z)}{I_0}$$

$$\beta = 2\pi/\lambda$$

$$Z_0 = \frac{\zeta\Omega}{2\pi}$$

$$\Omega = 2\ln\left(\frac{2L}{a}\right)$$

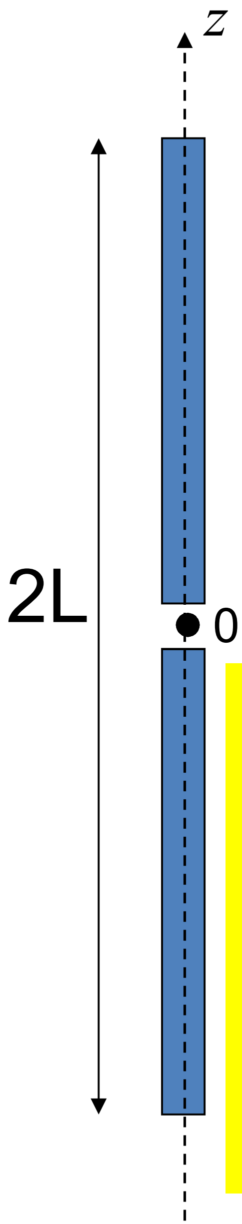


$$P_{\text{in}} = \frac{1}{2}R_{\text{in}}\left|I_0\right|^2 \qquad P_{rad} = \frac{1}{2}R_{rad}\left|I_0\right|^2$$

$$\text{I} \quad P_{rad} = \oint\!\!\oint_S \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2$$

This antenna radiates!

Hallen Formulation



$$Z_{\text{in}} = -jZ_o ctg(\beta L) = jX_{\text{in}}$$

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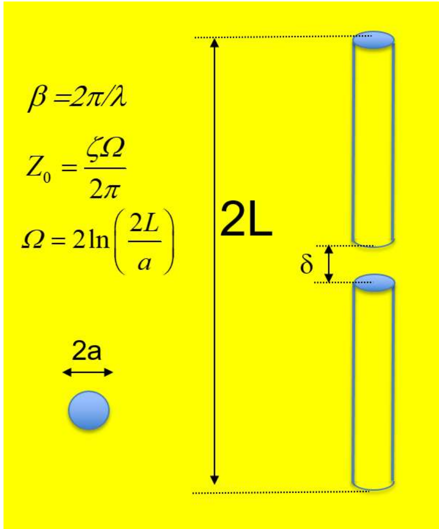


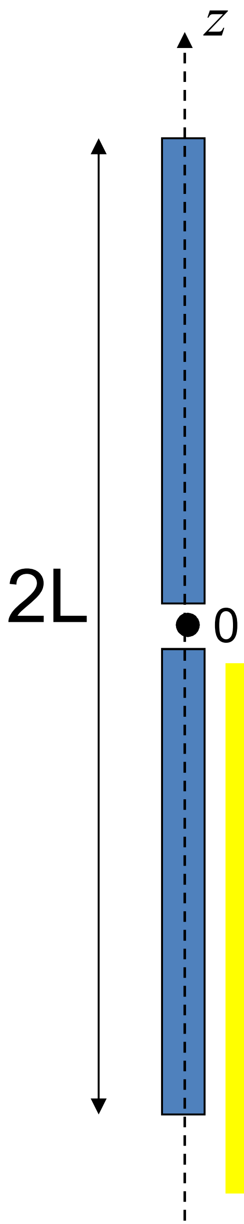
$$P_{\text{in}} = \frac{1}{2} R_{\text{in}} |I_0|^2$$

$$\text{II} \quad P_{rad} = \frac{1}{2} R_{rad} |I_0|^2$$

$$P_{rad} = \oiint_S \frac{1}{2\zeta} |\vec{\mathbf{E}}|^2$$

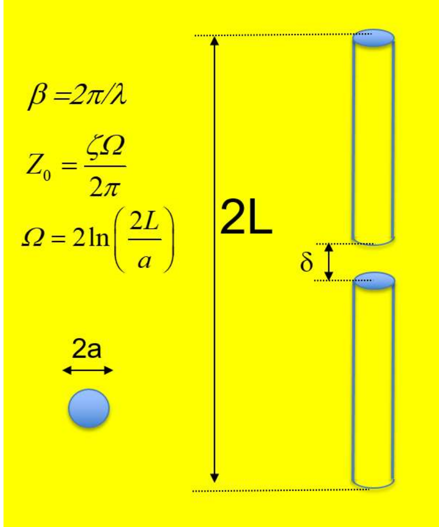
This antenna radiates!

Hallen Formulation



$$Z_{\text{in}} = -jZ_o ctg(\beta L) = jX_{\text{in}}$$

$$I(z) = I_0 \frac{\sin(\beta L - \beta|z|)}{\sin(\beta L)}$$



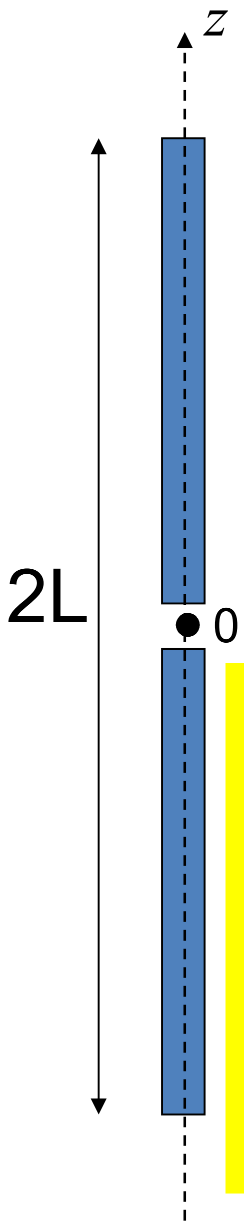
III $\quad P_{\text{in}} = \frac{1}{2} R_{\text{in}} |I_0|^2$

$$P_{rad} = \frac{1}{2} R_{rad} |I_0|^2$$

$$P_{rad} = \oint\!\!\oint_S \frac{1}{2\zeta} |\vec{\mathbf{E}}|^2$$

This antenna radiates!

Hallen Formulation



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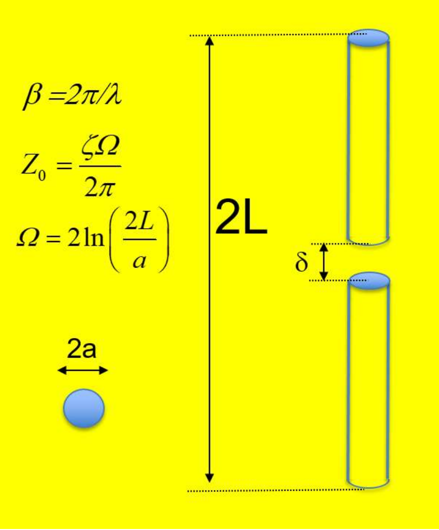

III $P_{\text{in}} = \frac{1}{2} R_{\text{in}} |I_0|^2$

$$P_{rad} = \frac{1}{2} R_{rad} |I_0|^2$$

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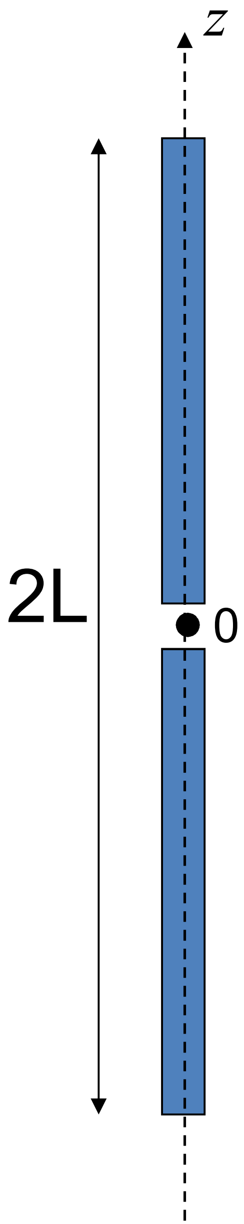
This antenna radiates!

Hallen Formulation



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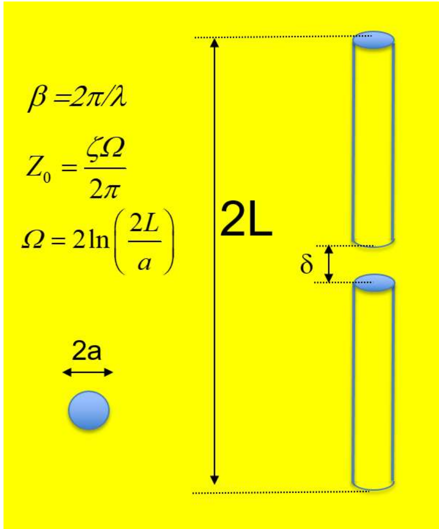

The Hallen model provides a wrong value of the input resistance R_{in}, due to the employed approximations

Notwithstanding, measurements carried out in laboratory show that the input reactance X_{in} provided by the Hallen model is quite accurate.

Measurements carried out in laboratory show also that the far field obtained by employing the expression I(z) provided by the Hallen model is very accurate.

From the analytical expression of the far field obtained by employing the expression I(z) provided by the Hallen model, we can obtain a quite accurate value of the radiation resistance R_{rad} (and thus a sound estimate of the input resistance R_{in}).