Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

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Color legend

🟪 New formulas, important considerations, important formulas, important concepts

🟨 Very important for the discussion

⬜ Memo

🟩 Mathematical tools to be exploited

🟦 Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases}$$

$$\frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x^+(z)=E^+e^{-jkz}=E^+e^{-j\beta z}e^{-\alpha z}$$

$$\zeta H_y^+(z)=E^+e^{-jkz}=E^+e^{-j\beta z}e^{-\alpha z}$$

$$\begin{cases} \varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0) \\ \mu(\omega_0)=\mu_1(\omega_0)-j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

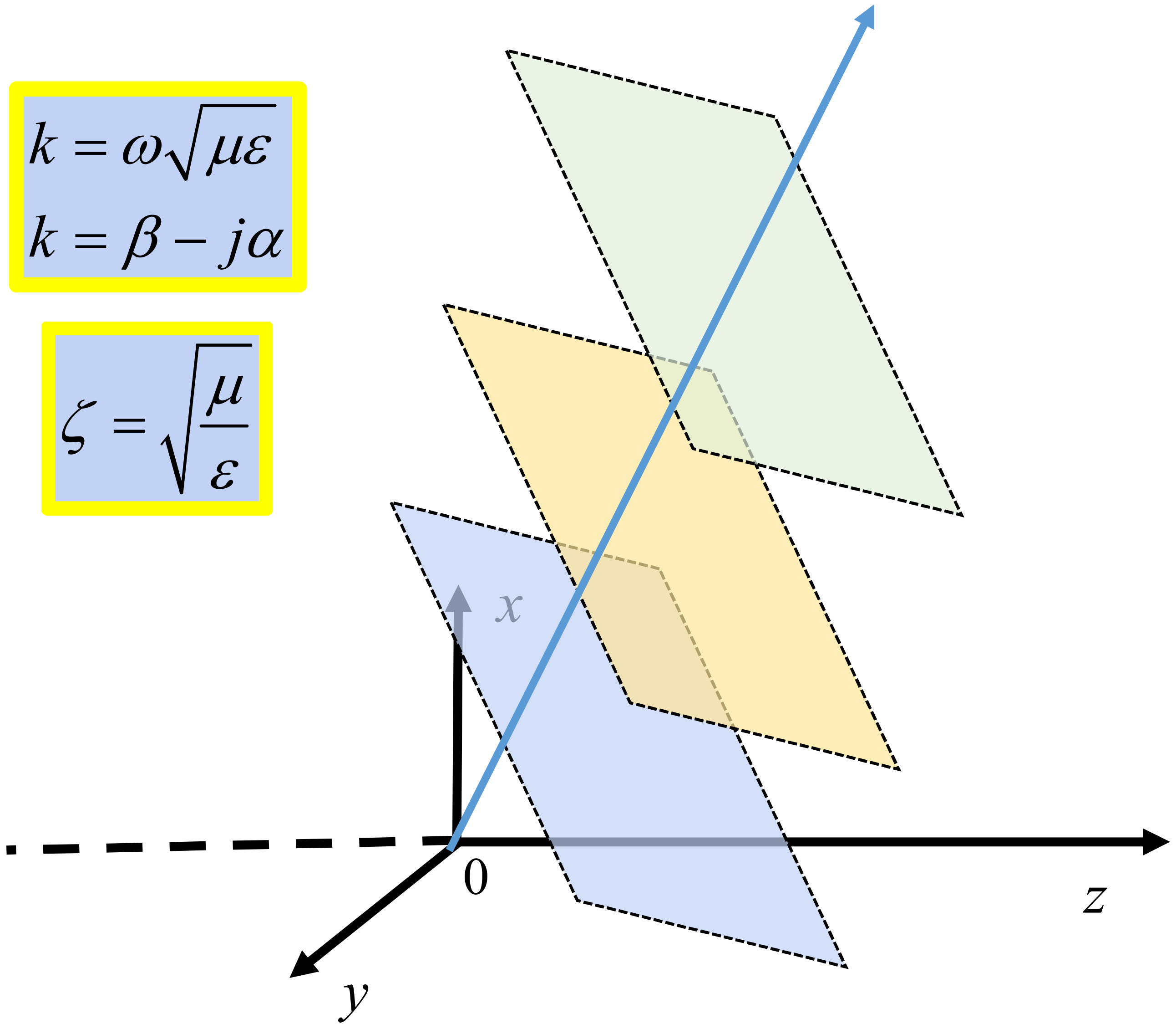
$$k(\omega_0)=\beta(\omega_0)-j\alpha(\omega_0)$$

Phasor Domain

$$k=\omega\sqrt{\mu\varepsilon}$$

$$k=\beta-j\alpha$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$



Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

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$$\begin{cases} \vec{\mathbf{k}}\times\vec{\mathbf{E}} = \omega\mu\overrightarrow{\mathbf{H}} \\ \vec{\mathbf{k}}\times\overrightarrow{\mathbf{H}} = -\omega\varepsilon\overrightarrow{\mathbf{E}} \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}}\cdot\overrightarrow{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

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$$\begin{cases} \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = k_z E_z = 0 \\ \vec{\mathbf{k}}\cdot\overrightarrow{\mathbf{H}} = k_z H_z = 0 \end{cases} \Rightarrow \quad E_z = H_z = 0$$

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$$\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}}$$

$$\Downarrow$$

$$\beta\hat{i}_z \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}}$$

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$\vec{\mathbf{k}}$:propagation vector

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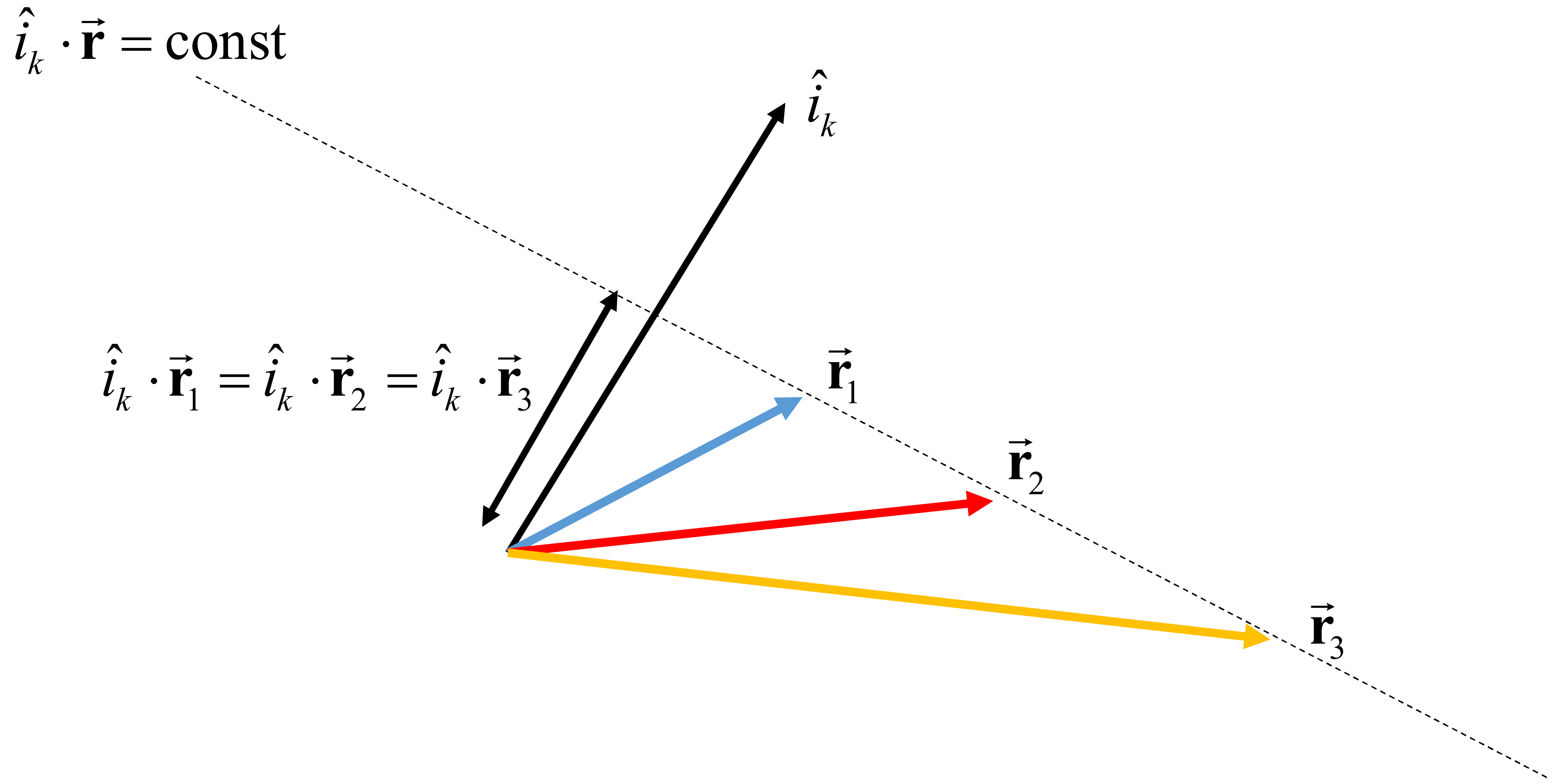

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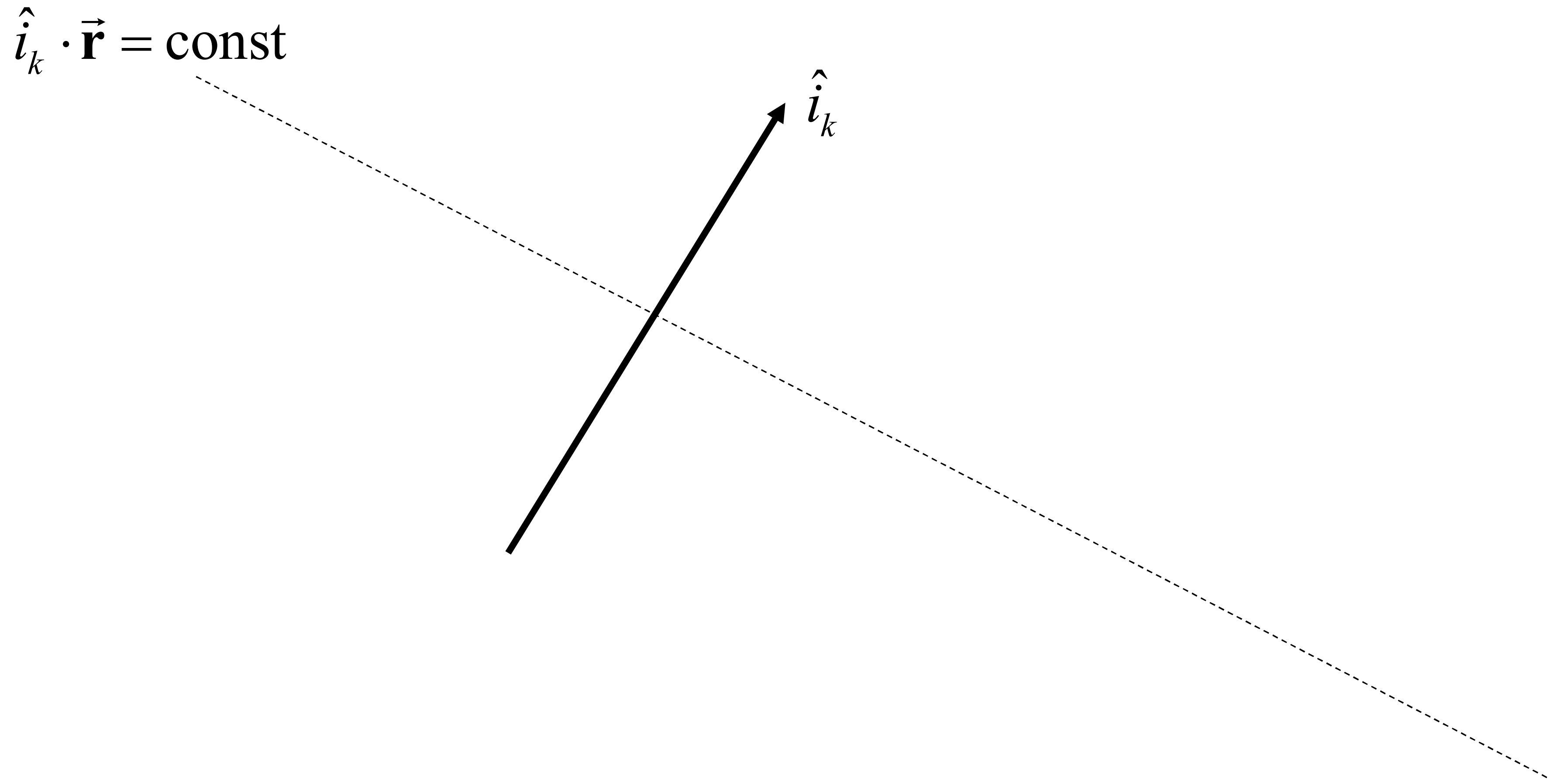

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = \text{const} \quad \Rightarrow \quad \text{equi-phase planes}$$

The wave propagates along $\vec{\mathbf{k}}$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = \text{const} \quad \Rightarrow \quad \text{planes over which the overall field is constant}$$

The field is constant in the plane orthogonal to the propagation direction $\vec{\mathbf{k}}$

The electromagnetic field is TEM

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$$\begin{cases} \hat{i}_k \times \vec{\mathbf{E}} = \zeta\overrightarrow{\mathbf{H}} \\ \zeta\overrightarrow{\mathbf{H}}\times\hat{i}_k = \vec{\mathbf{E}} \end{cases}$$

MEMO : Plane Waves (TD)

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta \vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$

Source-free

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- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$\Downarrow$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

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$\vec{\mathbf{k}}$:propagation vector

Medium

- Linear - Isotropic - Space nondispersive - Time nondispersive - Lossles
- Homogeneous (Time-invariant & Space-invariant)

$$k = \omega\sqrt{\mu\varepsilon} = \beta$$

$$\vec{\mathbf{k}} = \beta\hat{i}_k \quad \Rightarrow \quad \vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon = \beta^2$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-j\beta\hat{i}_k\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = \text{const} \quad \Rightarrow \quad \text{equi-phase planes}$$

The wave propagates along $\vec{\mathbf{k}}$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = \text{const} \quad \Rightarrow \quad \text{planes over which the overall field is constant}$$

The field is constant in the plane orthogonal to the propagation direction $\vec{\mathbf{k}}$

The electromagnetic field is TEM

$$\begin{cases} \hat{i}_k \times \vec{\mathbf{E}} = \zeta\overrightarrow{\mathbf{H}} \\ \zeta\overrightarrow{\mathbf{H}} \times \hat{i}_k = \vec{\mathbf{E}} \end{cases}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}^{+}e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}=\vec{\mathbf{E}}^{+}e^{-jk_x x}e^{-jk_y y}e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}^{+}e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}=\vec{\mathbf{H}}^{+}e^{-jk_x x}e^{-jk_y y}e^{-jk_z z}$$

$$\vec{\mathbf{k}}=k_x\hat{i}_x+k_y\hat{i}_y+k_z\hat{i}_z$$

$$\vec{\mathbf{r}}=x\hat{i}_x+y\hat{i}_y+z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}=k_x x+k_y y+k_z z$$

Source-free

$$\begin{cases}\vec{\mathbf{k}}\times\vec{\mathbf{E}}=\omega\mu\overrightarrow{\mathbf{H}}\\ \vec{\mathbf{k}}\times\overrightarrow{\mathbf{H}}=-\omega\varepsilon\vec{\mathbf{E}}\\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}}=0\\ \vec{\mathbf{k}}\cdot\overrightarrow{\mathbf{H}}=0\end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}}=\omega^2\mu\varepsilon$$

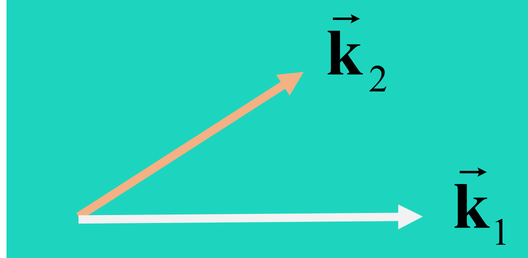
$\vec{\mathbf{k}}$:propagation vector

$$\vec{\mathbf{k}}=\vec{\mathbf{k}}_1-j\vec{\mathbf{k}}_2$$

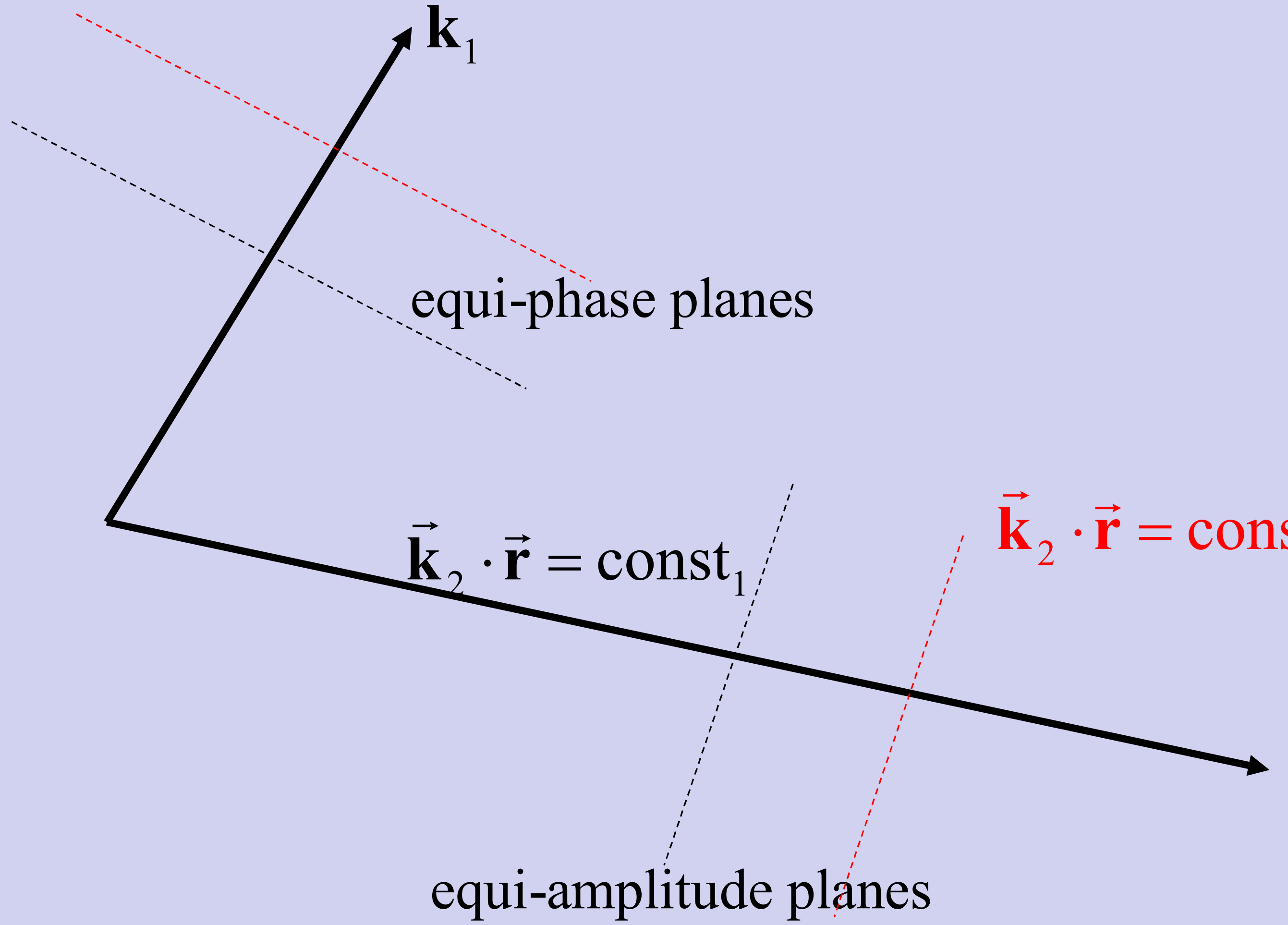
$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}^{+}e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}=\vec{\mathbf{E}}^{+}e^{-j(\vec{\mathbf{k}}_1-j\vec{\mathbf{k}}_2)\cdot\vec{\mathbf{r}}}=\vec{\mathbf{E}}^{+}\boxed{e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}}}\boxed{e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}}$$

Propagation

Attenuation



General expression of plane waves (PD)



$\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}} = \text{const}_2$

Propagation

$\vec{\mathbf{k}}_1$

$\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}} = \text{const}_1$

equi-phase planes

$\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}} = \text{const}_1$

$\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}} = \text{const}_2$

$\vec{\mathbf{k}}_2$

Attenuation

equi-amplitude planes

$$\vec{\mathbf{E}}\left(\vec{\mathbf{r}}\right) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \vec{\mathbf{k}}\times\vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}}\times\vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

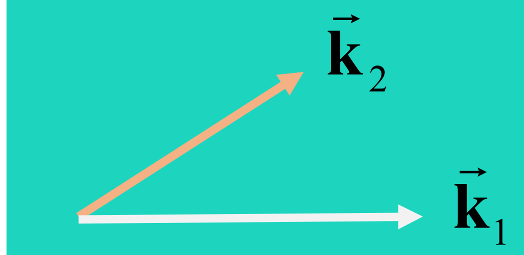
$\vec{\mathbf{k}}$:propagation vector

$$\vec{\mathbf{k}} = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-j\left(\vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2\right)\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} \boxed{e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}}}\,\boxed{e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}}$$

Propagation

Attenuation



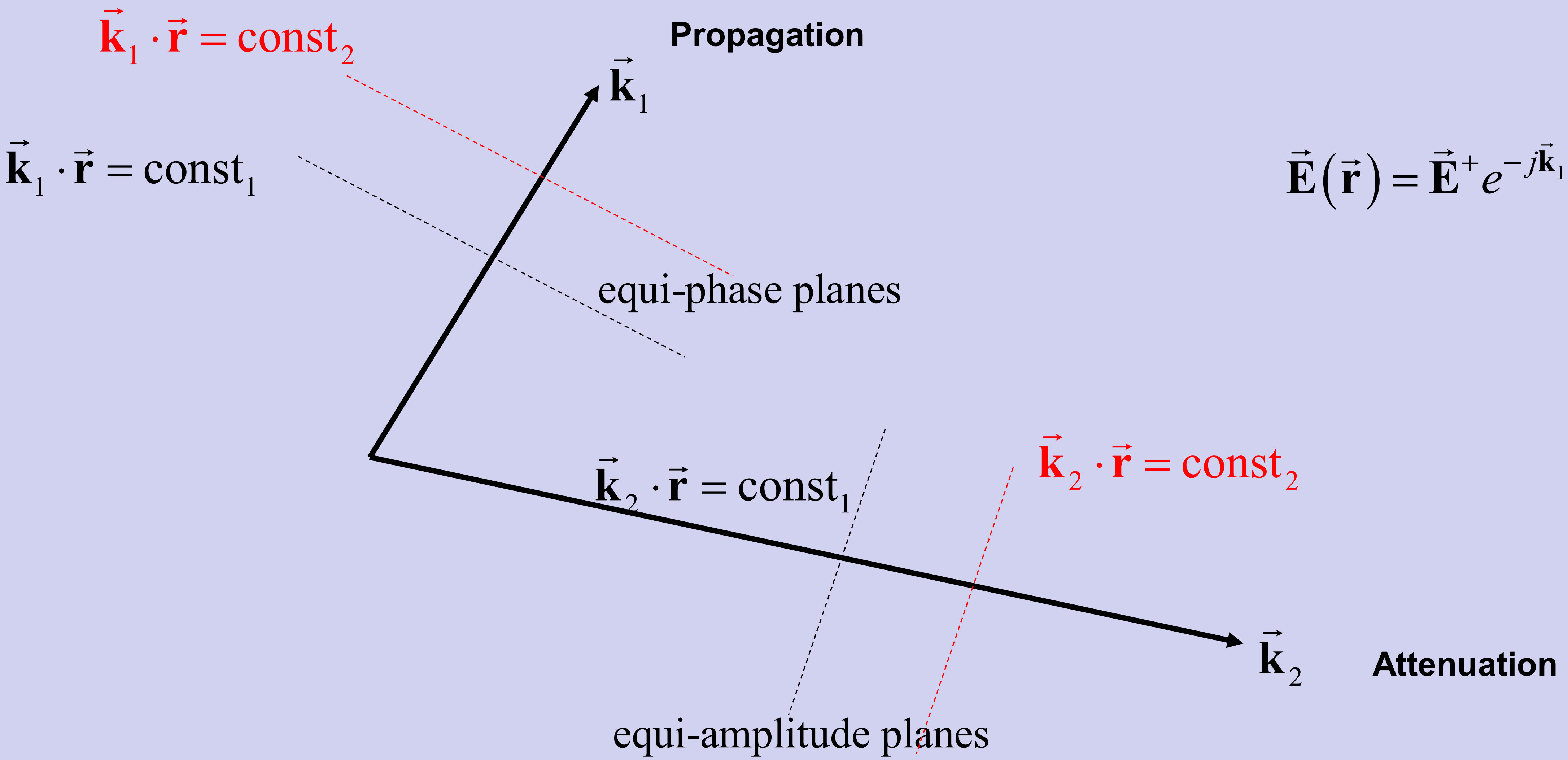
The wave propagates along $\vec{\mathbf{k}}_1$

The wave attenuates along $\vec{\mathbf{k}}_2$

$\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}} = \text{const} \Rightarrow$ equi-phase planes

$\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}} = \text{const} \Rightarrow$ equi-amplitude planes

General expression of plane waves (PD)



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$

When $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{k}}_2$ are proportional, equi-amplitude and equi-phase planes become coincident: the plane wave is said HOMOGENEOUS

More generally, equi-amplitude and equi-phase planes may be not coincident: in this the plane wave is said NOT-HOMOGENEOUS

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{H}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

Source-free

$$\left\{\begin{array}{l} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu \vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{array}\right.$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = \omega^2 \mu\varepsilon$$

$\vec{\mathbf{k}}$:propagation vector

Medium

- Linear - Isotropic - Space nondispersive - Time dispersive - Lossy

- Homogeneous (Time-invariant & Space-invariant)

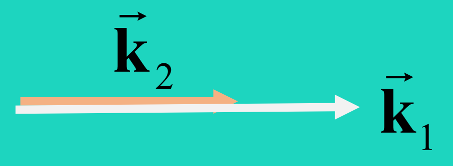
$$k = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$$

$$\vec{\mathbf{k}} = k\hat{i}_z \Rightarrow \vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = kz$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = k^2 = \omega^2 \mu\varepsilon$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^+ e^{-jkz} \quad = \vec{\mathbf{E}}^+ e^{-j\beta z} e^{-\alpha z}$$

$$\vec{\mathbf{k}} = (\beta - j\alpha)\hat{i}_z = \beta\hat{i}_z - j\alpha\hat{i}_z \quad = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$



HOMOGENEOUS PLANE-WAVE