Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Phasors and vector functions

Time domain → **Phasor domain**

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\hat{i}_x + f_y(x,y,z,t)\hat{i}_y + f_z(x,y,z,t)\hat{i}_z$$

$$\vec{\mathbf{F}}(x,y,z) = F_x(x,y,z)\hat{i}_x + F_y(x,y,z)\hat{i}_y + F_z(x,y,z)\hat{i}_z$$

$$f_x(x,y,z,t) = A_x(x,y,z)\cos\left(\omega_0 t + \alpha_x(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_x(x,y,z) = A_x(x,y,z)e^{j\alpha_x(x,y,z)}$$

$$f_y(x,y,z,t) = A_y(x,y,z)\cos\left(\omega_0 t + \alpha_y(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_y(x,y,z) = A_y(x,y,z)e^{j\alpha_y(x,y,z)}$$

$$f_z(x,y,z,t) = A_z(x,y,z)\cos\left(\omega_0 t + \alpha_z(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_z(x,y,z) = A_z(x,y,z)e^{j\alpha_z(x,y,z)}$$

$$\vec{\mathbf{f}}(x,y,z,t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x,y,z)$$

$$\vec{\mathbf{F}}(x,y,z) \longrightarrow \boxed{\text{P-to-T}} \longrightarrow \vec{\mathbf{f}}(x,y,z,t) = \mathrm{Re}\left\{\vec{\mathbf{F}}(x,y,z)e^{j\omega_0 t}\right\}$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$f_x(t) = A_x \cos(\omega_0 t + \alpha_x)$ → **T-to-P** → $F_x = A_x e^{j\alpha_x} = R_x + jI_x$

$f_y(t) = A_y \cos(\omega_0 t + \alpha_y)$ → **T-to-P** → $F_y = A_y e^{j\alpha_y} = R_y + jI_y$

$f_z(t) = A_z \cos(\omega_0 t + \alpha_z)$ → **T-to-P** → $F_z = A_z e^{j\alpha_z} = R_z + jI_z$

$\vec{\mathbf{f}}(t)$ → **T-to-P** → $\vec{\mathbf{F}}$

$\vec{\mathbf{F}}$ → **P-to-T** → $\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}} e^{j\omega_0 t}\right\}$

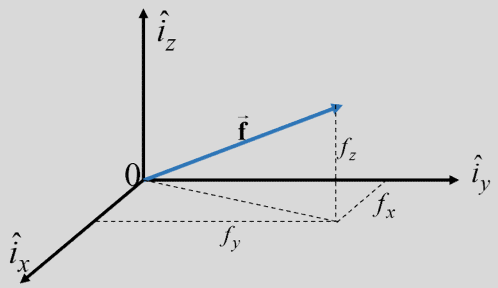
Complex vectors: graphical representation

Real numbers

f

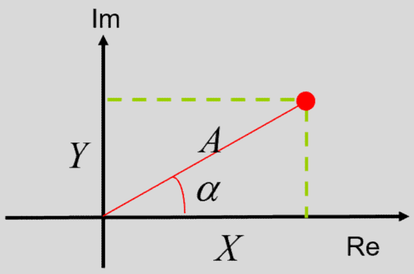
Real vectors (3 components)

$$\vec{\mathbf{f}} = f_x \hat{i}_x + f_y \hat{i}_y + f_z \hat{i}_z$$



Complex numbers

$$F = Ae^{j\alpha} = X + jY$$



Complex vectors (3 components)

$$\vec{\mathbf{F}} = F_x \hat{i}_x + F_y \hat{i}_y + F_z \hat{i}_z$$

?

Mathematical tools that we will exploit today

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}} \cdot \vec{\mathbf{F}}^*}$$

$$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha \;\; ; \;\; \sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

$$\cos(\alpha + \pi) = -\cos\alpha \;\; ; \;\; \cos(\alpha + 2\pi) = \cos\alpha$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

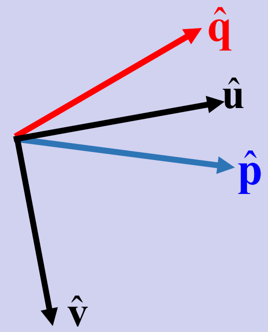
$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\hat{\mathbf{p}} = p_u\hat{\mathbf{u}} + p_v\hat{\mathbf{v}}$$

$$\hat{\mathbf{q}} = q_u\hat{\mathbf{u}} + q_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

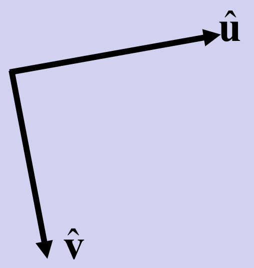
$$\vec{\mathbf{f}}(t) = \left|F_u\right|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + \left|F_v\right|\cos\left(\omega_0 t + \varphi_v\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = \left|F_u\right| e^{j\varphi_u} \\ F_v = \left|F_v\right| e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\left|F_u\right|^2 + \left|F_v\right|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$ and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- **Circular polarization**

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

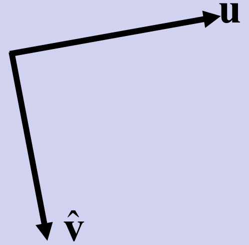
$$\vec{\mathbf{f}}(t) = \left|F_u\right|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + \left|F_v\right|\cos\left(\omega_0 t + \varphi_v\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = \left|F_u\right| e^{j\varphi_u} \\ F_v = \left|F_v\right| e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\left|F_u\right|^2 + \left|F_v\right|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



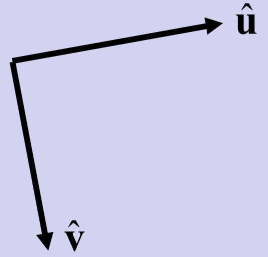
The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = \left|F_u\right|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}}$
- $F_u = 0$ and $F_v \neq 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = \left|F_v\right|\cos\left(\omega_0 t + \varphi_v\right)\hat{\mathbf{v}}$
- $\varphi_v - \varphi_u = n\pi$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = \left|F_u\right|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} \pm \left|F_v\right|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$

Linear Polarization

$\vec{\mathbf{F}} = F_u \hat{\mathbf{u}} + F_v \hat{\mathbf{v}}$

$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\, \hat{\mathbf{u}} + |F_v| \cos(\omega_0 t + \varphi_v)\, \hat{\mathbf{v}}$



Polarization plane

Linear polarization: the vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line.
To obtain linear polarization, one (**just one**) of the following three conditions must be enforced:

$$\left(F_u \neq 0 \text{ and } F_v = 0\right) \quad \underline{\textbf{or}} \quad \left(F_u = 0 \text{ and } F_v \neq 0\right) \quad \underline{\textbf{or}} \quad \left(\angle F_v - \angle F_u = n\pi\right)$$

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

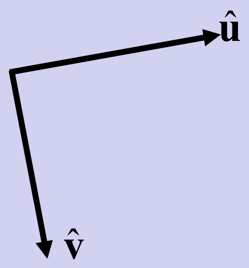
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$ and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- Linear polarization
- **Circular polarization**

Circular Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

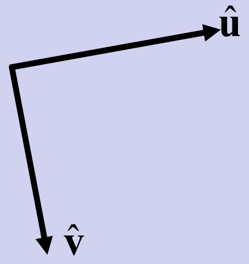
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} |F_u| = |F_v| = F \\ \varphi_v - \varphi_u = \dfrac{\pi}{2} + n\pi \end{cases}$$

Circular Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

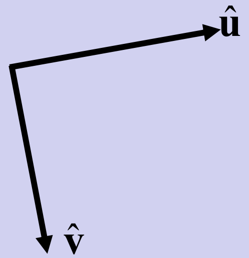
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} |F_u| = |F_v| = F \\ \varphi_v - \varphi_u = \pm\dfrac{\pi}{2} \end{cases}$$

$$\cos\left(\alpha + \frac{\pi}{2}\right) = -\sin\alpha \;;\; \cos\left(\alpha - \frac{\pi}{2}\right) = \sin\alpha$$

$$\vec{\mathbf{f}}(t) = F\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + F\cos\left(\omega_0 t + \varphi_u \pm \frac{\pi}{2}\right)\hat{\mathbf{v}}$$

$$\vec{\mathbf{f}}(t) = F\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \mp F\sin(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

Circular Polarization

$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = -\frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + F\sin\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos\left(\omega_0 t + \varphi_u\right)\right]^2 + F^2\left[\sin\left(\omega_0 t + \varphi_u\right)\right]^2 = F^2$$

$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = \frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} - F\sin\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos\left(\omega_0 t + \varphi_u\right)\right]^2 + F^2\left[\sin\left(\omega_0 t + \varphi_u\right)\right]^2 = F^2$$

The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus

Circular Polarization

$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = -\frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + F\sin\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos\left(\omega_0 t + \varphi_u\right)\right]^2 + F^2\left[\sin\left(\omega_0 t + \varphi_u\right)\right]^2 = F^2$$

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Circular Polarization

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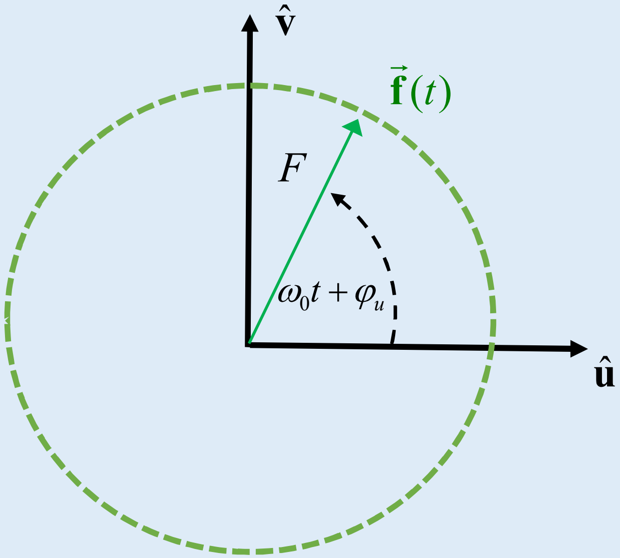
$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 + F^2\left[\sin(\omega_0 t + \varphi_u)\right]^2 = F^2$$



$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = \frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} - F\sin(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 + F^2\left[\sin(\omega_0 t + \varphi_u)\right]^2 = F^2$$

The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus

Circular Polarization

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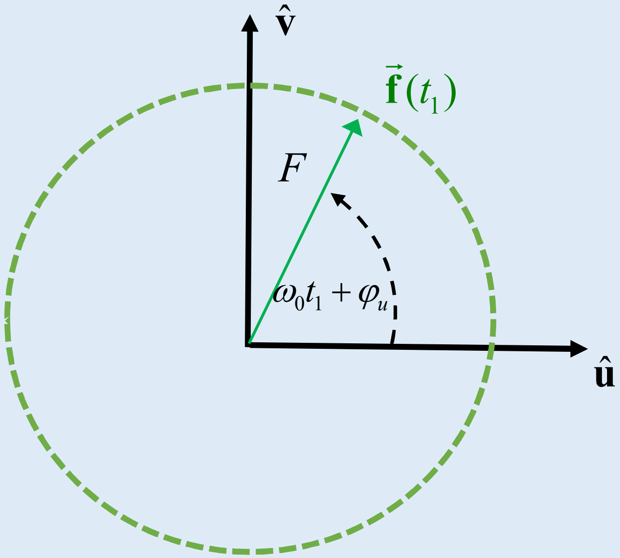
$$\vec{\mathbf{f}}(t) = F\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + F\sin\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos\left(\omega_0 t + \varphi_u\right)\right]^2 + F^2\left[\sin\left(\omega_0 t + \varphi_u\right)\right]^2 = F^2$$



$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = \frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} - F\sin\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos\left(\omega_0 t + \varphi_u\right)\right]^2 + F^2\left[\sin\left(\omega_0 t + \varphi_u\right)\right]^2 = F^2$$

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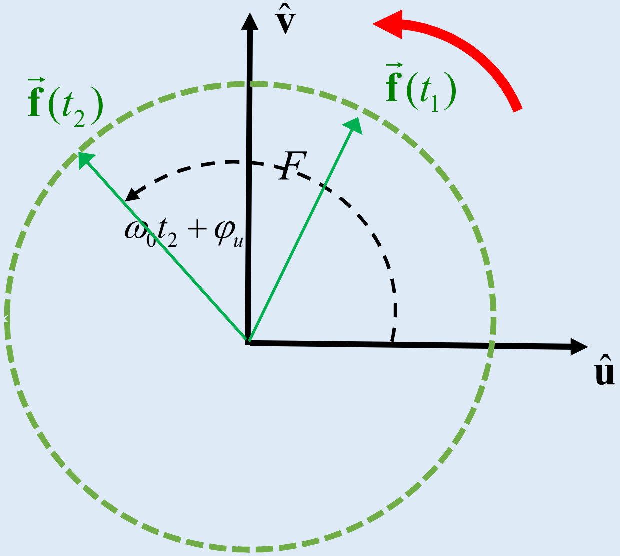
$$\vec{\mathbf{f}}(t) = F\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + F\sin(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 + F^2\left[\sin(\omega_0 t + \varphi_u)\right]^2 = F^2$$



$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = \frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} - F\sin(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

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The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus

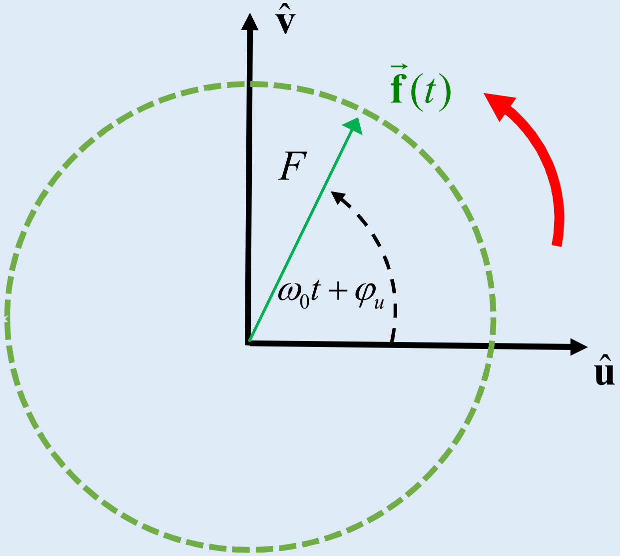
Its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = -\frac{\pi}{2}\right)$$

$$\vec{\mathbf{f}}(t) = F\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + F\sin\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = F^2\left[\cos\left(\omega_0 t + \varphi_u\right)\right]^2 + F^2\left[\sin\left(\omega_0 t + \varphi_u\right)\right]^2 \quad = F^2$$



The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = \frac{\pi}{2}\right)$$

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Circular Polarization

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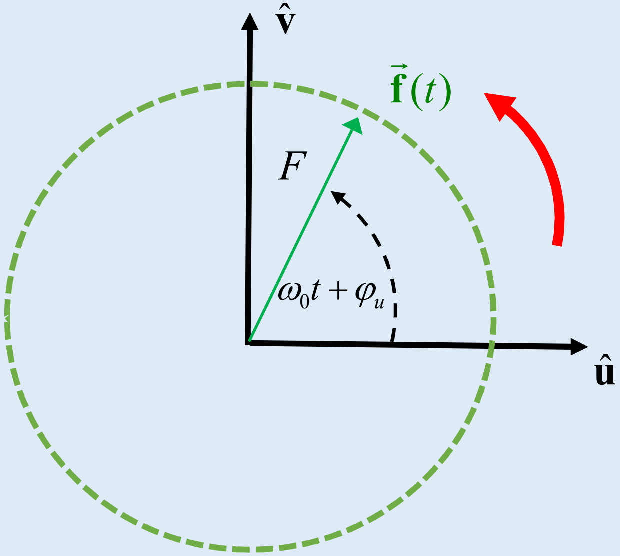

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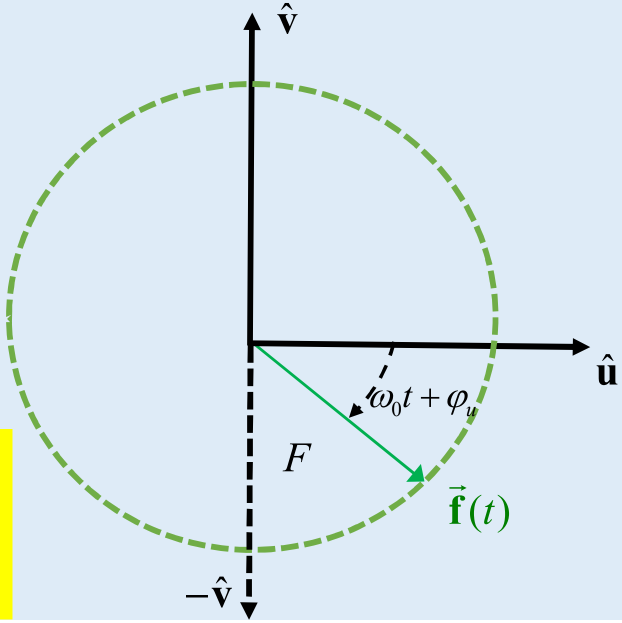

The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\left(|F_u| = |F_v| = F\right) \quad \text{and} \quad \left(\varphi_v - \varphi_u = -\frac{\pi}{2}\right)$$

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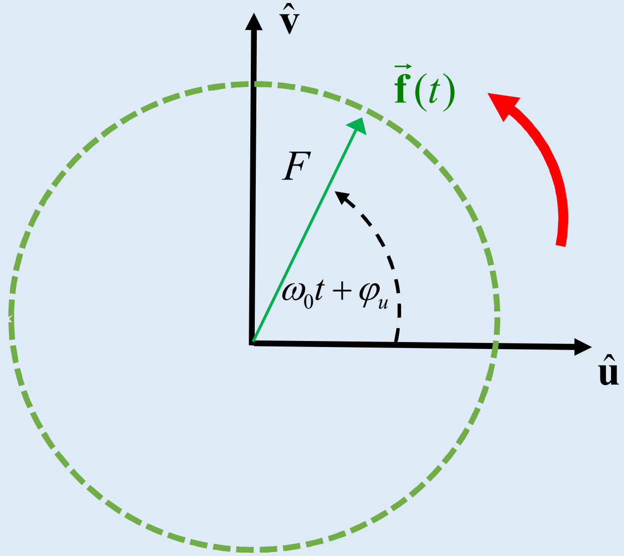

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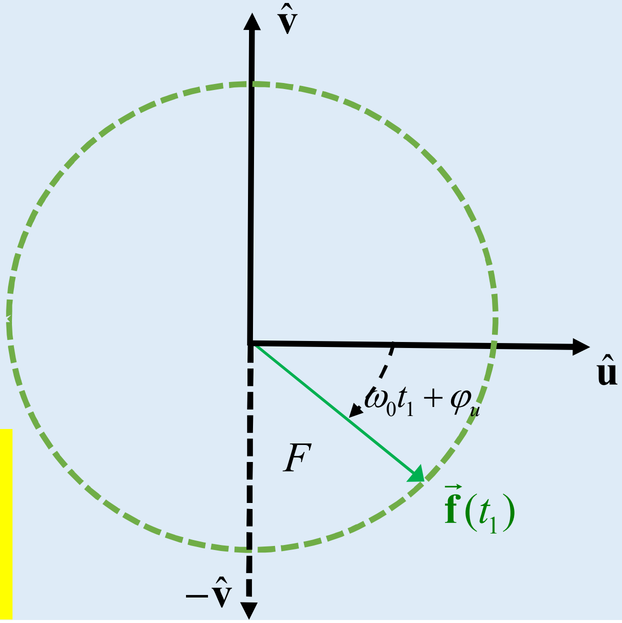

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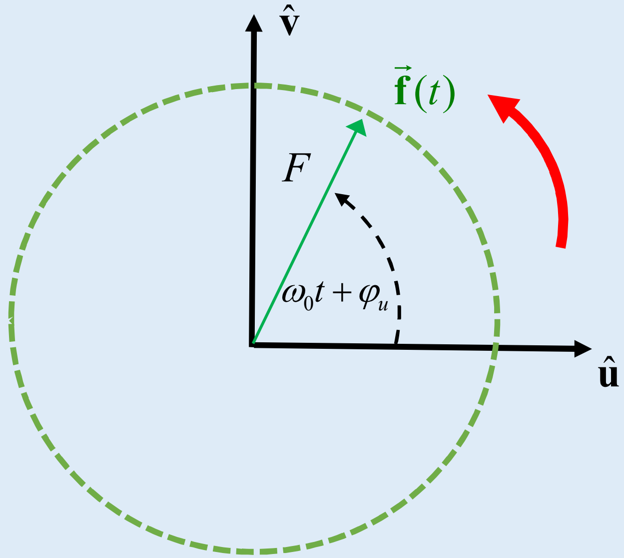

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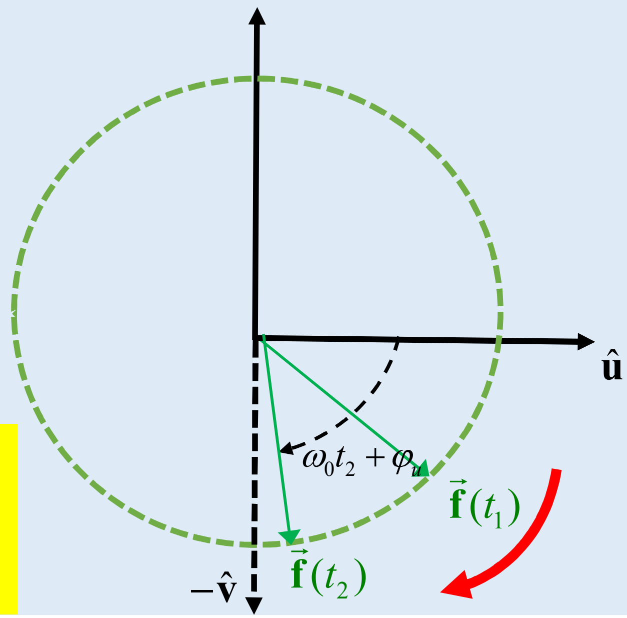


The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

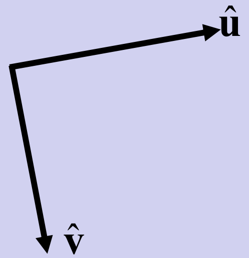
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

$$\left(|F_u| = |F_v| = F\right) \ \textbf{and} \ \left(\angle F_v - \angle F_u = \frac{\pi}{2} + n\pi\right)$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

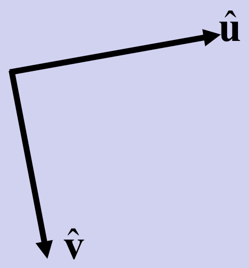
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

$(F_u \neq 0 \text{ and } F_v = 0)$ **or** $(F_u = 0 \text{ and } F_v \neq 0)$ **or** $(\angle F_v - \angle F_u = n\pi)$

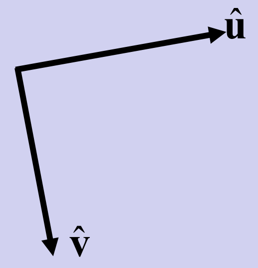
Field Polarization

$$\vec{\mathbf{F}} = F_u \hat{\mathbf{u}} + F_v \hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v| \cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

Polarization plane



Linear polarization: the vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line.
To obtain linear polarization, one (**just one**) of the following three conditions must be enforced:

$$(F_u \neq 0 \text{ and } F_v = 0) \quad \textbf{or} \quad (F_u = 0 \text{ and } F_v \neq 0) \quad \textbf{or} \quad (\angle F_v - \angle F_u = n\pi)$$

Circular polarization: the vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves with angular velocity ω_0 along a circle in the polarization plane.
To obtain circular polarization, the following two conditions must be **simultaneously** enforced:

$$|F_u| = |F_v| = F \quad \textbf{and} \quad \angle F_v - \angle F_u = \frac{\pi}{2} + n\pi$$

In the more general case, the tip of the vector $\vec{\mathbf{f}}(t)$ moves along an ellipse in the polarization plane. This case is referred to as **elliptical polarization.**

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

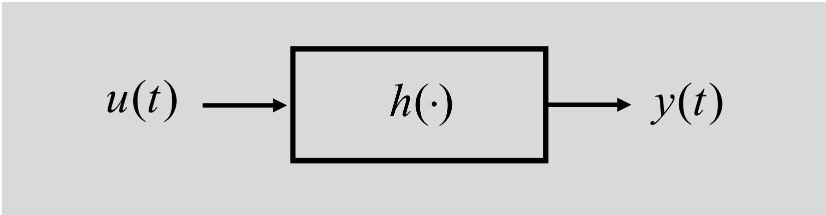
Mathematics

Memo: non-dispersive linear systems

Time-variant $y(t) = \alpha(t)u(t)$

Time-invariant $y(t) = \alpha u(t)$

Memo: dispersive linear systems



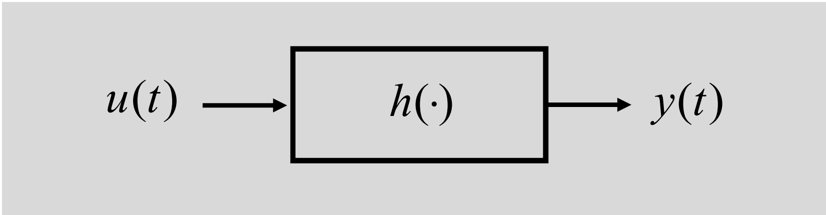
Time-variant

$$y(t)=\int d\tau\, h(t,\tau)\,u(\tau)$$

Time-invariant

$$y(t)=\int d\tau\, h(t-\tau)\,u(\tau)$$

Memo: linear systems

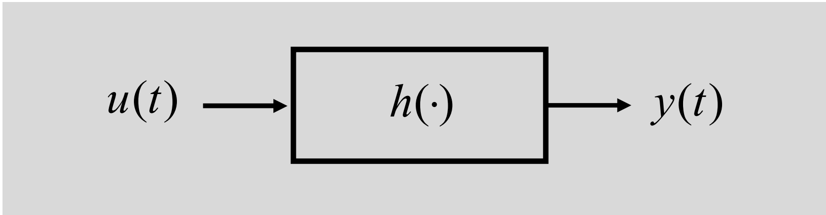


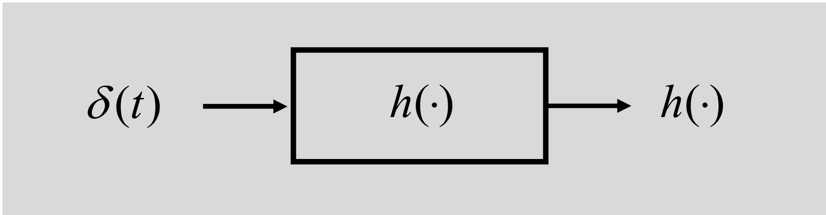
Time-variant	$y(t)=\int d\tau\, h(t,\tau)u(\tau)$		**Dispersive**
Time-invariant	$y(t)=\int d\tau\, h(t-\tau)u(\tau)$		
Time-variant	$y(t)=\alpha(t)u(t)$	$h(t,\tau)=\alpha(t)\delta(t-\tau)$	**Non-dispersive**
Time-invariant	$y(t)=\alpha\, u(t)$	$h(t-\tau)=\alpha\,\delta(t-\tau)$	

Memo: linear systems

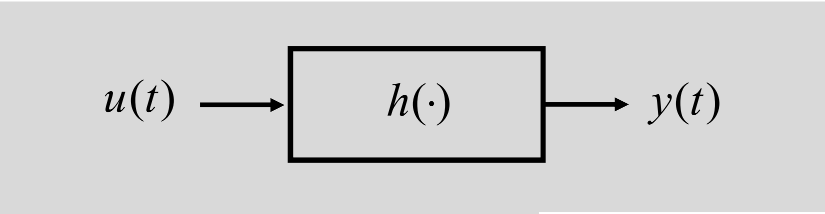
***h(·) is* referred to as impulse response**

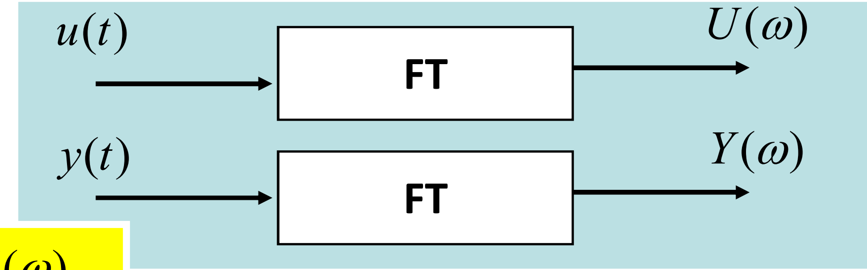
How can we calculate the impulse response *h(·)*?

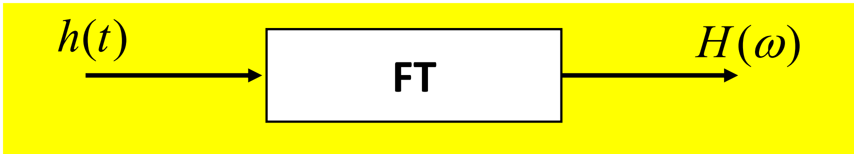




Memo: linear systems & Fourier domain







Time-variant $y(t)=\int d\tau\, h(t,\tau)u(\tau)$

Dispersive

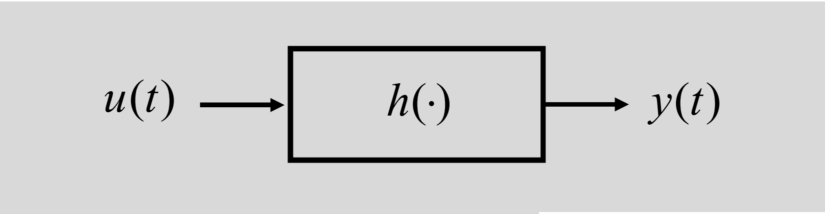
Time-invariant $y(t)=\int d\tau\, h(t-\tau)u(\tau)$ $\quad Y(\omega)=H(\omega)U(\omega)$

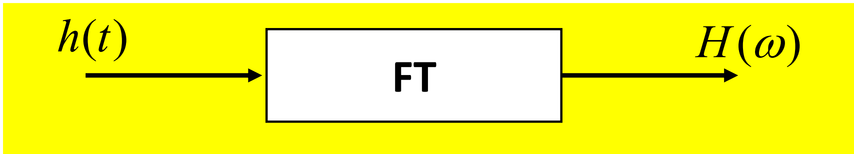
Time-variant $y(t)=\alpha(t)u(t)$

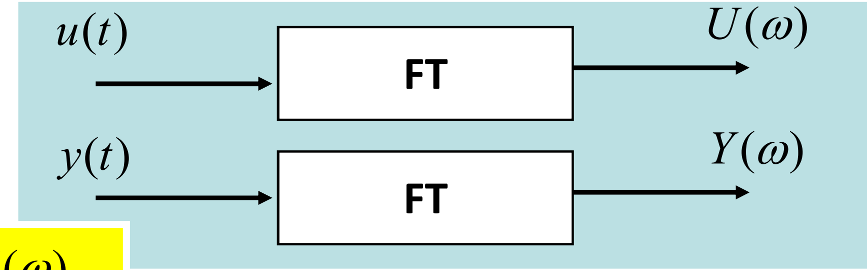
Non-dispersive

Time-invariant $y(t)=\alpha\, u(t)$ $\quad Y(\omega)=\alpha\, U(\omega)$

Memo: linear systems & Fourier domain







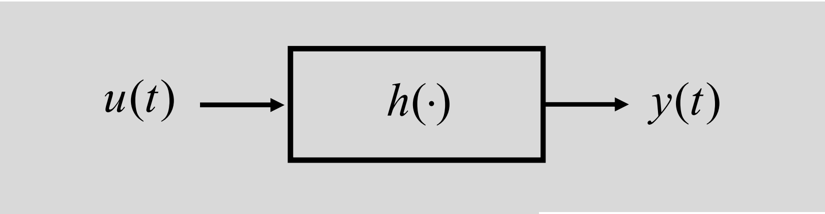
Dispersive

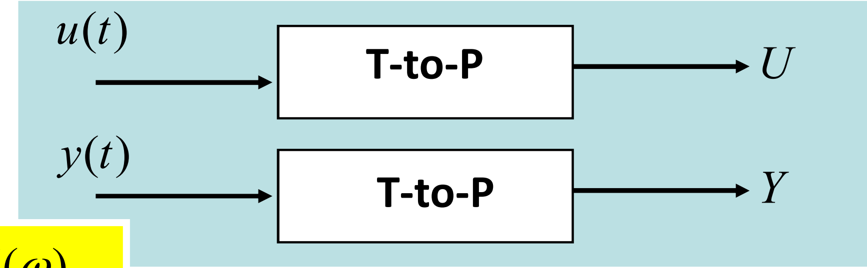
Time-invariant $y(t)=\int d\tau\, h(t-\tau)u(\tau)$ $\quad$ $Y(\omega)=H(\omega)U(\omega)$

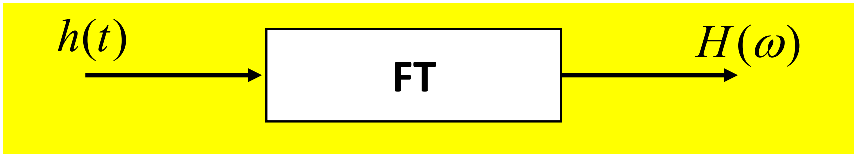
Non-dispersive

Time-invariant $y(t)=\alpha\, u(t)$ $\quad$ $Y(\omega)=\alpha\, U(\omega)$

Memo: linear systems & Phasor domain







Dispersive

Time-invariant $y(t)=\int d\tau\, h(t-\tau)u(\tau)$ $Y=H(\omega_0)U$

Non-dispersive

Time-invariant $y(t)=\alpha u(t)$ $Y=\alpha U$