Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\hat{i}_x + f_y(x,y,z,t)\hat{i}_y + f_z(x,y,z,t)\,\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x,y,z) = F_x(x,y,z)\hat{i}_x + F_y(x,y,z)\hat{i}_y + F_z(x,y,z)\hat{i}_z$$

$f_x(x,y,z,t) = A_x(x,y,z)\cos\left(\omega_0 t + \alpha_x(x,y,z)\right)$ → **T-to-P** → $F_x(x,y,z) = A_x(x,y,z)e^{j\alpha_x(x,y,z)}$

$f_y(x,y,z,t) = A_y(x,y,z)\cos\left(\omega_0 t + \alpha_y(x,y,z)\right)$ → **T-to-P** → $F_y(x,y,z) = A_y(x,y,z)e^{j\alpha_y(x,y,z)}$

$f_z(x,y,z,t) = A_z(x,y,z)\cos\left(\omega_0 t + \alpha_z(x,y,z)\right)$ → **T-to-P** → $F_z(x,y,z) = A_z(x,y,z)e^{j\alpha_z(x,y,z)}$

$\vec{\mathbf{f}}(x,y,z,t)$ → **T-to-P** → $\vec{\mathbf{F}}(x,y,z)$

$\vec{\mathbf{F}}(x,y,z)$ → **P-to-T** → $\vec{\mathbf{f}}(x,y,z,t) = \mathrm{Re}\left\{\vec{\mathbf{F}}(x,y,z)e^{j\omega_0 t}\right\}$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$f_x(t) = A_x \cos(\omega_0 t + \alpha_x)$ → **T-to-P** → $F_x = A_x e^{j\alpha_x} = R_x + jI_x$

$f_y(t) = A_y \cos(\omega_0 t + \alpha_y)$ → **T-to-P** → $F_y = A_y e^{j\alpha_y} = R_y + jI_y$

$f_z(t) = A_z \cos(\omega_0 t + \alpha_z)$ → **T-to-P** → $F_z = A_z e^{j\alpha_z} = R_z + jI_z$

$\vec{\mathbf{f}}(t)$ → **T-to-P** → $\vec{\mathbf{F}}$

$\vec{\mathbf{F}}$ → **P-to-T** → $\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}} e^{j\omega_0 t}\right\}$

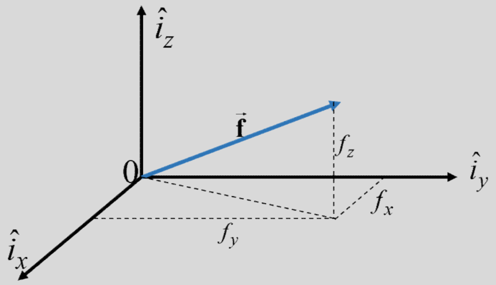
Complex vectors: graphical representation

Real numbers

$$f$$

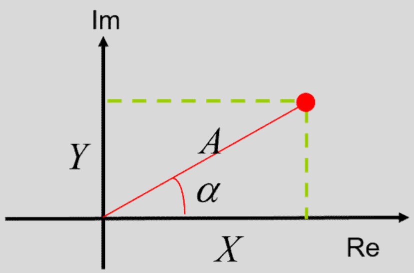
Real vectors (3 components)

$$\vec{\mathbf{f}} = f_x \hat{i}_x + f_y \hat{i}_y + f_z \hat{i}_z$$



Complex numbers

$$F = Ae^{j\alpha} = X + jY$$



Complex vectors (3 components)

$$\vec{\mathbf{F}} = F_x \hat{i}_x + F_y \hat{i}_y + F_z \hat{i}_z$$

?

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}} \cdot \vec{\mathbf{F}}^*}$$

$$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha \;\; ; \;\; \sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

$$\cos(\alpha + \pi) = -\cos\alpha \;\; ; \;\; \cos(\alpha + 2\pi) = \cos\alpha$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$f_x(t) = A_x \cos(\omega_0 t + \alpha_x)$ → **T-to-P** → $F_x = A_x e^{j\alpha_x} = R_x + jI_x$

$f_y(t) = A_y \cos(\omega_0 t + \alpha_y)$ → **T-to-P** → $F_y = A_y e^{j\alpha_y} = R_y + jI_y$

$f_z(t) = A_z \cos(\omega_0 t + \alpha_z)$ → **T-to-P** → $F_z = A_z e^{j\alpha_z} = R_z + jI_z$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = \left(A_x e^{j\alpha_x}\right)\hat{i}_x + \left(A_y e^{j\alpha_y}\right)\hat{i}_y + \left(A_z e^{j\alpha_z}\right)\hat{i}_z = (R_x + jI_x)\hat{i}_x + (R_y + jI_y)\hat{i}_y + (R_z + jI_z)\hat{i}_z$$

$$= \underbrace{\left[R_x\hat{i}_x + R_y\hat{i}_y + R_z\hat{i}_z\right]}_{F_p\hat{\mathbf{p}}} + j\underbrace{\left[I_x\hat{i}_x + I_y\hat{i}_y + I_z\hat{i}_z\right]}_{F_q\hat{\mathbf{q}}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

F_p and F_q are real!

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

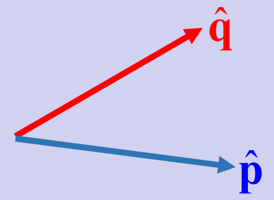
$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

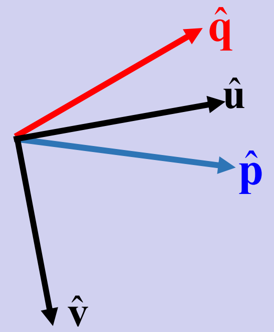
$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\hat{\mathbf{p}} = p_u\hat{\mathbf{u}} + p_v\hat{\mathbf{v}}$$

$$\hat{\mathbf{q}} = q_u\hat{\mathbf{u}} + q_v\hat{\mathbf{v}}$$

Polarization plane



$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}} = F_p\left(p_u\hat{\mathbf{u}} + p_v\hat{\mathbf{v}}\right) + jF_q\left(q_u\hat{\mathbf{u}} + q_v\hat{\mathbf{v}}\right)$$

$$= \underbrace{\left(F_p p_u + jF_q q_u\right)}_{F_u = |F_u| e^{j\varphi_u}}\hat{\mathbf{u}} + \underbrace{\left(F_p p_v + jF_q q_v\right)}_{F_v = |F_v| e^{j\varphi_v}}\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

F_u and F_v are complex!

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

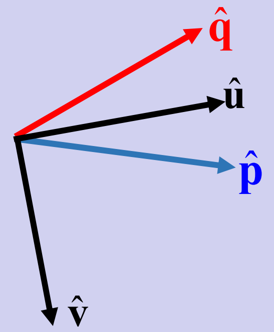
$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\hat{\mathbf{p}} = p_u\hat{\mathbf{u}} + p_v\hat{\mathbf{v}}$$

$$\hat{\mathbf{q}} = q_u\hat{\mathbf{u}} + q_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^*}$$

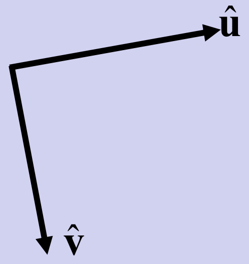
$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$



$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^*} = \sqrt{\left(F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}\right)\cdot\left(F_u^*\hat{\mathbf{u}} + F_v^*\hat{\mathbf{v}}\right)} = \sqrt{|F_u|^2 + |F_v|^2}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + |F_v|\cos\left(\omega_0 t + \varphi_v\right)\hat{\mathbf{v}}$$

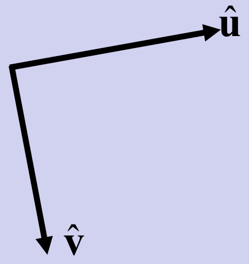
$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$



$$\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}} e^{j\omega_0 t}\right\} = \mathrm{Re}\left\{\left[F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}\right] e^{j\omega_0 t}\right\}$$

$$= \mathrm{Re}\left\{\left[|F_u| e^{j\varphi_u}\hat{\mathbf{u}} + |F_v| e^{j\varphi_v}\hat{\mathbf{v}}\right] e^{j\omega_0 t}\right\} = \mathrm{Re}\left\{|F_u| e^{j\varphi_u} e^{j\omega_0 t}\hat{\mathbf{u}} + |F_v| e^{j\varphi_v} e^{j\omega_0 t}\hat{\mathbf{v}}\right\}$$

$$= |F_u|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + |F_v|\cos\left(\omega_0 t + \varphi_v\right)\hat{\mathbf{v}}$$

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

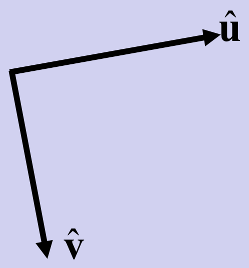
$$\vec{\mathbf{f}}(t) = |F_u|\cos\left(\omega_0 t + \varphi_u\right)\hat{\mathbf{u}} + |F_v|\cos\left(\omega_0 t + \varphi_v\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$ and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- **Circular polarization**

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

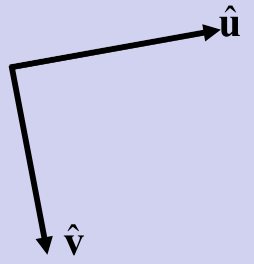
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$ and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- Circular polarization

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

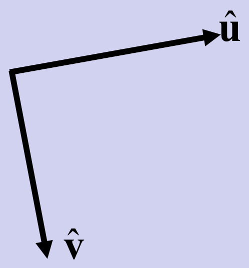
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

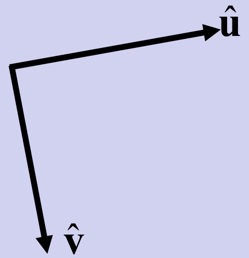
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

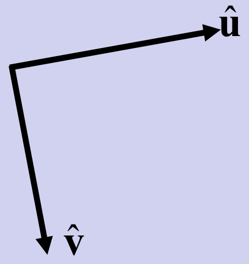
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane

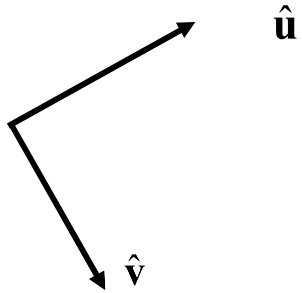


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$

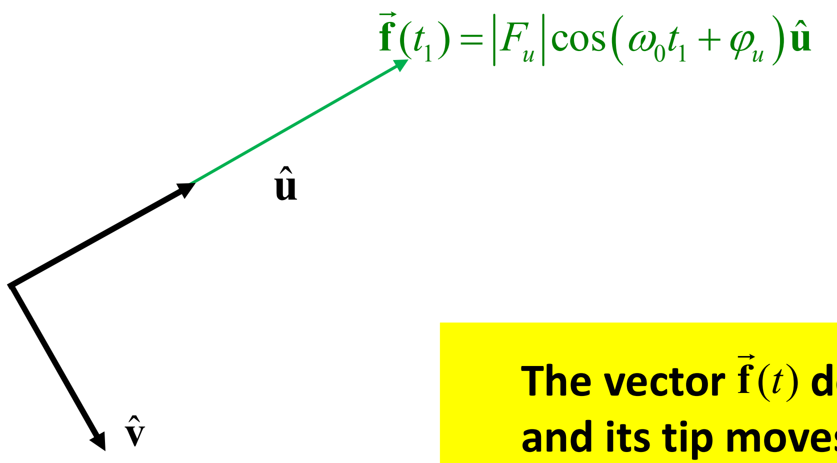
Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\, \hat{\mathbf{u}}$$



Linear Polarization

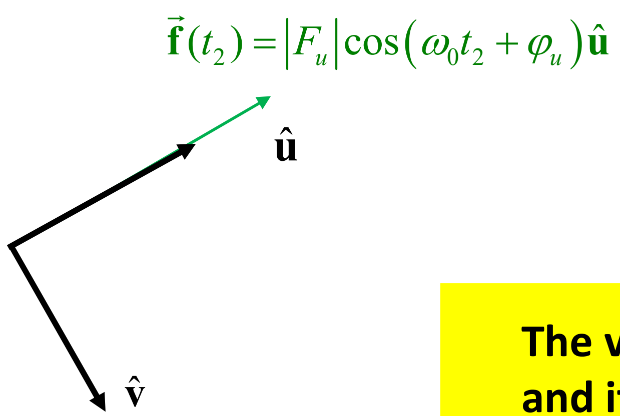
$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$

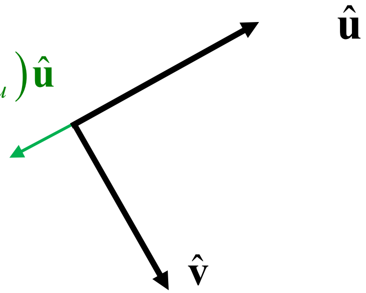


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$

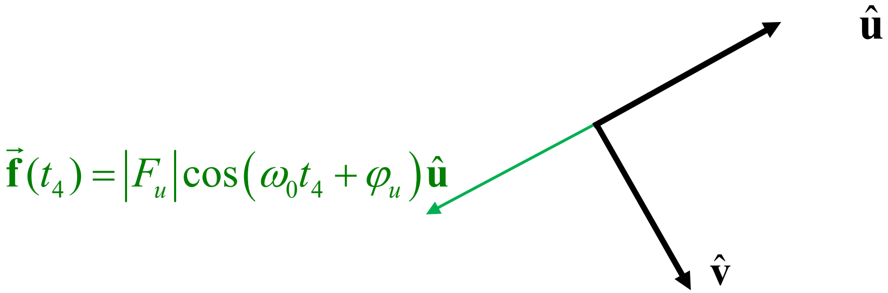
$$\vec{\mathbf{f}}(t_3) = |F_u| \cos(\omega_0 t_3 + \varphi_u)\,\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

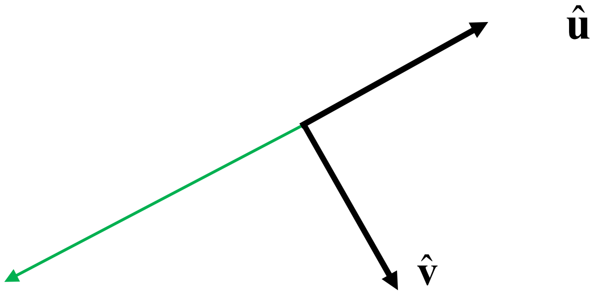
$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\, \hat{\mathbf{u}}$$



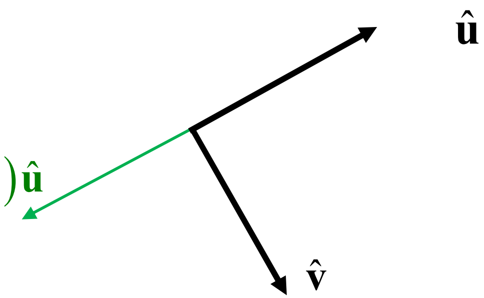
$$\vec{\mathbf{f}}(t_5) = |F_u| \cos(\omega_0 t_5 + \varphi_u)\, \hat{\mathbf{u}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$

$$\vec{\mathbf{f}}(t_6) = |F_u| \cos(\omega_0 t_6 + \varphi_u)\,\hat{\mathbf{u}}$$

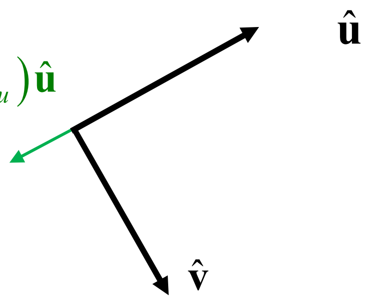


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos\left(\omega_0 t + \varphi_u\right) \hat{\mathbf{u}}$$

$$\vec{\mathbf{f}}(t_7) = |F_u| \cos\left(\omega_0 t_7 + \varphi_u\right) \hat{\mathbf{u}}$$

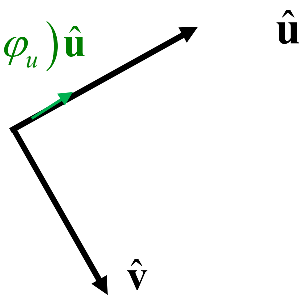


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$

$$\vec{\mathbf{f}}(t_8) = |F_u| \cos(\omega_0 t_8 + \varphi_u)\,\hat{\mathbf{u}}$$

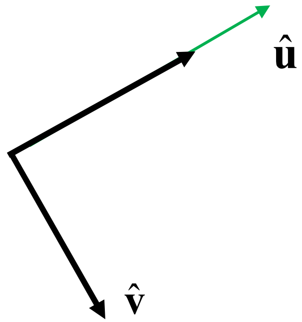


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}}$$

$$\vec{\mathbf{f}}(t_9) = |F_u| \cos(\omega_0 t_9 + \varphi_u)\,\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

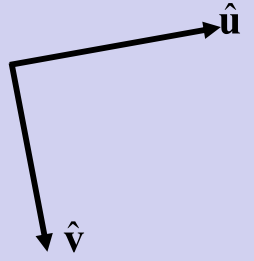
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

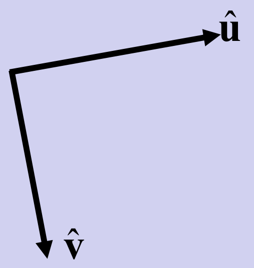
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$
- $F_u = 0$ and $F_v \neq 0$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

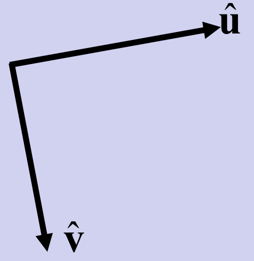
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



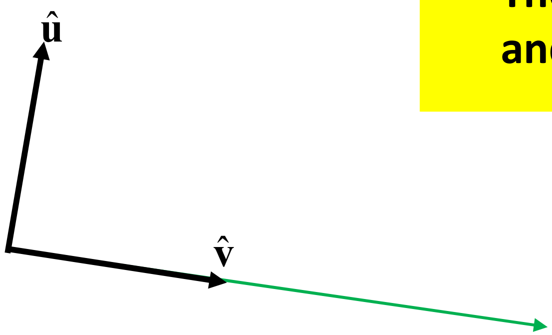
The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$
- $F_u = 0$ and $F_v \neq 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v| \cos(\omega_0 t + \varphi_v)\, \hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

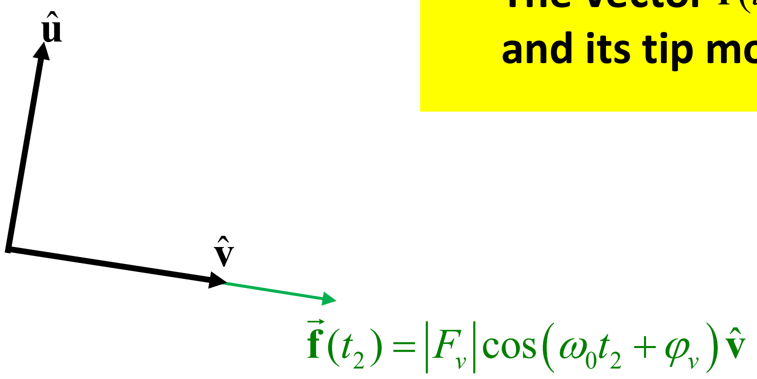


$$\vec{\mathbf{f}}(t_1) = |F_v| \cos(\omega_0 t_1 + \varphi_v)\, \hat{\mathbf{v}}$$

Linear Polarization

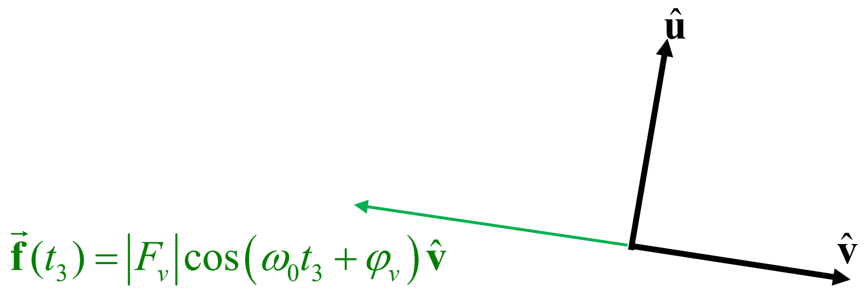
$$\vec{\mathbf{f}}(t) = |F_v| \cos(\omega_0 t + \varphi_v)\,\hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line



Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v| \cos(\omega_0 t + \varphi_v)\,\hat{\mathbf{v}}$$

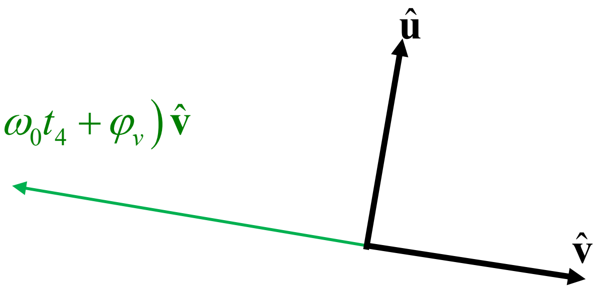


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v| \cos(\omega_0 t + \varphi_v)\,\hat{\mathbf{v}}$$

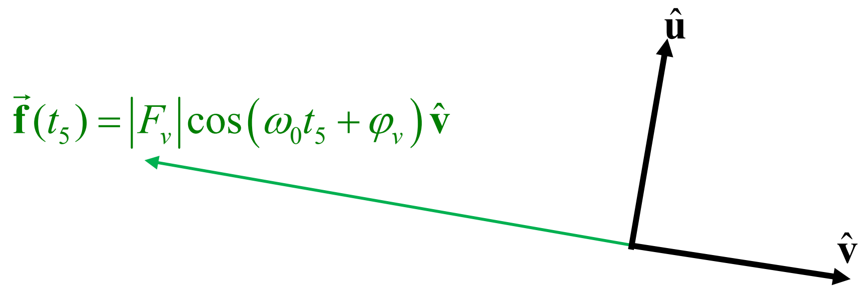
$$\vec{\mathbf{f}}(t_4) = |F_v| \cos(\omega_0 t_4 + \varphi_v)\,\hat{\mathbf{v}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

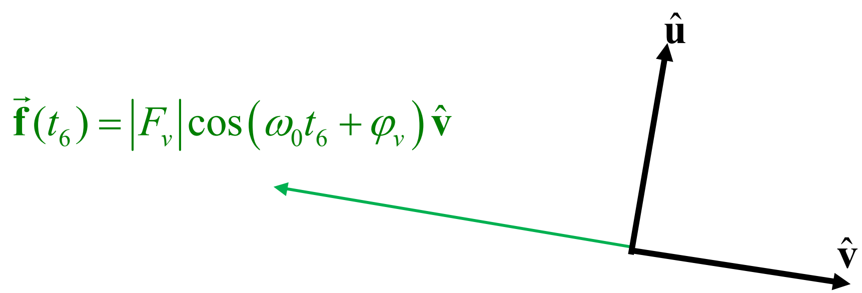
$$\vec{\mathbf{f}}(t) = |F_v|\cos(\omega_0 t + \varphi_v)\,\hat{\mathbf{v}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v| \cos\left(\omega_0 t + \varphi_v\right) \hat{\mathbf{v}}$$

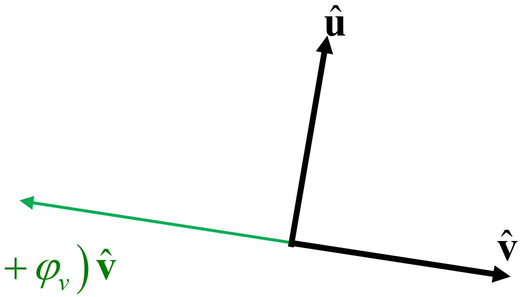


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v| \cos(\omega_0 t + \varphi_v)\, \hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

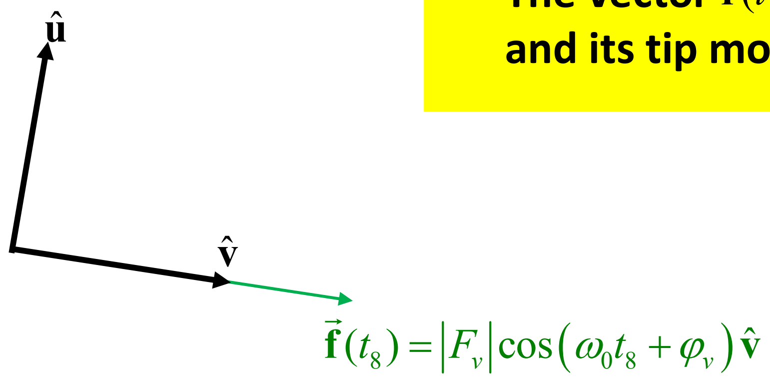


$$\vec{\mathbf{f}}(t_7) = |F_v| \cos(\omega_0 t_7 + \varphi_v)\, \hat{\mathbf{v}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v|\cos(\omega_0 t + \varphi_v)\,\hat{\mathbf{v}}$$

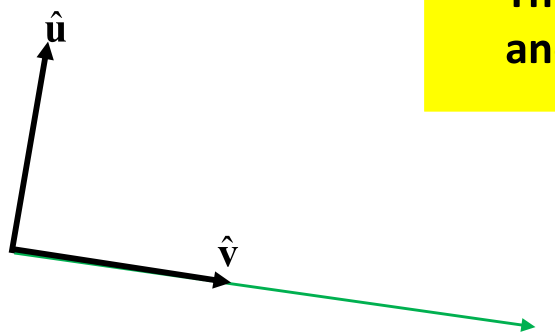
The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line



Linear Polarization

$$\vec{\mathbf{f}}(t) = |F_v| \cos(\omega_0 t + \varphi_v)\, \hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line



$$\vec{\mathbf{f}}(t_9) = |F_v| \cos(\omega_0 t_9 + \varphi_v)\, \hat{\mathbf{v}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

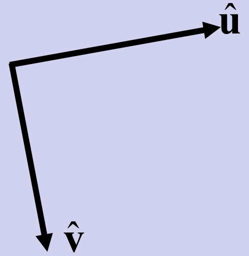
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ ⟹ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$
- $F_u = 0$ and $F_v \neq 0$ ⟹ $\vec{\mathbf{f}}(t) = |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

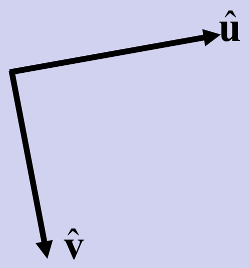
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Rightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$
- $F_u = 0$ and $F_v \neq 0$ $\Rightarrow$ $\vec{\mathbf{f}}(t) = |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$
- $\varphi_v - \varphi_u = n\pi$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

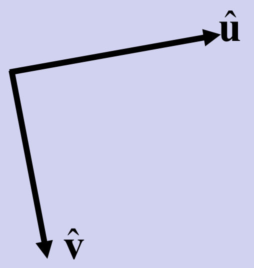
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\varphi_v - \varphi_u = n\pi$$

$$\cos(\alpha + \pi) = -\cos\alpha \;;\; \cos(\alpha + 2\pi) = \cos\alpha$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_u + n\pi)\hat{\mathbf{v}}$$

$$= |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = |F_u|^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 + |F_v|^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 = \left[|F_u|^2 + |F_v|^2\right]\left[\cos(\omega_0 t + \varphi_u)\right]^2$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t + \varphi_u)\right|$$

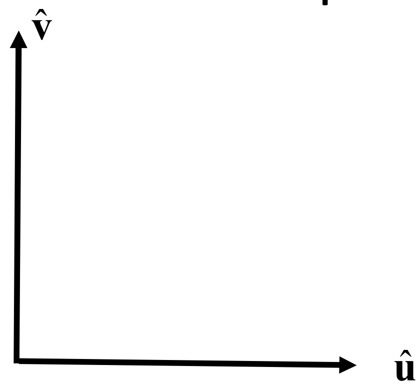
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = |F_u|^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 + |F_v|^2\left[\cos(\omega_0 t + \varphi_u)\right]^2 = \left[|F_u|^2 + |F_v|^2\right]\left[\cos(\omega_0 t + \varphi_u)\right]^2$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t + \varphi_u)\right|$$

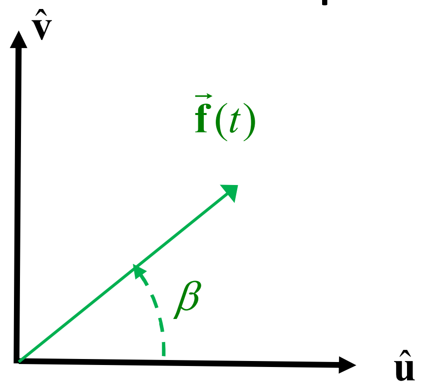
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\tan\beta = \pm\frac{|F_v|\cos(\omega_0 t + \varphi_u)}{|F_u|\cos(\omega_0 t + \varphi_u)} = \pm\frac{|F_v|}{|F_u|}$$

Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right| \left|\cos(\omega_0 t + \varphi_u)\right| \qquad \tan\beta = \pm\frac{|F_v|}{|F_u|}$$

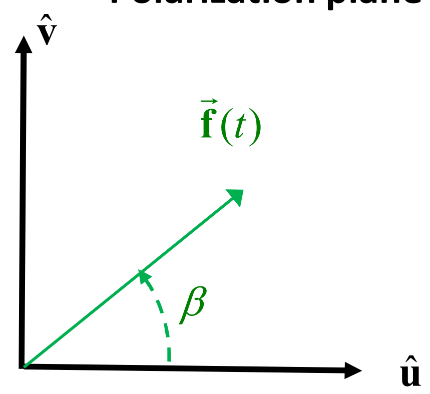
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t + \varphi_u)\right| \qquad \tan\beta = \pm\frac{|F_v|}{|F_u|}$$

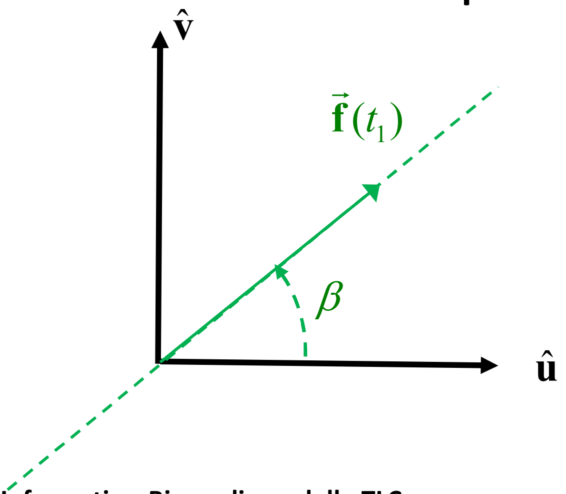
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} \left|\vec{\mathbf{f}}(t_1)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t_1 + \varphi_u)\right| \\ \tan\beta = \pm\dfrac{|F_v|}{|F_u|} \end{cases}$$

Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t + \varphi_u)\right| \qquad \tan\beta = \pm\frac{|F_v|}{|F_u|}$$

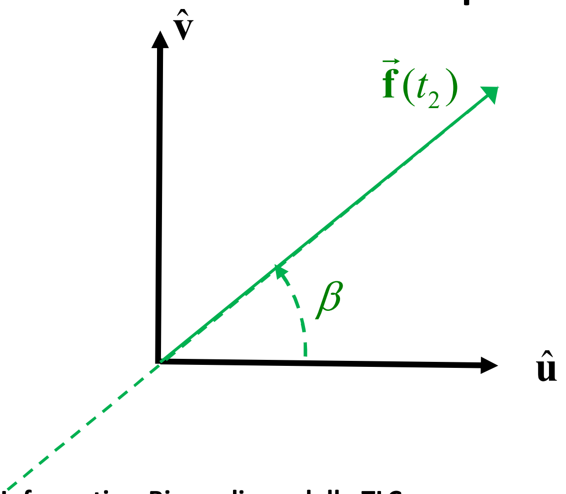
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} \left|\vec{\mathbf{f}}(t_2)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t_2 + \varphi_u)\right| \\ \tan\beta = \pm\dfrac{|F_v|}{|F_u|} \end{cases}$$

Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\,\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t + \varphi_u)\right| \qquad \tan\beta = \pm\frac{|F_v|}{|F_u|}$$

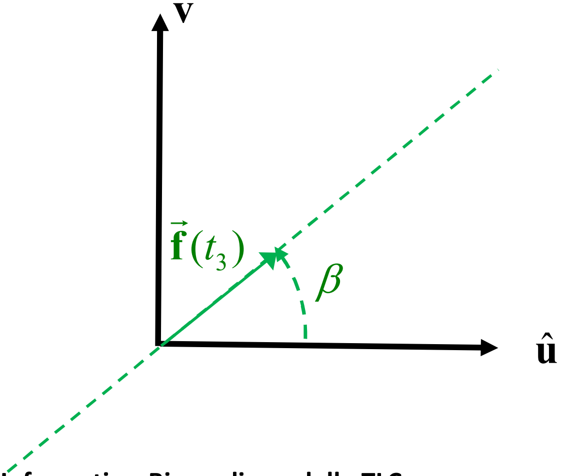
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} \left|\vec{\mathbf{f}}(t_2)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t_3 + \varphi_u)\right| \\ \tan\beta = \pm\dfrac{|F_v|}{|F_u|} \end{cases}$$

Linear Polarization

$$\varphi_v - \varphi_u = n\pi$$

$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t + \varphi_u)\right| \qquad \tan\beta = \pm\frac{|F_v|}{|F_u|}$$

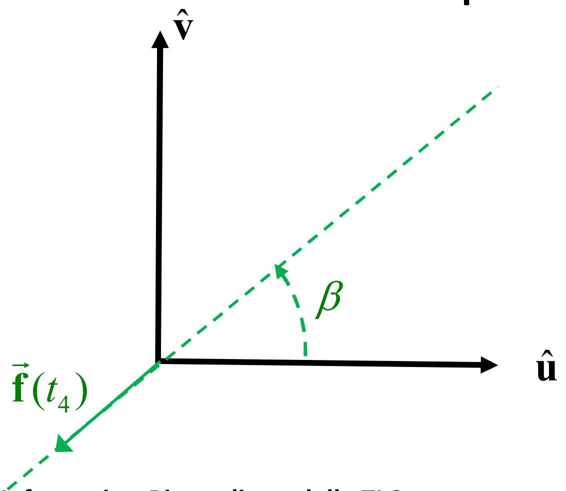
$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u|e^{j\varphi_u} \\ F_v = |F_v|e^{j\varphi_v} \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} \left|\vec{\mathbf{f}}(t_4)\right| = \left|\vec{\mathbf{F}}\right|\left|\cos(\omega_0 t_4 + \varphi_u)\right| \\ \tan\beta = \pm\dfrac{|F_v|}{|F_u|} \end{cases}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$$

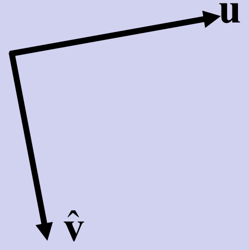
$$\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$

$$\begin{cases} F_u = |F_u| e^{j\varphi_u} \\ F_v = |F_v| e^{j\varphi_v} \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{|F_u|^2 + |F_v|^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



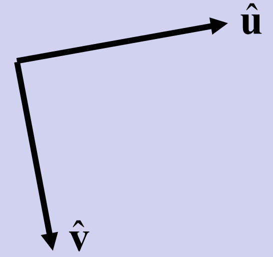
The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $F_u \neq 0$ and $F_v = 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}}$
- $F_u = 0$ and $F_v \neq 0$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_v|\cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$
- $\varphi_v - \varphi_u = n\pi$ $\Longrightarrow$ $\vec{\mathbf{f}}(t) = |F_u|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} \pm |F_v|\cos(\omega_0 t + \varphi_u)\hat{\mathbf{v}}$

Linear Polarization

$$\vec{\mathbf{F}} = F_u \hat{\mathbf{u}} + F_v \hat{\mathbf{v}}$$

$$\vec{\mathbf{f}}(t) = |F_u| \cos(\omega_0 t + \varphi_u)\hat{\mathbf{u}} + |F_v| \cos(\omega_0 t + \varphi_v)\hat{\mathbf{v}}$$



Polarization plane

Linear polarization: the vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line.
To obtain linear polarization, one (**just one**) of the following three conditions must be enforced:

$(F_u \neq 0 \text{ and } F_v = 0)$ **or** $(F_u = 0 \text{ and } F_v \neq 0)$ **or** $(\angle F_v - \angle F_u = n\pi)$