



Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

**a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo
anno**

Università degli Studi di Napoli “Parthenope”

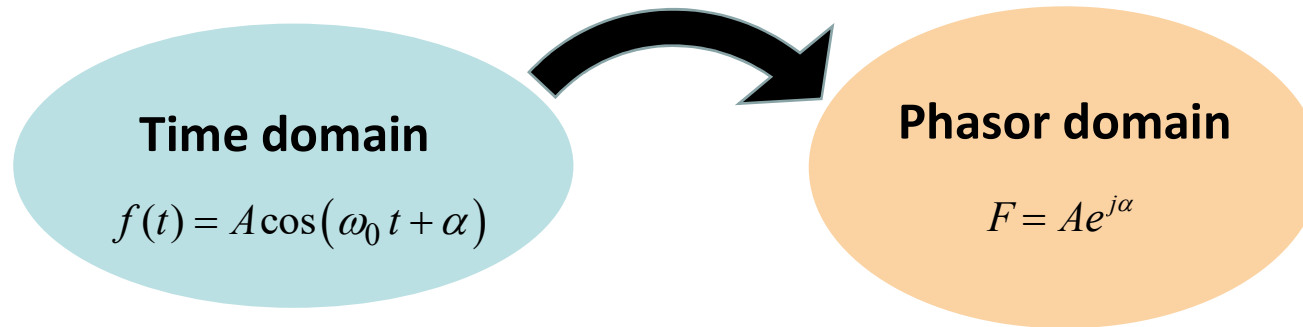
Stefano Perna

Maxwell equations

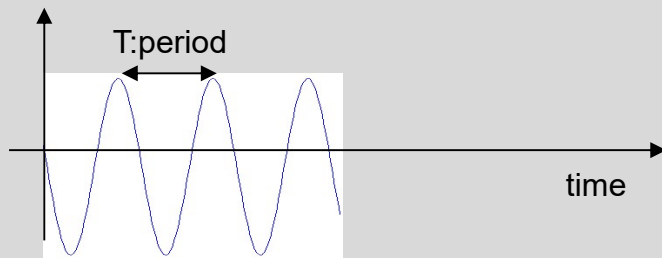
Time domain & Phasors



Phasors



Signals usually adopted in ICT applications



$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

$$f_0 : \text{frequency} = \frac{1}{T}$$

$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

Phasors

- Phasors and functions of n variables
- Phasors and vector functions
- **Phasors and vector functions of n variables**

- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

Phasors and vector functions of n variables

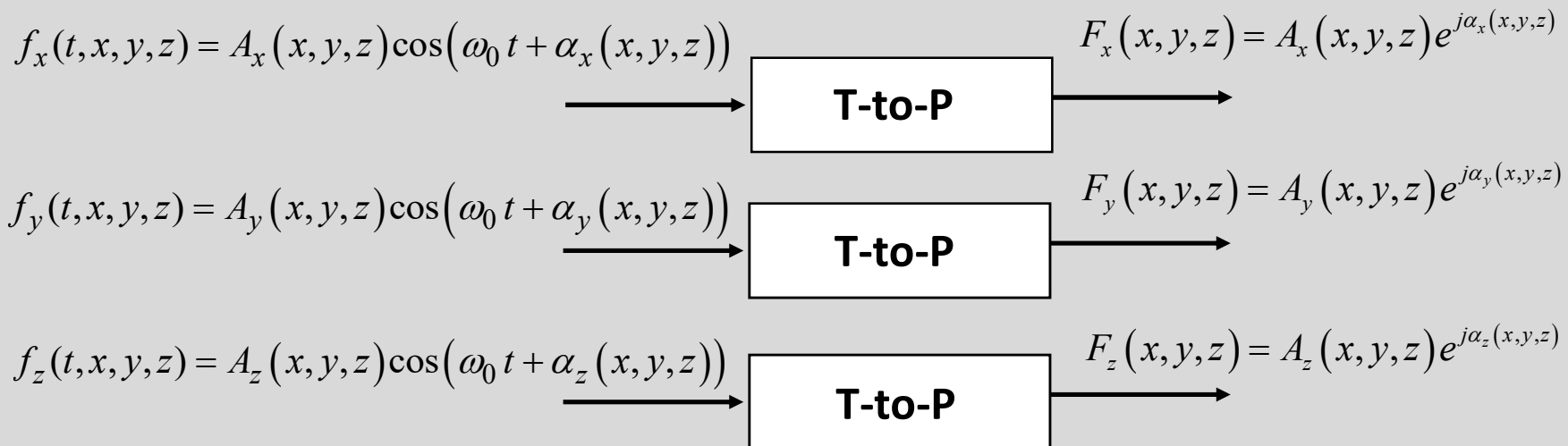
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$



Phasors and vector functions of n variables

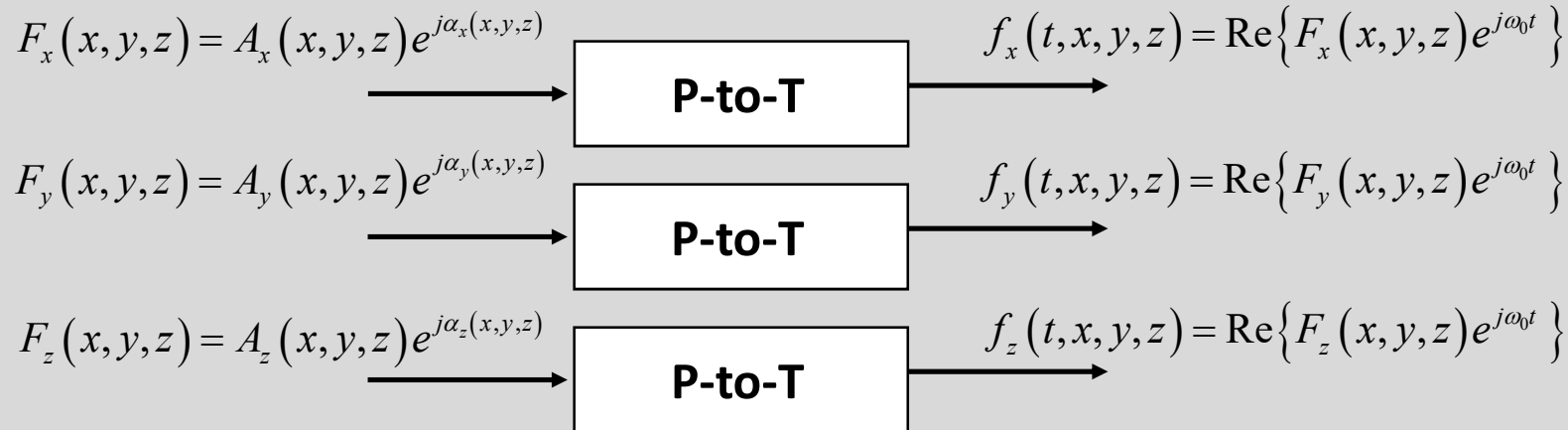
Time domain

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Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain



Phasors and vector functions of n variables

Time domain

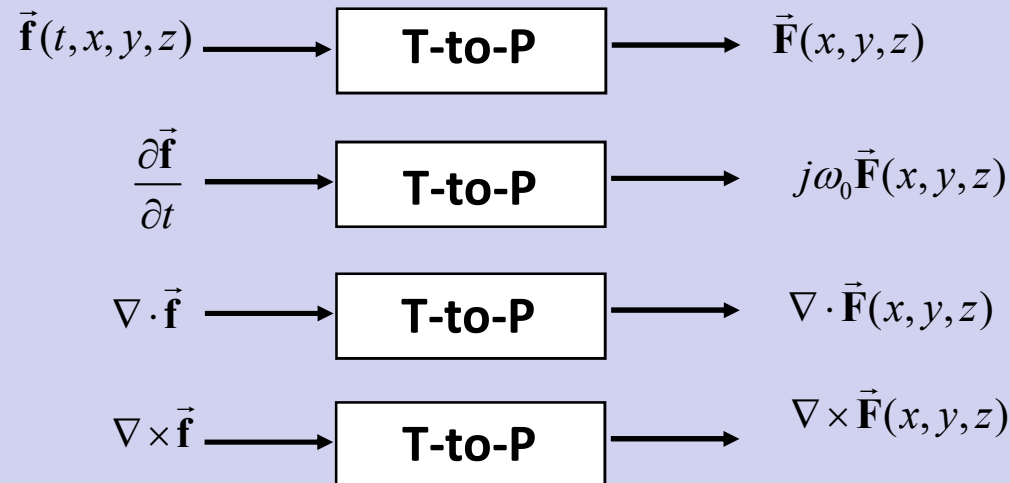
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



Phasors and vector functions of n variables

Time domain

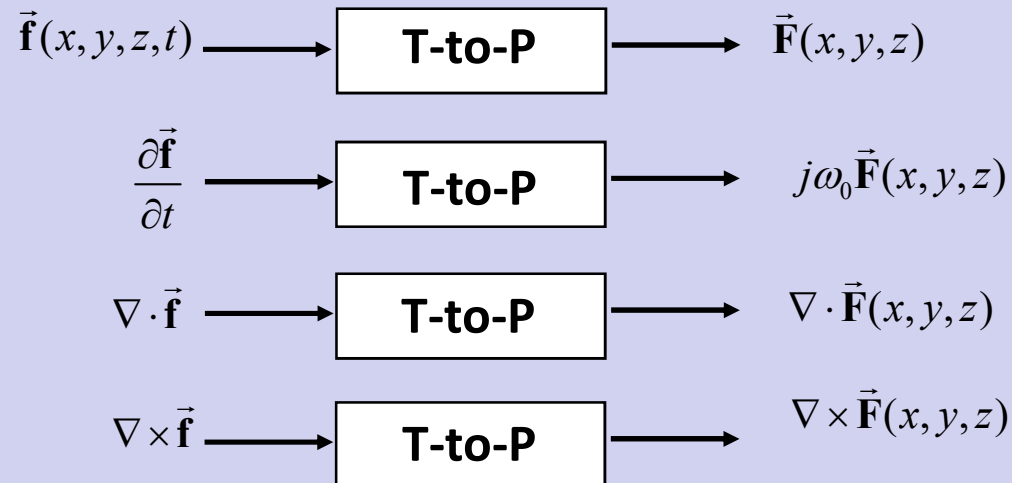
$$\vec{\mathbf{f}}(x, y, z, t) = f_x(x, y, z, t)\hat{i}_x + f_y(x, y, z, t)\hat{i}_y + f_z(x, y, z, t)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors





Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Phasor domain

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Phasor domain





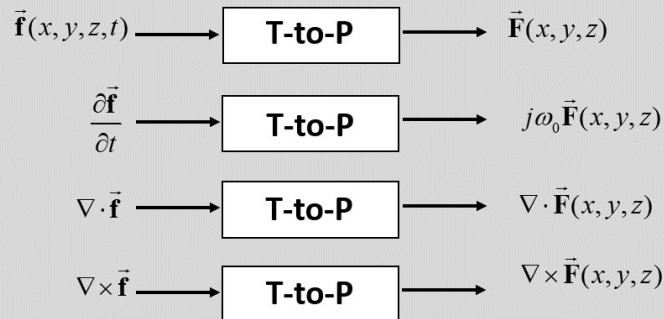
Maxwell equations

Time domain & Phasor domain

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Phasor domain





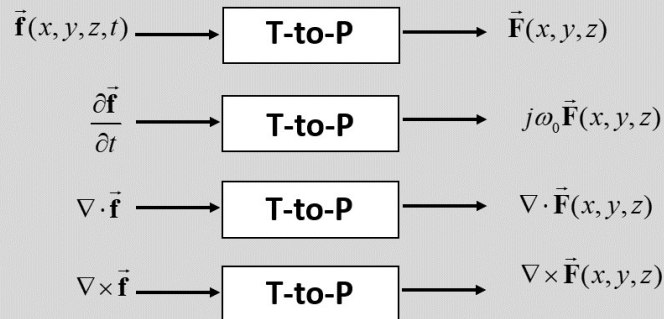
Maxwell equations

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Phasor domain





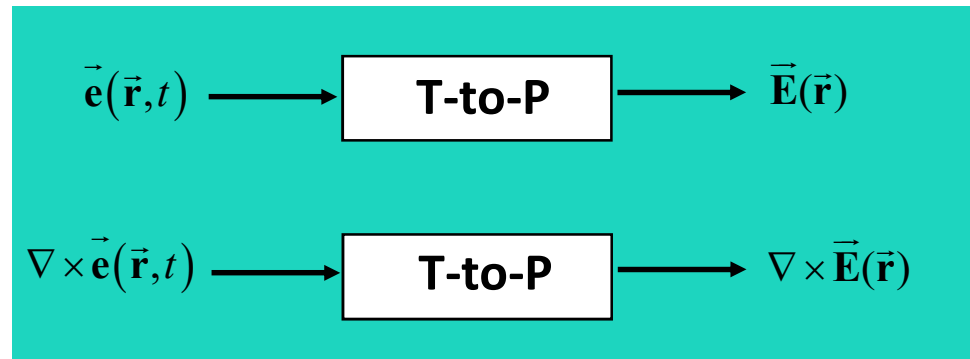
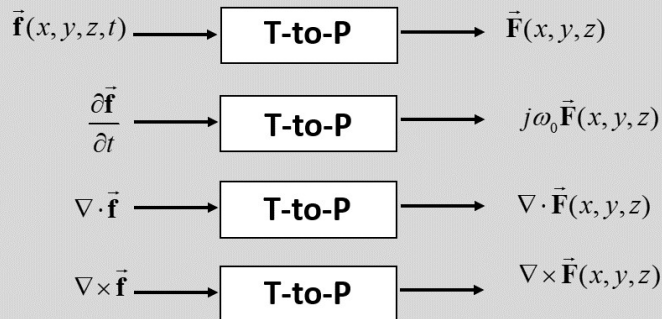
Maxwell equations

Time domain & Phasor domain

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Phasor domain





Maxwell equations

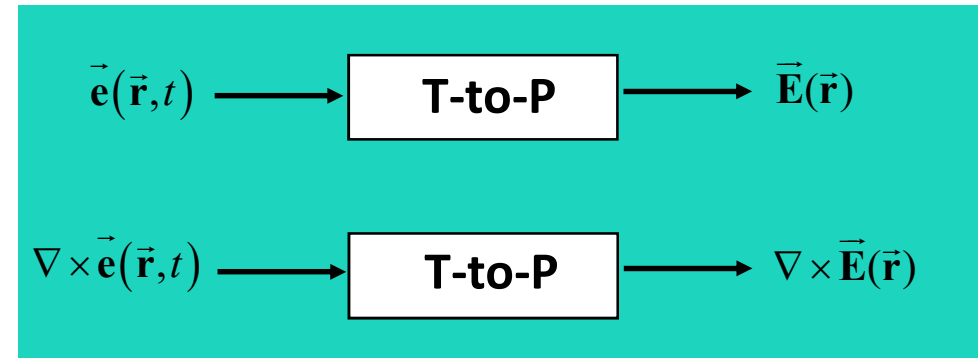
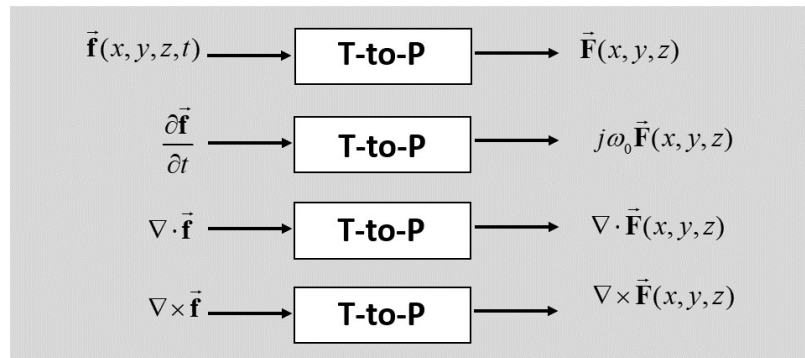
Time domain & Phasor domain

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Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) \end{array} \right.$$





Maxwell equations

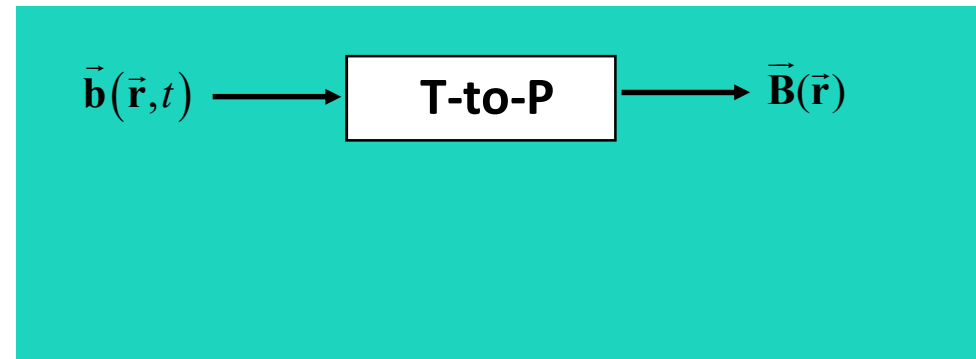
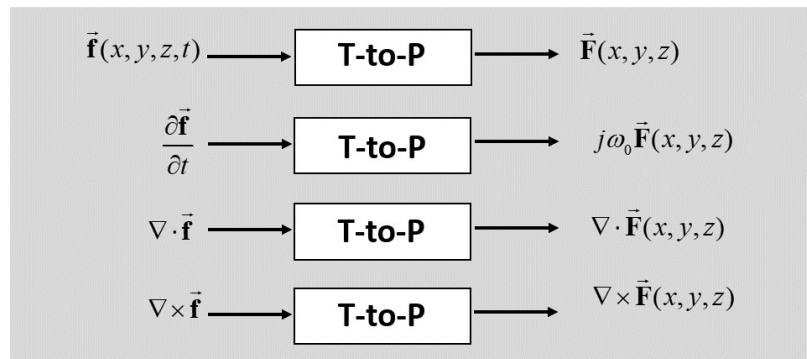
Time domain & Phasor domain

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Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vdots \end{array} \right.$$





Maxwell equations

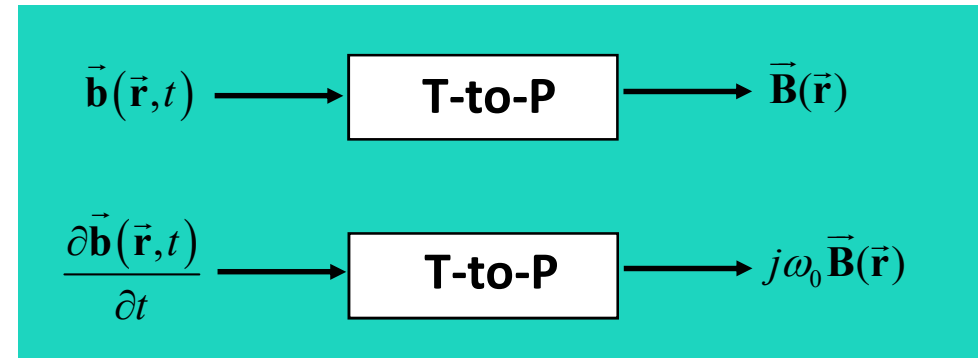
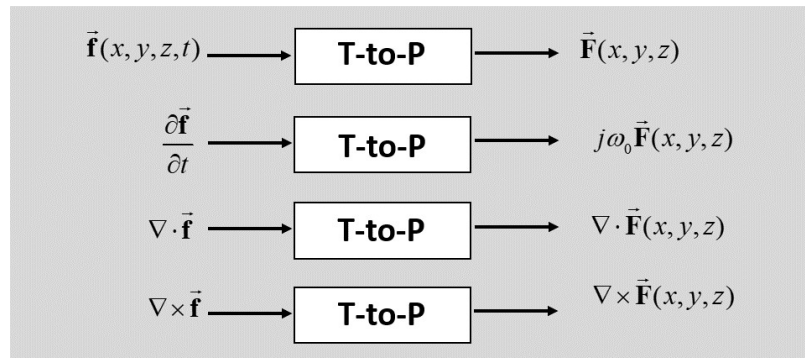
Time domain & Phasor domain

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Maxwell equations

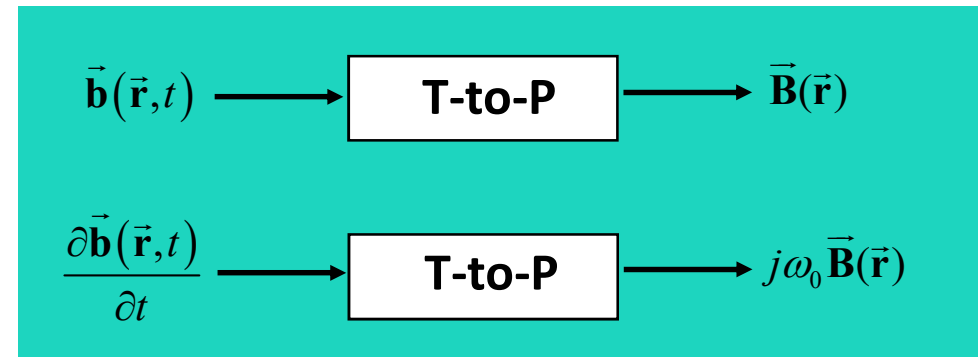
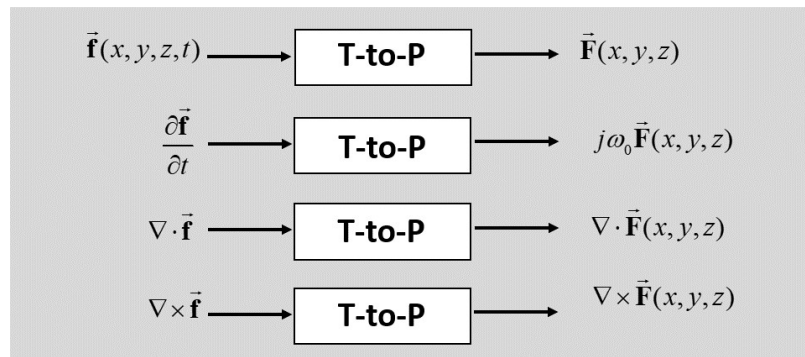
Time domain & Phasor domain

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Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

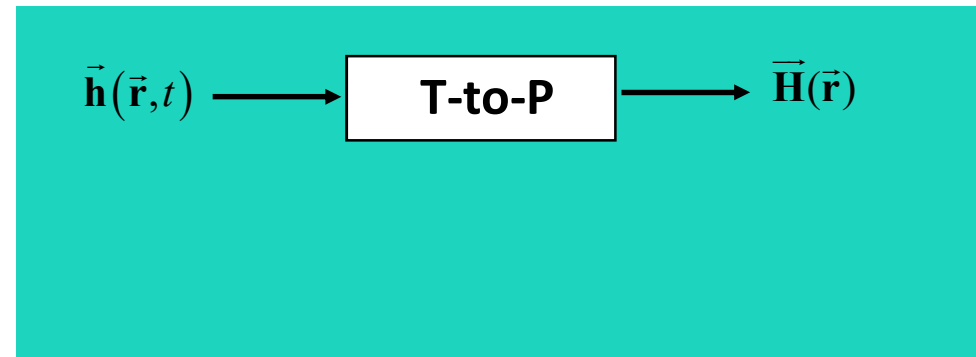
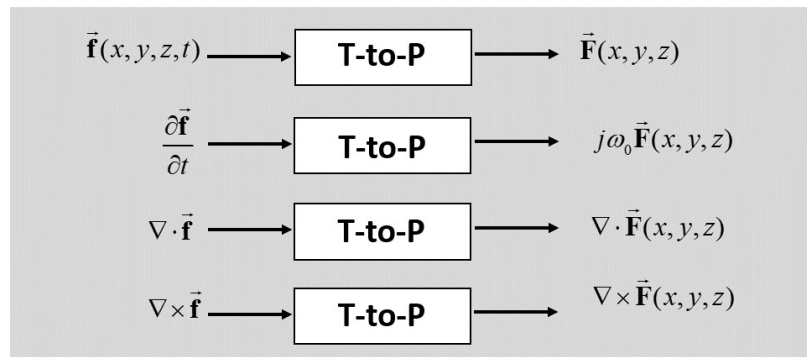
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Phasor domain

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Maxwell equations

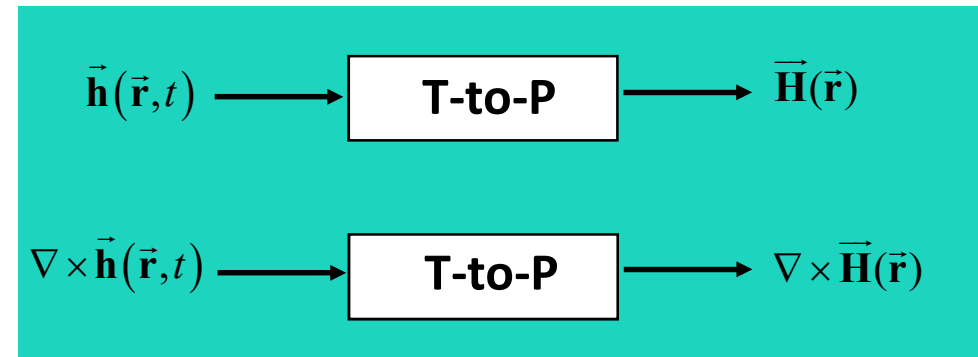
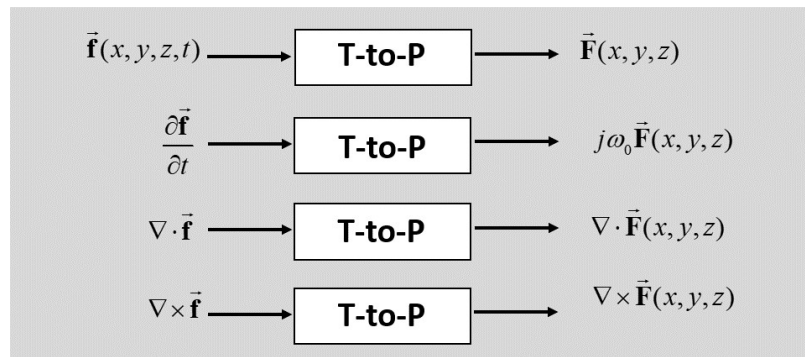
Time domain & Phasor domain

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Phasor domain

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Maxwell equations

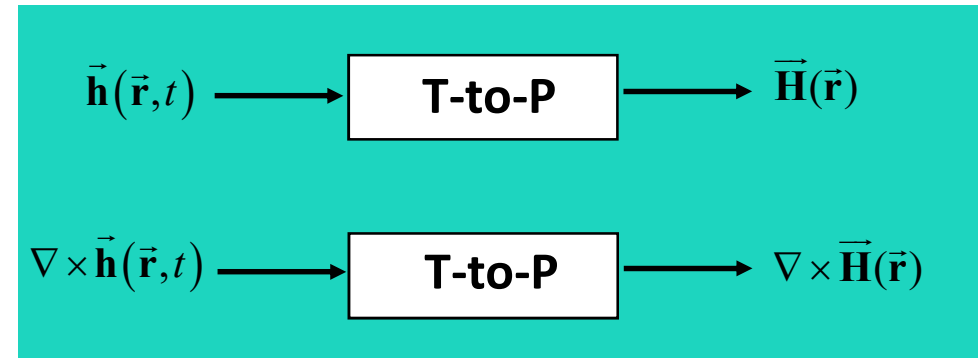
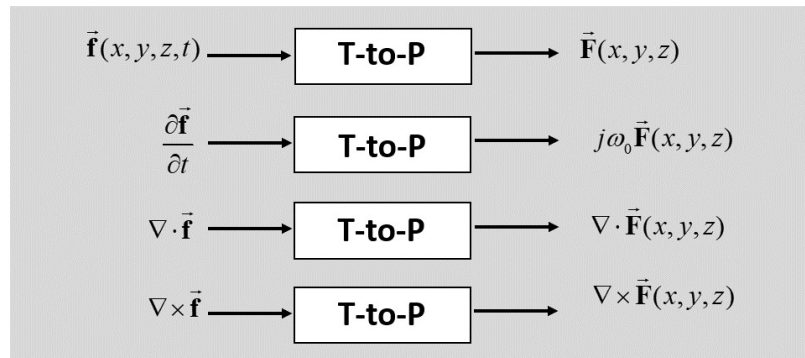
Time domain & Phasor domain

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Phasor domain

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Maxwell equations

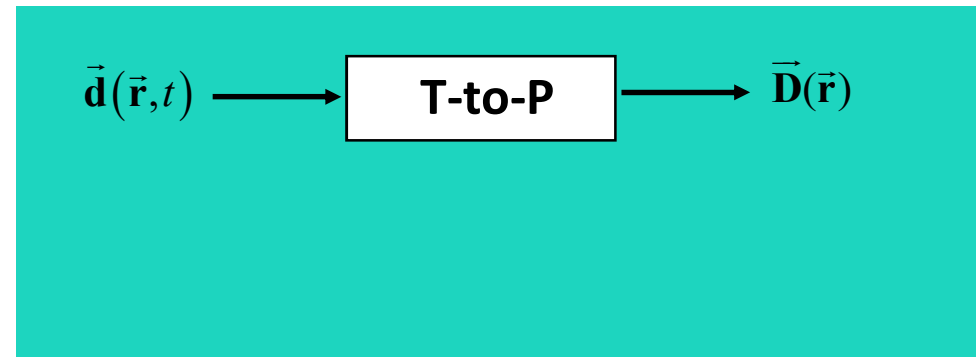
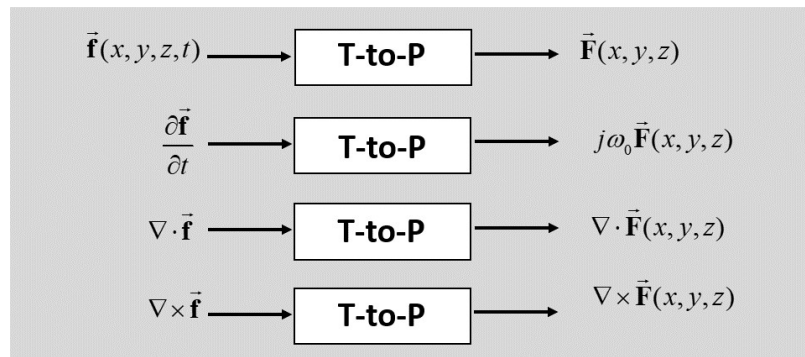
Time domain & Phasor domain

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Phasor domain

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Maxwell equations

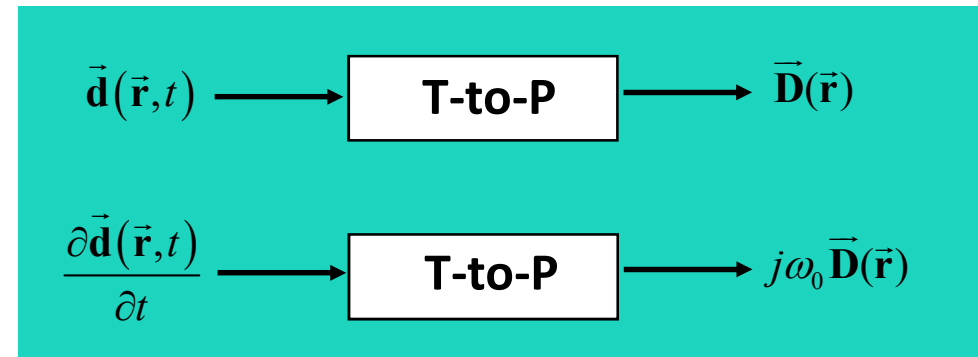
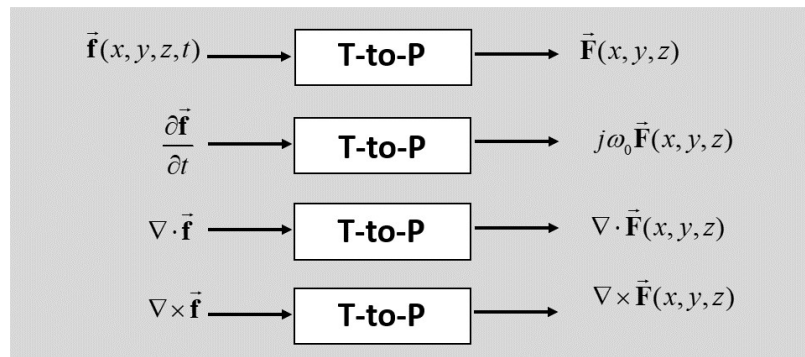
Time domain & Phasor domain

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Phasor domain

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Maxwell equations

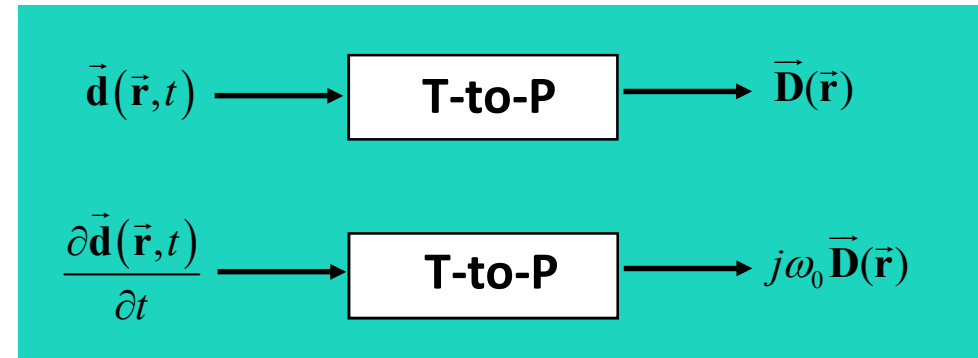
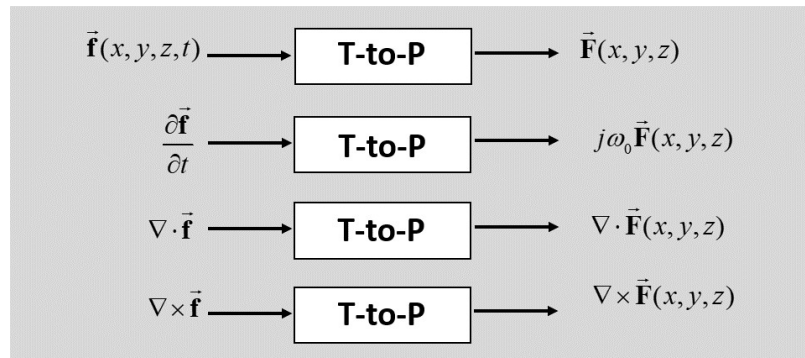
Time domain & Phasor domain

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Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

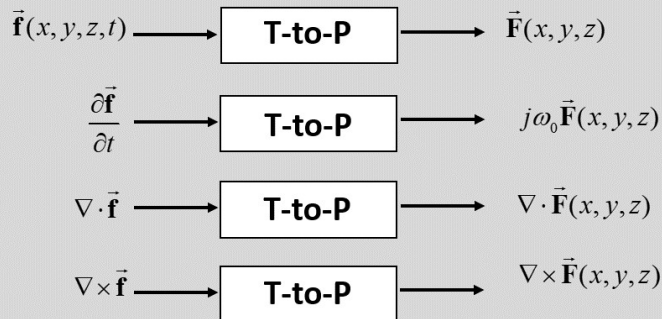
Time domain & Phasor domain

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Phasor domain

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Maxwell equations

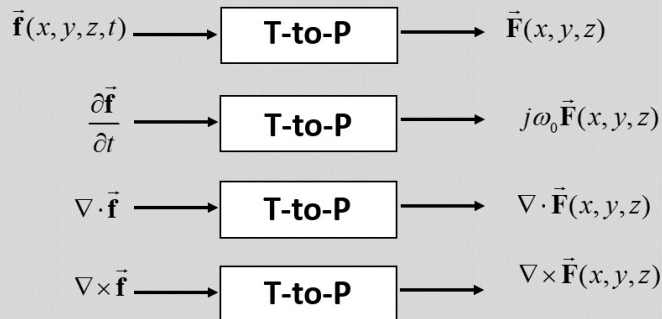
Time domain & Phasor domain

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Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

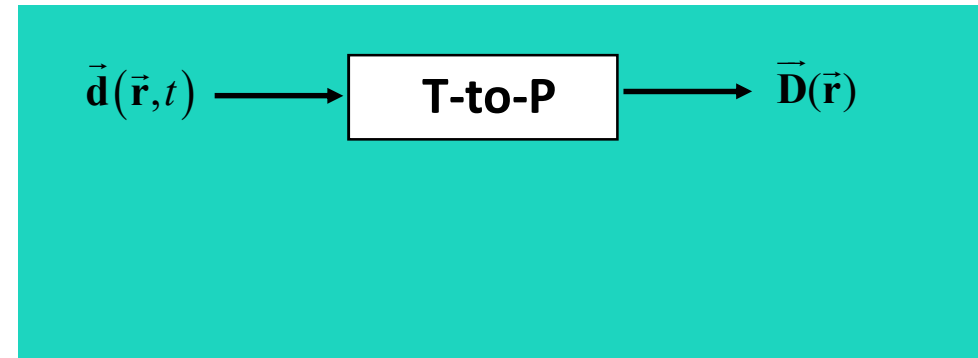
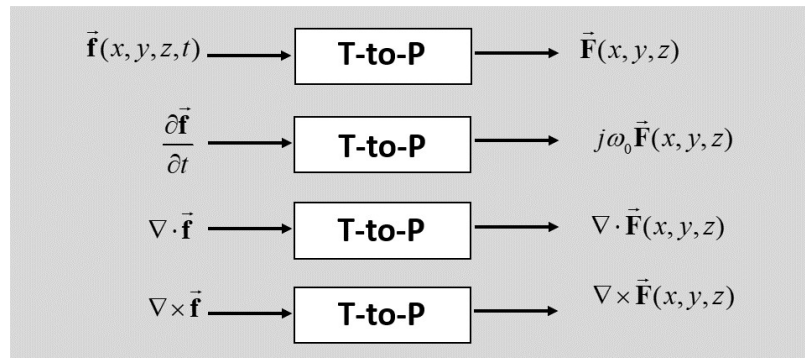
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$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

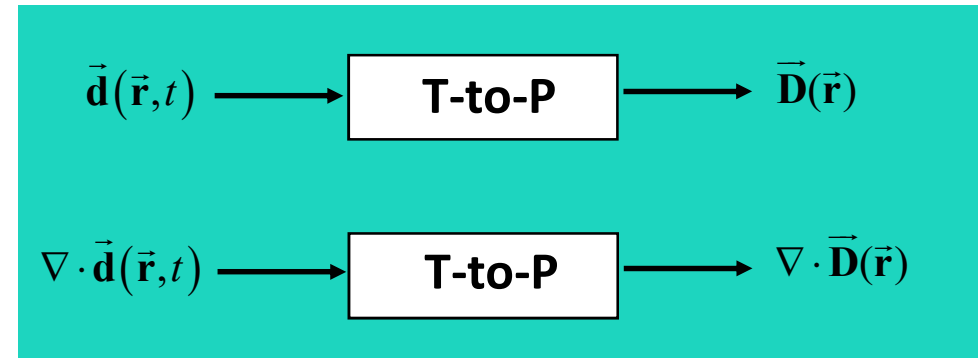
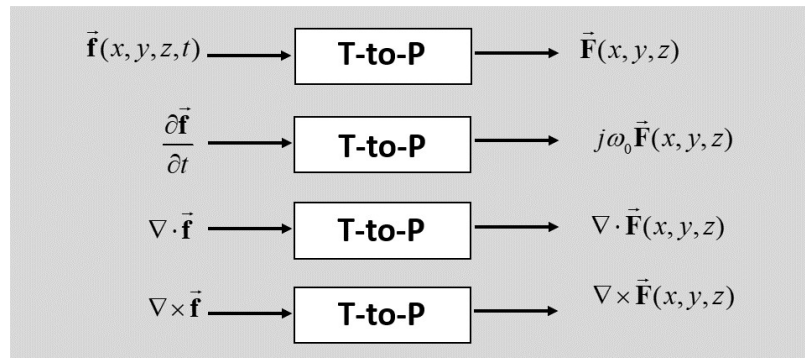
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

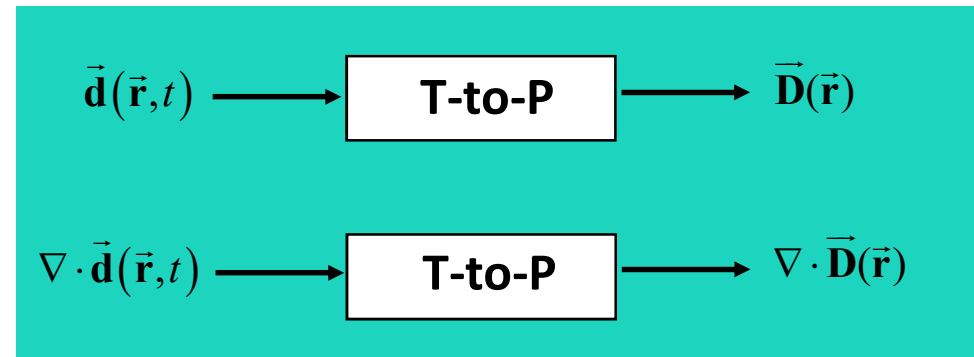
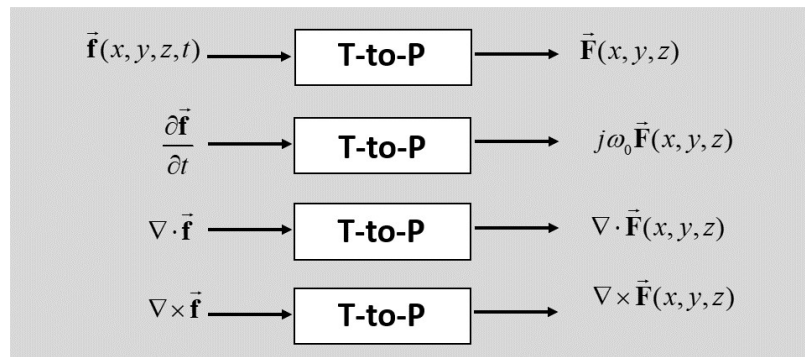
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

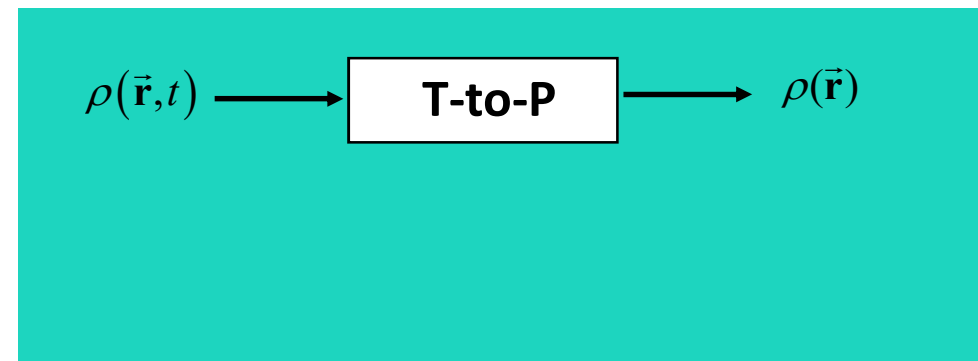
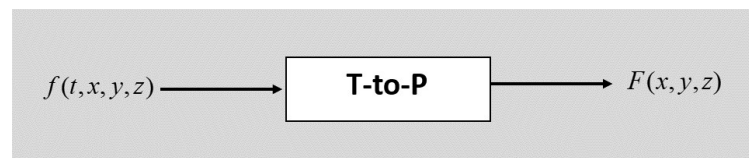
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

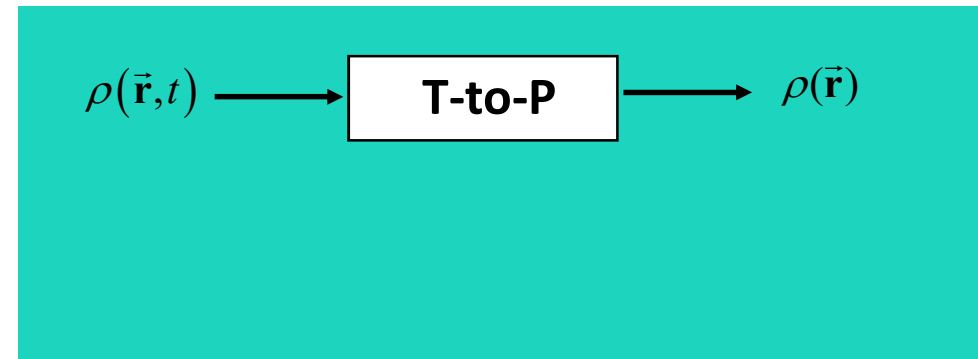
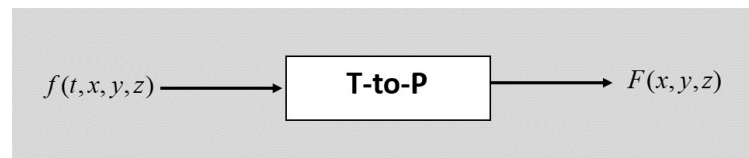
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

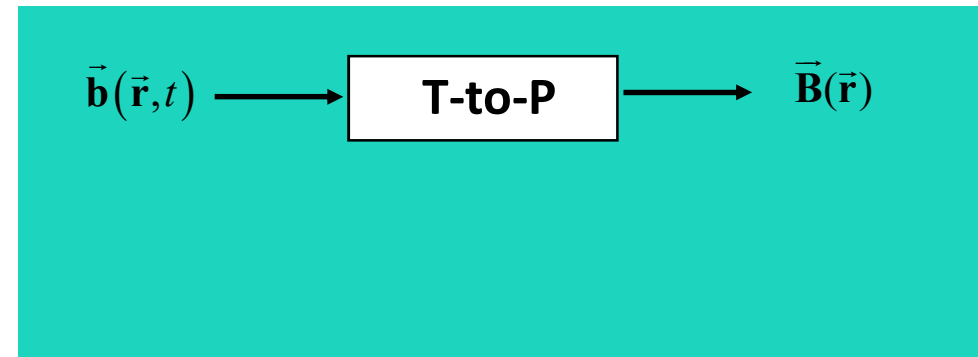
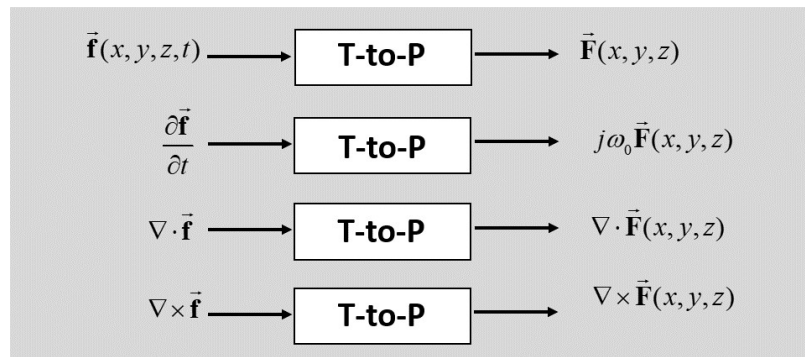
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

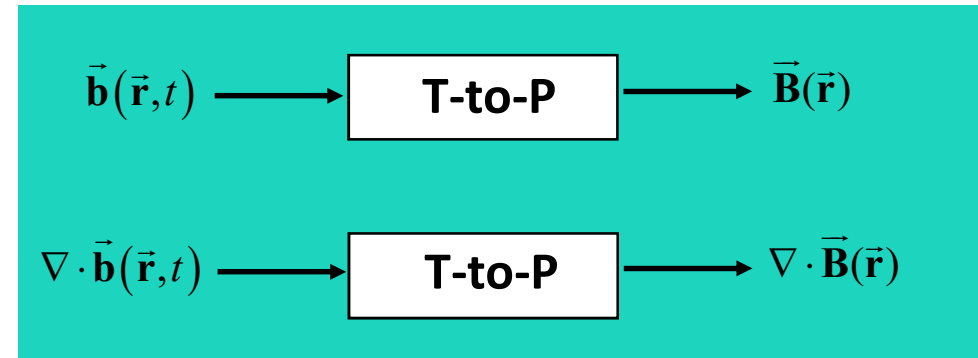
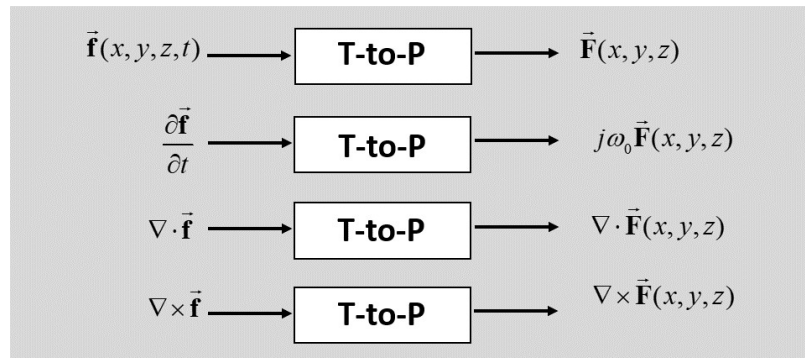
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

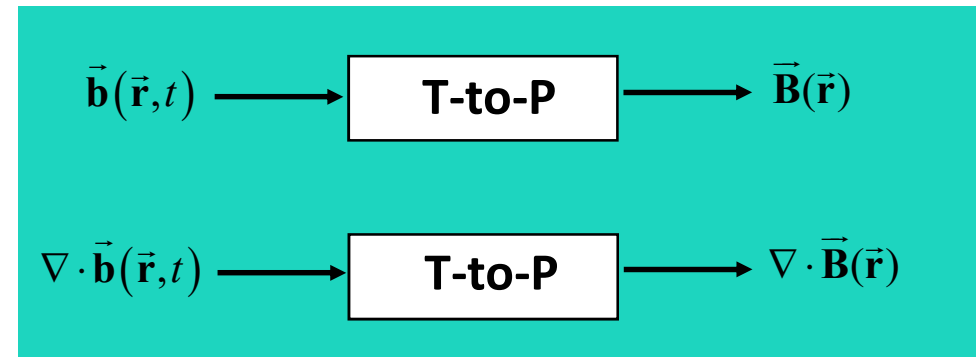
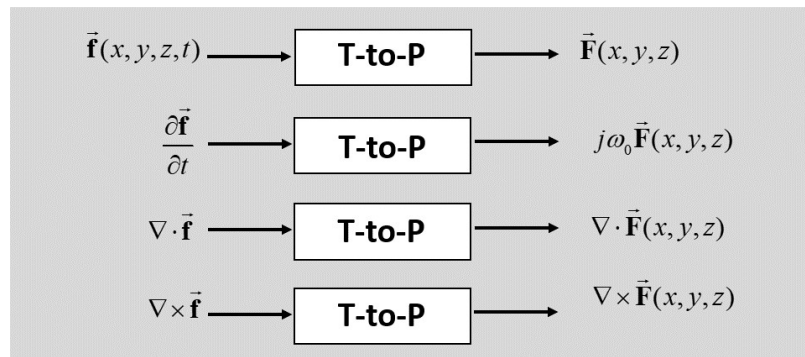
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

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Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$

$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$	Volt/m
$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t)$	Coulomb/m ²
$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t)$	Ampere/m
$\vec{\mathbf{b}}(\vec{\mathbf{r}}, t)$	Weber/m ²
$\vec{\mathbf{j}}(\vec{\mathbf{r}}, t)$	Ampere/m ²
$\rho(\vec{\mathbf{r}}, t)$	Coulomb/m ³



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r})$
$\vec{D}(\vec{r})$
$\vec{H}(\vec{r})$
$\vec{B}(\vec{r})$
$\vec{J}(\vec{r})$
$\rho(\vec{r})$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

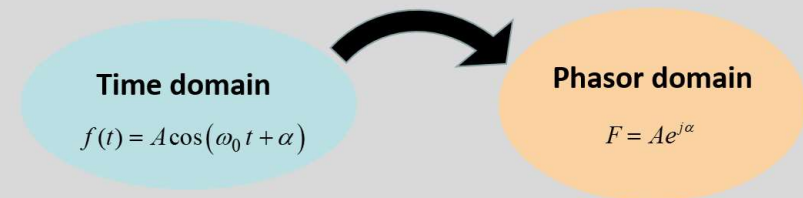
$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{E}(\vec{r})$

.. memo





Maxwell equations

Time domain & Phasor domain

Time domain

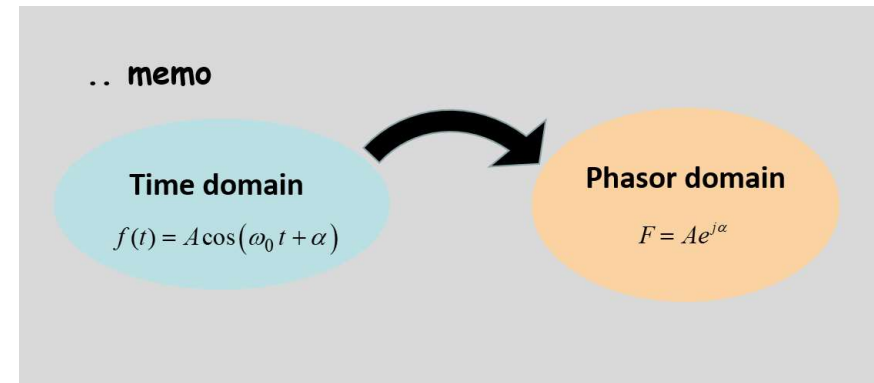
$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r})$ Volt/m





Maxwell equations

Time domain & Phasor domain

Time domain

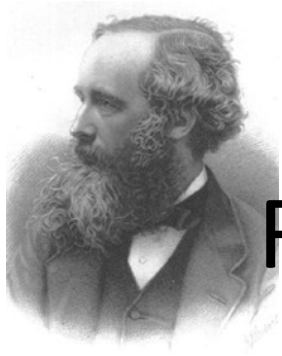
$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r})$	Volt/m
$\vec{D}(\vec{r})$	Coulomb/m ²
$\vec{H}(\vec{r})$	Ampere/m
$\vec{B}(\vec{r})$	Weber/m ²
$\vec{J}(\vec{r})$	Ampere/m ²
$\rho(\vec{r})$	Coulomb/m ³



Maxwell equations

Frequency domain & Phasor domain

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$

The Maxwell equations in the Fourier domain and Phasor domain are **formally** equivalent.

However, they exhibit noticeable differences:

- i) The dimensions of the involved quantities (f.i., $\vec{\mathbf{E}}$) are different in the two domains.
- ii) In the Frequency domain ω is an independent variable, whereas in the Phasor domain ω_0 is fixed.

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics



Maxwell equations

Time domain & Phasor domain

Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$$\frac{\partial \rho(\vec{r}, t)}{\partial t} + \nabla \cdot \vec{j}(\vec{r}, t) = 0$$

Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{cases}$$

$$j\omega \rho(\vec{r}, \omega) + \nabla \cdot \vec{J}(\vec{r}, \omega) = 0$$

Phasor domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{cases}$$

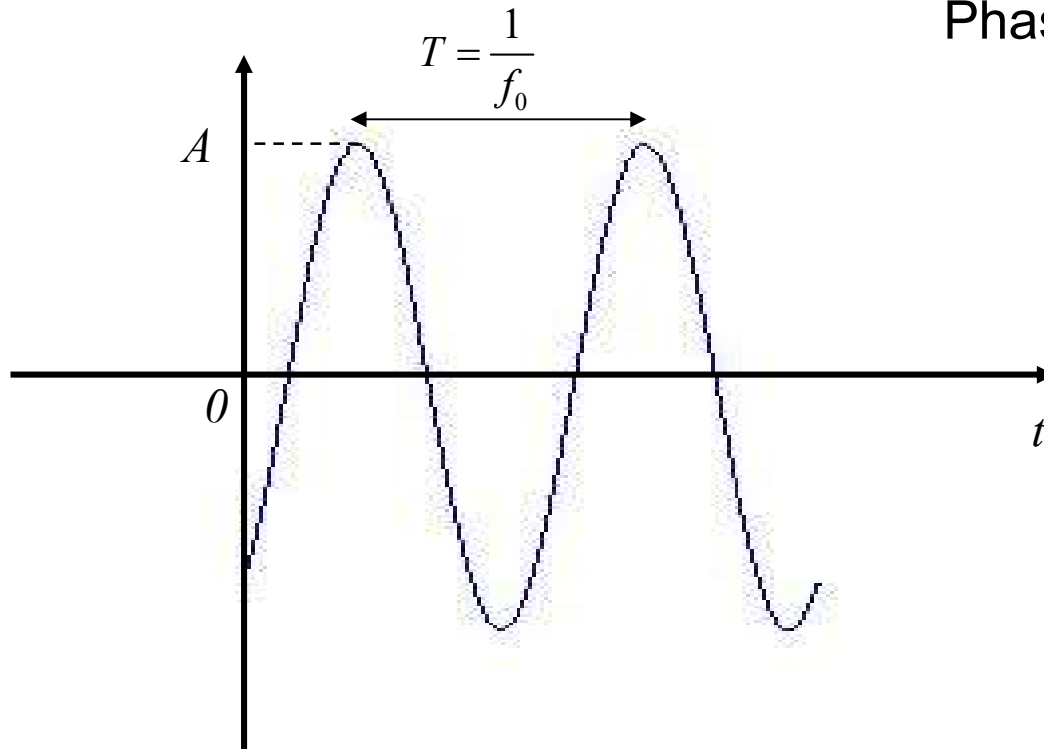
$$j\omega_0 \rho(\vec{r}) + \nabla \cdot \vec{J}(\vec{r}) = 0$$

Memo: Phasors

Phasors

$$v(t) = A \cos(2\pi f_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

Phasor (complex number)



Phasors

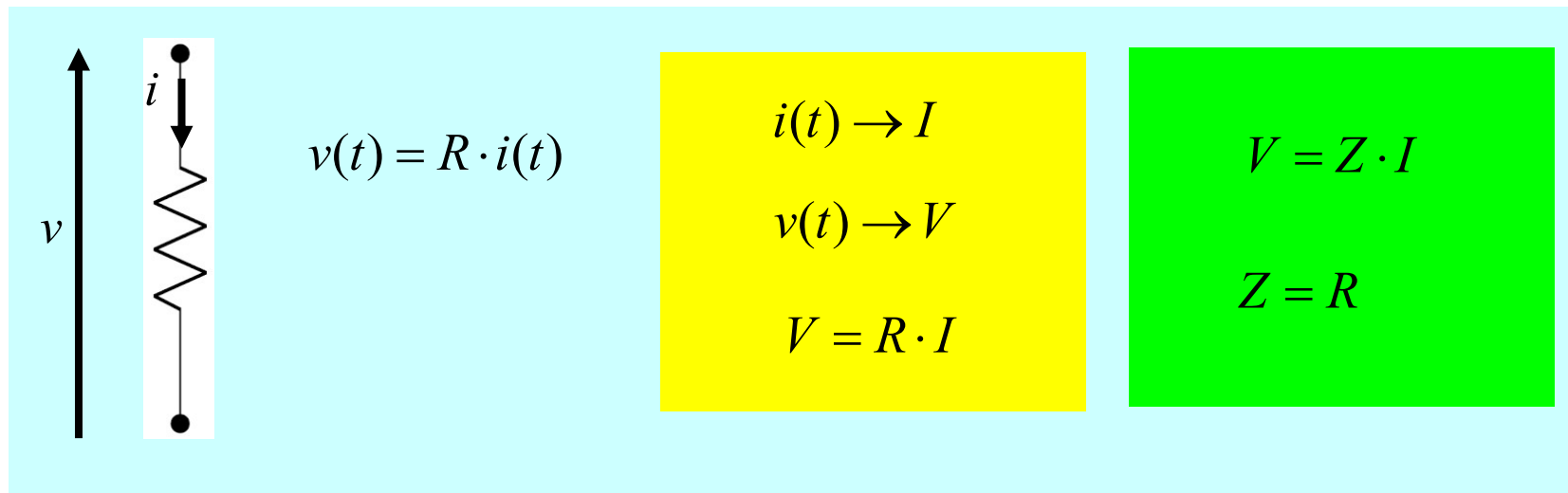
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$



Phasors

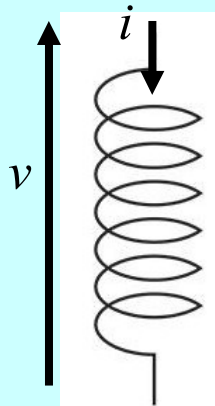
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$v(t) = L \frac{di(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = j\omega_0 LI$$

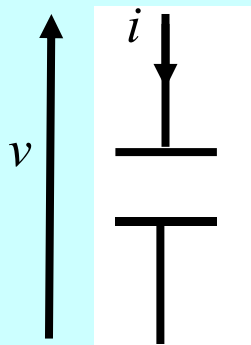
$$V = Z \cdot I$$

$$Z = j\omega_0 L$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$i(t) = C \frac{dv(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$I = j\omega_0 CV$$

$$V = Z \cdot I$$

$$Z = -j \frac{1}{\omega_0 C}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

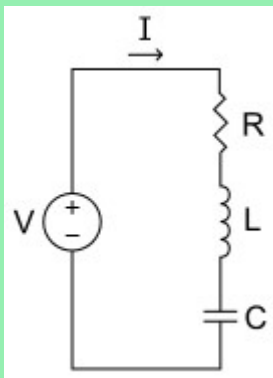
$$\langle v(t) \cdot i(t) \rangle = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \operatorname{Re}\{V \cdot I^*\}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

$$\langle v(t) \cdot i(t) \rangle = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \operatorname{Re}\{V \cdot I^*\}$$



$$P = \frac{1}{2} V \cdot I^* = P_1 + jP_2$$

$$P = \frac{1}{2} V \cdot I^* = \frac{1}{2} (Z_R + Z_L + Z_C) I \cdot I^* = \frac{1}{2} \left(R + j\omega_0 L - \frac{j}{\omega_0 C} \right) |I|^2$$

$$P_1 = \frac{1}{2} R |I|^2 ; \quad P_2 = \frac{1}{2} \left(\omega_0 L - \frac{1}{\omega_0 C} \right) |I|^2$$

Phasors and vector functions of n variables

$$\vec{\mathbf{f}}_1(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_1(x, y, z)$$

$$\vec{\mathbf{f}}_2(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_2(x, y, z)$$

Phasors and vector functions of n variables

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

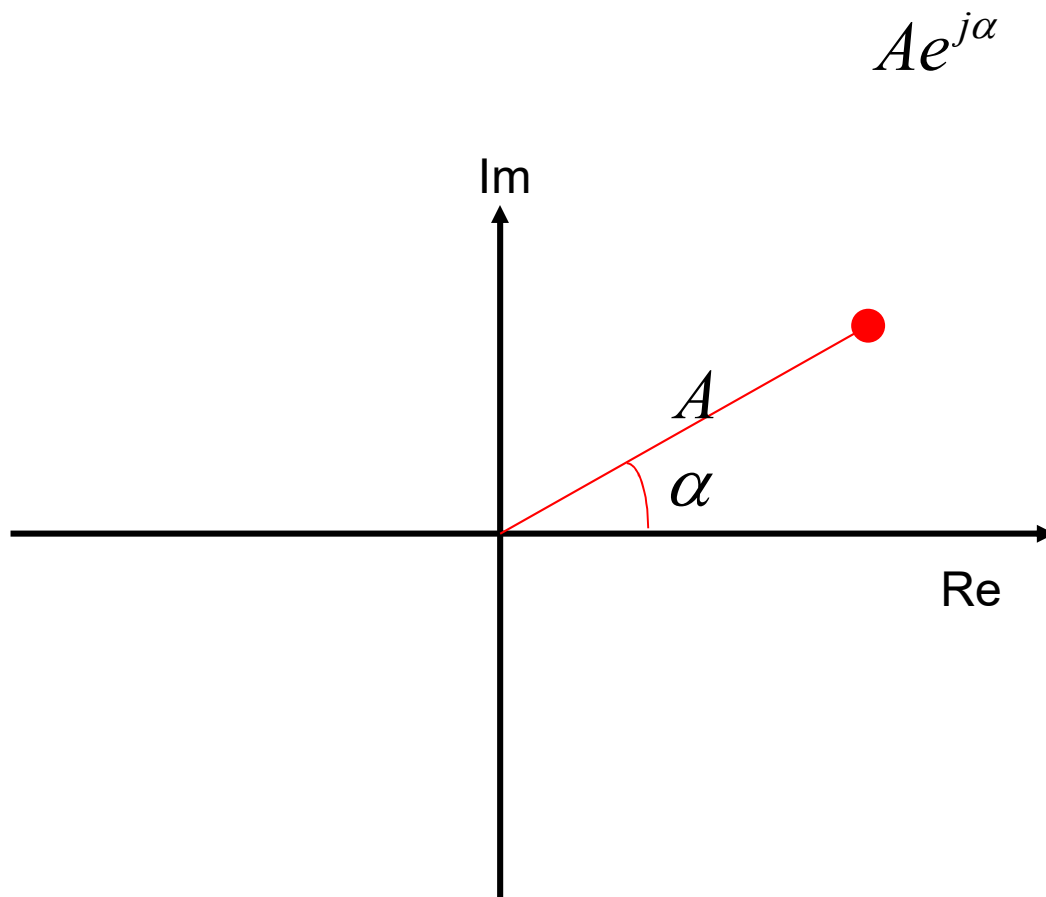
$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \cdot \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \times \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

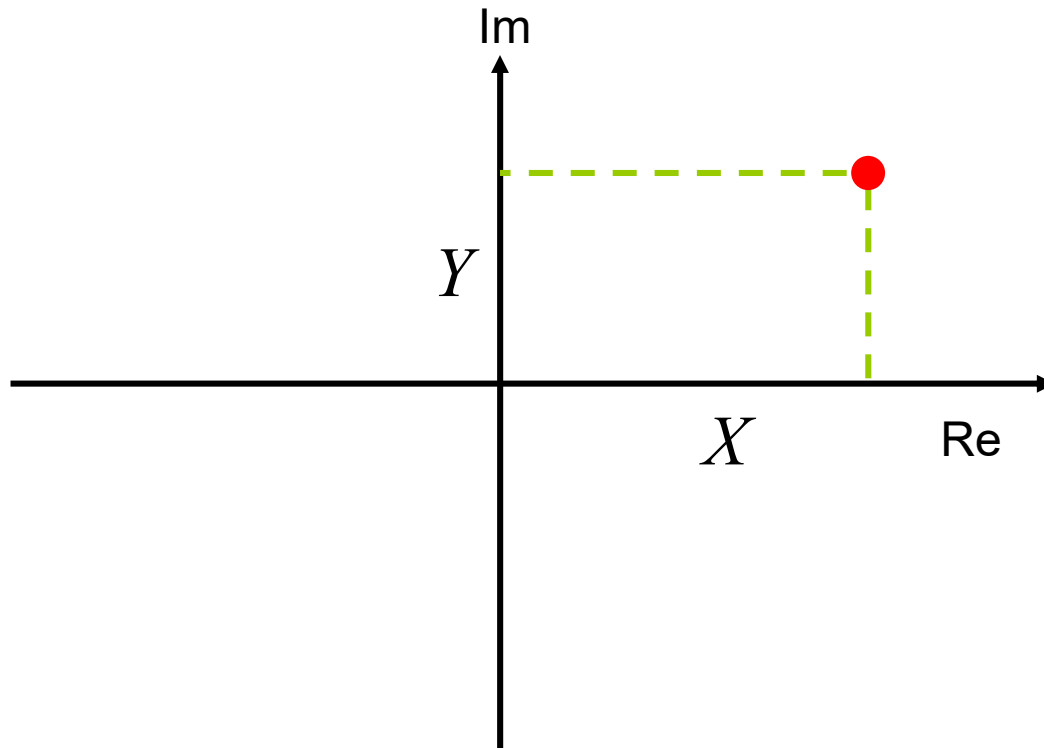
Memo: complex numbers

Complex numbers



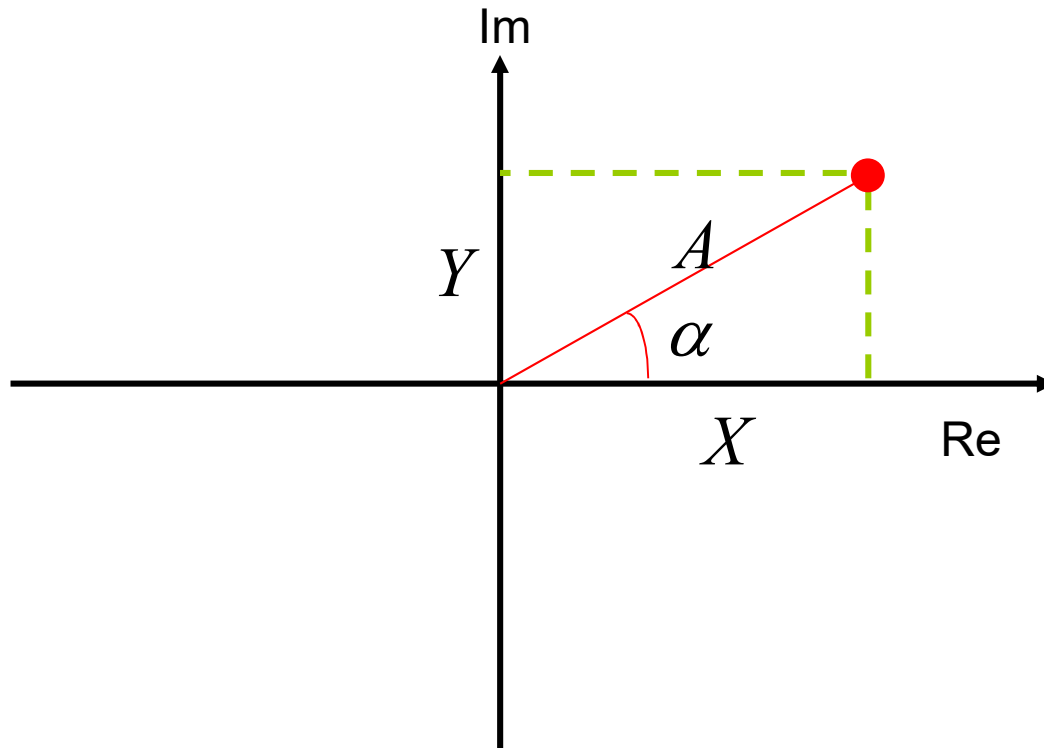
Complex numbers

$$Ae^{j\alpha} = X + jY$$



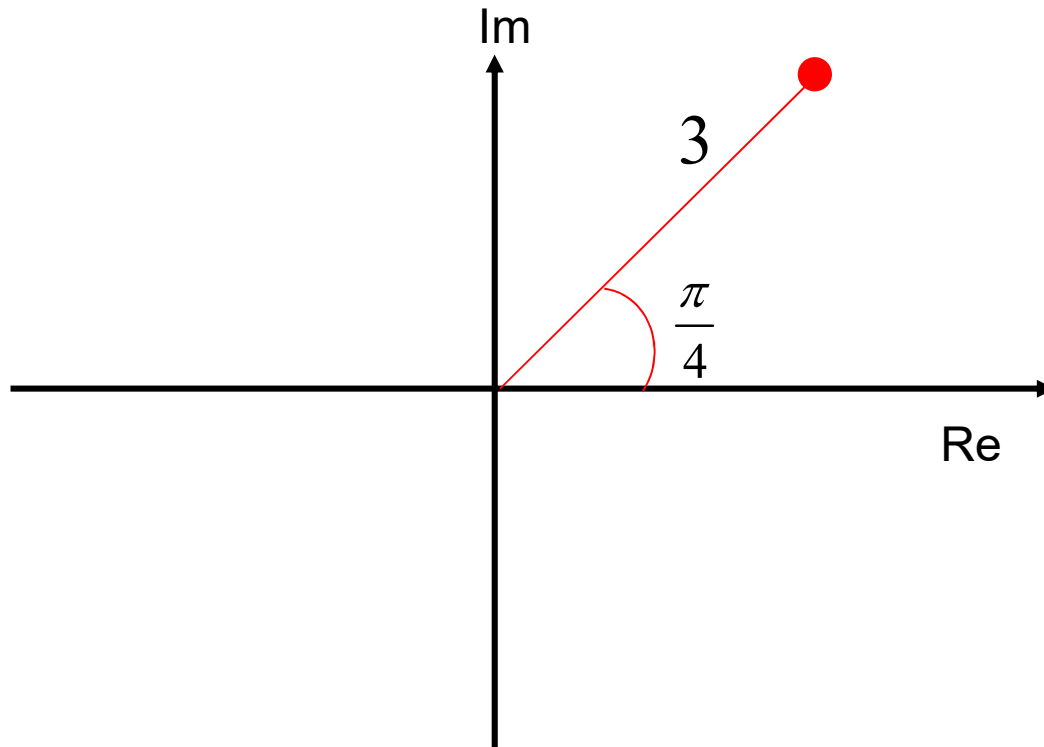
Complex numbers

$$Ae^{j\alpha} = X + jY$$

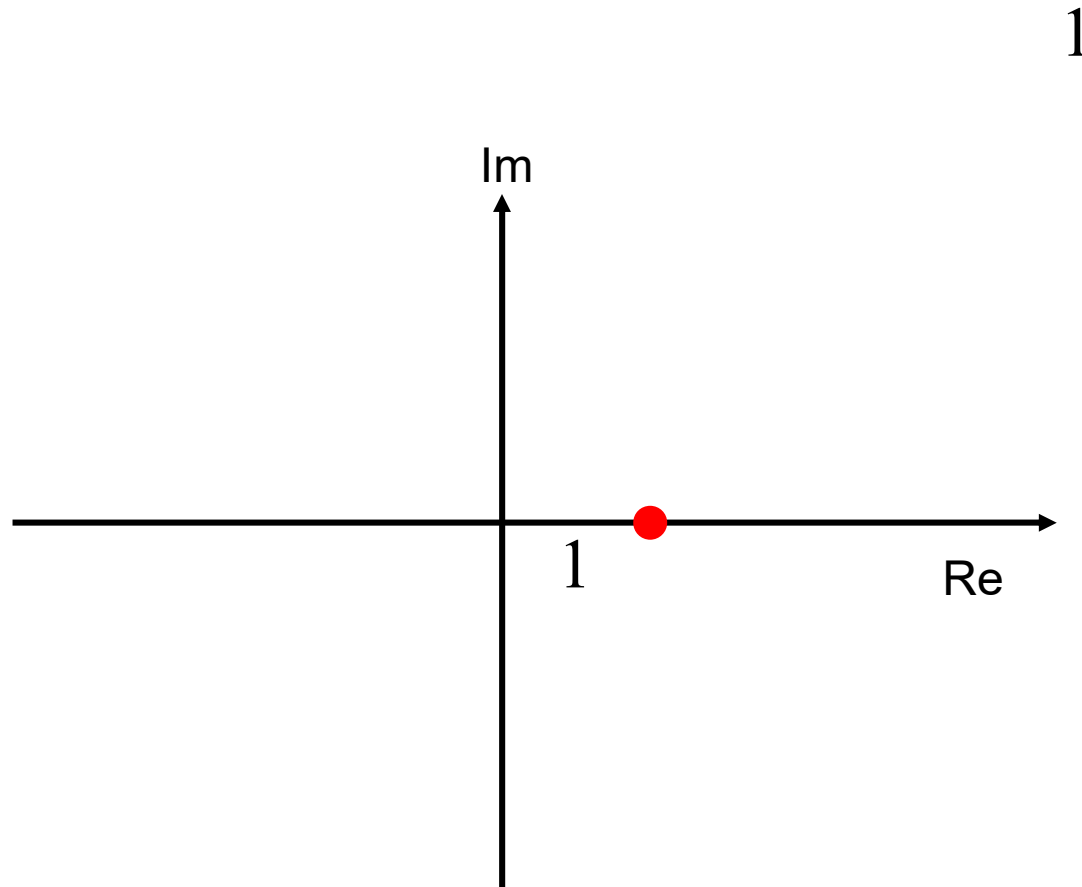


Complex numbers: some examples

$$3e^{j\frac{\pi}{4}}$$

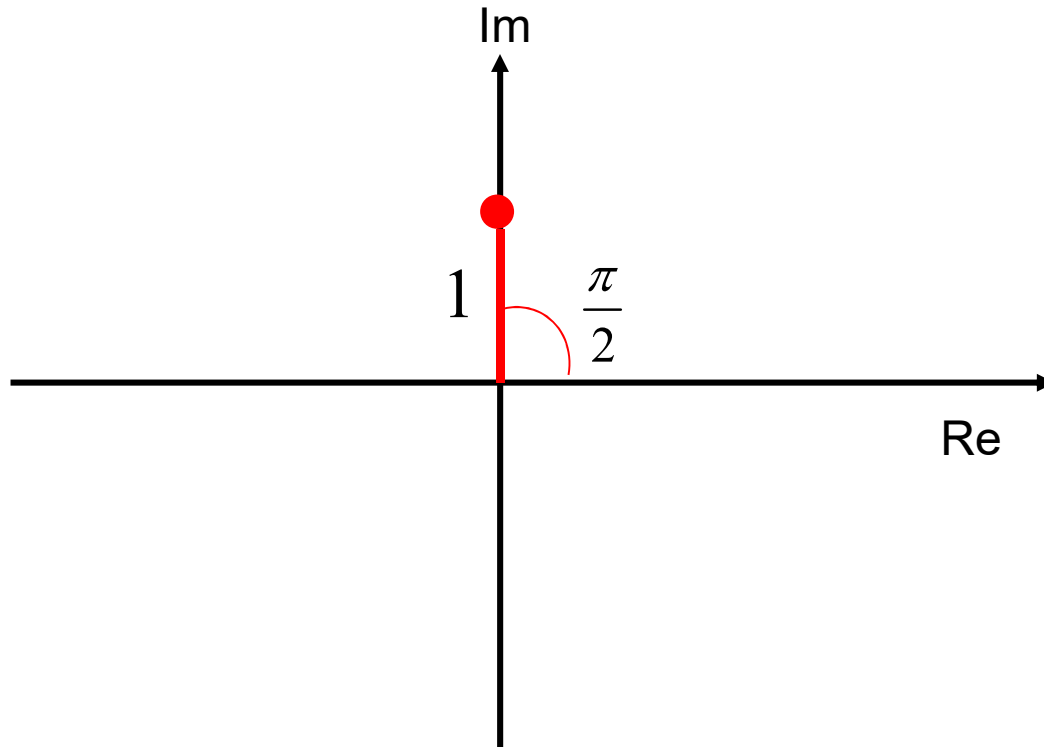


Complex numbers: some examples



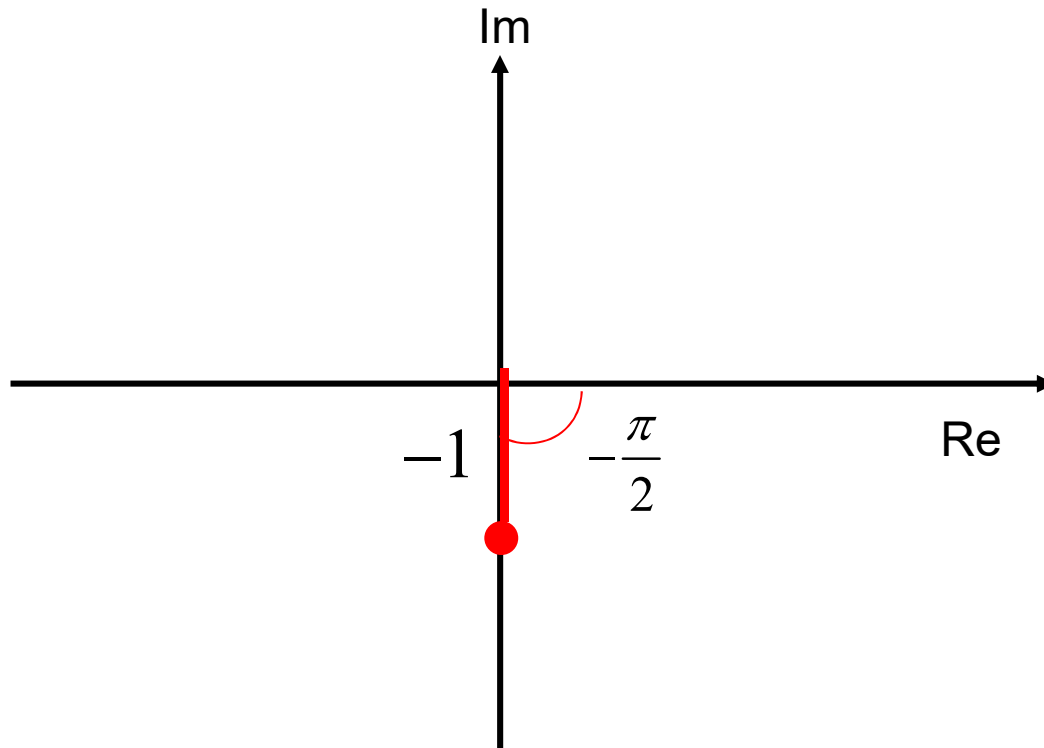
Complex numbers: some examples

$$j = e^{j\frac{\pi}{2}} = e^{j\left(\frac{\pi}{2} + 2\pi\right)}$$



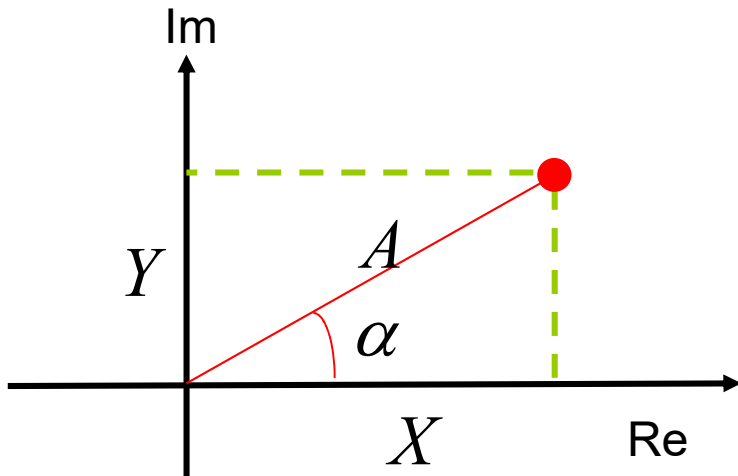
Complex numbers: some examples

$$-j = e^{-j\frac{\pi}{2}}$$



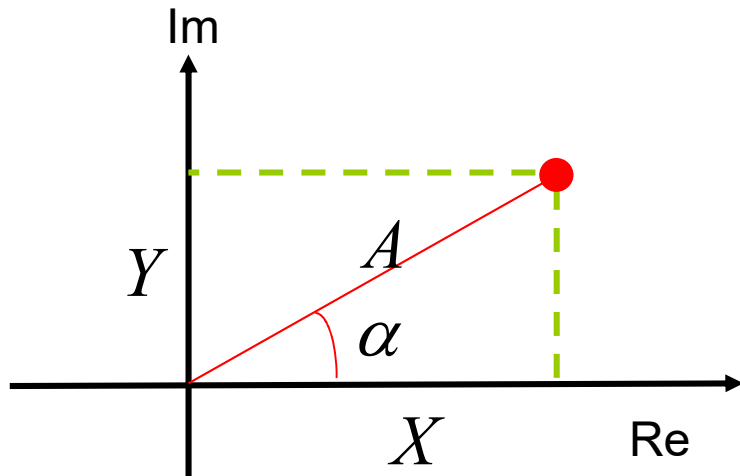
Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



Complex numbers: conversion formulas

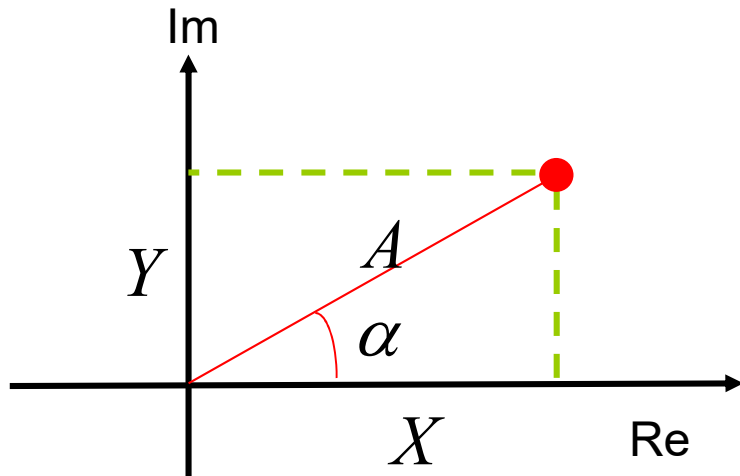
$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



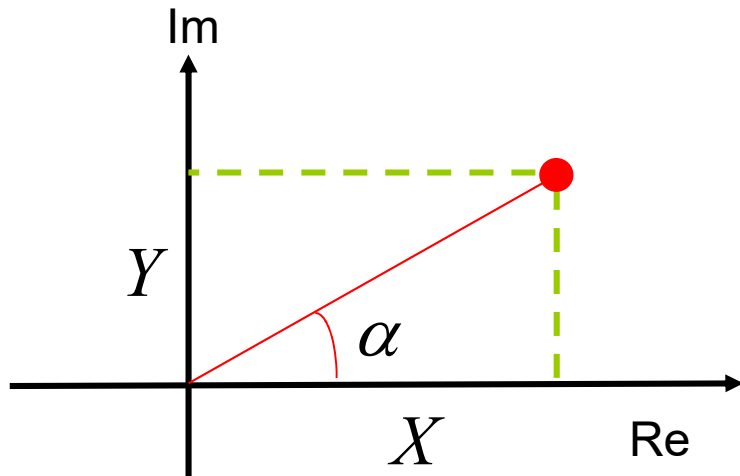
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$e^{j\alpha} = \cos \alpha + j \sin \alpha$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

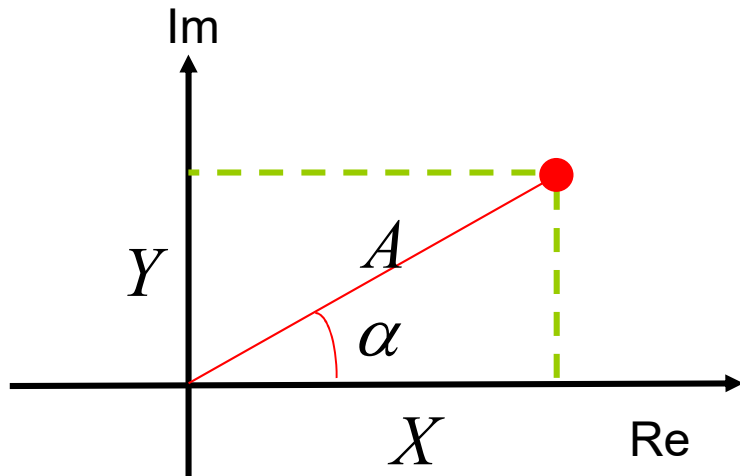
Eulero's formulas

$$\cos \alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



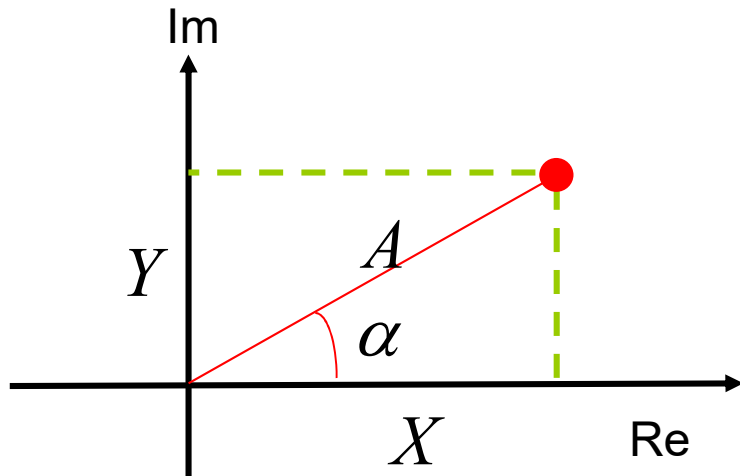
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$X + jY \rightarrow Ae^{j\alpha}$$

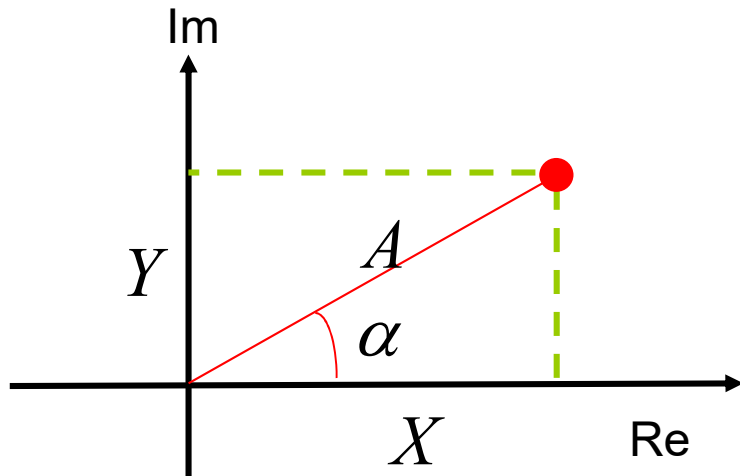
$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

Complex numbers: conversion formulas

Some examples

$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$2e^{j\frac{3}{4}\pi} = -\sqrt{2} + j\sqrt{2}$$

Complex numbers: conversion formulas

Some examples

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

$$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$\sqrt{3} - j = 2e^{-j\frac{\pi}{6}}$$

Complex numbers: summation

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 + Z_2 = (X_1 + X_2) + j(Y_1 + Y_2)$$

Complex numbers: product

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = (X_1 \cdot X_2 - Y_1 \cdot Y_2) + j(X_1 \cdot Y_2 + Y_1 \cdot X_2)$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = 3e^{j\frac{\pi}{6}}$$

$$z_2 = 4e^{j\frac{\pi}{4}}$$

$$z_1 \cdot z_2 = 12e^{j\left(\frac{\pi}{6} + \frac{\pi}{4}\right)} = 12e^{j\left(\frac{5}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = \sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$z_2 = -\sqrt{2} + j\sqrt{2} = 2e^{j\frac{3\pi}{4}}$$

$$z_1 \cdot z_2 = 4e^{j\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)} = 4e^{j\left(\frac{11}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

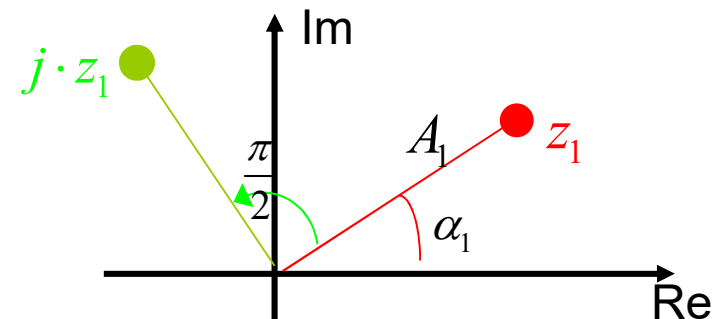
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$j \cdot z_1 = A_1 e^{j\left(\alpha_1 + \frac{\pi}{2}\right)}$$

$$z_2 = j = e^{j\frac{\pi}{2}}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

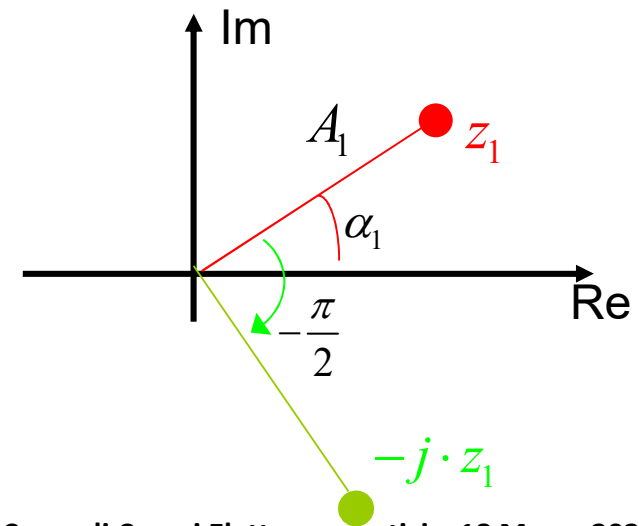
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-j \cdot z_1 = A_1 e^{j\left(\alpha_1 - \frac{\pi}{2}\right)}$$

$$z_2 = -j = e^{-j\frac{\pi}{2}}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-z_1 = A_1 e^{j(\alpha_1 + \pi)}$$

$$z_2 = -1 = e^{j\pi}$$

