



Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

**a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo
anno**

Università degli Studi di Napoli “Parthenope”

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Equazioni di Maxwell

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$



		Unità di misura
$\vec{e}(\vec{r}, t)$:	Campo elettrico	Volt/m
$\vec{d}(\vec{r}, t)$:	Induzione elettrica	Coulomb/m ²
$\vec{h}(\vec{r}, t)$:	Campo magnetico	Ampere/m
$\vec{b}(\vec{r}, t)$:	Induzione magnetica	Weber/m ²
$\vec{j}(\vec{r}, t)$:	Densità di corrente	Ampere/m ²
$\rho(\vec{r}, t)$:	Densità di carica	Coulomb/m ³

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics



Maxwell equations

Differential form

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) + \frac{\partial \rho(\vec{\mathbf{r}}, t)}{\partial t} = 0$$

Integral form

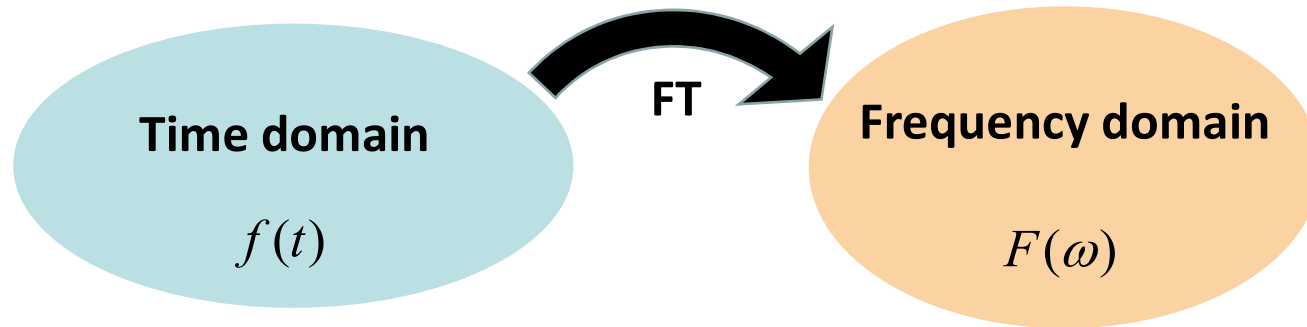
$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + i(t) \\ \oiint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = q(t) \\ \oiint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$

$$\oiint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Maxwell equations: Time domain, Frequency domain, Phasors



Frequency domain

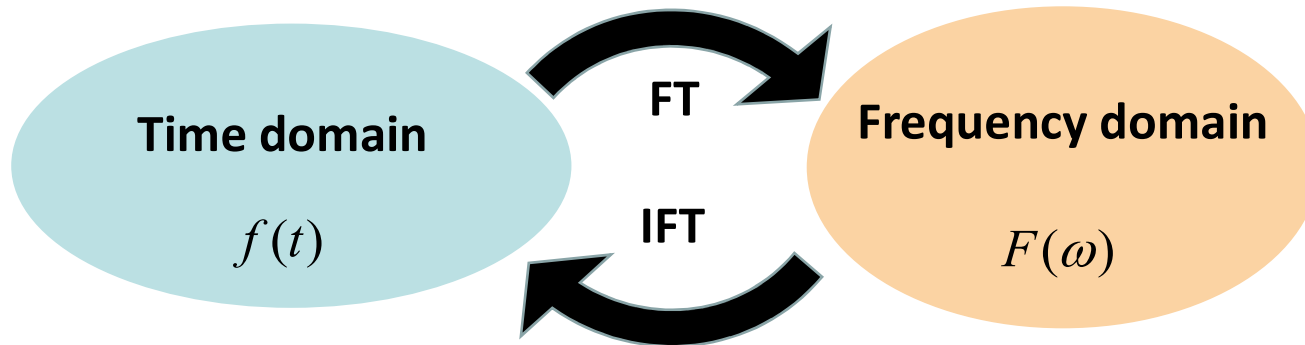


$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

- 1) How to jump back from the Frequency domain to the Time domain
- 2) Time domain derivative and Fourier Transform

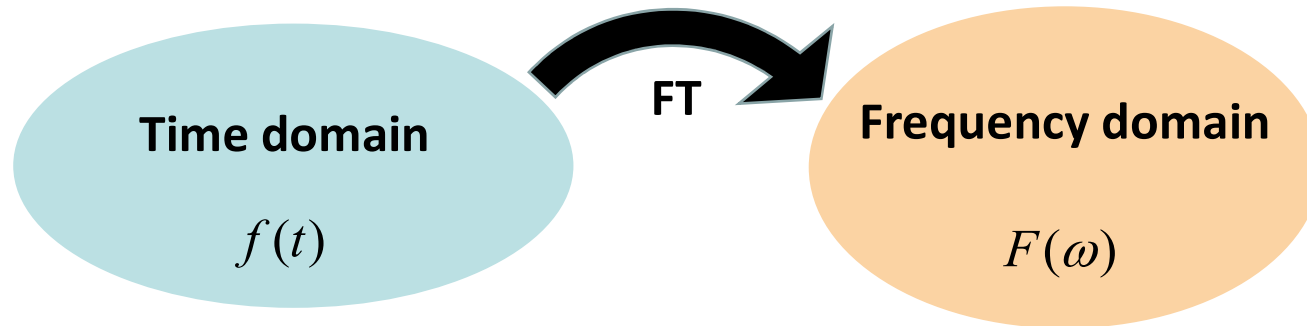
Frequency domain



$$f(t) \longrightarrow \boxed{\text{FT}} \longrightarrow F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

$$F(\omega) \longrightarrow \boxed{\text{IFT}} \longrightarrow f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$$

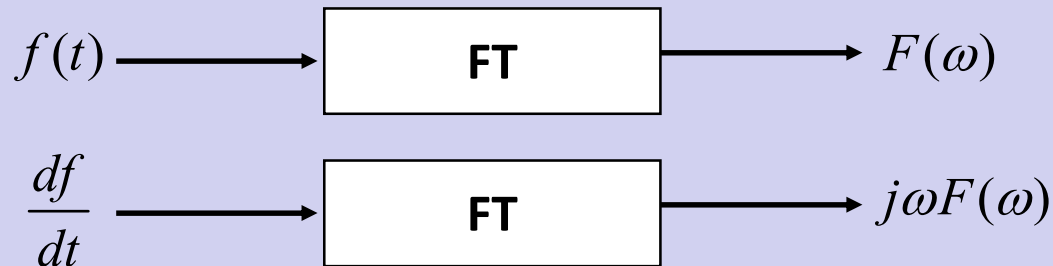
Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform

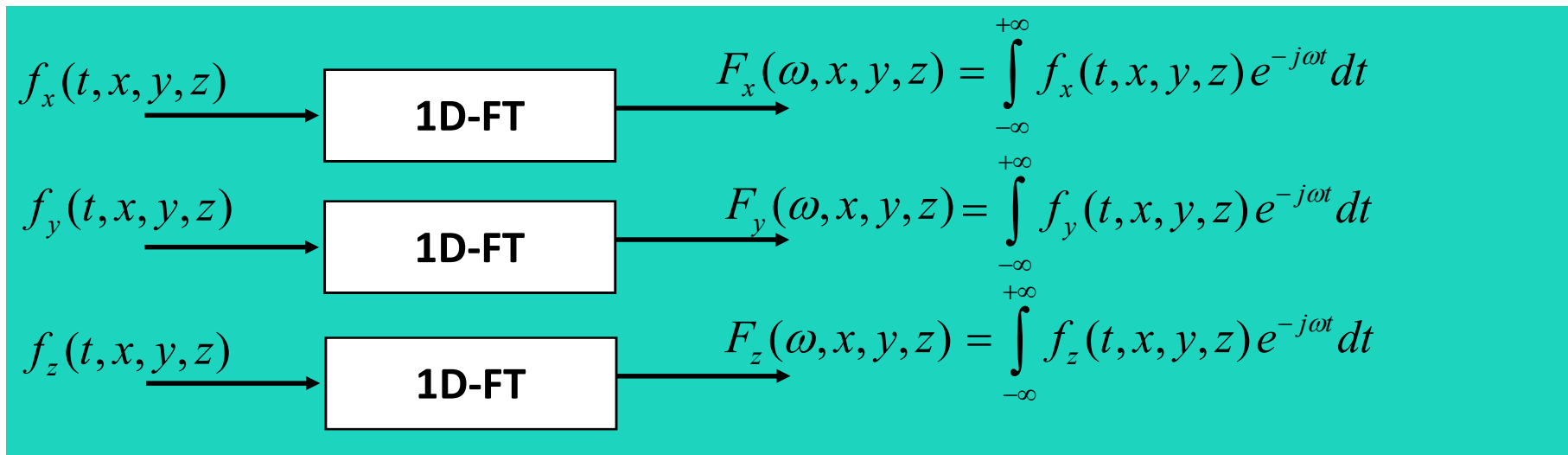
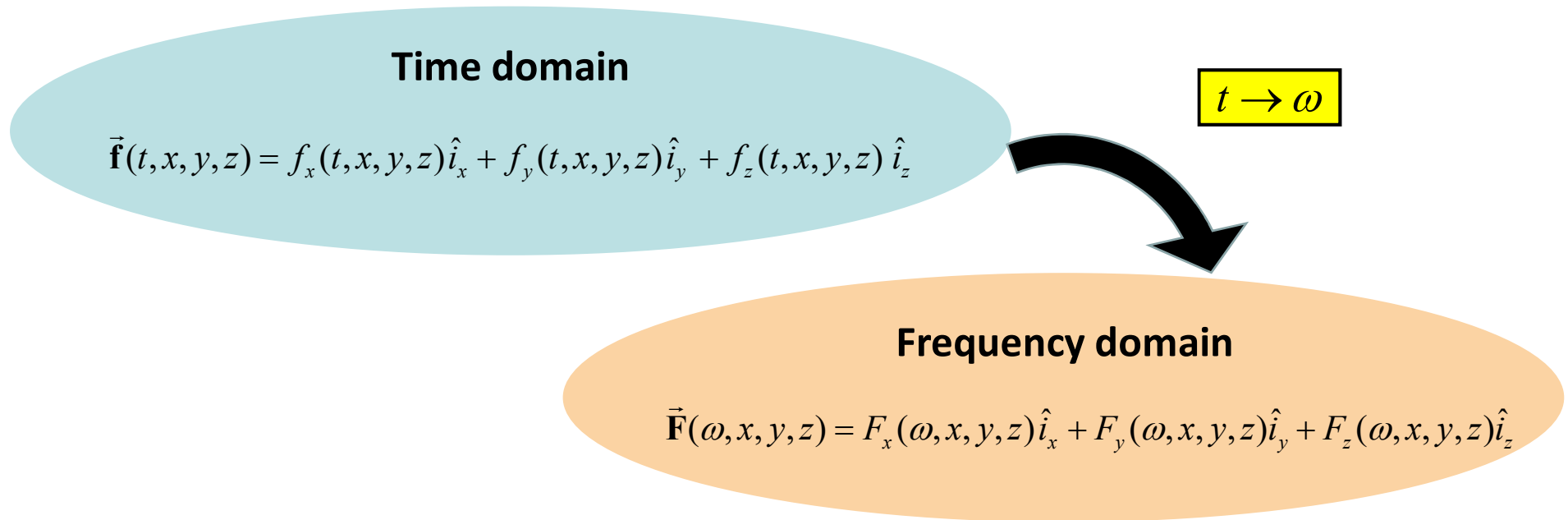


Frequency domain

- **Fourier Transform and functions of n variables**
- **Fourier Transform and vector functions**
- **Fourier Transform and vector functions of n variables**

- 1) How to jump back from the Frequency domain to the Time domain**
- 2) Time domain derivative and Fourier Transform**

Fourier Transform and vector functions of n variables



Fourier Transform and vector functions of n variables

Time domain

$$\vec{f}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

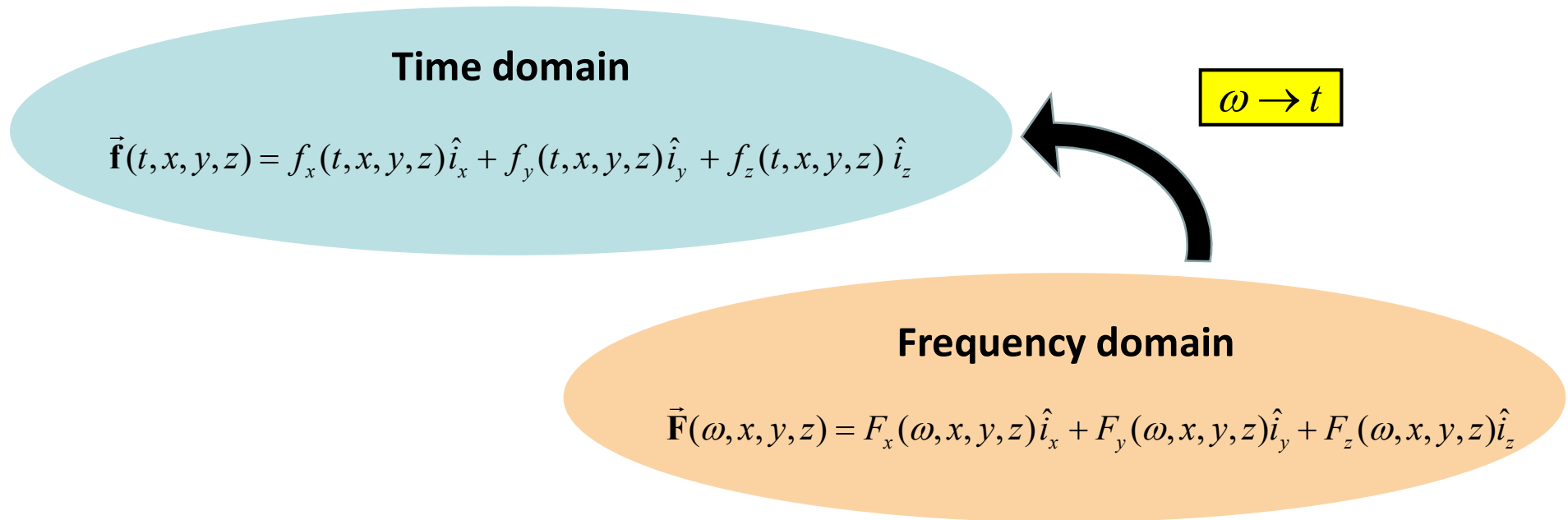
$t \rightarrow \omega$

Frequency domain

$$\vec{F}(\omega, x, y, z) = F_x(\omega, x, y, z)\hat{i}_x + F_y(\omega, x, y, z)\hat{i}_y + F_z(\omega, x, y, z)\hat{i}_z$$

$$\vec{f}(t, x, y, z) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{F}(\omega, x, y, z)$$

Fourier Transform and vector functions of n variables



1) How to jump back from the Spectral domain to the Time domain

$$F_x(\omega, x, y, z) \xrightarrow{\text{1D-IFT}} f_x(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_x(\omega, x, y, z) e^{j\omega t} d\omega$$

$$F_y(\omega, x, y, z) \xrightarrow{\text{1D-IFT}} f_y(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_y(\omega, x, y, z) e^{j\omega t} d\omega$$

$$F_z(\omega, x, y, z) \xrightarrow{\text{1D-IFT}} f_z(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_z(\omega, x, y, z) e^{j\omega t} d\omega$$

Fourier Transform and vector functions of n variables

Time domain

$$\vec{f}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{F}(\omega, x, y, z) = F_x(\omega, x, y, z)\hat{i}_x + F_y(\omega, x, y, z)\hat{i}_y + F_z(\omega, x, y, z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$$\vec{f}(t, x, y, z) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{F}(\omega, x, y, z)$$

$$\frac{\partial \vec{f}}{\partial t} \longrightarrow \boxed{\text{FT}} \longrightarrow j\omega \vec{F}(\omega, x, y, z)$$

$$\nabla \cdot \vec{f} \longrightarrow \boxed{\text{FT}} \longrightarrow \nabla \cdot \vec{F}(\omega, x, y, z)$$

$$\nabla \times \vec{f} \longrightarrow \boxed{\text{FT}} \longrightarrow \nabla \times \vec{F}(\omega, x, y, z)$$

Fourier Transform and vector functions of n variables

Time domain

$$\vec{f}(x, y, z, t) = f_x(x, y, z, t)\hat{i}_x + f_y(x, y, z, t)\hat{i}_y + f_z(x, y, z, t)\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{F}(x, y, z, \omega) = F_x(x, y, z, \omega)\hat{i}_x + F_y(x, y, z, \omega)\hat{i}_y + F_z(x, y, z, \omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$$\vec{f}(x, y, z, t) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{F}(x, y, z, \omega)$$

$$\frac{\partial \vec{f}}{\partial t} \longrightarrow \boxed{\text{FT}} \longrightarrow j\omega \vec{F}(x, y, z, \omega)$$

$$\nabla \cdot \vec{f} \longrightarrow \boxed{\text{FT}} \longrightarrow \nabla \cdot \vec{F}(x, y, z, \omega)$$

$$\nabla \times \vec{f} \longrightarrow \boxed{\text{FT}} \longrightarrow \nabla \times \vec{F}(x, y, z, \omega)$$



Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain



Maxwell equations

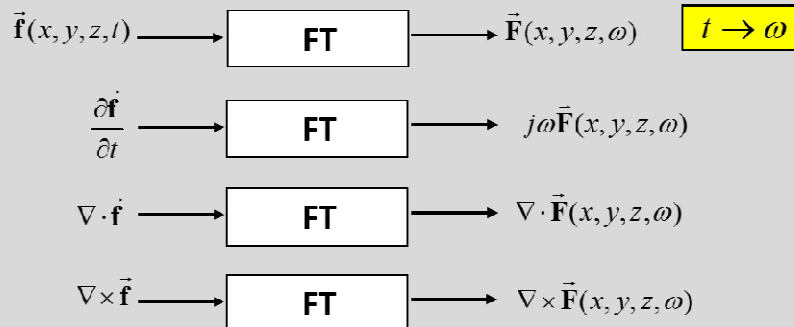
Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain





Maxwell equations

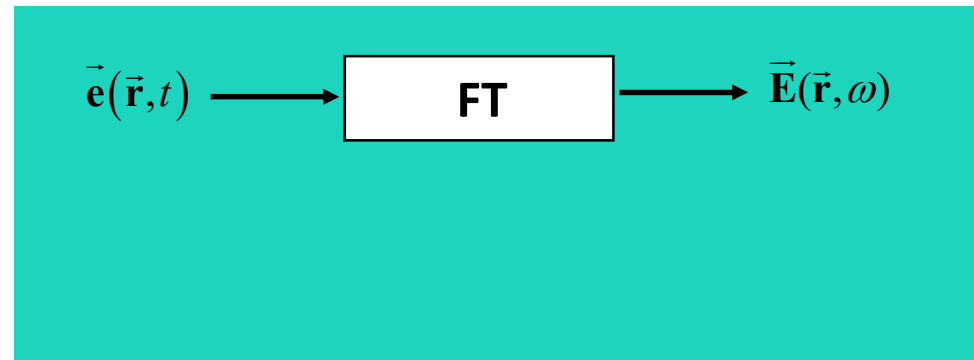
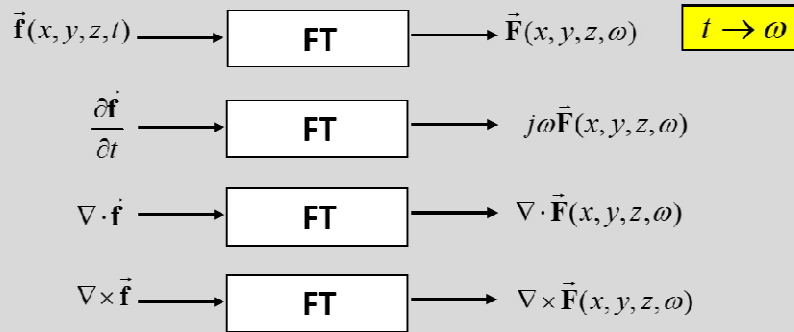
Time domain & Frequency domain

Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

Frequency domain





Maxwell equations

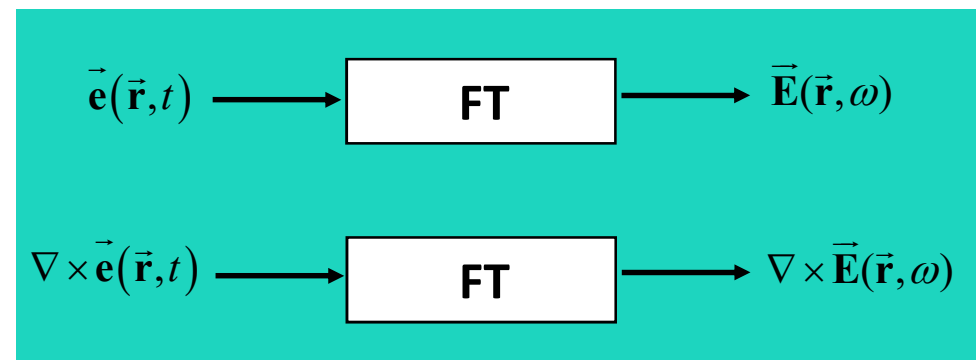
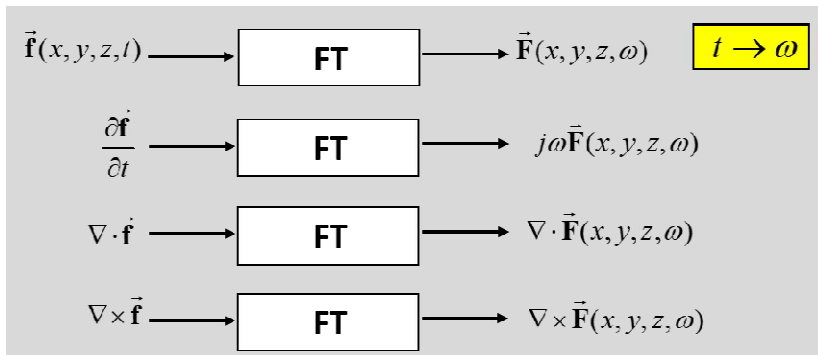
Time domain & Frequency domain

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$t \rightarrow \omega$

Frequency domain





Maxwell equations

Time domain & Frequency domain

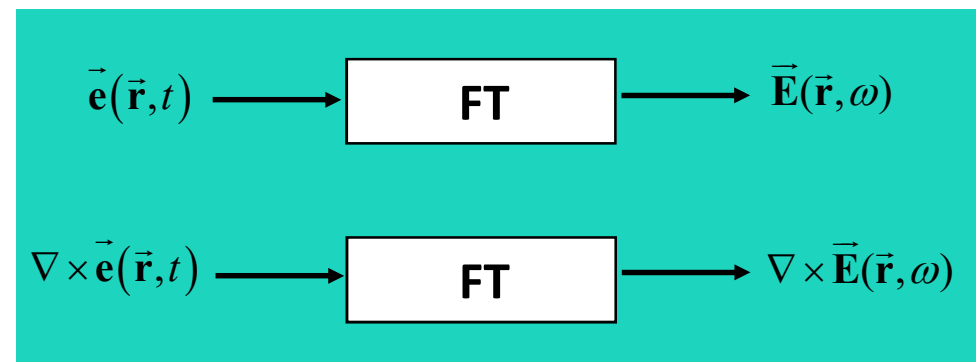
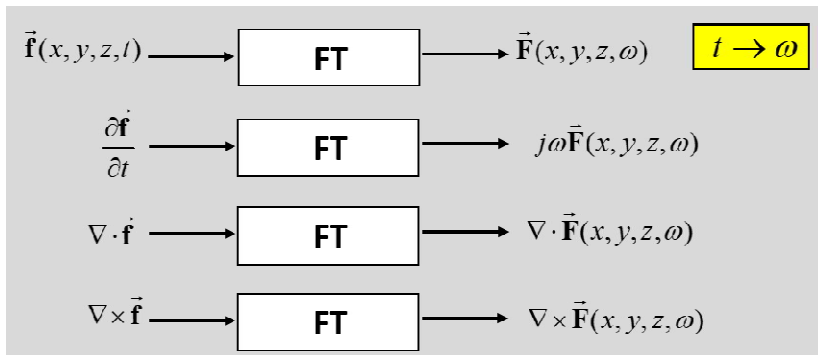
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

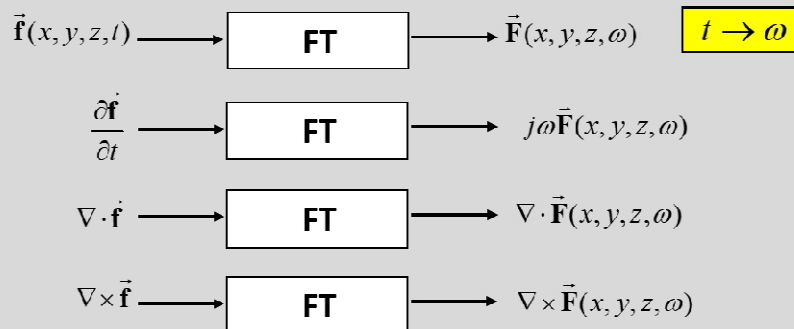
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

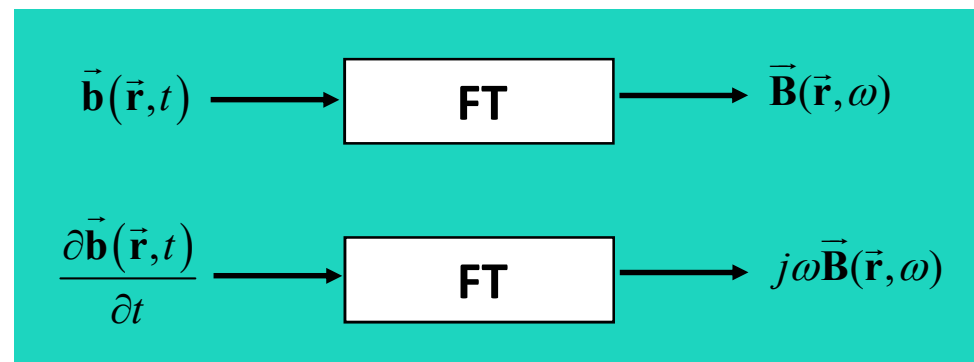
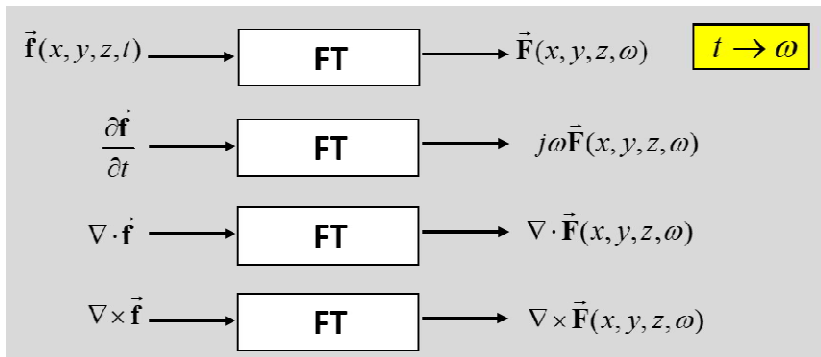
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

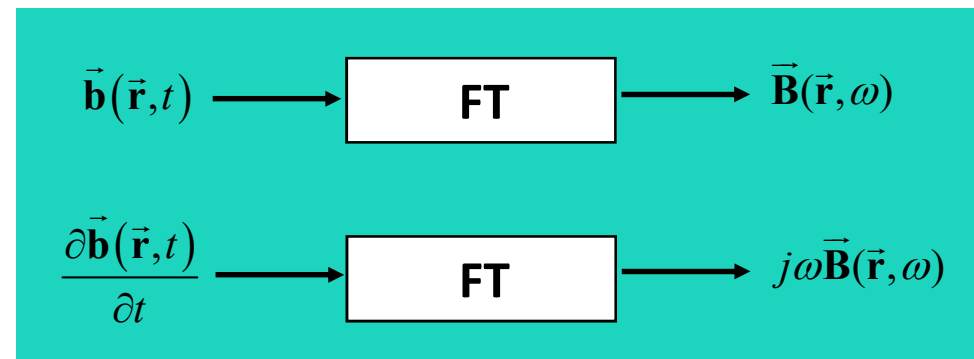
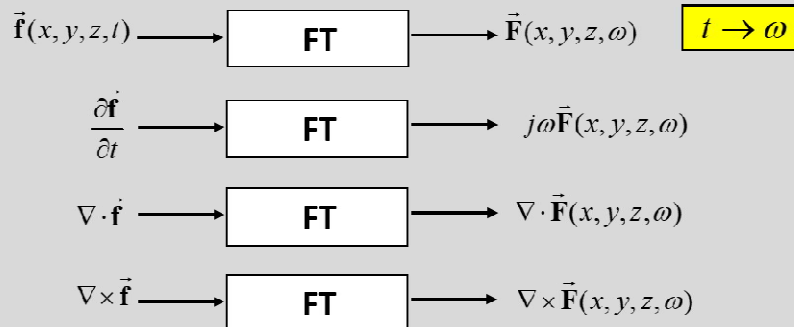
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

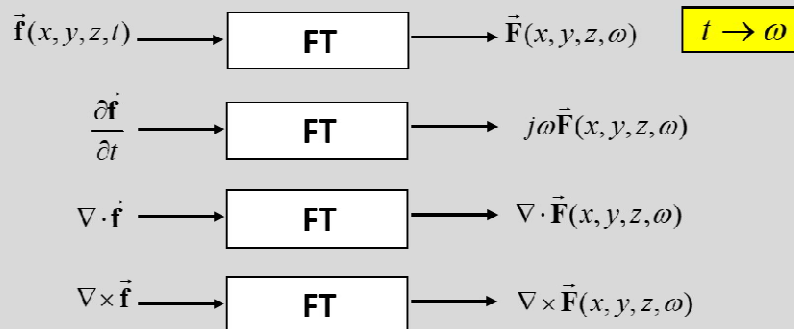
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

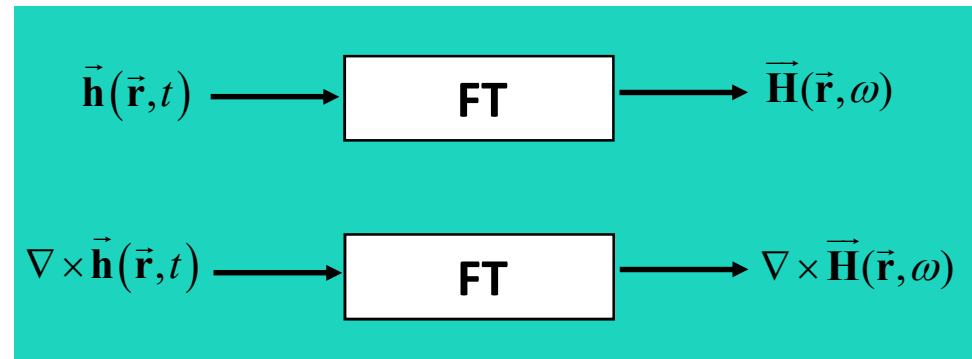
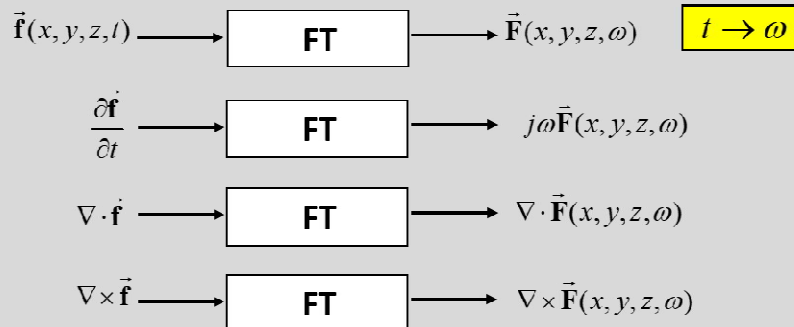
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

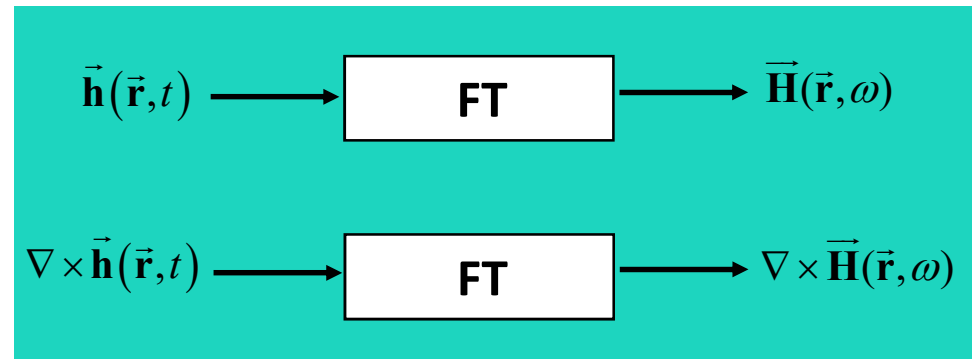
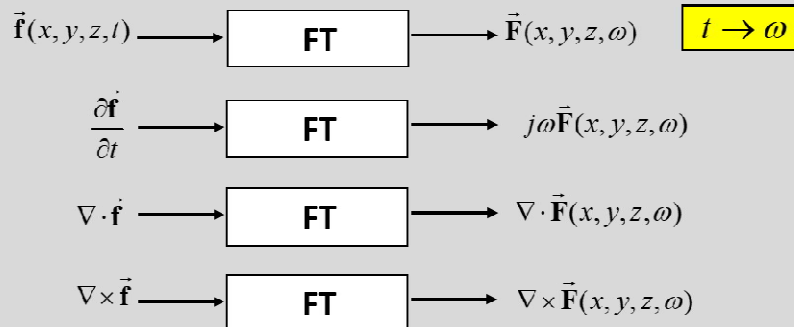
Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) \end{cases}$$





Maxwell equations

Time domain & Frequency domain

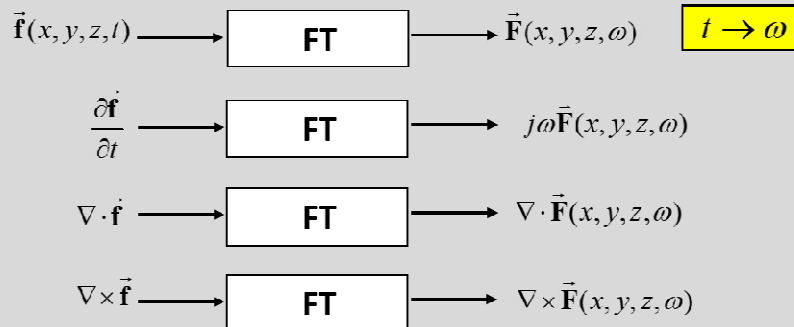
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

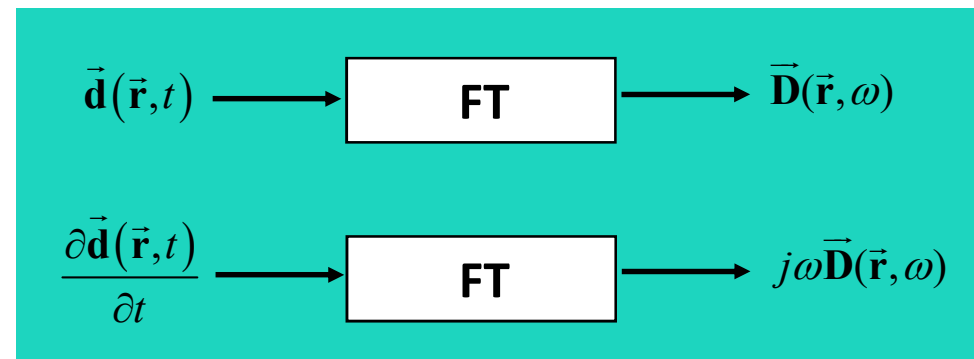
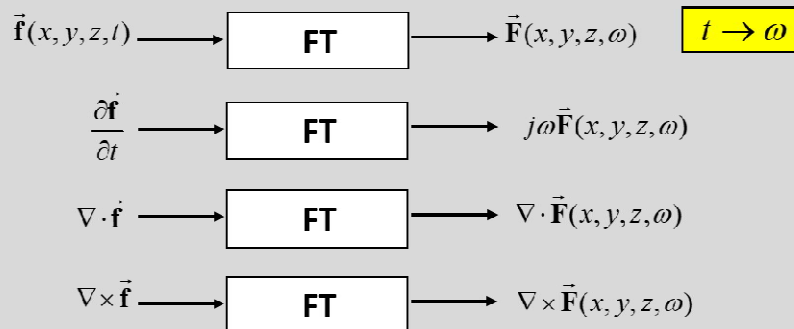
Time domain

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$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

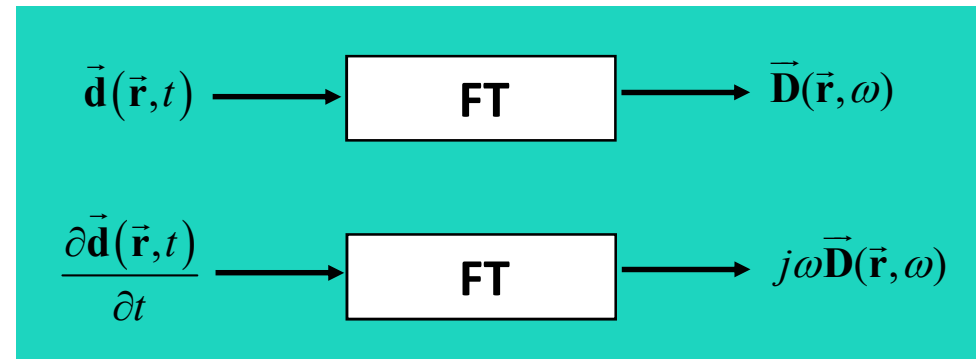
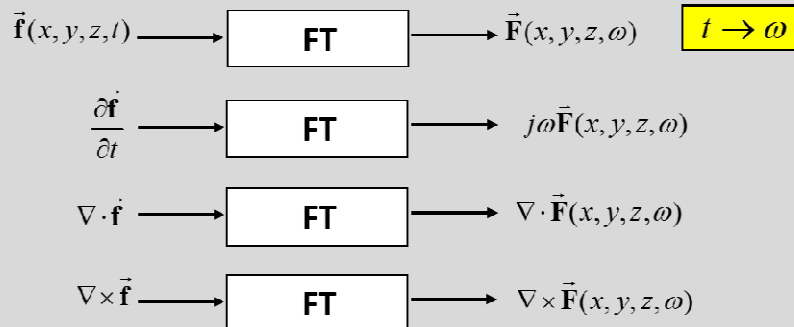
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

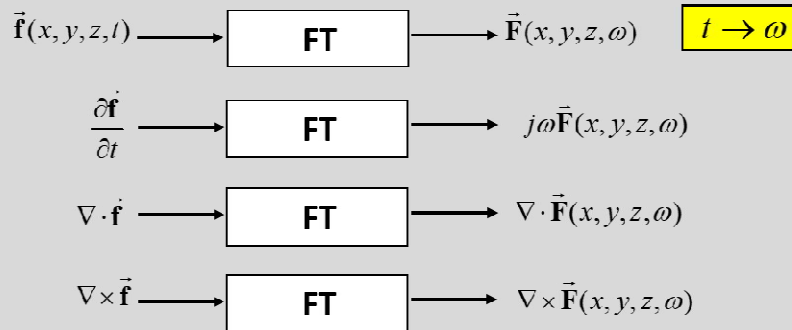
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

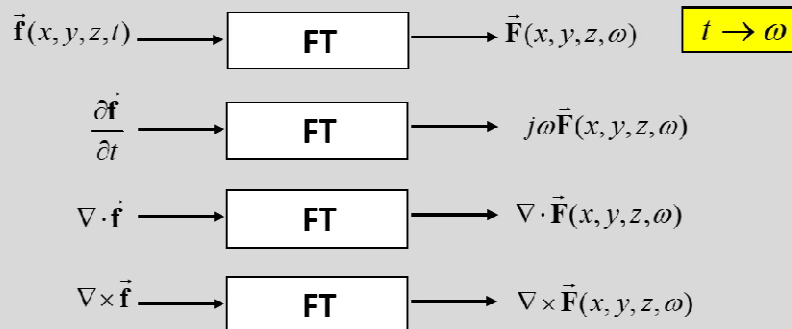
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

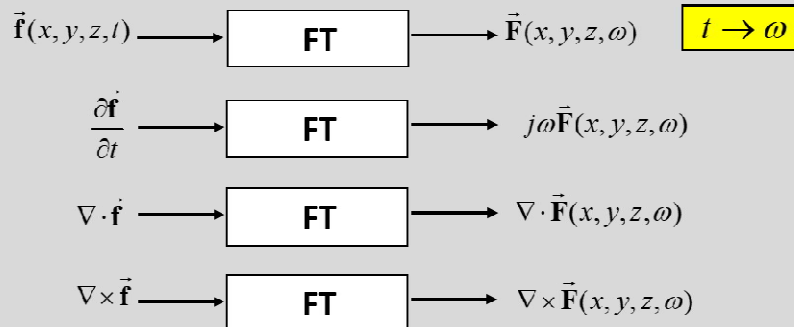
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

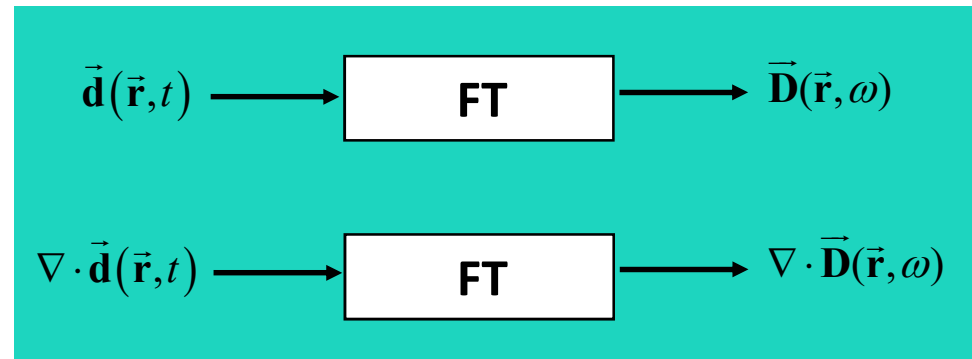
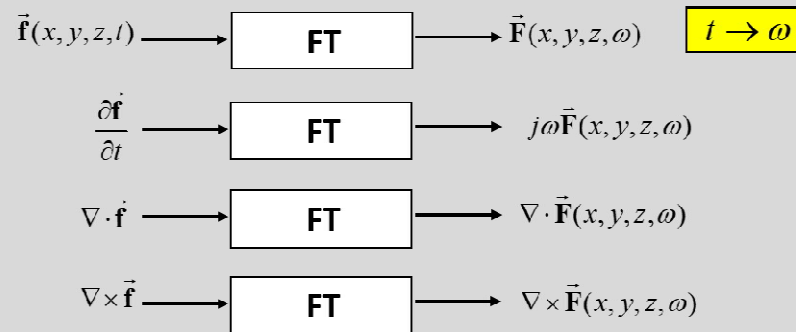
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

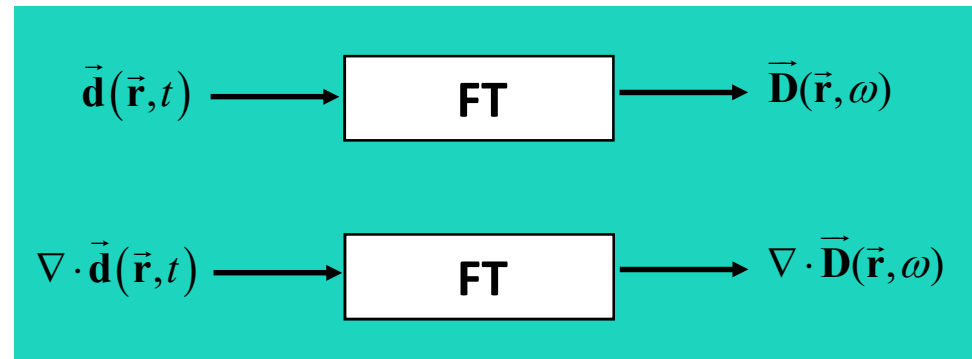
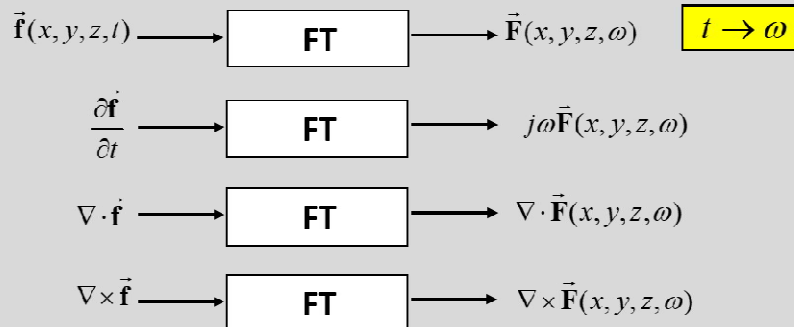
Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) \end{cases}$$





Maxwell equations

Time domain & Frequency domain

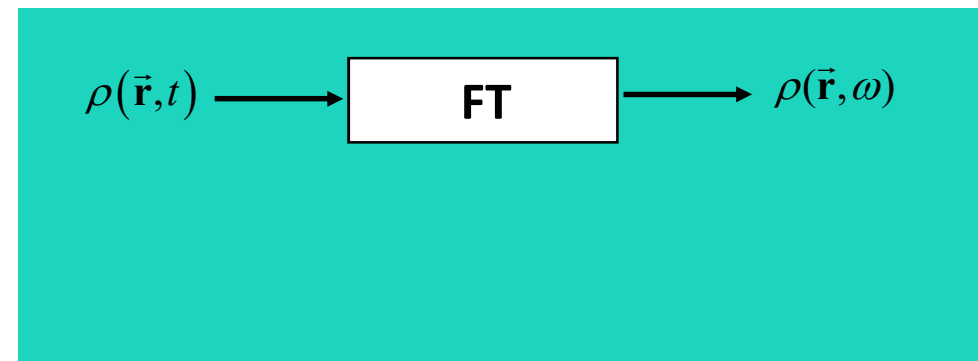
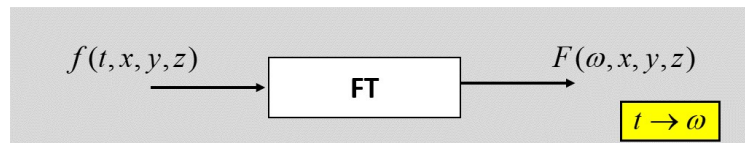
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

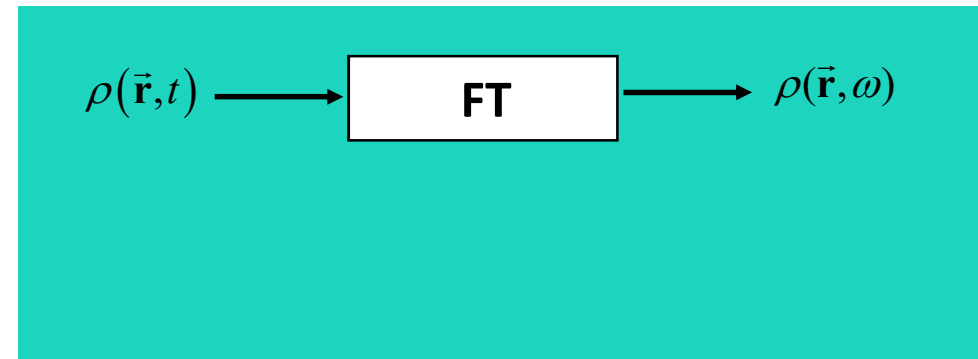
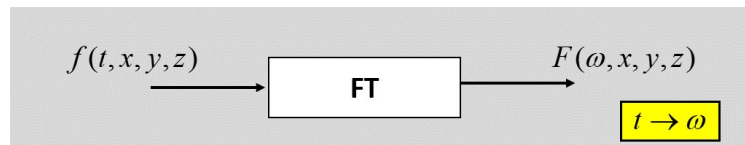
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

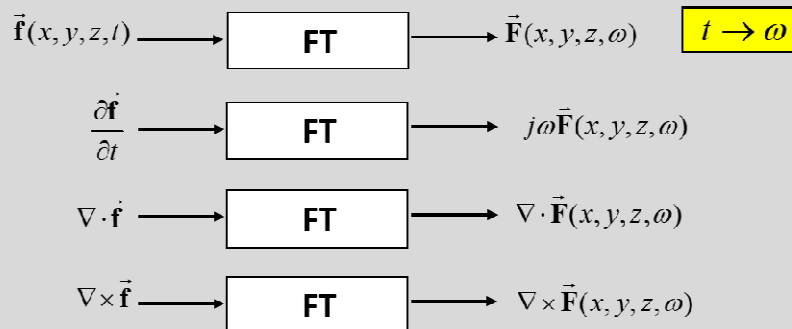
Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \end{array} \right.$$





Maxwell equations

Time domain & Frequency domain

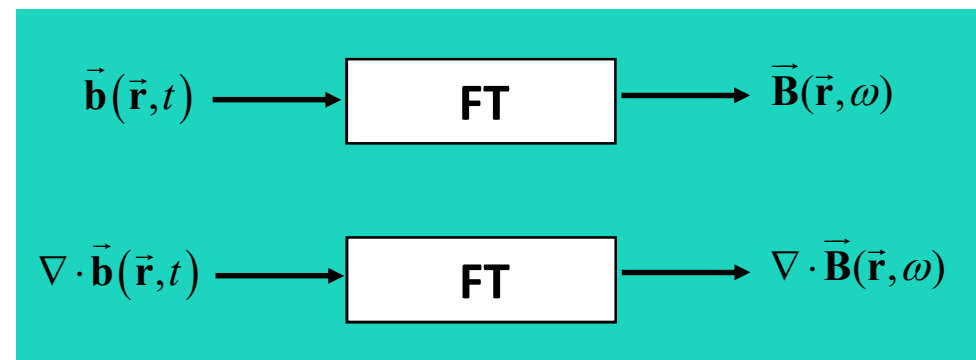
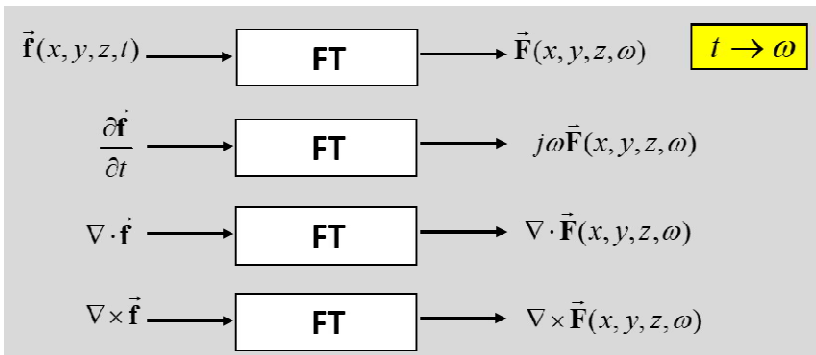
Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \end{cases}$$





Maxwell equations

Time domain & Frequency domain

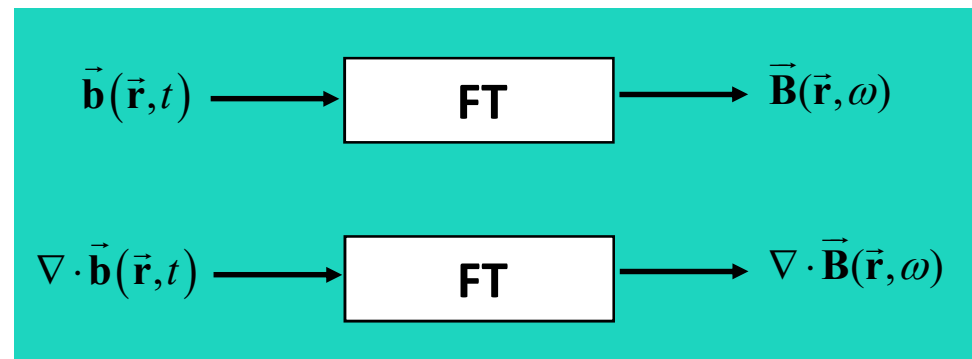
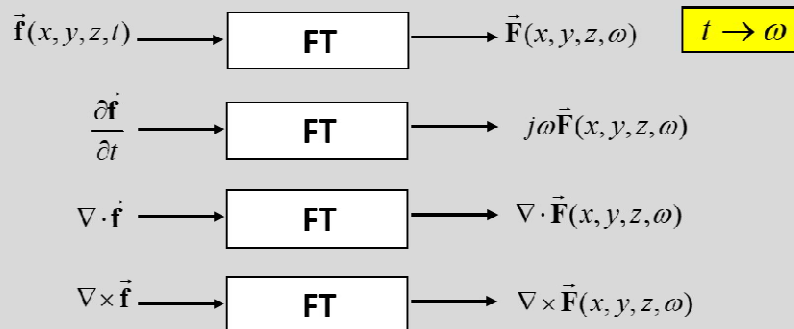
Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{cases}$$





Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	$t \rightarrow \omega$	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³



Maxwell equations

Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	<div style="border: 1px solid black; background-color: yellow; display: inline-block; padding: 2px 10px;"> $t \rightarrow \omega$ </div>	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r}, \omega)$
$\vec{D}(\vec{r}, \omega)$
$\vec{H}(\vec{r}, \omega)$
$\vec{B}(\vec{r}, \omega)$
$\vec{J}(\vec{r}, \omega)$
$\rho(\vec{r}, \omega)$

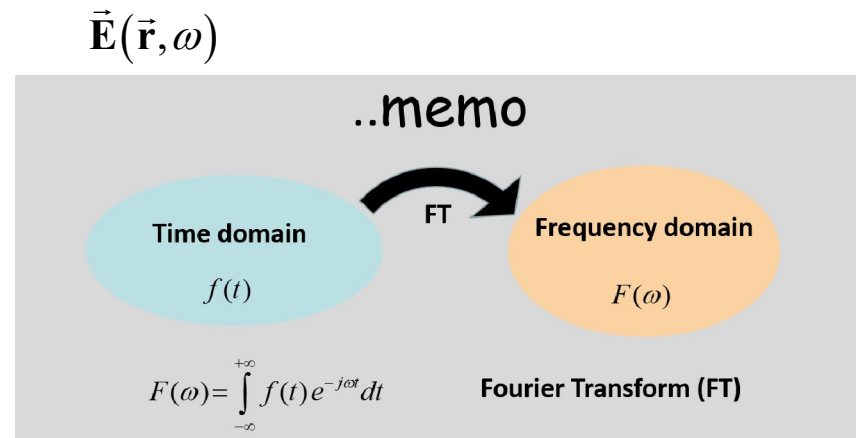


Maxwell equations

Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	<div style="background-color: yellow; border: 1px solid black; padding: 2px; display: inline-block;">$t \rightarrow \omega$</div>	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

- $\vec{e}(\vec{r}, t)$ Volt/m
- $\vec{d}(\vec{r}, t)$ Coulomb/m²
- $\vec{h}(\vec{r}, t)$ Ampere/m
- $\vec{b}(\vec{r}, t)$ Weber/m²
- $\vec{j}(\vec{r}, t)$ Ampere/m²
- $\rho(\vec{r}, t)$ Coulomb/m³





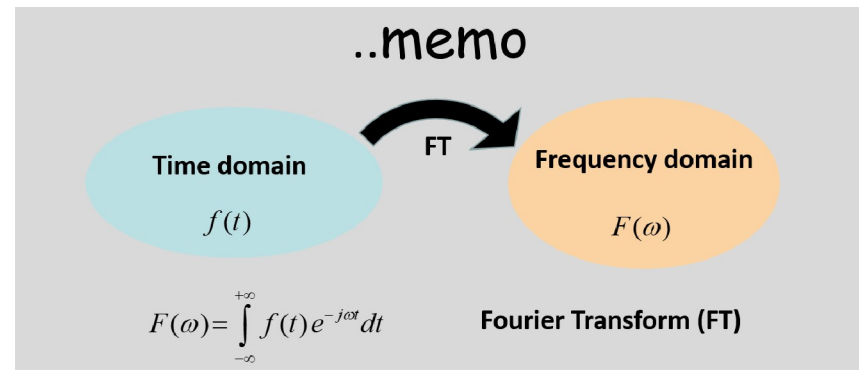
Maxwell equations

Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{aligned} \nabla \times \vec{e}(\vec{r}, t) &= -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) &= \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) &= \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) &= 0 \end{aligned} \right.$	<div style="background-color: yellow; border: 1px solid black; padding: 2px; display: inline-block;">$t \rightarrow \omega$</div>	$\left\{ \begin{aligned} \nabla \times \vec{E}(\vec{r}, \omega) &= -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) &= j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) &= \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) &= 0 \end{aligned} \right.$

- $\vec{e}(\vec{r}, t)$ Volt/m
- $\vec{d}(\vec{r}, t)$ Coulomb/m²
- $\vec{h}(\vec{r}, t)$ Ampere/m
- $\vec{b}(\vec{r}, t)$ Weber/m²
- $\vec{j}(\vec{r}, t)$ Ampere/m²
- $\rho(\vec{r}, t)$ Coulomb/m³

$\vec{E}(\vec{r}, \omega)$ (Volt x s) /m





Maxwell equations

Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	<div style="border: 1px solid black; background-color: yellow; display: inline-block; padding: 2px 10px;"> $t \rightarrow \omega$ </div>	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r}, \omega)$	(Volt x s) /m
$\vec{D}(\vec{r}, \omega)$	(Coulomb x s)/m ²
$\vec{H}(\vec{r}, \omega)$	(Ampere x s)/m
$\vec{B}(\vec{r}, \omega)$	(Weber x s)/m ²
$\vec{J}(\vec{r}, \omega)$	(Ampere x s)/m ²
$\rho(\vec{r}, \omega)$	(Coulomb x s)/m ³



Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$

$$\frac{\partial \rho(\vec{\mathbf{r}}, t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = 0$$



Maxwell equations

Time domain & Frequency domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

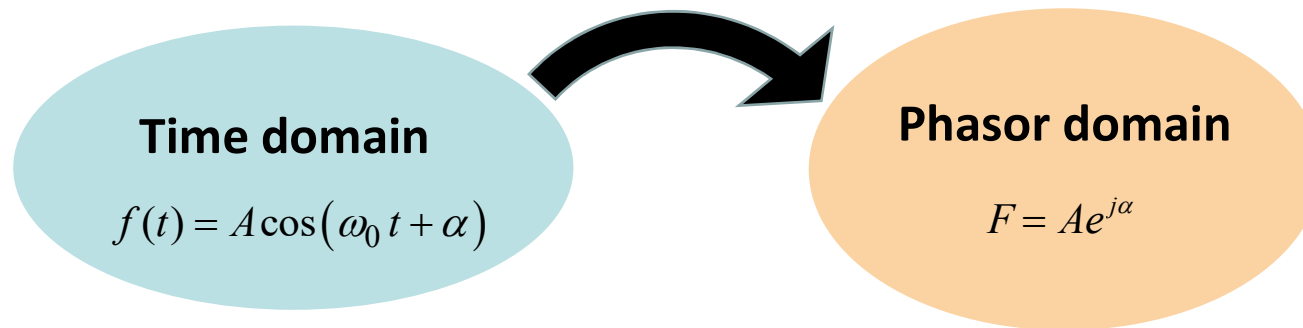
Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$

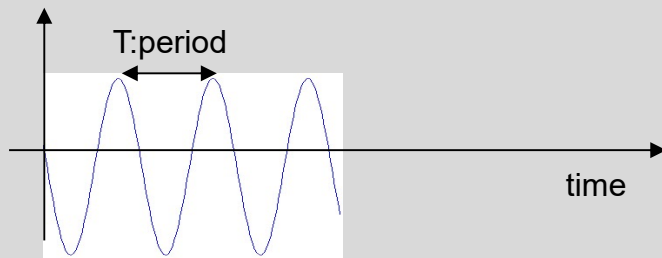
$$\frac{\partial \rho(\vec{\mathbf{r}}, t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = 0$$

$$j\omega \rho(\vec{\mathbf{r}}, \omega) + \nabla \cdot \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) = 0$$

Phasors



Signals usually adopted in ICT applications

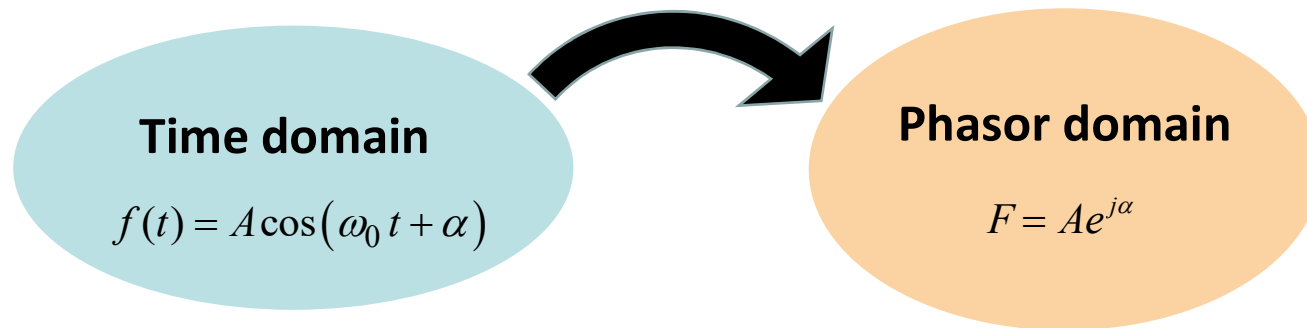


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

$$f_0 : \text{frequency} = \frac{1}{T}$$

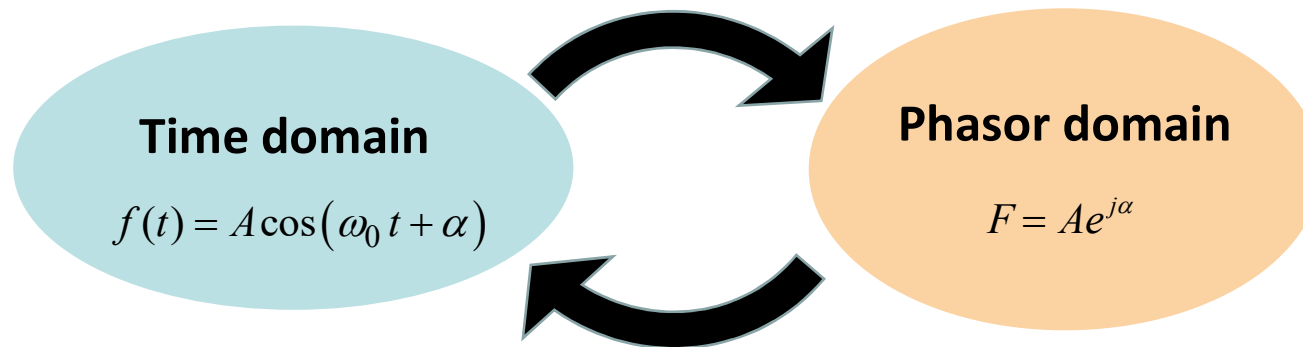
$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

Phasors



- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

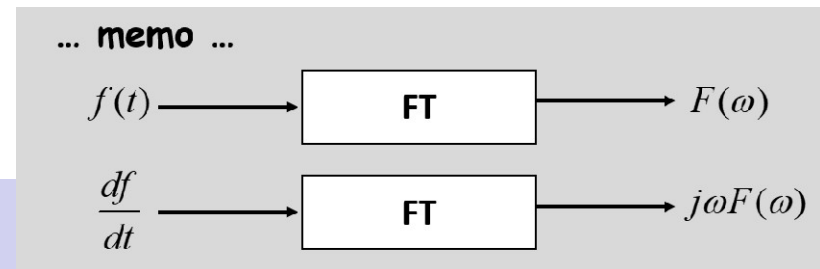
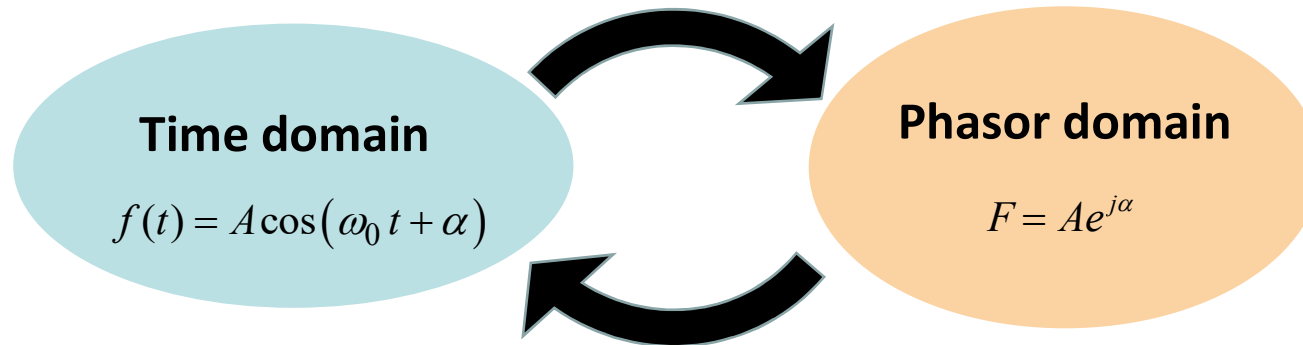
Phasors



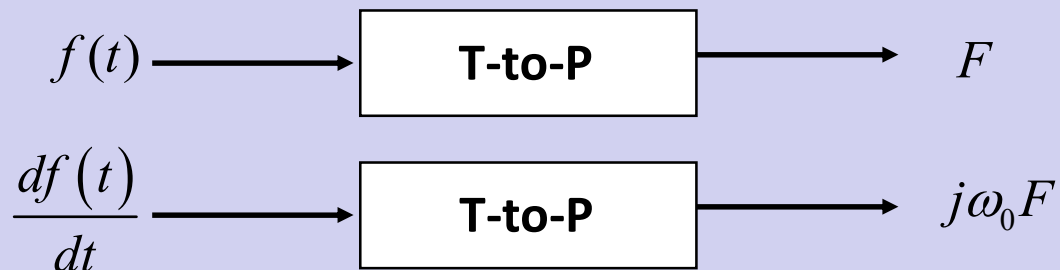
$$f(t) = A \cos(\omega_0 t + \alpha) \longrightarrow \text{T-to-P} \longrightarrow F = A e^{j\alpha}$$

$$F = A e^{j\alpha} \longrightarrow \text{P-to-T} \longrightarrow f(t) = \text{Re}\{F e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

Phasors



2) Time domain derivative and Phasors



ω_0 now is fixed!

Phasors

- Phasors and functions of n variables
- Phasors and vector functions
- Phasors and vector functions of n variables

- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

Phasors

- Phasors and functions of n variables
- Phasors and vector functions
- **Phasors and vector functions of n variables**

- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

$$f_x(t, x, y, z) = A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z))$$

$$f_y(t, x, y, z) = A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))$$

$$f_z(t, x, y, z) = A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))$$

Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

$$f_x(t, x, y, z) = A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z))$$

$$f_y(t, x, y, z) = A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))$$

$$f_z(t, x, y, z) = A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))$$

Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

$$f_x(t, x, y, z) = A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z))$$



T-to-P



$$f_y(t, x, y, z) = A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))$$



T-to-P



$$f_z(t, x, y, z) = A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))$$



T-to-P



Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



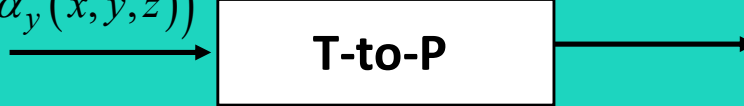
Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

$$f_x(t, x, y, z) = A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z)) \quad \longrightarrow \quad F_x(x, y, z) = A_x(x, y, z)e^{j\alpha_x(x, y, z)}$$



$$f_y(t, x, y, z) = A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))$$



$$f_z(t, x, y, z) = A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))$$



Phasors and vector functions of n variables

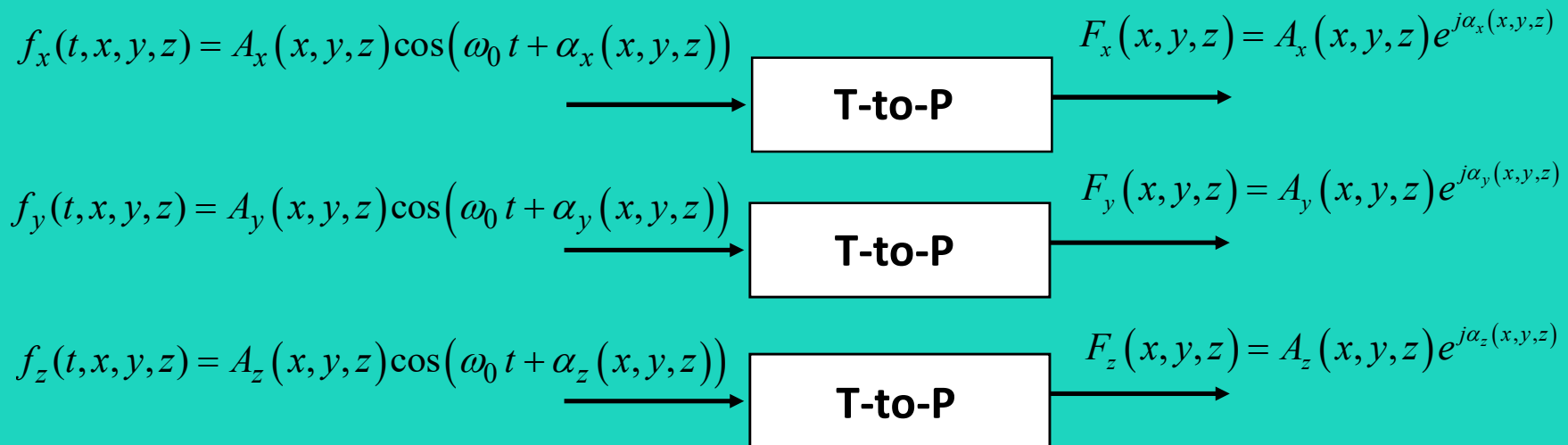
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$



Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

$$\vec{\mathbf{f}}(t, x, y, z) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x, y, z)$$

$$\begin{aligned}\vec{\mathbf{f}}(t, x, y, z) &= f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z = \\ &= A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z))\hat{i}_x + \\ &A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))\hat{i}_y + \\ &A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))\hat{i}_z\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{F}}(x, y, z) &= F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z = \\ &= A_x(x, y, z)e^{j\alpha_x(x, y, z)}\hat{i}_x + A_y(x, y, z)e^{j\alpha_y(x, y, z)}\hat{i}_y + A_z(x, y, z)e^{j\alpha_z(x, y, z)}\hat{i}_z\end{aligned}$$

Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain

$$F_x(x, y, z) = A_x(x, y, z)e^{j\alpha_x(x, y, z)}$$



P-to-T



$$F_y(x, y, z) = A_y(x, y, z)e^{j\alpha_y(x, y, z)}$$



P-to-T



$$F_z(x, y, z) = A_z(x, y, z)e^{j\alpha_z(x, y, z)}$$



P-to-T



Phasors and vector functions of n variables

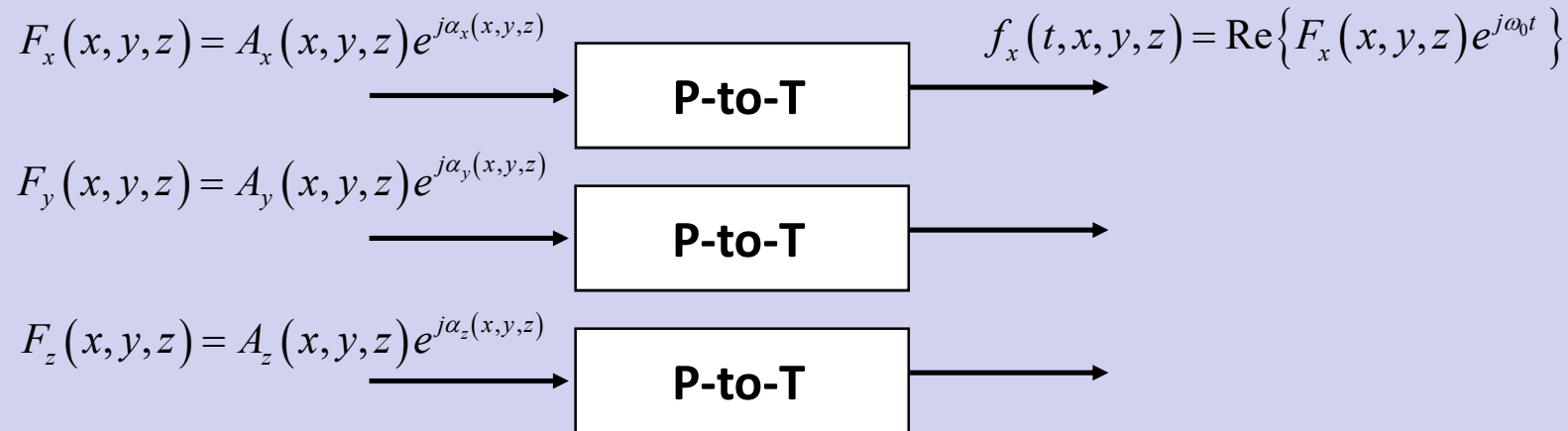
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain



Phasors and vector functions of n variables

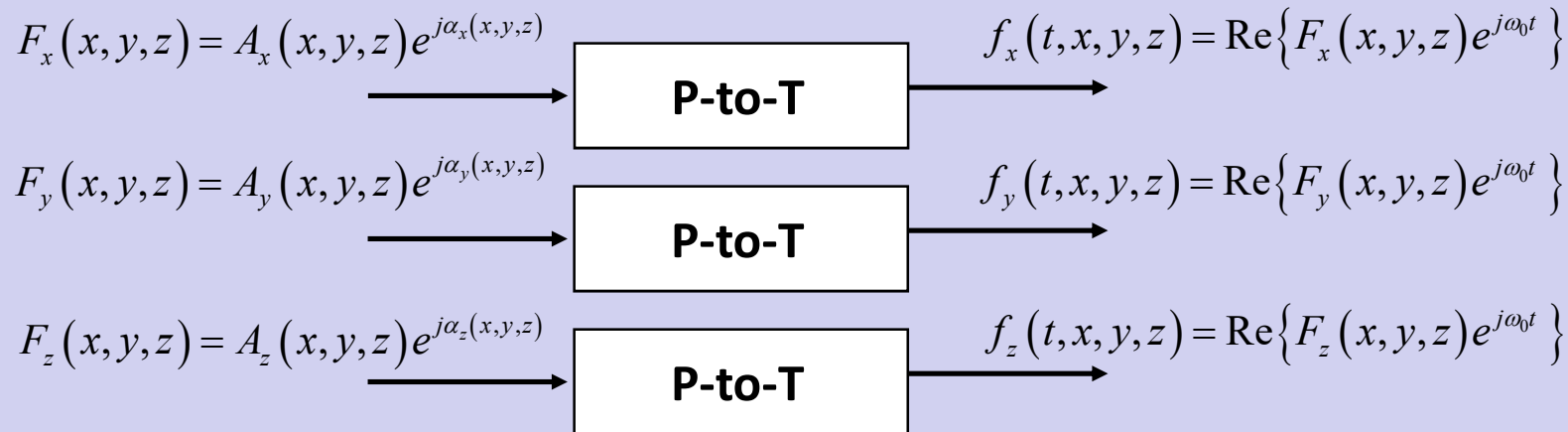
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain



Phasors and vector functions of n variables

Time domain

$$\vec{f}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

Phasor domain

$$\vec{F}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain

$$\vec{F}(x, y, z) \longrightarrow \boxed{\text{P-to-T}} \longrightarrow \vec{f}(t, x, y, z) = \text{Re}\left\{\vec{F}(x, y, z)e^{j\omega_0 t}\right\}$$

Phasors and vector functions of n variables

Time domain

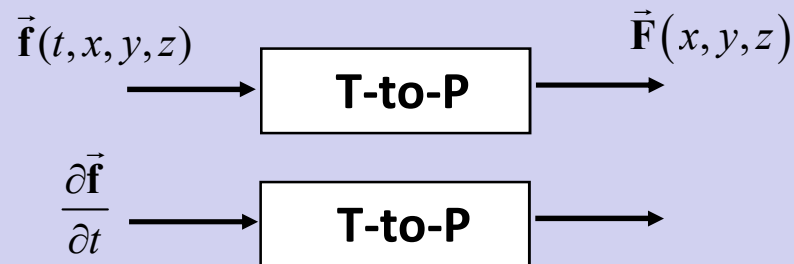
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



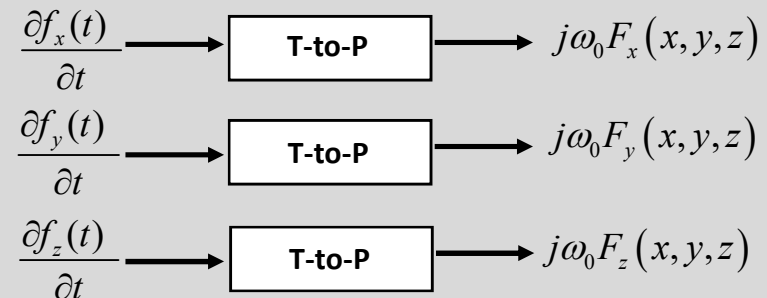
Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



$$\frac{\partial \vec{\mathbf{f}}(t, \vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$



Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t, x, y, z) \xrightarrow{\text{T-to-P}} \vec{\mathbf{F}}(x, y, z)$$

$$\frac{\partial \vec{\mathbf{f}}}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 \vec{\mathbf{F}}(x, y, z) = j\omega_0 F_x(x, y, z)\hat{i}_x + j\omega_0 F_y(x, y, z)\hat{i}_y + j\omega_0 F_z(x, y, z)\hat{i}_z$$

$$\frac{\partial \vec{\mathbf{f}}(t, \vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$$\frac{\partial f_x(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_x(x, y, z)$$

$$\frac{\partial f_y(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_y(x, y, z)$$

$$\frac{\partial f_z(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_z(x, y, z)$$

Phasors and vector functions of n variables

Time domain

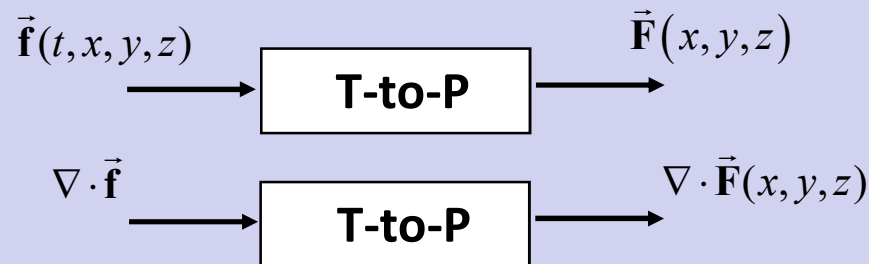
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



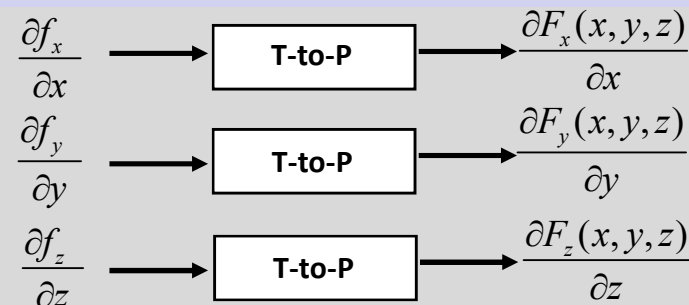
Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



$$\nabla \cdot \vec{\mathbf{f}}(t, \vec{\mathbf{r}}) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$



Phasors and vector functions of n variables

Time domain

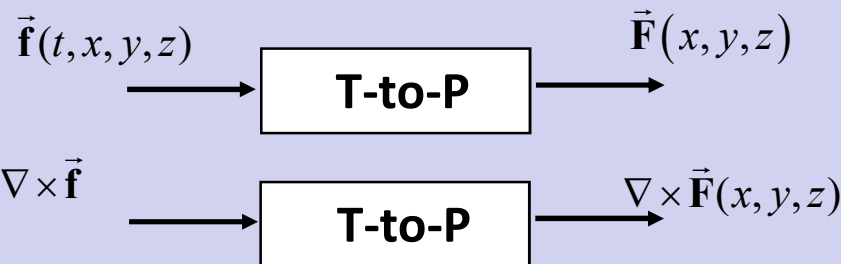
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



$$\nabla \times \vec{\mathbf{f}}(t, \vec{\mathbf{r}}) = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} \right) \hat{i}_y + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \right) \hat{i}_z$$

Phasors and vector functions of n variables

Time domain

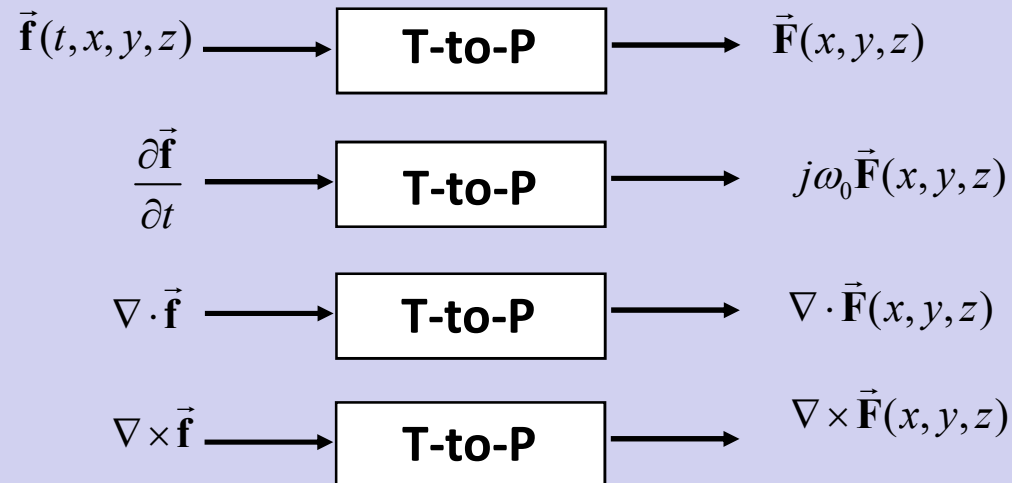
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(x, y, z, t) = f_x(x, y, z, t)\hat{i}_x + f_y(x, y, z, t)\hat{i}_y + f_z(x, y, z, t)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors

