Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 – Laurea “Triennale” – Secondo semestre – Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Equazioni di Maxwell

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

		Unità di misura
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$:	Campo elettrico	Volt/m
$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)$:	Induzione elettrica	Coulomb/m^2
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$:	Campo magnetico	Ampere/m
$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)$:	Induzione magnetica	Weber/m^2
$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$:	Densità di corrente	Ampere/m^2
$\rho(\vec{\mathbf{r}},t)$:	Densità di carica	Coulomb/m^3

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

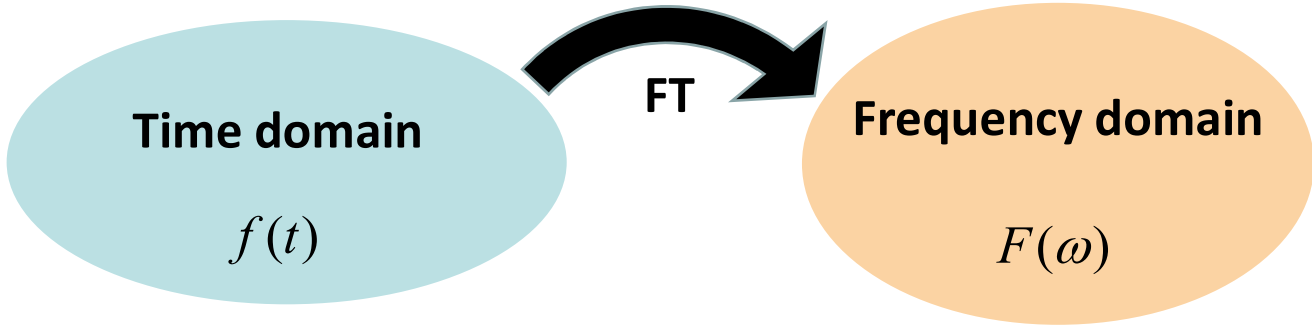
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = \dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0 \end{cases}$$

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Maxwell equations: Time domain, Frequency domain, Phasors

Frequency domain



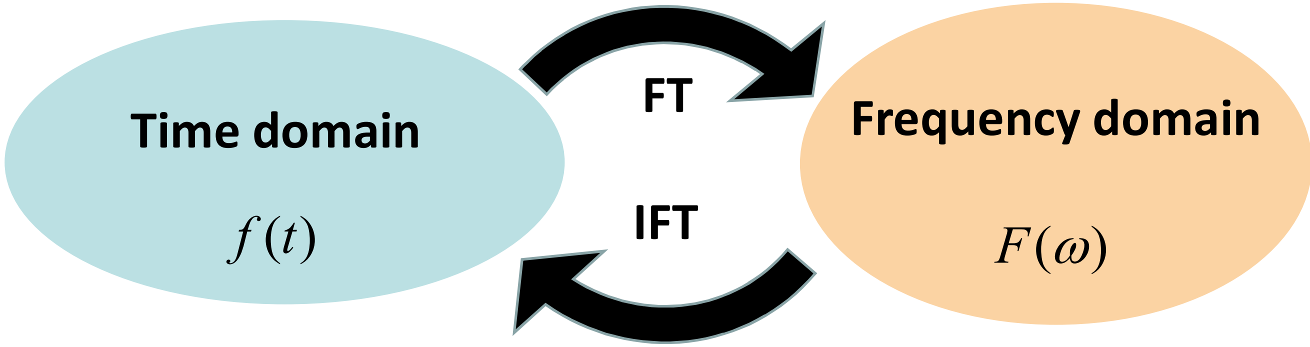
$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

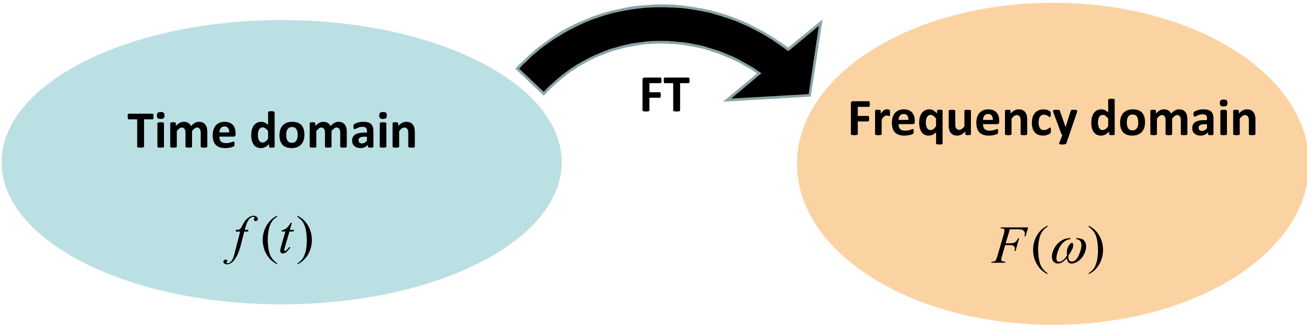
Frequency domain



$f(t)$ → **FT** → $$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

$F(\omega)$ → **IFT** → $$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$$

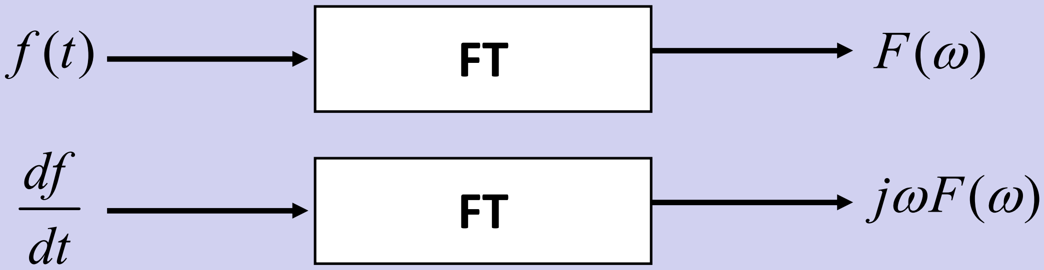
Frequency domain



$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



Frequency domain

- Fourier Transform and functions of *n* variables

- Fourier Transform and vector functions

- Fourier Transform and vector functions of *n* variables

Frequency domain

- Fourier Transform and functions of *n* variables
- Fourier Transform and vector functions
- Fourier Transform and vector functions of *n* variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Frequency domain

- Fourier Transform and functions of *n* variables

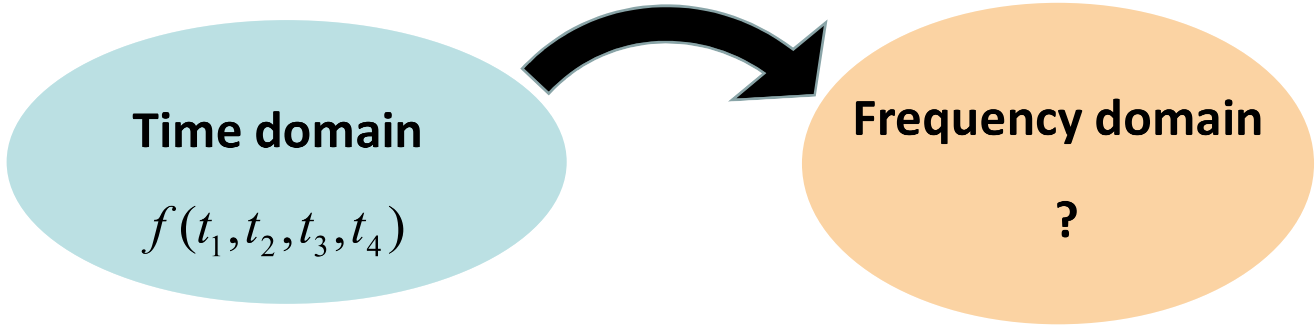
- Fourier Transform and vector functions

- Fourier Transform and vector functions of *n* variables

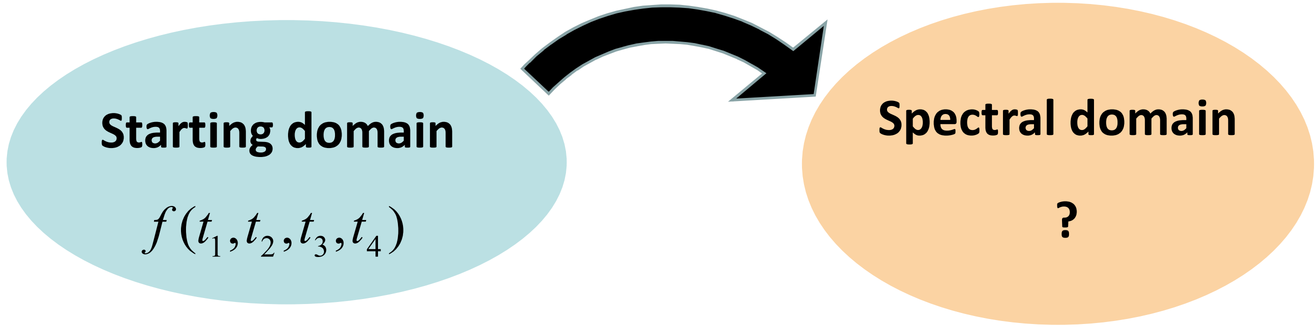
1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

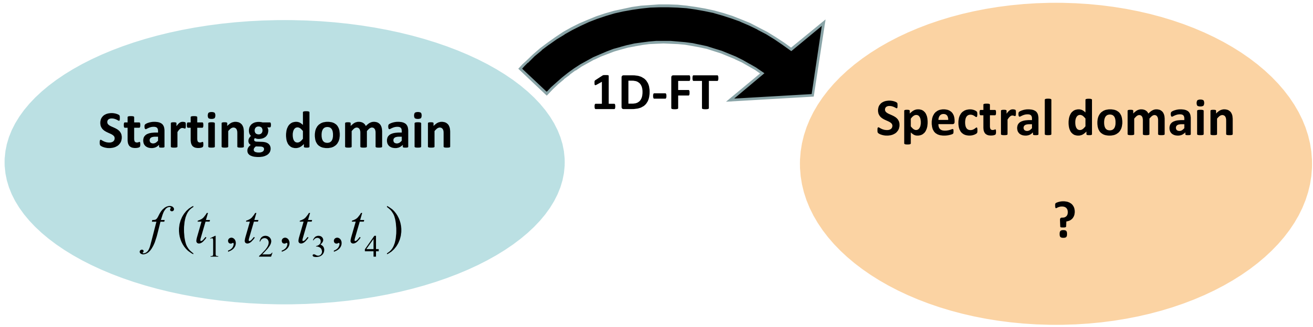
Fourier Transform and functions of *n* variables



Fourier Transform and functions of *n* variables

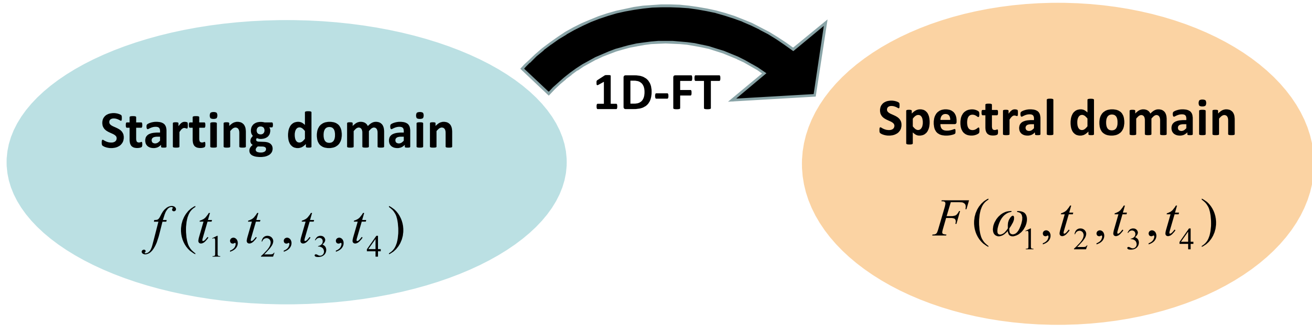


Fourier Transform and functions of *n* variables



One Dimensional Fourier Transform (1D-FT)

Fourier Transform and functions of *n* variables



One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1,t_2,t_3,t_4)=\int_{-\infty}^{+\infty} f(t_1,t_2,t_3,t_4)\,e^{-j\omega_1 t_1}\,dt_1$$

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

Spectral domain

$F(\omega_1, t_2, t_3, t_4)$

One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1, t_2, t_3, t_4) = \int_{-\infty}^{+\infty} f(t_1, t_2, t_3, t_4)\, e^{-j\omega_1 t_1}\, dt_1$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1,t_2,t_3,t_4)$

1D-IFT

Spectral domain

$F(\omega_1,t_2,t_3,t_4)$

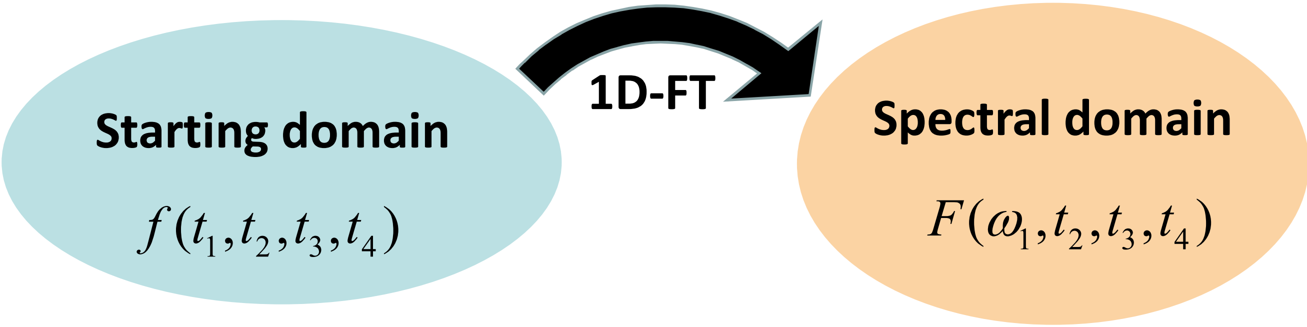
One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1,t_2,t_3,t_4)=\int_{-\infty}^{+\infty} f(t_1,t_2,t_3,t_4)\,e^{-j\omega_1 t_1}\,dt_1$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t_1,t_2,t_3,t_4)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega_1,t_2,t_3,t_4)\,e^{j\omega_1 t_1}\,d\omega_1 \quad \textbf{1D-IFT}$$

Fourier Transform and functions of *n* variables



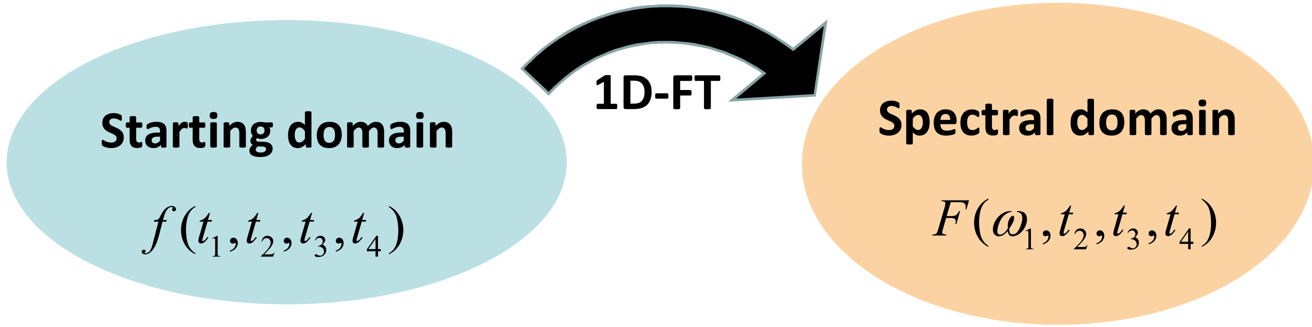
$f(t_1,t_2,t_3,t_4)$ → **1D-FT** → $$F(\omega_1,t_2,t_3,t_4)=\int_{-\infty}^{+\infty} f(t_1,t_2,t_3,t_4)\,e^{-j\omega_1 t_1}\,dt_1$$

$F(\omega_1,t_2,t_3,t_4)$ → **1D-IFT** → $$f(t_1,t_2,t_3,t_4)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega_1,t_2,t_3,t_4)\,e^{j\omega_1 t_1}\,d\omega_1$$

$$\frac{\partial f(t_1,t_2,t_3,t_4)}{\partial t_1}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\boxed{j\omega_1 F(\omega_1,t_2,t_3,t_4}) e^{j\omega_1 t_1}d\omega_1$$

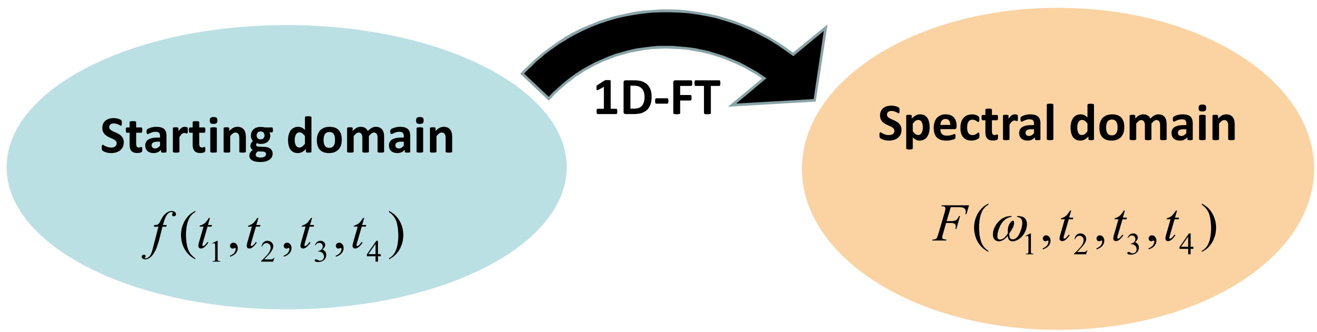
$$\frac{\partial f(t_1,t_2,t_3,t_4)}{\partial t_2}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\boxed{\frac{\partial F(\omega_1,t_2,t_3,t_4)}{\partial t_2}} e^{j\omega_1 t_1}d\omega_1$$

Fourier Transform and functions of *n* variables



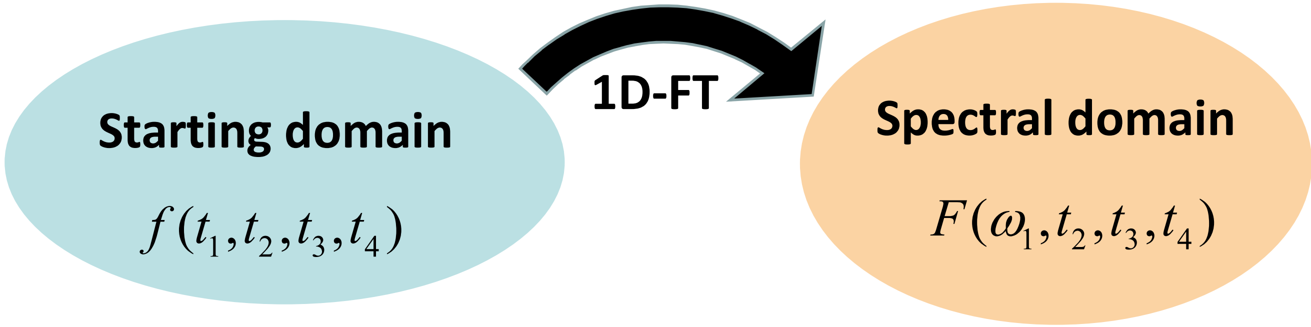
2) Time domain derivative and Fourier Transform

Fourier Transform and functions of *n* variables

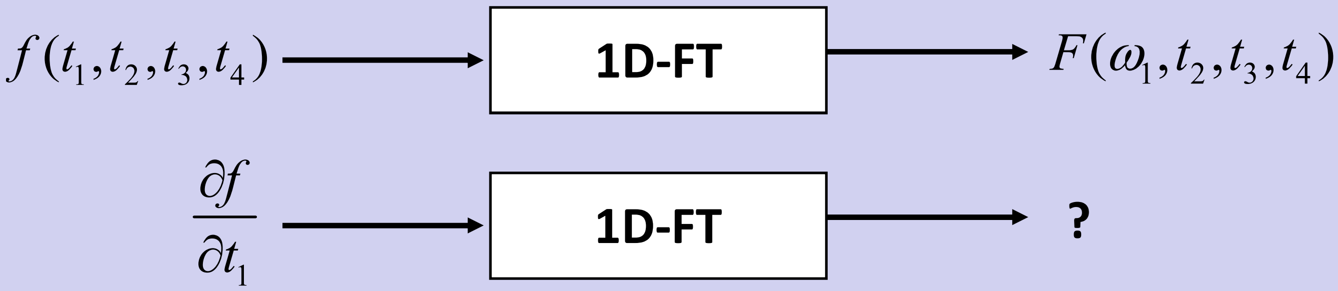


2) Time domain derivative and Fourier Transform

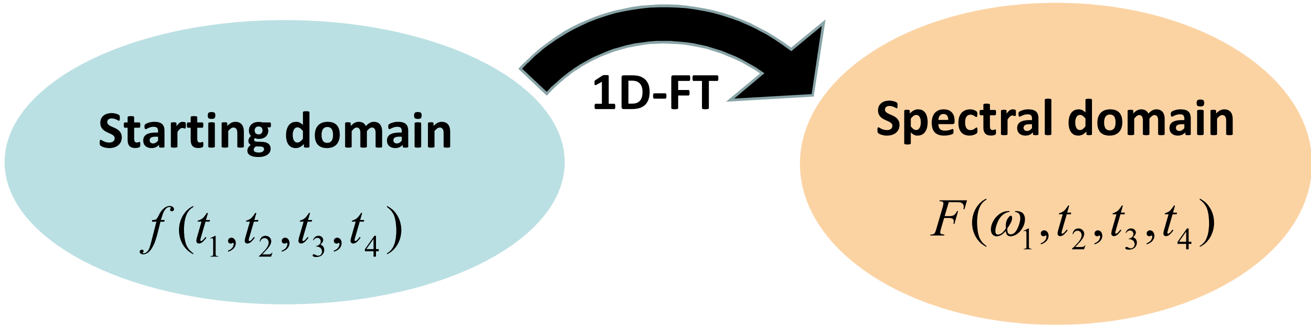
Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform

$$f(t_1,t_2,t_3,t_4) \longrightarrow \boxed{\text{1D-FT}} \longrightarrow F(\omega_1,t_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_1} \longrightarrow \boxed{\text{1D-FT}} \longrightarrow j\omega_1 F(\omega_1,t_2,t_3,t_4)$$

Fourier Transform and functions of *n* variables

Starting domain $f(t_1,t_2,t_3,t_4)$ → **1D-FT** → **Spectral domain** $F(\omega_1,t_2,t_3,t_4)$

2) Time domain derivative and Fourier Transform

$f(t_1,t_2,t_3,t_4)$ → **1D-FT** → $F(\omega_1,t_2,t_3,t_4)$

$\dfrac{\partial f}{\partial t_1}$ → **1D-FT** → $j\omega_1 F(\omega_1,t_2,t_3,t_4)$

$\dfrac{\partial f}{\partial t_2}$ → **1D-FT** → **?**

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

1D-FT

Spectral domain

$F(\omega_1, t_2, t_3, t_4)$

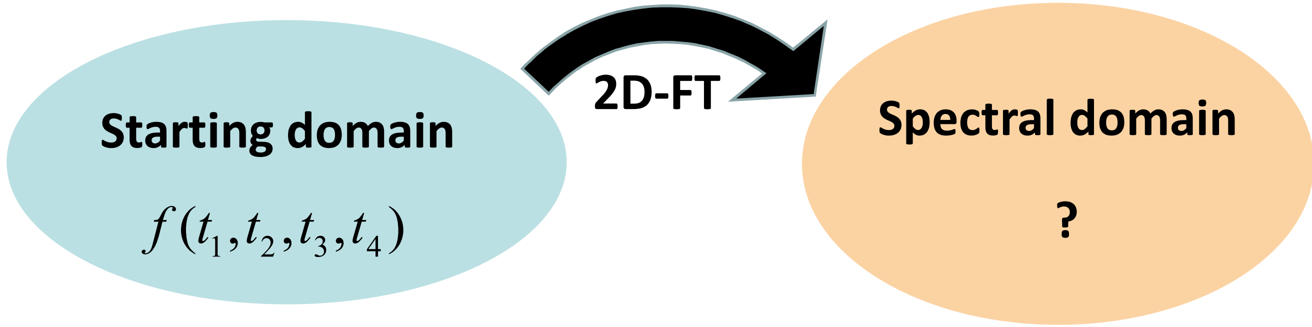
2) Time domain derivative and Fourier Transform

$f(t_1, t_2, t_3, t_4) \longrightarrow$ **1D-FT** $\longrightarrow F(\omega_1, t_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_1} \longrightarrow$ **1D-FT** $\longrightarrow j\omega_1 F(\omega_1, t_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_2} \longrightarrow$ **1D-FT** $\longrightarrow \dfrac{\partial F(\omega_1, t_2, t_3, t_4)}{\partial t_2}$

Fourier Transform and functions of *n* variables



Two Dimensional Fourier Transform (2D-FT)

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1,t_2,t_3,t_4)$

2D-FT

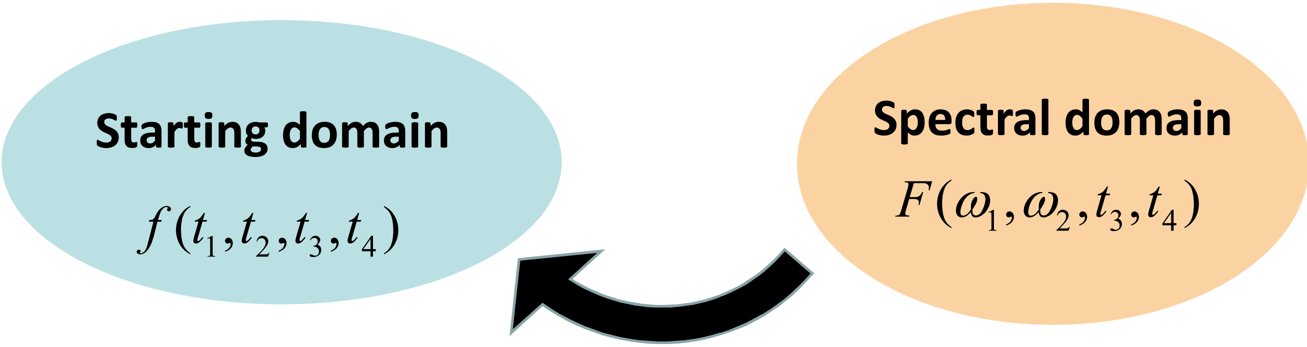
Spectral domain

$F(\omega_1,\omega_2,t_3,t_4)$

Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1,\omega_2,t_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1,t_2,t_3,t_4)\,e^{-j\omega_1 t_1}e^{-j\omega_2 t_2}$$

Fourier Transform and functions of *n* variables



Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1,\omega_2,t_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2}$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1,t_2,t_3,t_4)$

2D-IFT

Spectral domain

$F(\omega_1,\omega_2,t_3,t_4)$

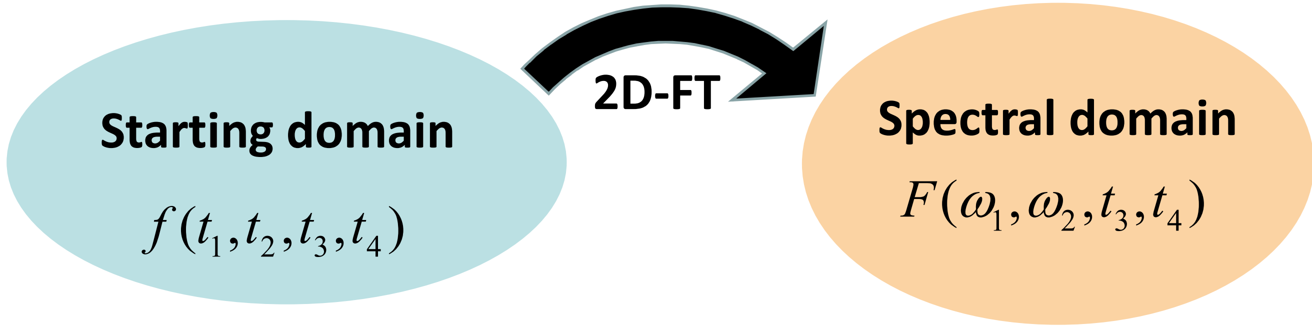
Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1,\omega_2,t_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2}$$

1) How to jump back from the Spectral domain to the Time domain

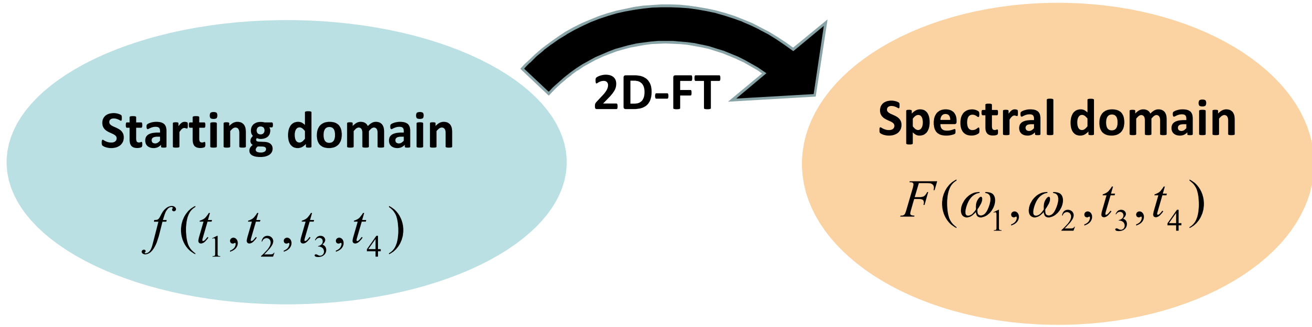
$$f(t_1,t_2,t_3,t_4)=\left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega_1,\omega_2,t_3,t_4) e^{j\omega_1 t_1} e^{j\omega_2 t_2} d\omega_1 d\omega_2$$ **2D-IFT**

Fourier Transform and functions of *n* variables

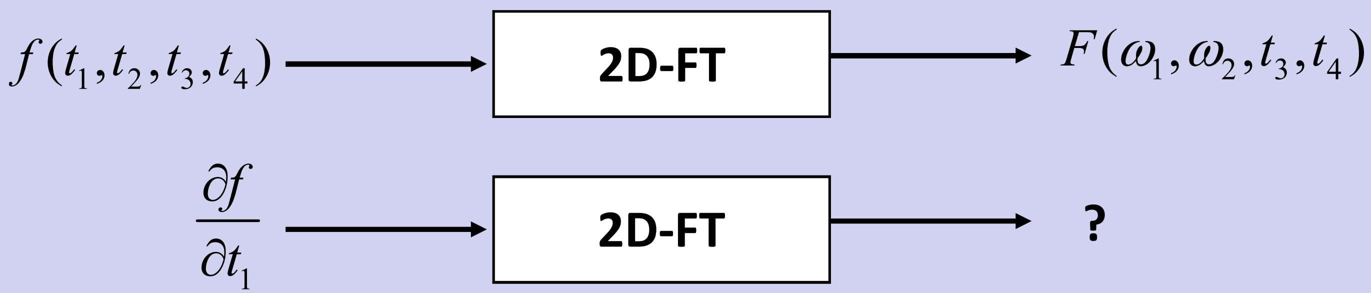


2) Time domain derivative and Fourier Transform

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

2D-FT

Spectral domain

$F(\omega_1, \omega_2, t_3, t_4)$

2) Time domain derivative and Fourier Transform

$$f(t_1, t_2, t_3, t_4) \longrightarrow \boxed{\textbf{2D-FT}} \longrightarrow F(\omega_1, \omega_2, t_3, t_4)$$

$$\frac{\partial f}{\partial t_1} \longrightarrow \boxed{\textbf{2D-FT}} \longrightarrow j\omega_1 F(\omega_1, \omega_2, t_3, t_4)$$

Fourier Transform and functions of *n* variables

Starting domain $f(t_1, t_2, t_3, t_4)$ → **2D-FT** → **Spectral domain** $F(\omega_1, \omega_2, t_3, t_4)$

2) Time domain derivative and Fourier Transform

$f(t_1, t_2, t_3, t_4)$ → **2D-FT** → $F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_1}$ → **2D-FT** → $j\omega_1 F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_2}$ → **2D-FT** → **?**

Fourier Transform and functions of *n* variables

Starting domain $f(t_1, t_2, t_3, t_4)$ → **2D-FT** → **Spectral domain** $F(\omega_1, \omega_2, t_3, t_4)$

2) Time domain derivative and Fourier Transform

$$f(t_1, t_2, t_3, t_4) \longrightarrow \boxed{\text{2D-FT}} \longrightarrow F(\omega_1, \omega_2, t_3, t_4)$$

$$\frac{\partial f}{\partial t_1} \longrightarrow \boxed{\text{2D-FT}} \longrightarrow j\omega_1 F(\omega_1, \omega_2, t_3, t_4)$$

$$\frac{\partial f}{\partial t_2} \longrightarrow \boxed{\text{2D-FT}} \longrightarrow j\omega_2 F(\omega_1, \omega_2, t_3, t_4)$$

Fourier Transform and functions of *n* variables

Starting domain $f(t_1, t_2, t_3, t_4)$ → **2D-FT** → **Spectral domain** $F(\omega_1, \omega_2, t_3, t_4)$

2) Time domain derivative and Fourier Transform

$f(t_1, t_2, t_3, t_4)$ → **2D-FT** → $F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_1}$ → **2D-FT** → $j\omega_1 F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_2}$ → **2D-FT** → $j\omega_2 F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_3}$ → **2D-FT** → **?**

Fourier Transform and functions of *n* variables

Starting domain $f(t_1,t_2,t_3,t_4)$ $\xrightarrow{\text{2D-FT}}$ **Spectral domain** $F(\omega_1,\omega_2,t_3,t_4)$

2) Time domain derivative and Fourier Transform

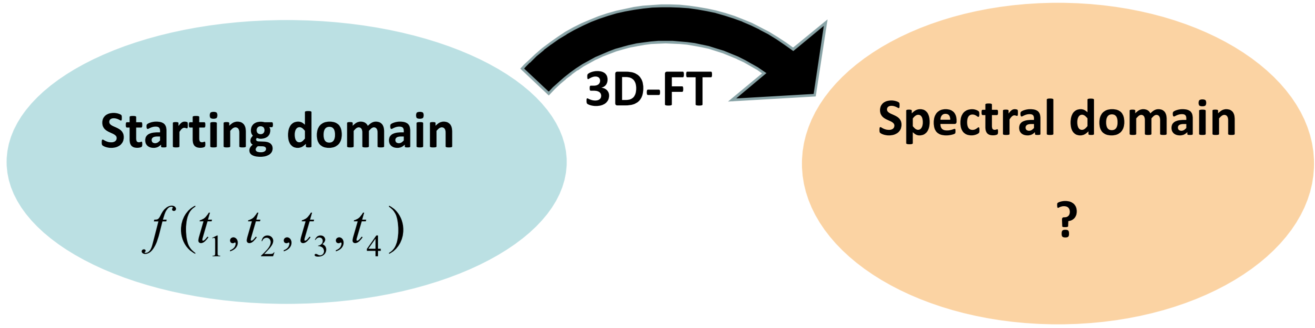
$$f(t_1,t_2,t_3,t_4) \longrightarrow \boxed{\textbf{2D-FT}} \longrightarrow F(\omega_1,\omega_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_1} \longrightarrow \boxed{\textbf{2D-FT}} \longrightarrow j\omega_1 F(\omega_1,\omega_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_2} \longrightarrow \boxed{\textbf{2D-FT}} \longrightarrow j\omega_2 F(\omega_1,\omega_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_3} \longrightarrow \boxed{\textbf{2D-FT}} \longrightarrow \frac{\partial F(\omega_1,\omega_2,t_3,t_4)}{\partial t_3}$$

Fourier Transform and functions of *n* variables



Three Dimensional Fourier Transform (3D-FT)

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1,t_2,t_3,t_4)$

3D-FT

Spectral domain

$F(\omega_1,\omega_2,\omega_3,t_4)$

Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1,\omega_2,\omega_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3\, f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

Spectral domain

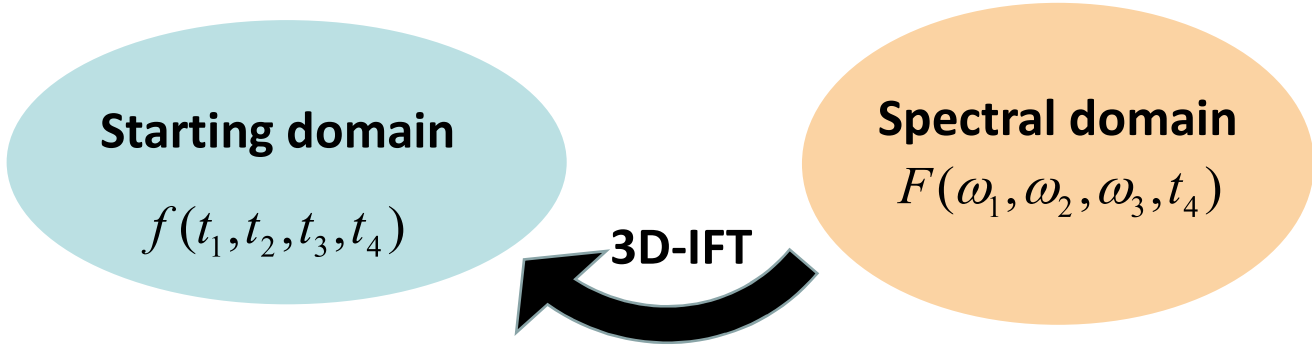
$F(\omega_1, \omega_2, \omega_3, t_4)$

Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1, \omega_2, \omega_3, t_4) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 f(t_1, t_2, t_3, t_4) e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables



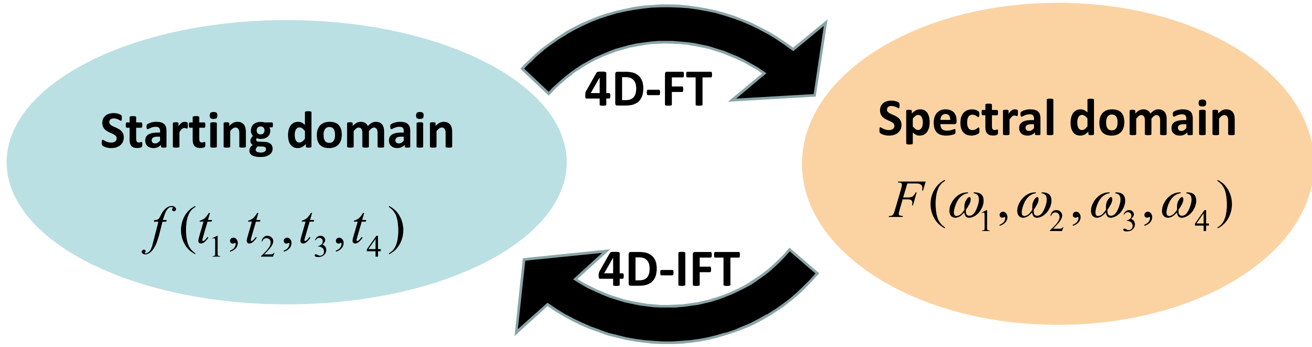
Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1,\omega_2,\omega_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t_1,t_2,t_3,t_4)=\left(\frac{1}{2\pi}\right)^3 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega_1,\omega_2,\omega_3,t_4) e^{j\omega_1 t_1} e^{j\omega_2 t_2} e^{j\omega_3 t_3} d\omega_1 d\omega_2 d\omega_3$$ **3D-IFT**

Fourier Transform and functions of *n* variables



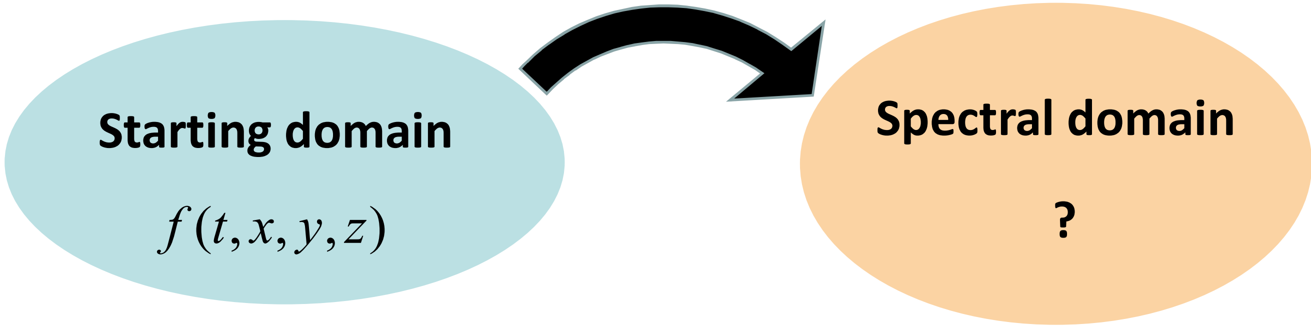
Four Dimensional Fourier Transform (4D-FT)

$$F(\omega_1,\omega_2,\omega_3,\omega_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 dt_4 f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3} e^{-j\omega_4 t_4}$$

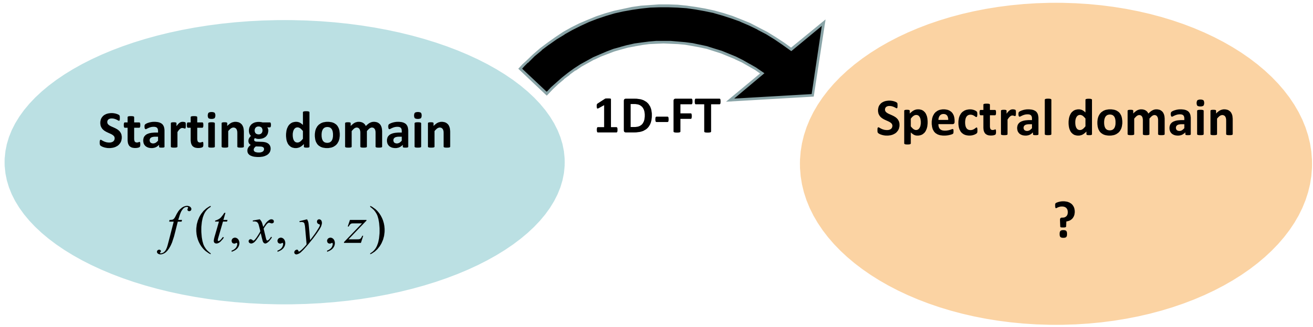
1) How to jump back from the Spectral domain to the Time domain

$$f(t_1,t_2,t_3,t_4)=\left(\frac{1}{2\pi}\right)^4 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega_1,\omega_2,\omega_3,\omega_4) e^{j\omega_1 t_1} e^{j\omega_2 t_2} e^{j\omega_3 t_3} e^{j\omega_4 t_4} d\omega_1 d\omega_2 d\omega_3 d\omega_4$$ **4D-IFT**

Fourier Transform and functions of *n* variables



Fourier Transform and functions of *n* variables



One Dimensional Fourier Transform (1D-FT)

Fourier Transform and functions of *n* variables

Starting domain

$f(t,x,y,z)$

1D-FT →

Spectral domain

$F(\omega,x,y,z)$

One Dimensional Fourier Transform (1D-FT)

$$F(\omega,x,y,z)=\int_{-\infty}^{+\infty} f(t,x,y,z)\,e^{-j\omega t}\,dt$$

Fourier Transform and functions of *n* variables

Starting domain

$f(t, x, y, z)$

Spectral domain

$F(\omega, x, y, z)$

One Dimensional Fourier Transform (1D-FT)

$$F(\omega, x, y, z) = \int_{-\infty}^{+\infty} f(t, x, y, z)\, e^{-j\omega t}\, dt$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables

Starting domain

$f(t,x,y,z)$

1D-IFT

Spectral domain

$F(\omega,x,y,z)$

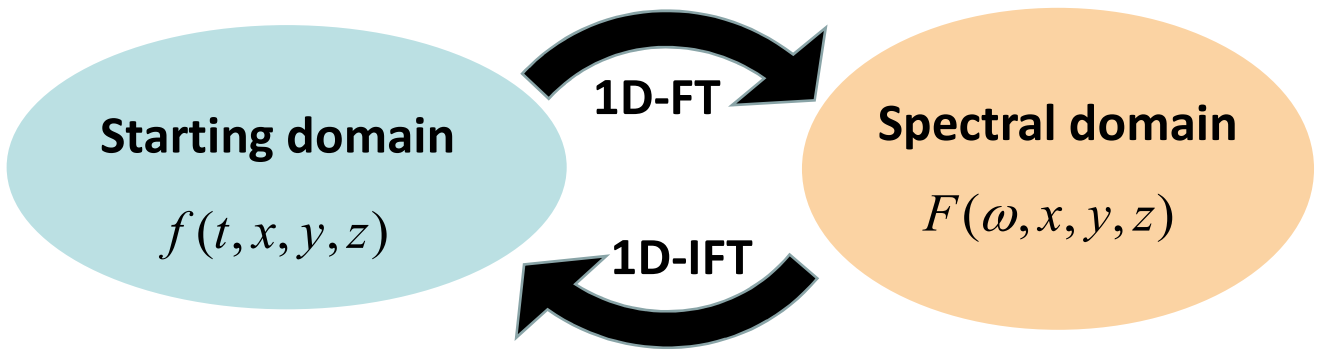
One Dimensional Fourier Transform (1D-FT)

$$F(\omega,x,y,z)=\int_{-\infty}^{+\infty} f(t,x,y,z)\,e^{-j\omega t}\,dt$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t,x,y,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega,x,y,z)\,e^{j\omega t}\,d\omega$$ **1D-IFT**

Fourier Transform and functions of *n* variables



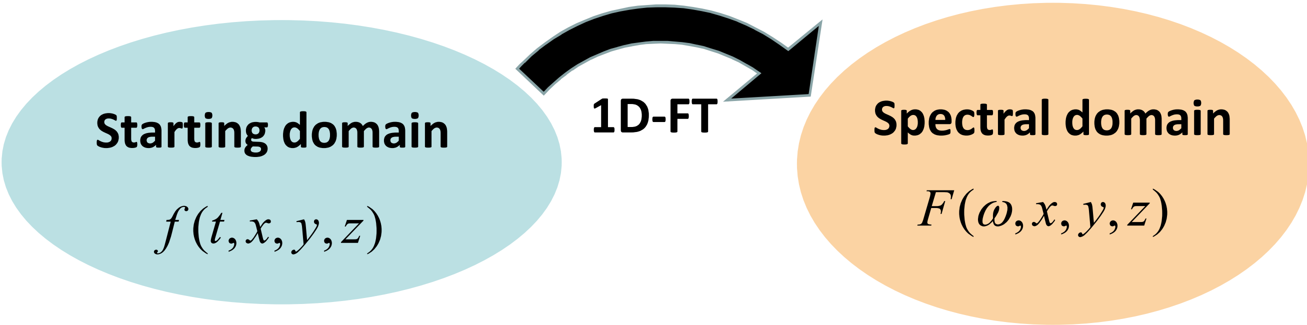
$f(t,x,y,z)$ → **1D-FT** →

$$F(\omega,x,y,z) = \int_{-\infty}^{+\infty} f(t,x,y,z)\, e^{-j\omega t}\, dt$$

$F(\omega,x,y,z)$ → **1D-IFT** →

$$f(t,x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega,x,y,z)\, e^{j\omega t}\, d\omega$$

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform

Fourier Transform and functions of *n* variables

Starting domain

$f(t,x,y,z)$

1D-FT

Spectral domain

$F(\omega,x,y,z)$

2) Time domain derivative and Fourier Transform

$f(t,x,y,z)$ → **1D-FT** → $F(\omega,x,y,z)$

$\frac{\partial f}{\partial t}$ → **1D-FT** → **?**

Fourier Transform and functions of *n* variables

Starting domain

$f(t,x,y,z)$

1D-FT

Spectral domain

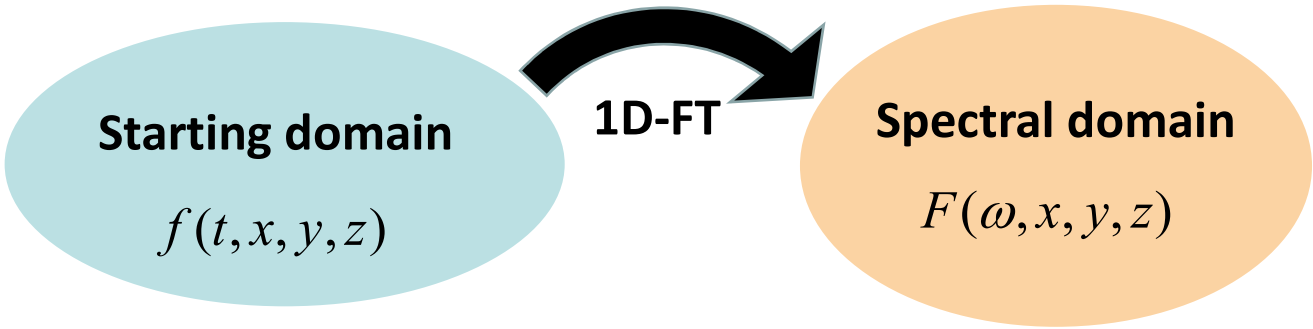
$F(\omega,x,y,z)$

2) Time domain derivative and Fourier Transform

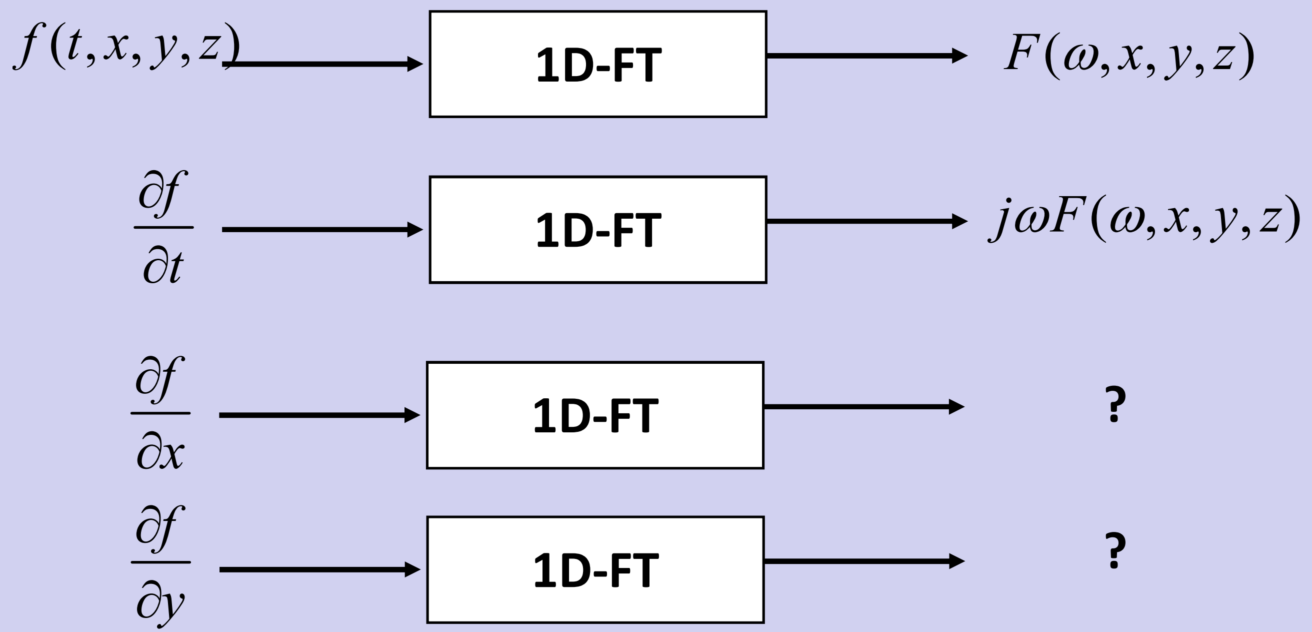
$f(t,x,y,z) \rightarrow$ **1D-FT** $\rightarrow F(\omega,x,y,z)$

$\dfrac{\partial f}{\partial t} \rightarrow$ **1D-FT** $\rightarrow j\omega F(\omega,x,y,z)$

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables

Starting domain $f(t,x,y,z)$ → **1D-FT** → **Spectral domain** $F(\omega,x,y,z)$

2) Time domain derivative and Fourier Transform

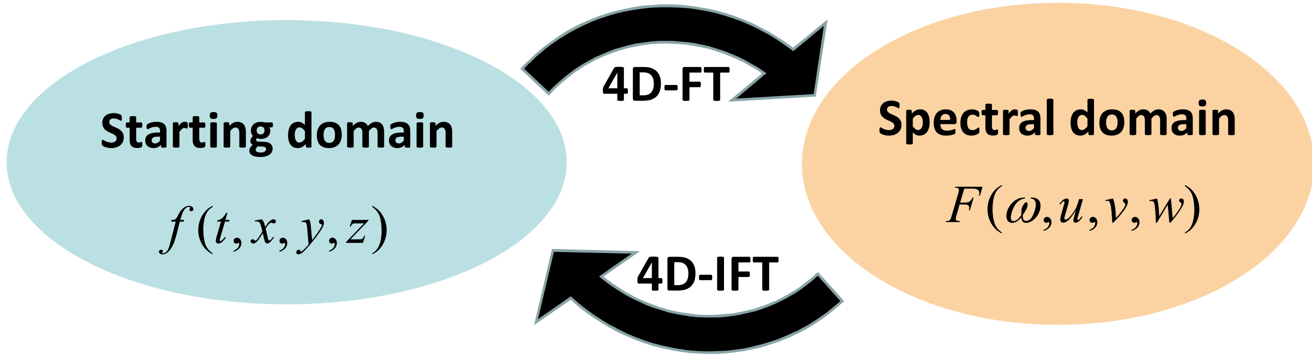
$f(t,x,y,z)$ → **1D-FT** → $F(\omega,x,y,z)$

$\dfrac{\partial f}{\partial t}$ → **1D-FT** → $j\omega F(\omega,x,y,z)$

$\dfrac{\partial f}{\partial x}$ → **1D-FT** → $\dfrac{\partial F(\omega,x,y,z)}{\partial x}$

$\dfrac{\partial f}{\partial y}$ → **1D-FT** → $\dfrac{\partial F(\omega,x,y,z)}{\partial y}$

Fourier Transform and functions of *n* variables



Four Dimensional Fourier Transform (4D-FT)

$$F(\omega,u,v,w)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t,x,y,z)\,e^{-j\omega t}e^{-jux}e^{-jvy}e^{-jwz}\,dt\,dx\,dy\,dz$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t,x,y,z)=\left(\frac{1}{2\pi}\right)^4\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega,u,v,w)\,e^{j\omega t}e^{jux}e^{jvy}e^{jwz}\,d\omega\,du\,dv\,dw$$ **4D-IFT**

Frequency domain

- Fourier Transform and functions of n variables
- **Fourier Transform and vector functions**
- Fourier Transform and vector functions of n variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

?

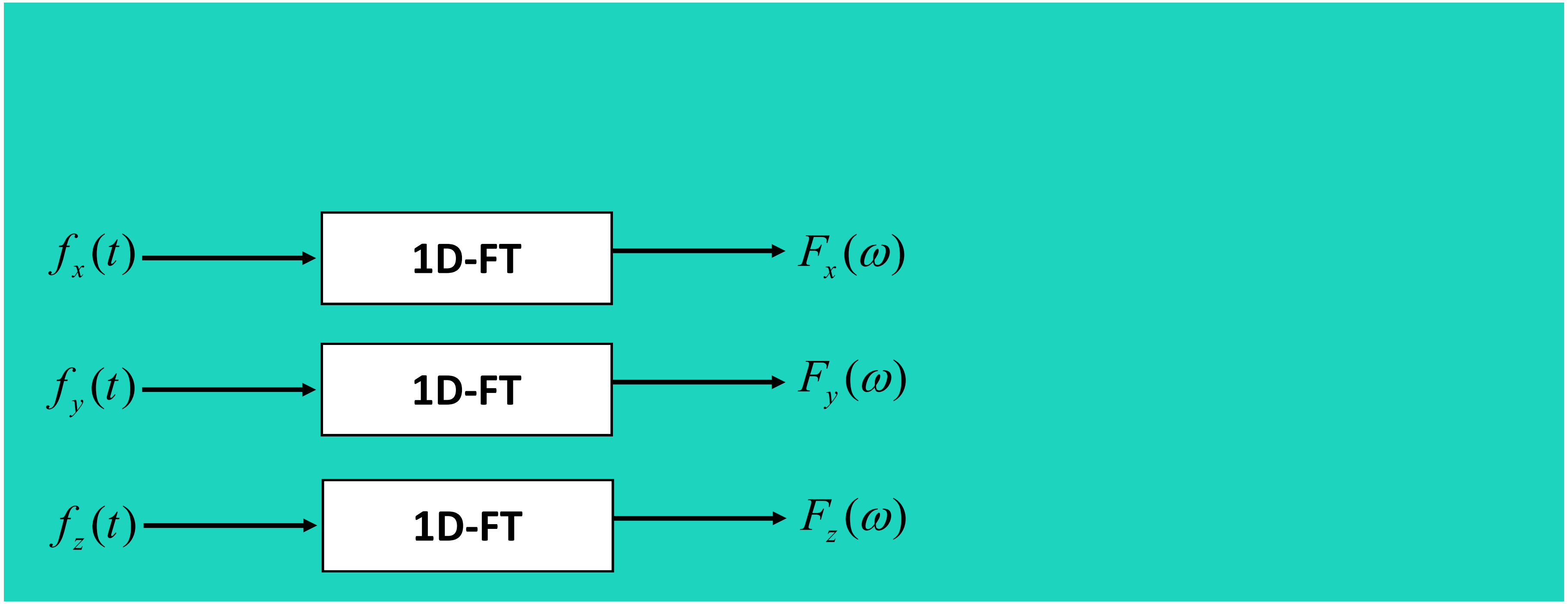
Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

?



Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

?

$f_x(t) \rightarrow$ **1D-FT** $\rightarrow F_x(\omega) = \int_{-\infty}^{+\infty} f_x(t) e^{-j\omega t} dt$

$f_y(t) \rightarrow$ **1D-FT** $\rightarrow F_y(\omega) = \int_{-\infty}^{+\infty} f_y(t) e^{-j\omega t} dt$

$f_z(t) \rightarrow$ **1D-FT** $\rightarrow F_z(\omega) = \int_{-\infty}^{+\infty} f_z(t) e^{-j\omega t} dt$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

$f_x(t)$ → **1D-FT** → $F_x(\omega) = \int_{-\infty}^{+\infty} f_x(t)e^{-j\omega t}dt$

$f_y(t)$ → **1D-FT** → $F_y(\omega) = \int_{-\infty}^{+\infty} f_y(t)e^{-j\omega t}dt$

$f_z(t)$ → **1D-FT** → $F_z(\omega) = \int_{-\infty}^{+\infty} f_z(t)e^{-j\omega t}dt$

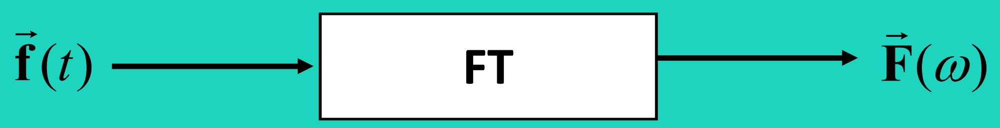
Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$



Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and vector functions

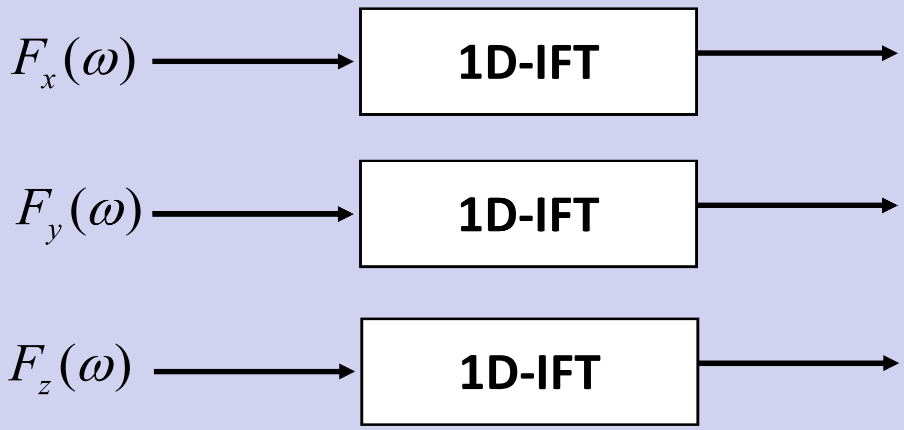
Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain



Fourier Transform and vector functions

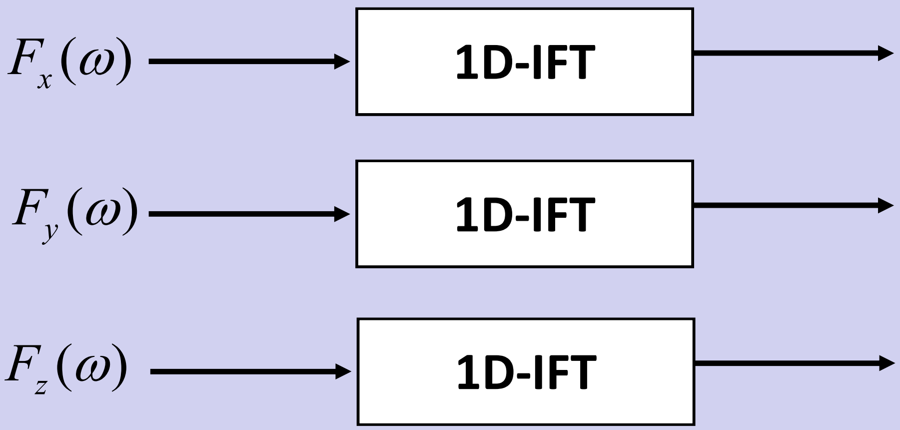
Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain



$$f_x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_x(\omega)e^{j\omega t}d\omega$$

$$f_y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_y(\omega)e^{j\omega t}d\omega$$

$$f_z(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_z(\omega)e^{j\omega t}d\omega$$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\dfrac{df_x(t)}{dt}$ → **1D-FT** → **?**

$\dfrac{df_y(t)}{dt}$ → **1D-FT** → **?**

$\dfrac{df_z(t)}{dt}$ → **1D-FT** → **?**

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\dfrac{df_x(t)}{dt}$ → **1D-FT** → $j\omega F_x(\omega)$

$\dfrac{df_y(t)}{dt}$ → **1D-FT** → $j\omega F_y(\omega)$

$\dfrac{df_z(t)}{dt}$ → **1D-FT** → $j\omega F_z(\omega)$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\frac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → $j\omega F_x(\omega)\hat{i}_x + j\omega F_y(\omega)\hat{i}_y + j\omega F_z(\omega)\hat{i}_z$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\frac{df_x(t)}{dt}$ → **1D-FT** → $j\omega F_x(\omega)$

$\frac{df_y(t)}{dt}$ → **1D-FT** → $j\omega F_y(\omega)$

$\frac{df_z(t)}{dt}$ → **1D-FT** → $j\omega F_z(\omega)$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → $j\omega\vec{\mathbf{F}}(\omega) = j\omega F_x(\omega)\hat{i}_x + j\omega F_y(\omega)\hat{i}_y + j\omega F_z(\omega)\hat{i}_z$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\dfrac{df_x(t)}{dt}$ → **1D-FT** → $j\omega F_x(\omega)$

$\dfrac{df_y(t)}{dt}$ → **1D-FT** → $j\omega F_y(\omega)$

$\dfrac{df_z(t)}{dt}$ → **1D-FT** → $j\omega F_z(\omega)$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow \vec{\mathbf{F}}(\omega)$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\textbf{FT}} \longrightarrow j\omega\vec{\mathbf{F}}(\omega)$$

Frequency domain

- Fourier Transform and functions of n variables

- Fourier Transform and vector functions

- Fourier Transform and vector functions of n variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Frequency domain

- Fourier Transform and functions of n variables

- Fourier Transform and vector functions

- Fourier Transform and vector functions of n variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

Frequency domain

?

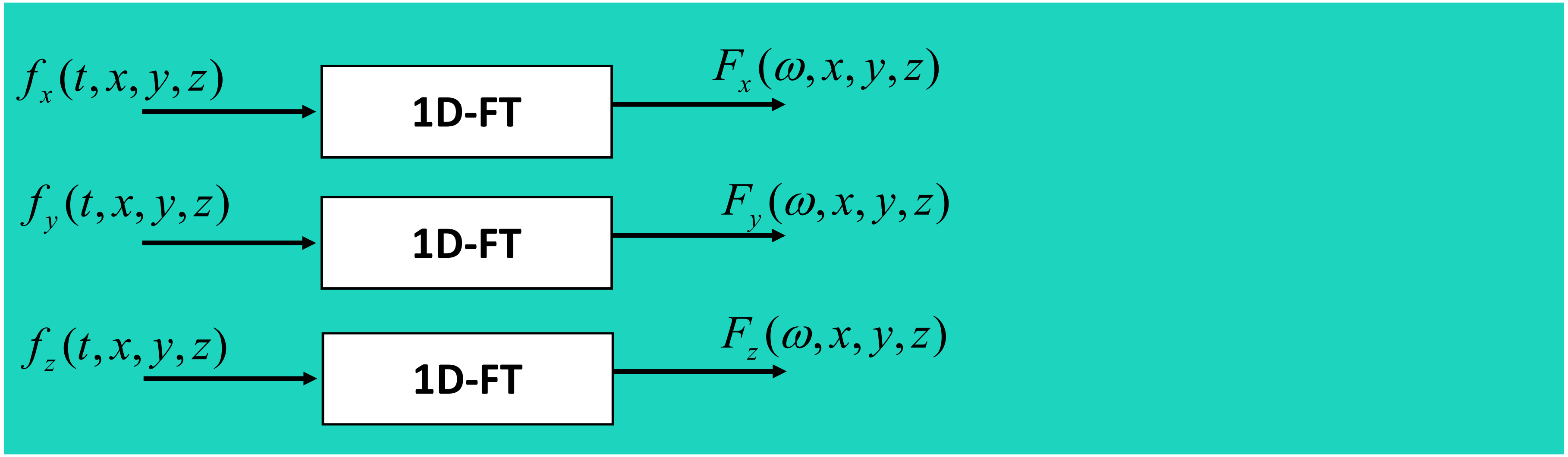
Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

Frequency domain

?



Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

Frequency domain

?

$f_x(t,x,y,z)$ → **1D-FT** → $F_x(\omega,x,y,z) = \int_{-\infty}^{+\infty} f_x(t,x,y,z)\, e^{-j\omega t}\, dt$

$f_y(t,x,y,z)$ → **1D-FT** → $F_y(\omega,x,y,z) = \int_{-\infty}^{+\infty} f_y(t,x,y,z)\, e^{-j\omega t}\, dt$

$f_z(t,x,y,z)$ → **1D-FT** → $F_z(\omega,x,y,z) = \int_{-\infty}^{+\infty} f_z(t,x,y,z)\, e^{-j\omega t}\, dt$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

$f_x(t,x,y,z)$ → **1D-FT** → $F_x(\omega,x,y,z) = \int_{-\infty}^{+\infty} f_x(t,x,y,z)\,e^{-j\omega t}\,dt$

$f_y(t,x,y,z)$ → **1D-FT** → $F_y(\omega,x,y,z) = \int_{-\infty}^{+\infty} f_y(t,x,y,z)\,e^{-j\omega t}\,dt$

$f_z(t,x,y,z)$ → **1D-FT** → $F_z(\omega,x,y,z) = \int_{-\infty}^{+\infty} f_z(t,x,y,z)\,e^{-j\omega t}\,dt$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$\omega \to t$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and vector functions of *n* variables

Time domain

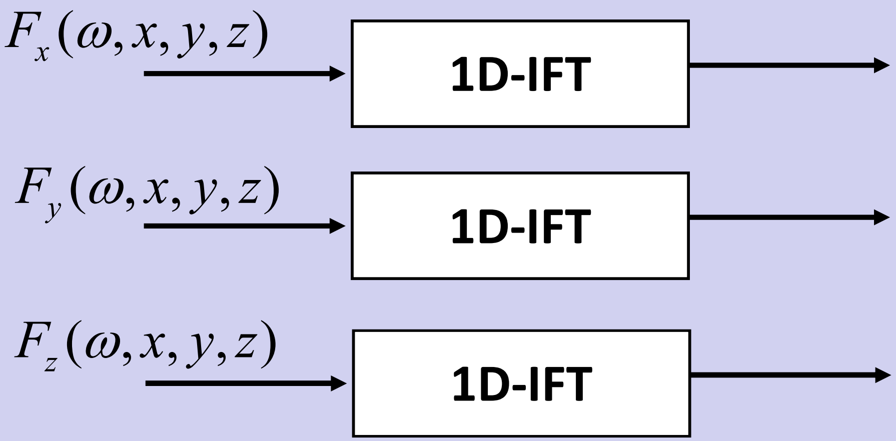
$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$\omega \rightarrow t$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain



Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$\omega \rightarrow t$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain

$F_x(\omega,x,y,z)$ → **1D-IFT** → $f_x(t,x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_x(\omega,x,y,z)e^{j\omega t}d\omega$

$F_y(\omega,x,y,z)$ → **1D-IFT** → $f_y(t,x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_y(\omega,x,y,z)e^{j\omega t}d\omega$

$F_z(\omega,x,y,z)$ → **1D-IFT** → $f_z(t,x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_z(\omega,x,y,z)e^{j\omega t}d\omega$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \rightarrow \omega$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ → **FT** → **?**

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z) \rightarrow$ **FT** $\rightarrow \vec{\mathbf{F}}(\omega,x,y,z)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t} \rightarrow$ **FT** $\rightarrow$ **?**

$$\frac{\partial \vec{\mathbf{f}}(t,\vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ → **FT** → **?**

$$\frac{\partial \vec{\mathbf{f}}(t,\mathbf{r})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$\dfrac{\partial f_x}{\partial t}$ → **1D-FT** → **?**

$\dfrac{\partial f_y}{\partial t}$ → **1D-FT** → **?**

$\dfrac{\partial f_z}{\partial t}$ → **1D-FT** → **?**

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ → **FT** → **?**

$$\frac{\partial \vec{\mathbf{f}}(t,\vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$\dfrac{\partial f_x}{\partial t}$ → **1D-FT** → $j\omega F_x(\omega,x,y,z)$

$\dfrac{\partial f_y}{\partial t}$ → **1D-FT** → $j\omega F_y(\omega,x,y,z)$

$\dfrac{\partial f_z}{\partial t}$ → **1D-FT** → $j\omega F_z(\omega,x,y,z)$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ → **FT** → $j\omega F_x(\omega,x,y,z)\hat{i}_x + j\omega F_y(\omega,x,y,z)\hat{i}_y + j\omega F_z(\omega,x,y,z)\hat{i}_z$

$$\frac{\partial \vec{\mathbf{f}}(t,\vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$\dfrac{\partial f_x}{\partial t}$ → **1D-FT** → $j\omega F_x(\omega,x,y,z)$

$\dfrac{\partial f_y}{\partial t}$ → **1D-FT** → $j\omega F_y(\omega,x,y,z)$

$\dfrac{\partial f_z}{\partial t}$ → **1D-FT** → $j\omega F_z(\omega,x,y,z)$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z) \longrightarrow$ **FT** $\longrightarrow \vec{\mathbf{F}}(\omega,x,y,z)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t} \longrightarrow$ **FT** $\longrightarrow j\omega\vec{\mathbf{F}}(\omega,x,y,z) = j\omega F_x(\omega,x,y,z)\hat{i}_x + j\omega F_y(\omega,x,y,z)\hat{i}_y + j\omega F_z(\omega,x,y,z)\hat{i}_z$

$$\frac{\partial \vec{\mathbf{f}}(t,\vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$\dfrac{\partial f_x}{\partial t} \longrightarrow$ **1D-FT** $\longrightarrow j\omega F_x(\omega,x,y,z)$

$\dfrac{\partial f_y}{\partial t} \longrightarrow$ **1D-FT** $\longrightarrow j\omega F_y(\omega,x,y,z)$

$\dfrac{\partial f_z}{\partial t} \longrightarrow$ **1D-FT** $\longrightarrow j\omega F_z(\omega,x,y,z)$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\nabla \cdot \vec{\mathbf{f}}$ → **FT** → **?**

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

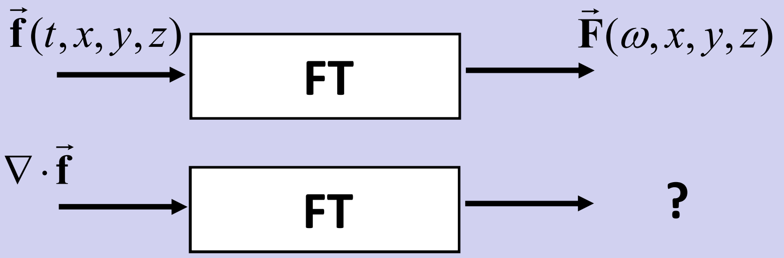
$t \to \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \to \omega$



$$\nabla \cdot \vec{\mathbf{f}}\left(t,\vec{\mathbf{r}}\right) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\nabla \cdot \vec{\mathbf{f}}$ → **FT** → **?**

$$\nabla \cdot \vec{\mathbf{f}}\left(t,\vec{\mathbf{r}}\right) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

$\frac{\partial f_x}{\partial x}$ → **1D-FT** → **?**

$\frac{\partial f_y}{\partial y}$ → **1D-FT** → **?**

$\frac{\partial f_z}{\partial z}$ → **1D-FT** → **?**

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \to \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \to \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\nabla\cdot\vec{\mathbf{f}}$ → **FT** → **?**

$$\nabla\cdot\vec{\mathbf{f}}\left(t,\vec{\mathbf{r}}\right) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

$\frac{\partial f_x}{\partial x}$ → **1D-FT** → $\frac{\partial F_x(\omega,x,y,z)}{\partial x}$

$\frac{\partial f_y}{\partial y}$ → **1D-FT** → $\frac{\partial F_y(\omega,x,y,z)}{\partial y}$

$\frac{\partial f_z}{\partial z}$ → **1D-FT** → $\frac{\partial F_z(\omega,x,y,z)}{\partial z}$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z)=f_x(t,x,y,z)\hat{i}_x+f_y(t,x,y,z)\hat{i}_y+f_z(t,x,y,z)\,\hat{i}_z$$

$t\rightarrow\omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z)=F_x(\omega,x,y,z)\hat{i}_x+F_y(\omega,x,y,z)\hat{i}_y+F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t\rightarrow\omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\nabla\cdot\vec{\mathbf{f}}$ → **FT** → $\nabla\cdot\vec{\mathbf{F}}(\omega,x,y,z)$

$$\nabla\cdot\vec{\mathbf{f}}\left(t,\vec{\mathbf{r}}\right)=\frac{\partial f_x}{\partial x}+\frac{\partial f_y}{\partial y}+\frac{\partial f_z}{\partial z}$$

$\frac{\partial f_x}{\partial x}$ → **1D-FT** → $\frac{\partial F_x(\omega,x,y,z)}{\partial x}$

$\frac{\partial f_y}{\partial y}$ → **1D-FT** → $\frac{\partial F_y(\omega,x,y,z)}{\partial y}$

$\frac{\partial f_z}{\partial z}$ → **1D-FT** → $\frac{\partial F_z(\omega,x,y,z)}{\partial z}$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ → **FT** → $\vec{\mathbf{F}}(\omega,x,y,z)$

$\nabla \times \vec{\mathbf{f}}$ → **FT** → $\nabla \times \vec{\mathbf{F}}(\omega,x,y,z)$

$$\nabla \times \vec{\mathbf{f}}\,(t,\vec{\mathbf{r}}) = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\hat{i}_z$$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(t,x,y,z) = f_x(t,x,y,z)\hat{i}_x + f_y(t,x,y,z)\hat{i}_y + f_z(t,x,y,z)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(\omega,x,y,z) = F_x(\omega,x,y,z)\hat{i}_x + F_y(\omega,x,y,z)\hat{i}_y + F_z(\omega,x,y,z)\hat{i}_z$$

2) Time domain derivative and Fourier Transform $t \rightarrow \omega$

$\vec{\mathbf{f}}(t,x,y,z)$ →	**FT**	→ $\vec{\mathbf{F}}(\omega,x,y,z)$
$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ →	**FT**	→ $j\omega\vec{\mathbf{F}}(\omega,x,y,z)$
$\nabla \cdot \vec{\mathbf{f}}$ →	**FT**	→ $\nabla \cdot \vec{\mathbf{F}}(\omega,x,y,z)$
$\nabla \times \vec{\mathbf{f}}$ →	**FT**	→ $\nabla \times \vec{\mathbf{F}}(\omega,x,y,z)$

Fourier Transform and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\hat{i}_x + f_y(x,y,z,t)\hat{i}_y + f_z(x,y,z,t)\,\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{\mathbf{F}}(x,y,z,\omega) = F_x(x,y,z,\omega)\hat{i}_x + F_y(x,y,z,\omega)\hat{i}_y + F_z(x,y,z,\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$\vec{\mathbf{f}}(x,y,z,t)$ → **FT** → $\vec{\mathbf{F}}(x,y,z,\omega)$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ → **FT** → $j\omega\vec{\mathbf{F}}(x,y,z,\omega)$

$\nabla \cdot \vec{\mathbf{f}}$ → **FT** → $\nabla \cdot \vec{\mathbf{F}}(x,y,z,\omega)$

$\nabla \times \vec{\mathbf{f}}$ → **FT** → $\nabla \times \vec{\mathbf{F}}(x,y,z,\omega)$