Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 – Laurea "Triennale" – Secondo semestre – Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Equazioni di Maxwell

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

		Unità di misura
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$:	Campo elettrico	Volt/m
$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)$:	Induzione elettrica	Coulomb/m^2
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$:	Campo magnetico	Ampere/m
$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)$:	Induzione magnetica	Weber/m^2
$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$:	Densità di corrente	Ampere/m^2
$\rho(\vec{\mathbf{r}},t)$:	Densità di carica	Coulomb/m^3

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

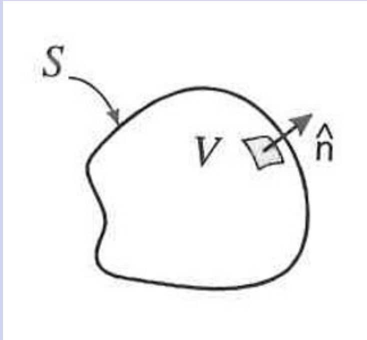
Memo

Mathematical tools to be exploited

Mathematics

Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



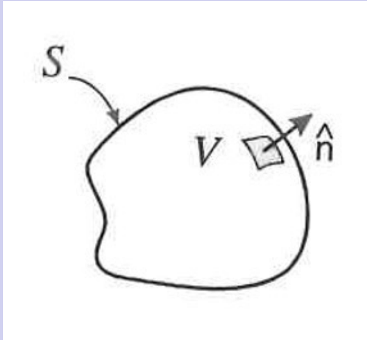
- Scalar quantity

- Its value **DOES NOT DEPEND** on the coordinate system we have chosen

- Its analytical expression **DEPENDS** on the coordinate system we have chosen

Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oint\!\!\!\oint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

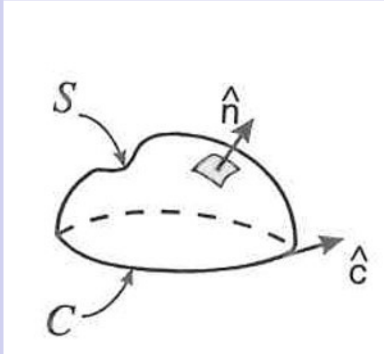


Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oint\!\!\!\oint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

Curl

$$\left(\nabla\times\vec{\mathbf{A}}\left(\vec{\mathbf{r}}\right)\right)\cdot\hat{\mathbf{n}}=\lim_{S\to 0}\frac{1}{S}\oint_C dc\,\vec{\mathbf{A}}\left(\vec{\mathbf{r}}\right)\cdot\hat{\mathbf{c}}$$



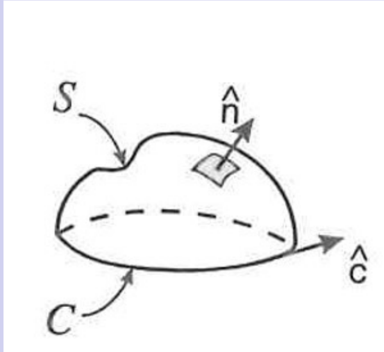
- Vector quantity

- Its value **DOES NOT DEPEND** on the coordinate system we have chosen

- Its analytical expression **DEPENDS** on the coordinate system we have chosen

Curl

$$\left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \lim_{S \to 0} \frac{1}{S} \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



Stokes theorem

$$\iint_S dS \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$

Cartesian Coordinates

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t)=A_x(x,y,z,t)\hat{i}_x + A_y(x,y,z,t)\hat{i}_y + A_z(x,y,z,t)\hat{i}_z$$

$$\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\hat{i}_y+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\hat{i}_z$$

Spherical Coordinates

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t)=A_r(r,\vartheta,\varphi,t)\hat{i}_r+A_\vartheta(r,\vartheta,\varphi,t)\hat{i}_\vartheta+A_\varphi(r,\vartheta,\varphi,t)\hat{i}_\varphi$$

$$\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right)+\frac{1}{r\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\vartheta\right)+\frac{1}{r\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{1}{r\sin\vartheta}\left[\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\varphi\right)-\frac{\partial A_\vartheta}{\partial\varphi}\right]\hat{i}_r+\frac{1}{r}\left[\frac{1}{\sin\vartheta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}\left(rA_\varphi\right)\right]\hat{i}_\vartheta+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\vartheta\right)-\frac{\partial A_r}{\partial\vartheta}\right]\hat{i}_\varphi$$

Color legend

New formulas, important considerations,
important formulas, important concepts

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Memo

Mathematical tools to be exploited

Mathematics

Maxwell equations

James Clerk Maxwell 1831-1879

Differential form

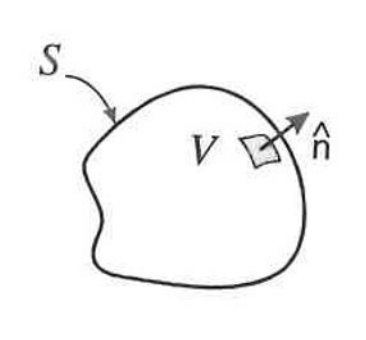
$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

Maxwell equations: integral form

… mathematical tools that we will exploit today…

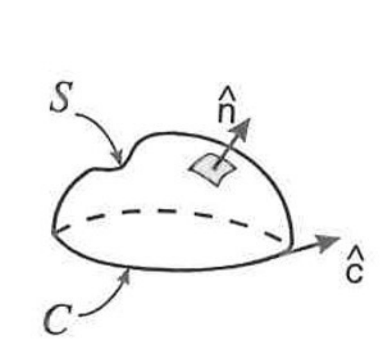
I) Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



II) Stokes theorem

$$\iint_S dS \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



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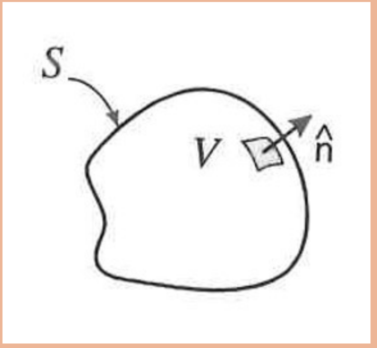

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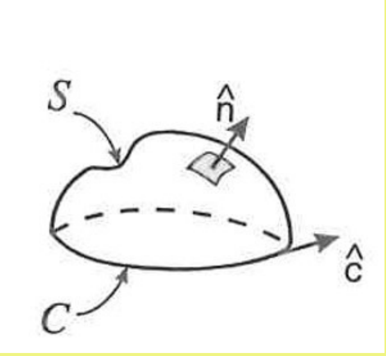
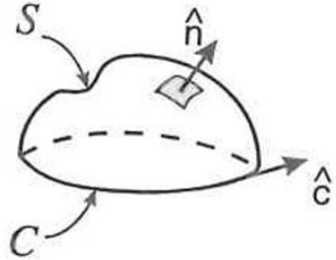


Maxwell equations: integral form

$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

Maxwell equations: integral form



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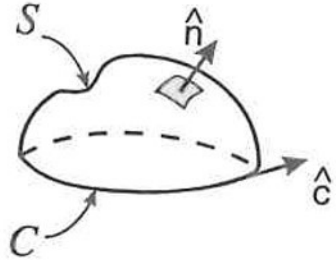
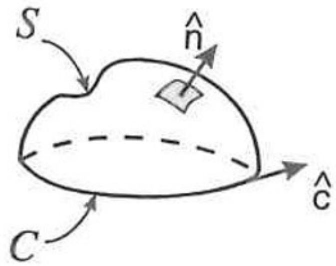


$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \Rightarrow \iint_S dS\left(\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right)\cdot \hat{\mathbf{n}} = -\iint_S dS \frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$



$$\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\quad\Rightarrow\quad\iint_S dS\left(\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=-\iint_S dS\,\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}$$

Maxwell equations: integral form

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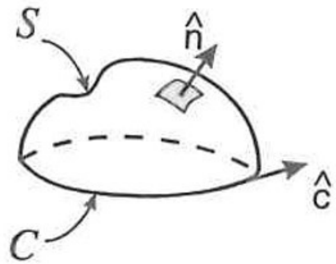

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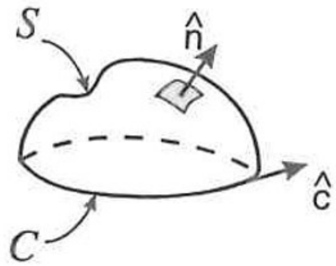
$$\downarrow$$

$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} \quad =$$

Maxwell equations: integral form



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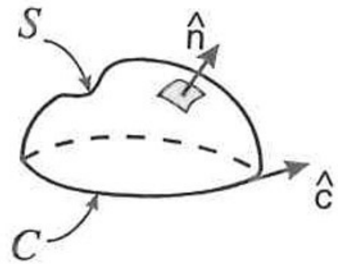
$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form

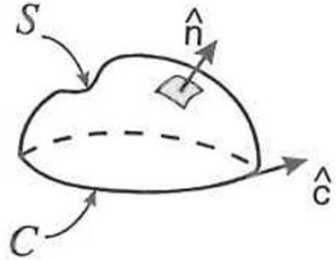


$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \quad \Rightarrow$$

Lenz-Neumann law

$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form



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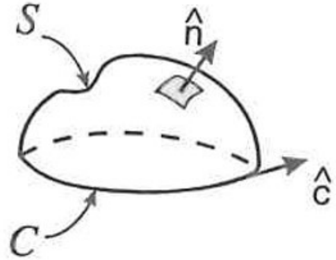
….considerations

Stationary fields $\left(\frac{d}{dt} = 0 \right) \Rightarrow \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}} = 0$

Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

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$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

….considerations

Stationary fields $\left(\frac{d}{dt} = 0\right)$ ➡ $\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}} = 0$ **Kirchhoff's second law**

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

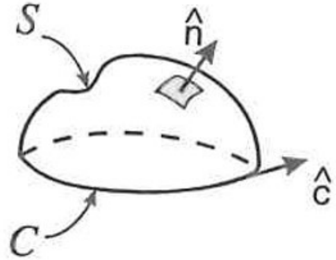
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \end{cases}$$

Maxwell equations: integral form

$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

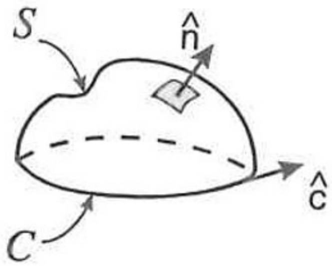


$$\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iint_S dS\left(\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=\iint_S dS\,\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}+\iint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$



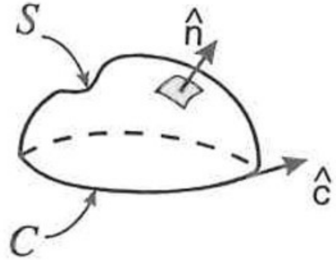
$$\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t) \Rightarrow \iint_S dS\left(\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=\iint_S dS\,\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}+\iint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

$\downarrow$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$



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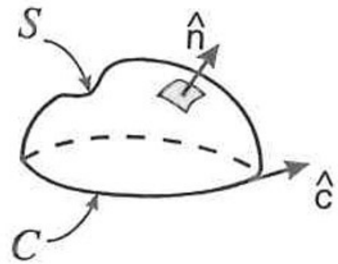
$$\downarrow$$

$$\oint_C dc\,\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}}=$$

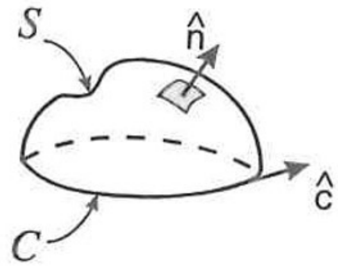
Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iint_S dS\left(\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right)\cdot \hat{\mathbf{n}} = \iint_S dS\,\frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\cdot \hat{\mathbf{n}} + \iint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}}$$

$$\oint_C dc\,\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = \frac{d}{dt}\iint_S dS\,\vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form



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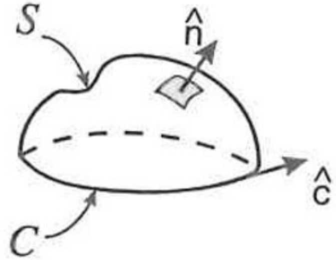
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \quad + i(t)$$

Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t)$$

➡

Ampere-Faraday law

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = \frac{d}{dt}\iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + i(t)$$

Maxwell equations

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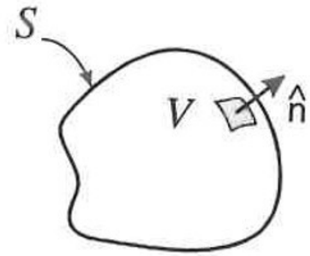
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \end{cases}$$

Maxwell equations: integral form

$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form



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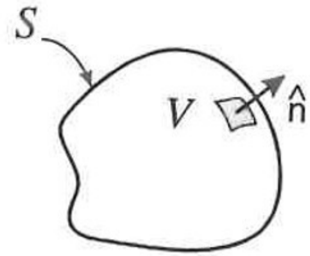
Maxwell equations: integral form

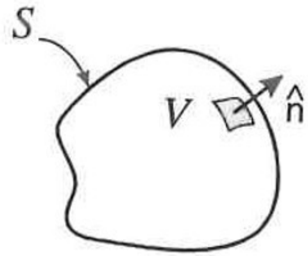


$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV \, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV \, \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oint\!\!\oint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

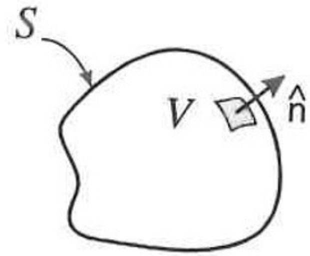


$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

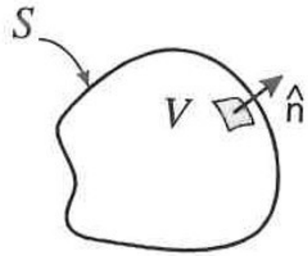


$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

$$\downarrow$$

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \;=$$

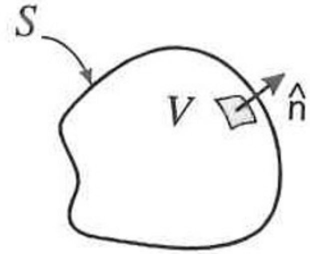
Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV \, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV \, \rho(\vec{\mathbf{r}},t)$$

$$\oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t)$$

Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{d}}\left(\vec{\mathbf{r}},t\right) = \rho\left(\vec{\mathbf{r}},t\right)$$

➡

Coulomb law

$$\oiint_S dS\,\vec{\mathbf{d}}\left(\vec{\mathbf{r}},t\right)\cdot\hat{\mathbf{n}} = q(t)$$

Maxwell equations

Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

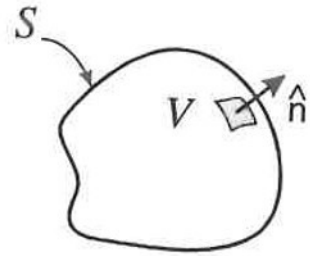
Integral form

$$
\begin{cases}
\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\
\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \\
\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t)
\end{cases}
$$

Maxwell equations: integral form

$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

Maxwell equations: integral form

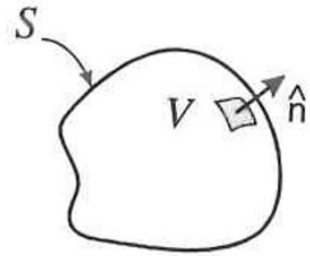


$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \iiint_V dV \; \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

Maxwell equations: integral form

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

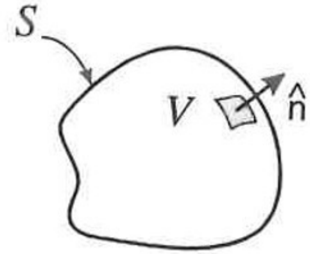


$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

$$\downarrow$$

$$\oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0$$

Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0 \end{cases}$$

… mathematical tools …

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right) = 0$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\Rightarrow \quad \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = 0$$

Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

Current density equation

Integral form

Current density equation

Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t}=0$$

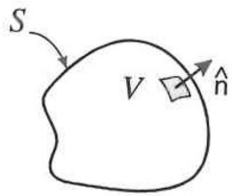
Current density equation

Integral form

Current density equation

Gauss theorem

$$\iiint_V dV\,\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}})=\oiint_S dS\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot \hat{\mathbf{n}}$$



Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

Current density equation

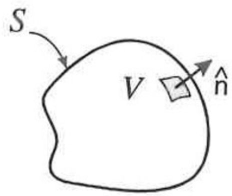
Integral form

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Current density equation

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



Maxwell equations

Integral form

$$\oiint_S dS\,\vec{\mathbf{j}}\left(\vec{\mathbf{r}},t\right)\cdot\hat{\mathbf{n}}+\frac{dq(t)}{dt}=0$$

Current density equation

Maxwell equations

Integral form

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Current density equation

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ ➡ $\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}} = 0$ **Kirchhoff's first law**

Maxwell equations

Differential form

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla\cdot\vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial\rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

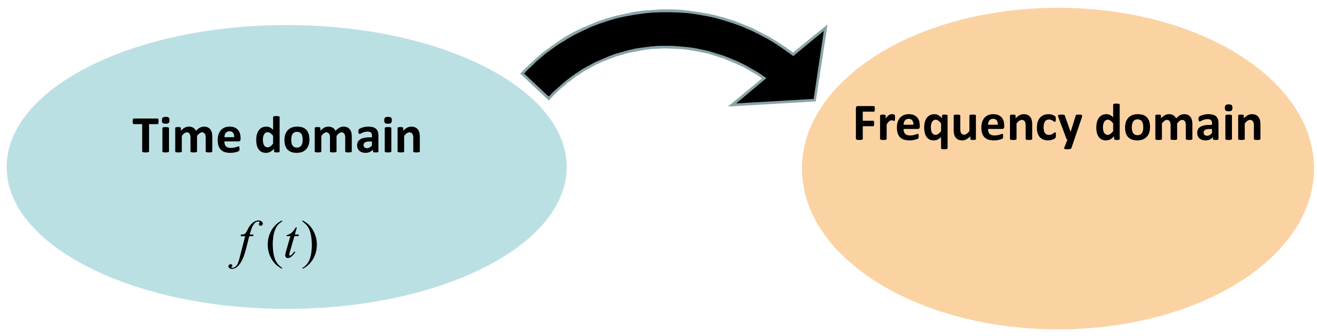
Integral form

$$\begin{cases} \oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\dfrac{d}{dt}\iint_S dS\,\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} \\ \oint_C dc\,\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = \dfrac{d}{dt}\iint_S dS\,\vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + i(t) \\ \oiint_S dS\,\vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = q(t) \\ \oiint_S dS\,\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0 \end{cases}$$

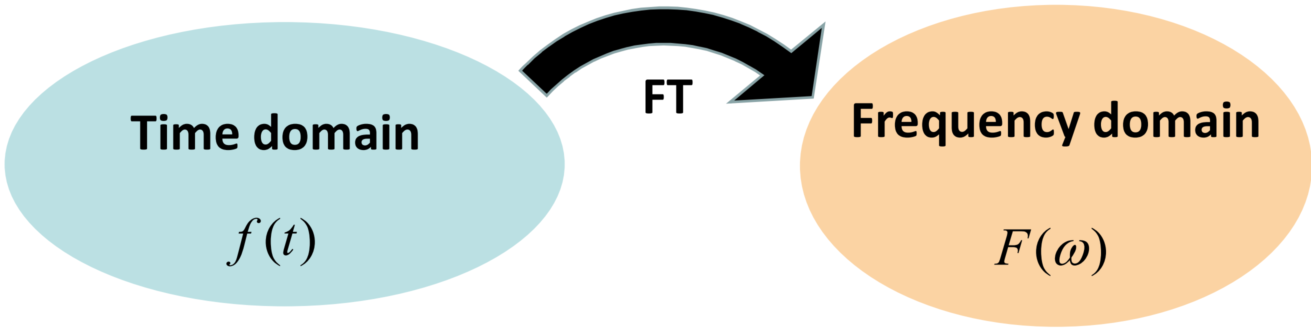
$$\oiint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Maxwell equations: Time domain, Frequency domain, Phasors

Frequency domain



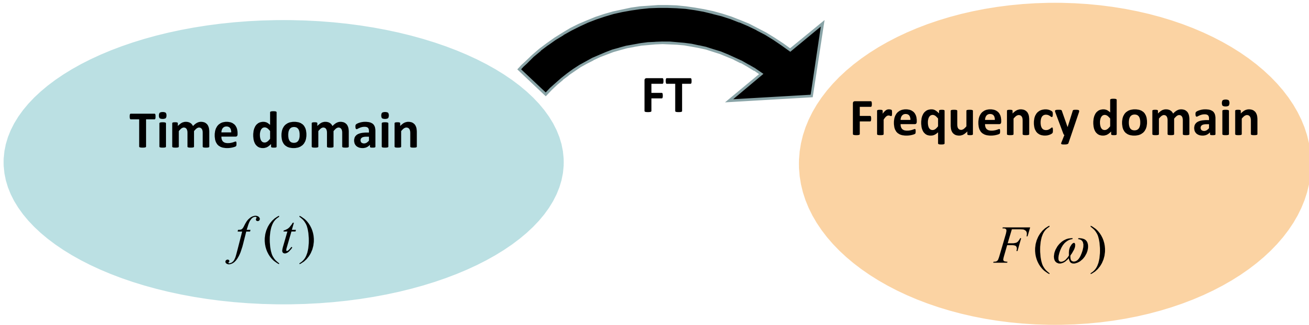
Frequency domain



$$F(\omega)=\int_{-\infty}^{+\infty} f(t)\,e^{-j\omega t}\,dt$$

Fourier Transform (FT)

Frequency domain



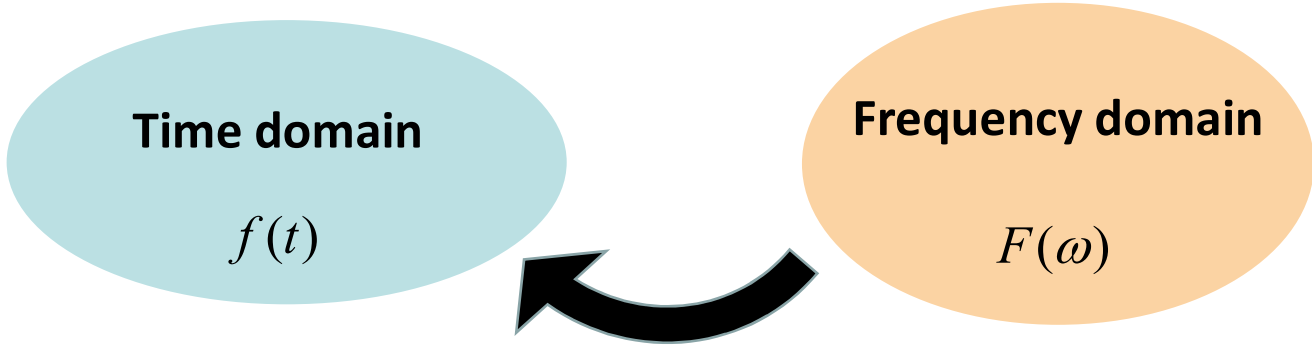
$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt$$

Fourier Transform (FT)

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Frequency domain

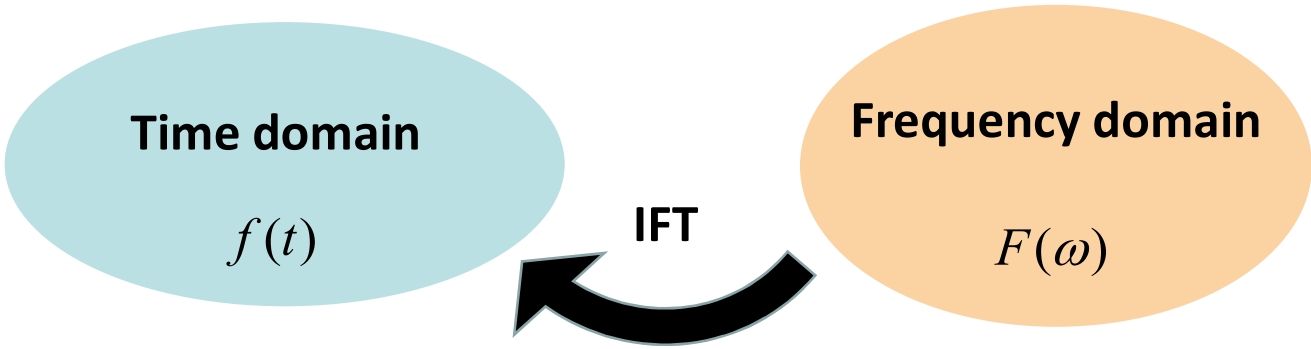


$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt$$

Fourier Transform (FT)

1) How to jump back from the Spectral domain to the Time domain

Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

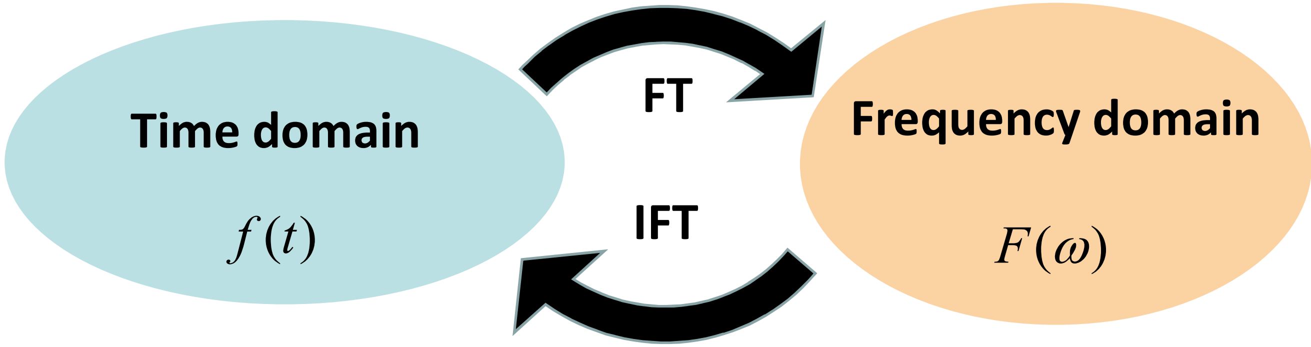
Fourier Transform (FT)

1) How to jump back from the Spectral domain to the Time domain

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$$

Inverse Fourier Transform (IFT)

Frequency domain



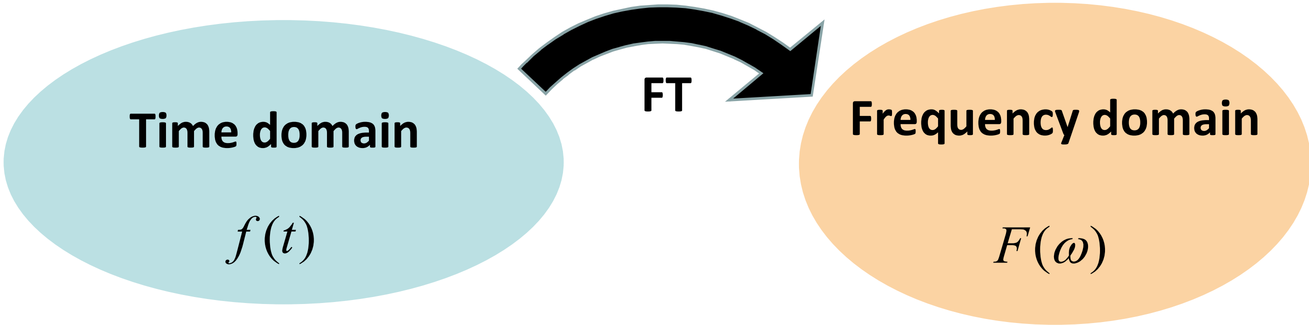
$f(t) \rightarrow$ **FT** $\rightarrow$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt$$

$F(\omega) \rightarrow$ **IFT** $\rightarrow$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

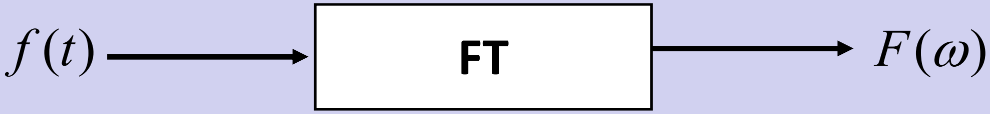
Frequency domain



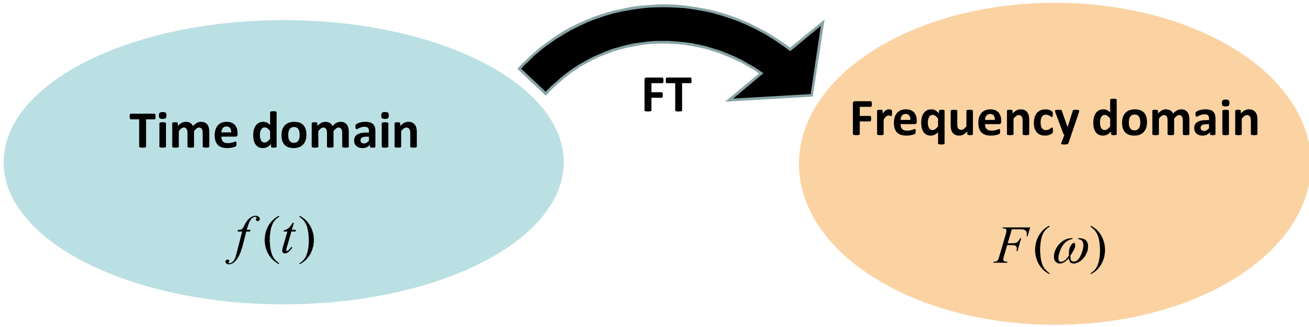
$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



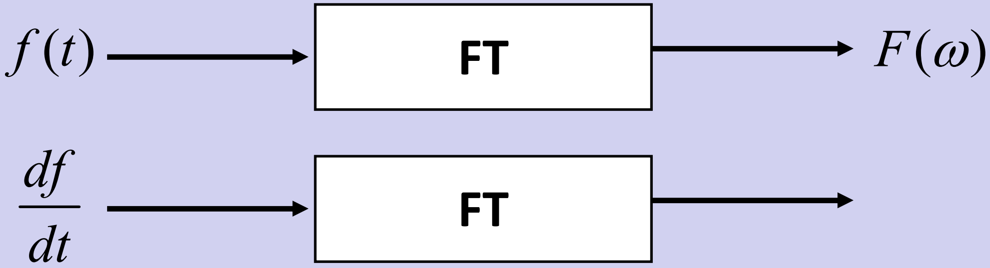
Frequency domain



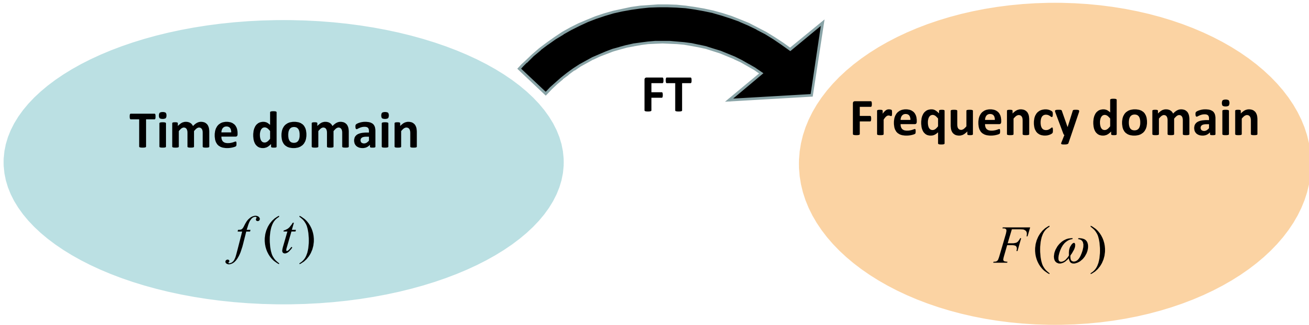
$$F(\omega)=\int_{-\infty}^{+\infty} f(t)\,e^{-j\omega t}\,dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



Frequency domain



$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform