Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Time domain (TD)

Spectral domains

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Incidence

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

$$-\zeta h_x(z,t) = e_y^+(z-ct) - e_y^-(z+ct)$$

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$\Downarrow$

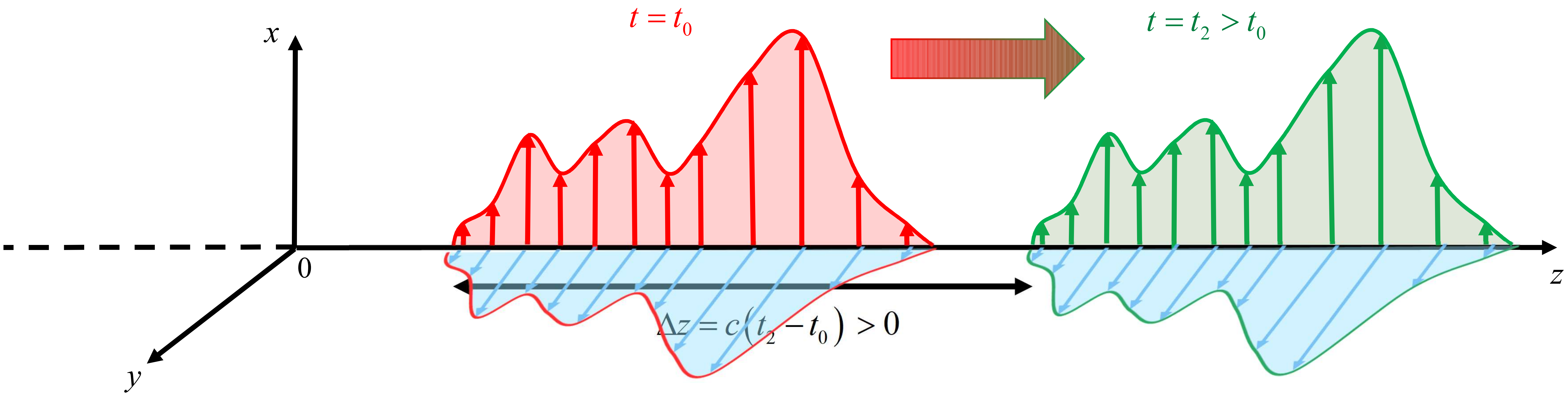
$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

Plane Waves (TD)

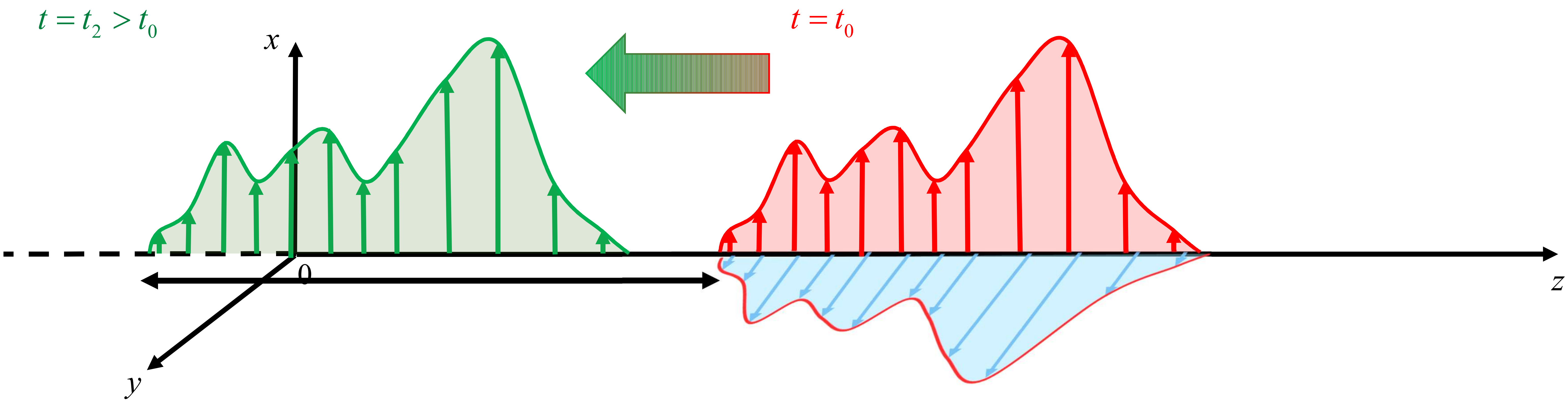


The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the z-axis

$$\begin{cases} e^{+}\left(z-ct\right) \\ h^{+}\left(z-ct\right) \end{cases}$$ is referred to as electromagnetic <u>**progressive**</u> **plane wave**

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^{-}(z+ct) \\ h^{-}(z+ct) \end{cases}$$ is referred to as electromagnetic **<u>regressive</u> plane wave**

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves

Time domain (TD)

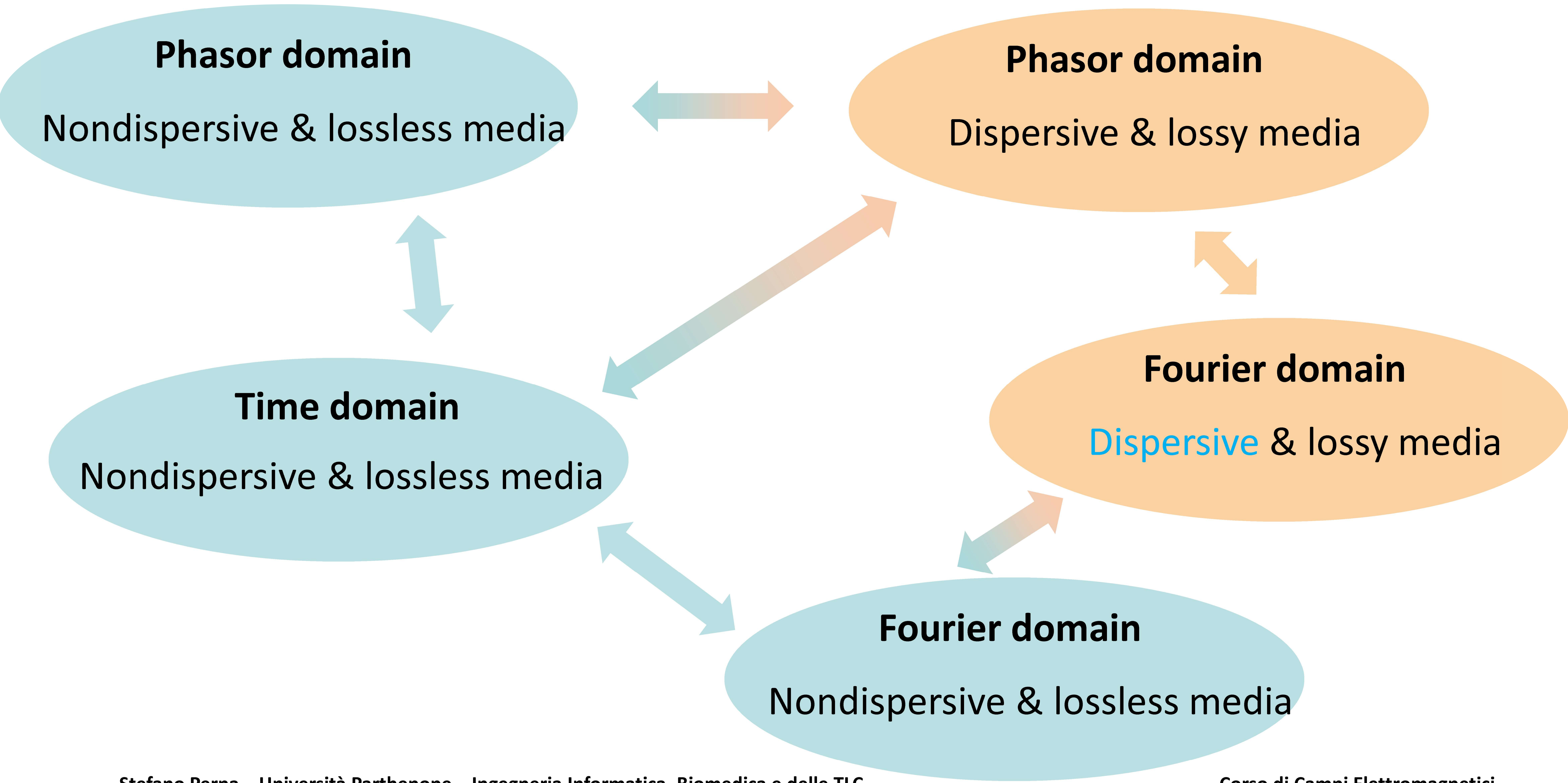
Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Rationale


Phasor domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Time domain
Nondispersive & lossless media
Fourier domain
Dispersive & lossy media
Fourier domain
Nondispersive & lossless media

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$-\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

Source-free

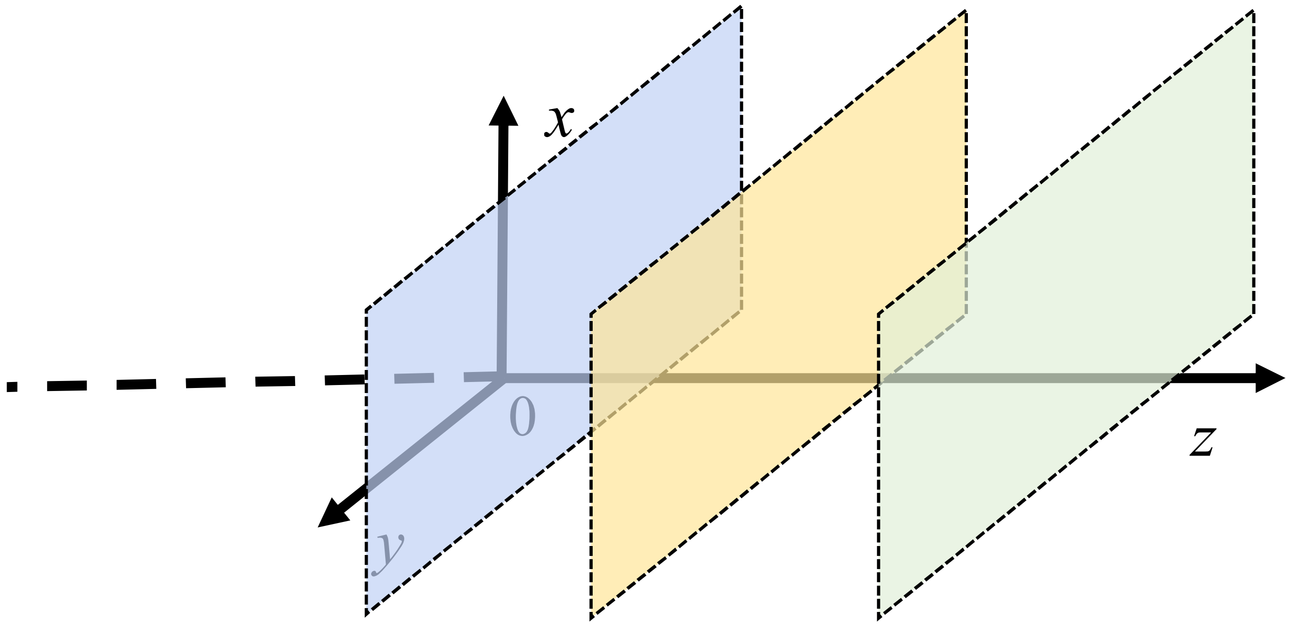
Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**



Plane Waves (Phasor Domain)

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\{E_y,H_x\}$, $\{E_x,H_y\}$ **Independent each other**

$$\{E_x,H_y\}$$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}=\beta(\omega_0)-j\alpha(\omega_0)$$

$$E_x^+(z)=E^+e^{-jkz}$$

$$\zeta H_y^+(z)=E^+e^{-jkz}$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\mathrm{Re}\left\{\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}}=\frac{1}{\mathrm{Re}\left\{\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}}=v_p(\omega_0)$$

$$E^+e^{-jkz}\quad\rightarrow\quad e^+(z,t)=e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu(\omega_0)=\mu_1(\omega_0)-j\mu_2(\omega_0)\\ \sigma:\ real\end{cases}$$

$$E_x^+(z)=E^+e^{-j\beta z}$$

$$\zeta H_y^+(z)=E^+e^{-j\beta z}$$

$$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu\varepsilon}}=c$$

Time nondispersive & lossless

$$\begin{cases}\varepsilon:real\\ \mu:real\\ \sigma=0\end{cases}$$

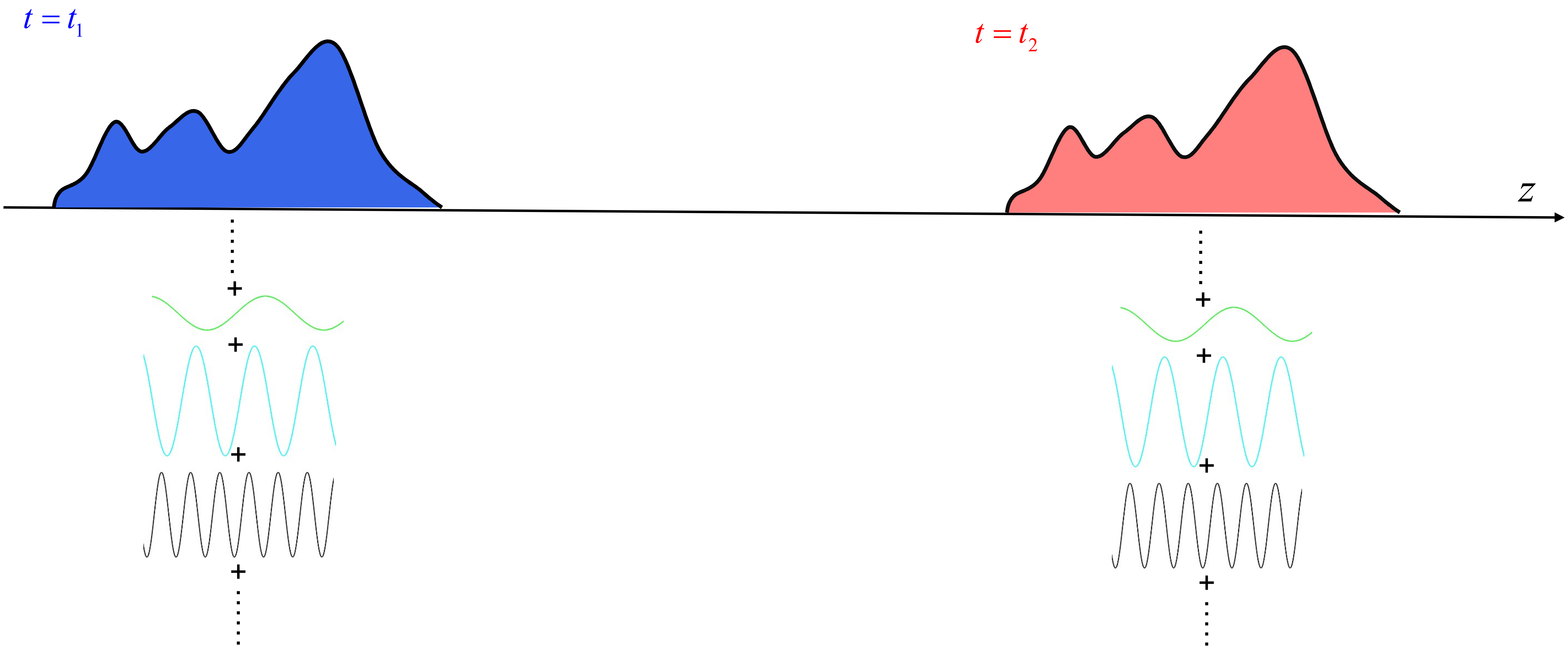
$$\boxed{c=\frac{1}{\sqrt{\mu\varepsilon}}}$$

$$E^+e^{-j\beta z}\rightarrow e_x^+(z,t)=\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)=e_x^+\left(z-v_p t\right)$$

Plane Waves

Time nondispersive & lossless medium

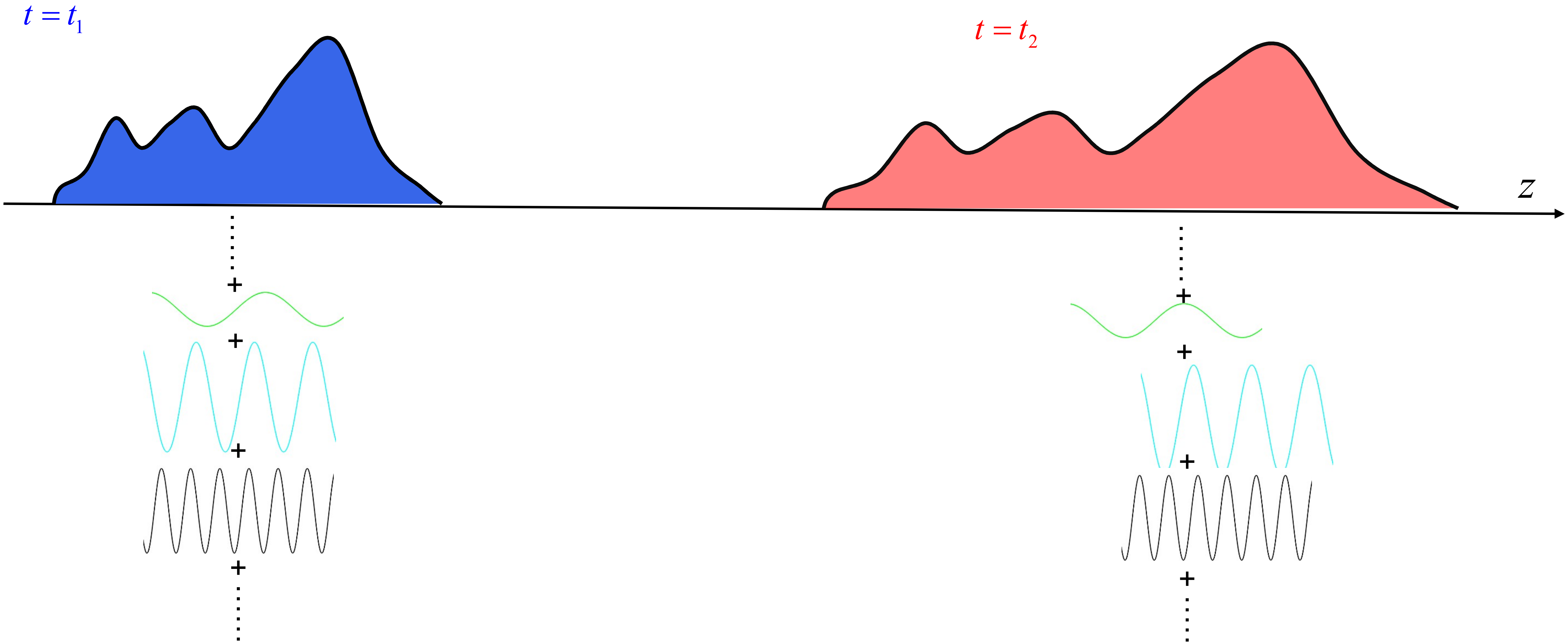
$$\boxed{v_p = c}$$



Plane Waves

Time dispersive medium

$$\boxed{v_p = v_p(\omega_0)}$$



Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\boxed{v_p = \frac{\omega_0}{\beta} = c}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

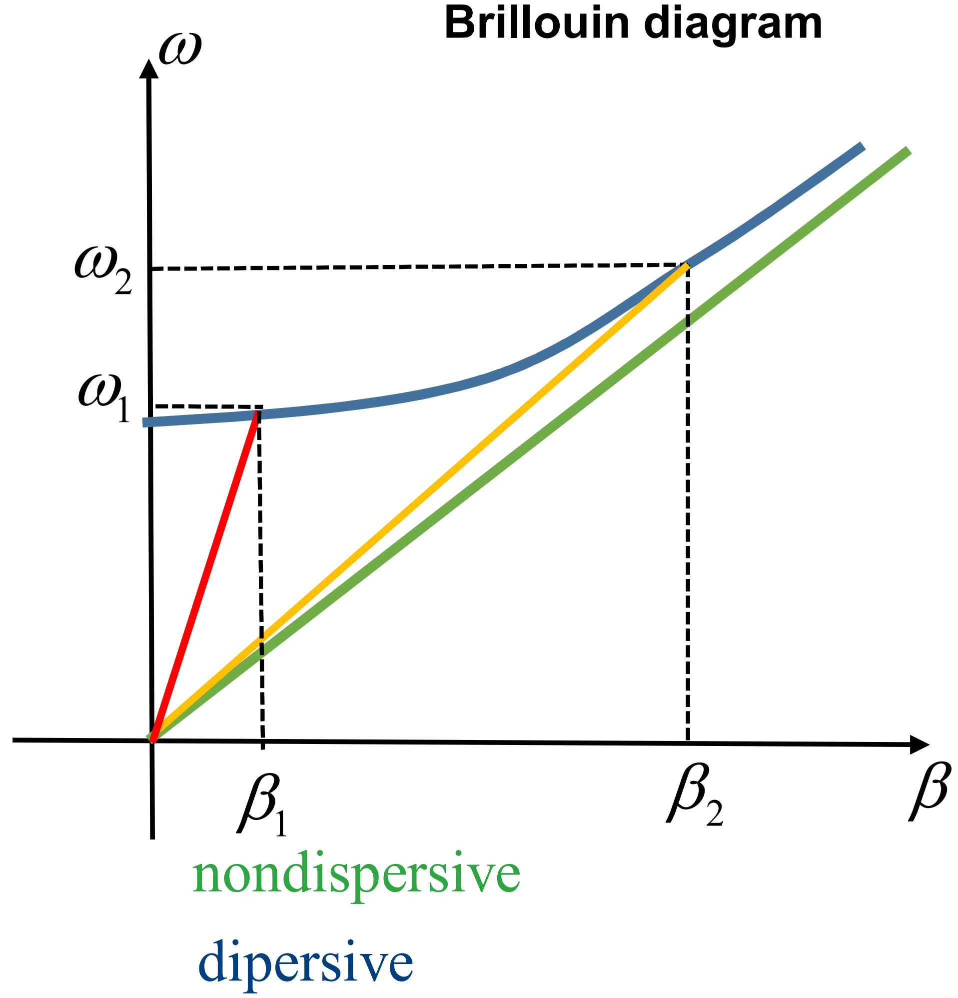
Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

Brillouin diagram



$$\boxed{v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)}$$

$$\boxed{v_p = \frac{\omega_0}{\beta} = c}$$

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- **Fourier Domain (FD)**
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega) e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega) e^{-j\beta z}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega) e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E^+(\omega) e^{j\omega\left(t-\frac{z}{c}\right)} d\omega$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$e_x^+(z=0,t) = \frac{1}{2\pi}\int E^+(\omega) e^{j\omega t} d\omega = f(t)$$

$$e_x^+(z>0,t) = \frac{1}{2\pi}\int E^+(\omega) e^{j\omega\left(t-\frac{z}{c}\right)} d\omega = f\left(t-\frac{z}{c}\right) = f\left[-\frac{1}{c}(z-ct)\right] = f\left[(z-ct)\right]$$

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

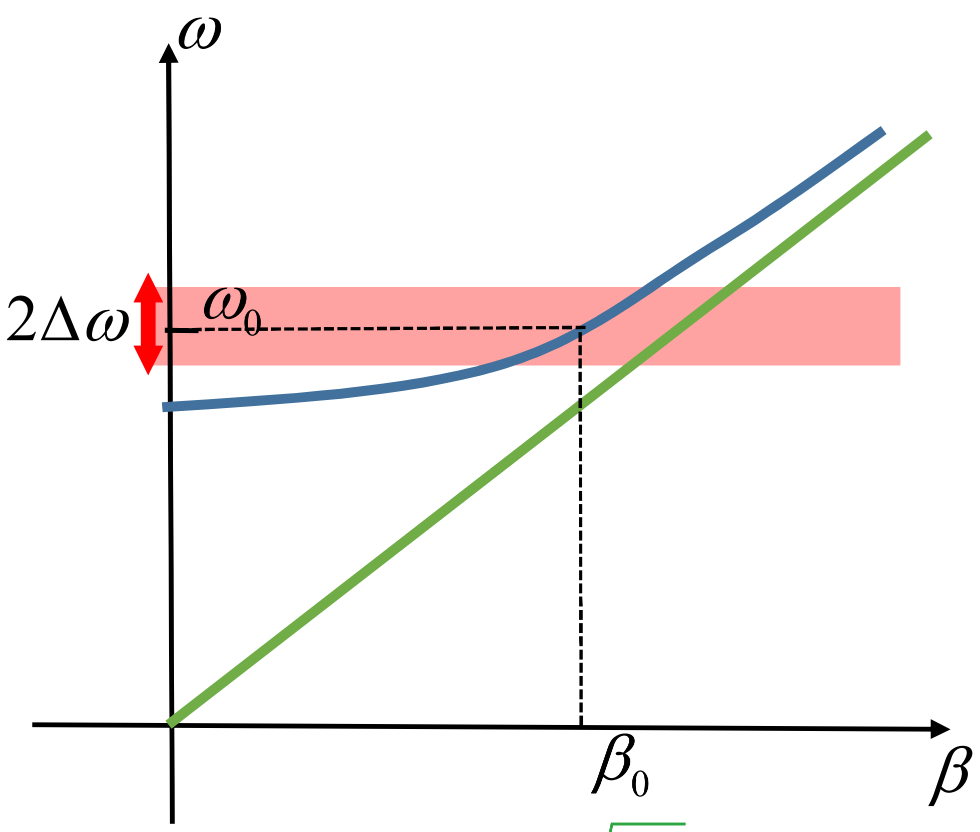
Plane Waves : dispersion

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$



nondispersive : $\beta = \omega\sqrt{\mu\varepsilon}$

dipersive : $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2} + \cdots$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

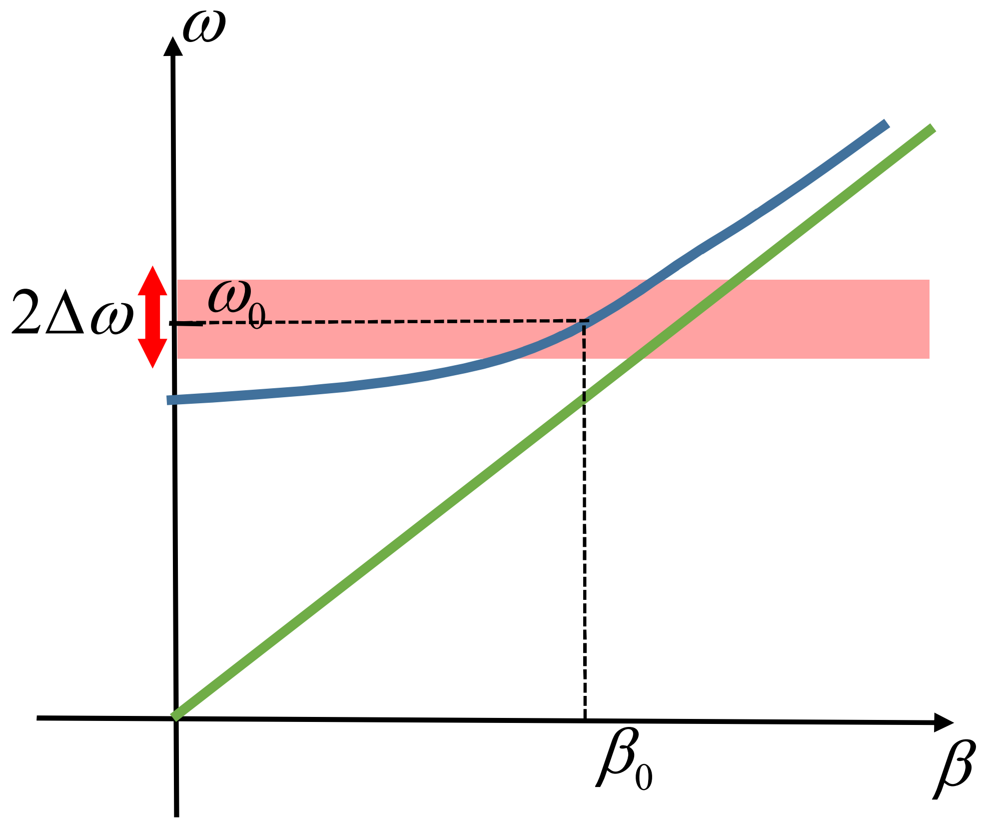
Plane Waves : dispersion

$$e_x^+(z,t)=\frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

$$\beta_0=\beta(\omega_0)$$

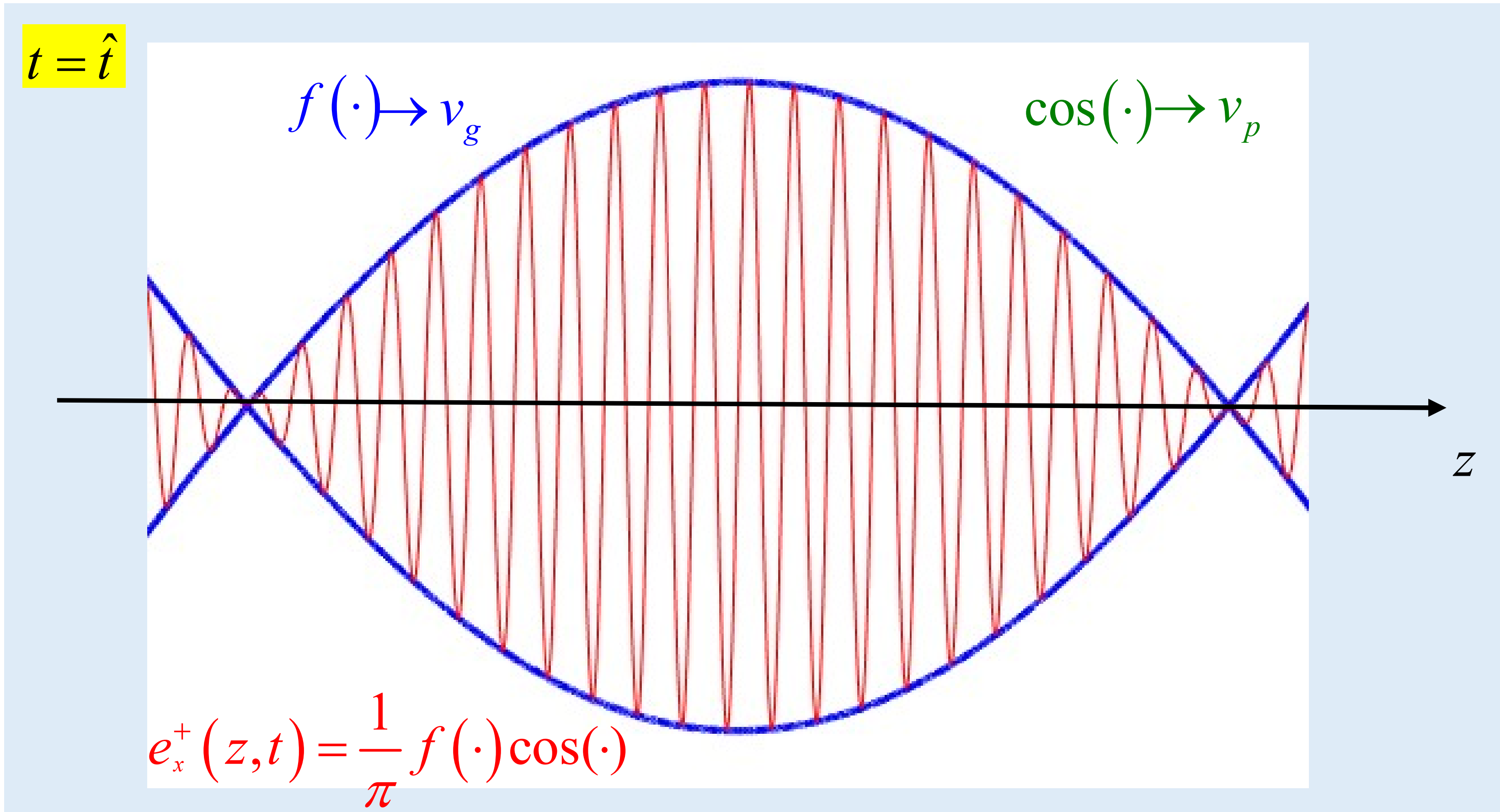
$$v_g=\frac{1}{\beta'(\omega_0)}$$

$$v_p=\frac{\omega_0}{\beta_0}$$

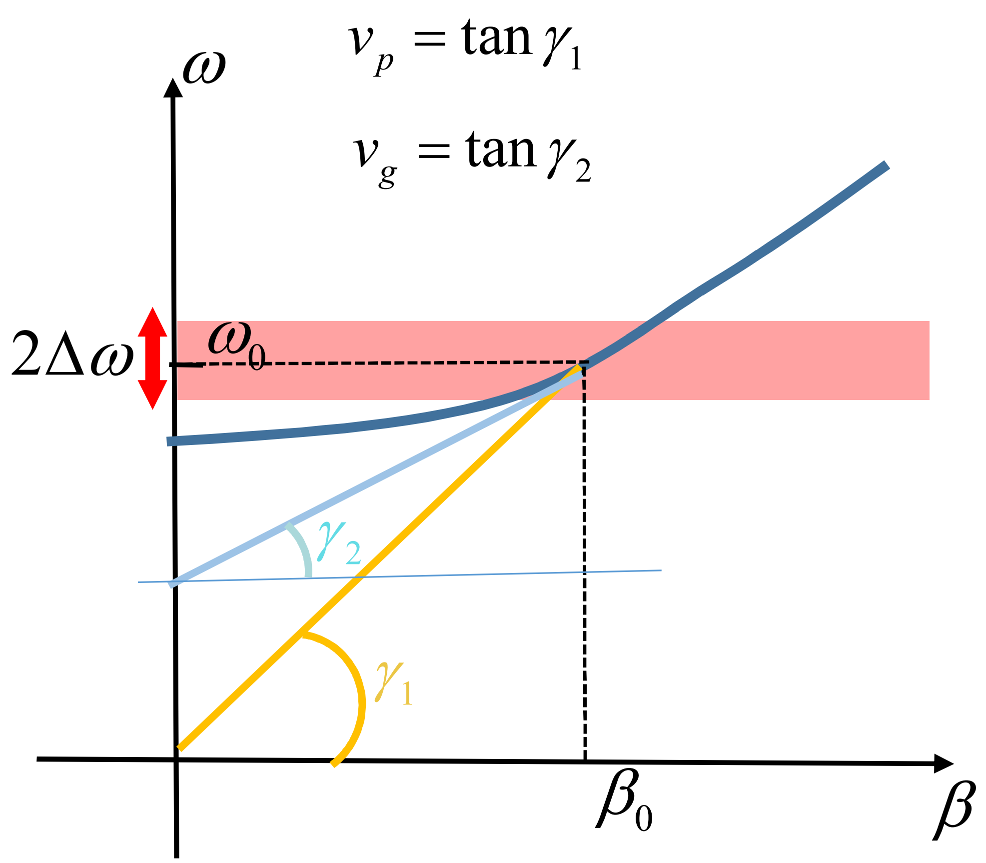


nondispersive : $\beta=\omega\sqrt{\mu\varepsilon}$

dipersive : $\beta=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}$



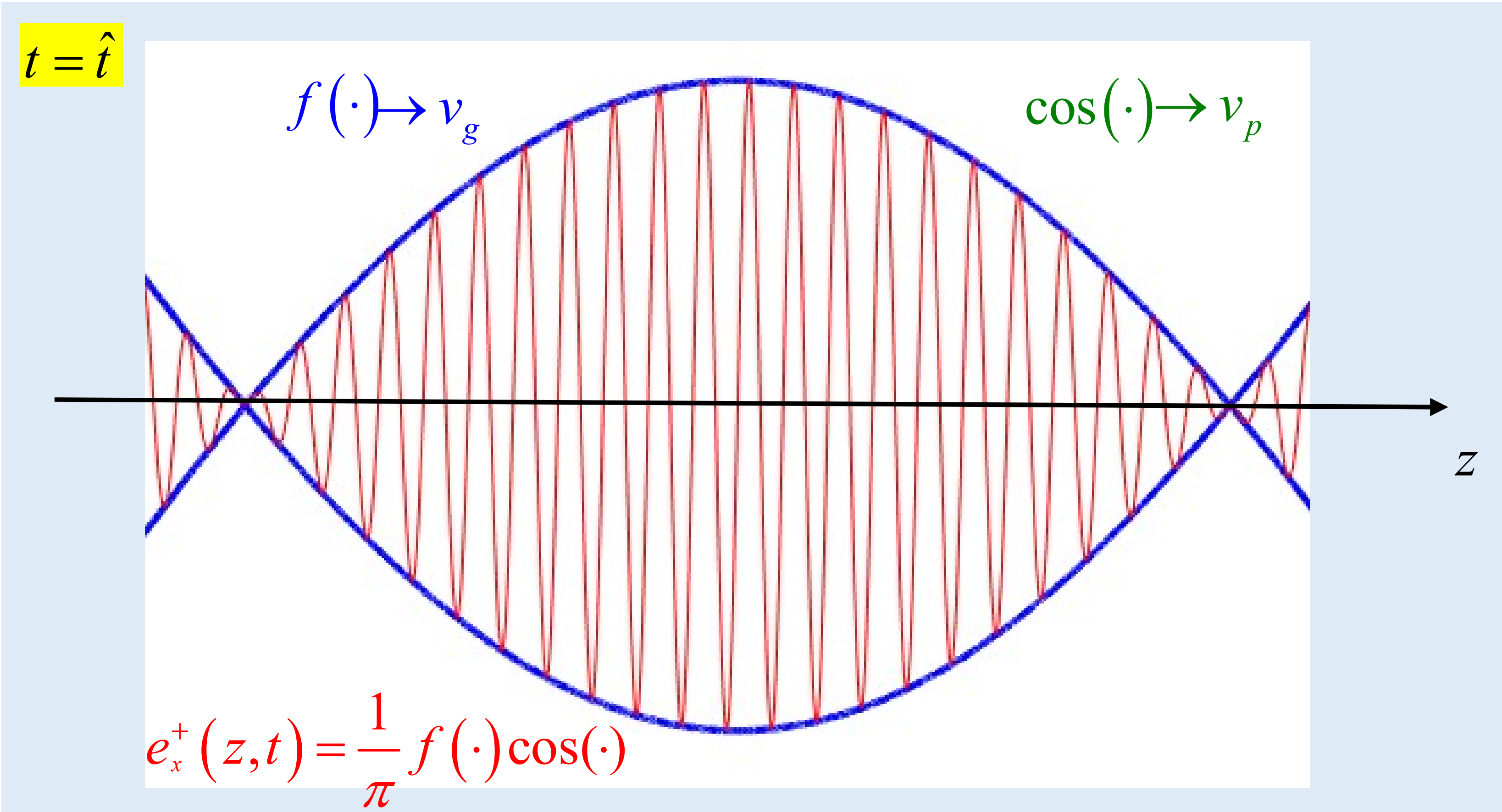
Plane Waves : dispersion



dipersive : $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$e_x^+(z,t) = \frac{1}{\pi} f\left(t - \frac{z}{v_g}\right)\cos\left[\omega_0\left(t - \frac{z}{v_p}\right)\right]$$

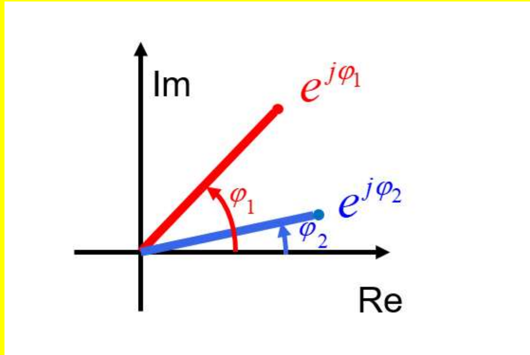
$$\beta_0 = \beta(\omega_0)$$
$$v_g = \frac{1}{\beta'(\omega_0)} = \omega'(\beta_0)$$
$$v_p = \frac{\omega_0}{\beta_0}$$



Plane Waves : dispersion

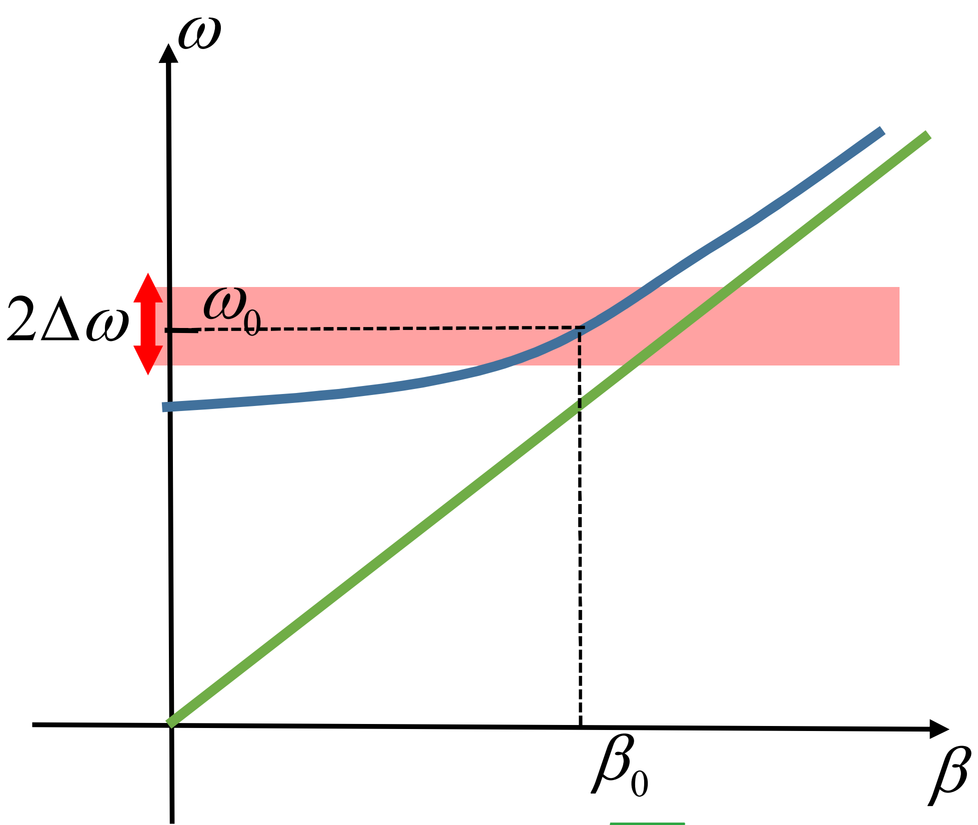
$$e^{-j\beta(\omega)z} = e^{-j\beta(\omega_0)z} e^{-j\beta'(\omega_0)(\omega-\omega_0)z} \cancel{e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z}} \cdots$$

$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2}\beta''(\omega_0)\Delta\omega^2 z << 2\pi$$

Channel & carrier frequency — Bandwidth — Distance



$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2} + \cdots$$

$\text{nondispersive}: \beta = \omega\sqrt{\mu\varepsilon}$

$\text{dipersive}: \beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- **Fourier Domain (FD)**
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

General expression of plane waves (PD)

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \{E_x, H_y\} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \{E_x, H_y\} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\left\{E_x, H_y\right\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
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$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$
$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

Source-free

Medium
- **Linear**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$
$\left\{E_x, H_y\right\}$
Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$
$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

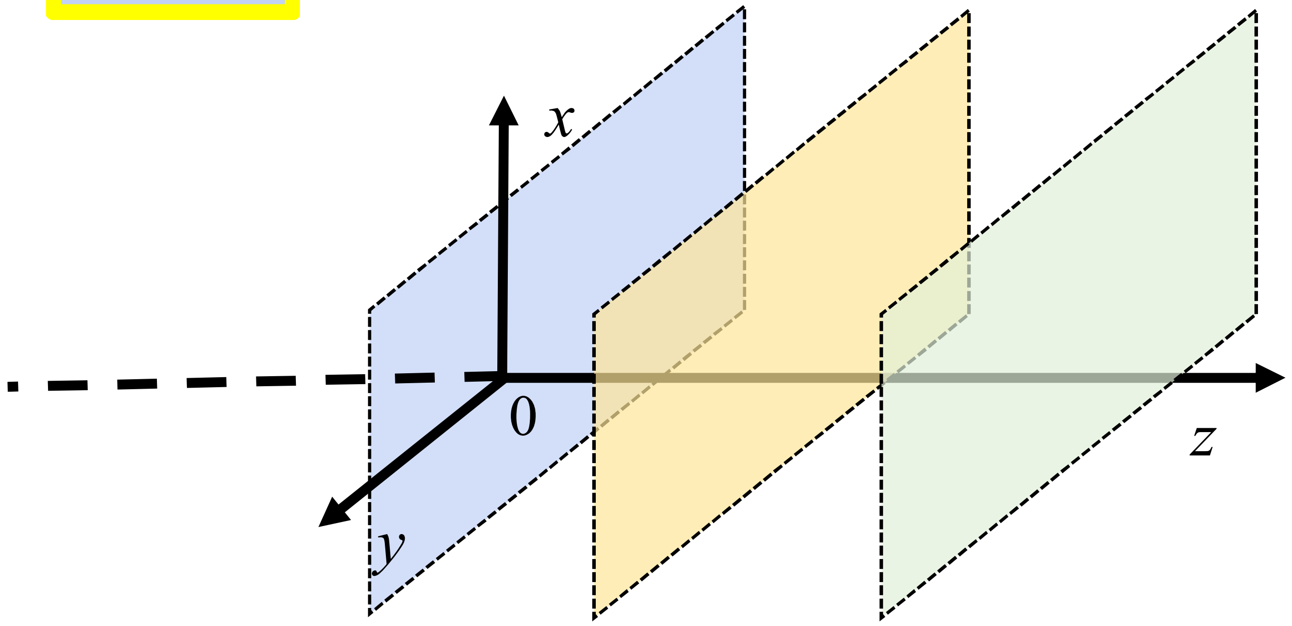
$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z) = E^+ e^{-jkz} = E^+ e^{-j\beta z} e^{-\alpha z}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz} = E^+ e^{-j\beta z} e^{-\alpha z}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$
$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain



Source-free

Medium
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- Time dispersive
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- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
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$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z) = E^+ e^{-jkz} = E^+ e^{-j\beta z} e^{-\alpha z}$$
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$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

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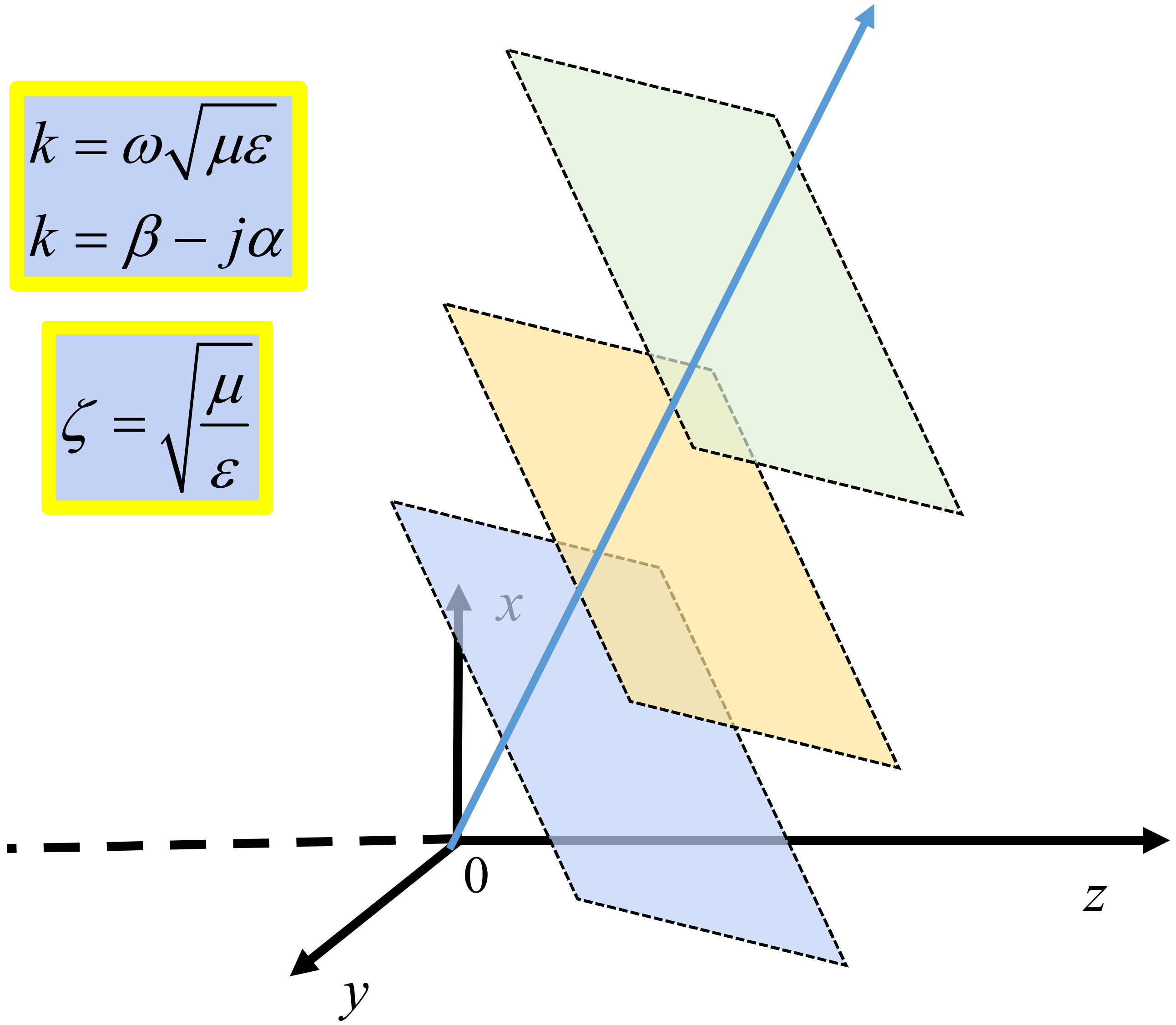
Phasor Domain



Source-free

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\frac{\partial}{\partial x} \to -jk_x \qquad \frac{\partial}{\partial y} \to -jk_y \qquad \frac{\partial}{\partial z} \to -jk_z$$

$$\frac{\partial}{\partial x}\vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \frac{\partial}{\partial x}\vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = -jk_x \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = -jk_x \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

$$\frac{\partial}{\partial x} \to -jk_x \qquad \frac{\partial}{\partial y} \to -jk_y \qquad \frac{\partial}{\partial z} \to -jk_z$$

$$\Downarrow$$

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ -jk_x & -jk_y & -jk_z \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{\mathbf{k}}\times\vec{\mathbf{E}}$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases} \rightarrow \begin{cases} -j\vec{\mathbf{k}}\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}}\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \ldots\ldots \\ \ldots\ldots \end{cases}$$

$$\frac{\partial}{\partial x} \rightarrow -jk_x \qquad \frac{\partial}{\partial y} \rightarrow -jk_y \qquad \frac{\partial}{\partial z} \rightarrow -jk_z$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ -jk_x & -jk_y & -jk_z \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{\mathbf{k}}\times\vec{\mathbf{E}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases} \rightarrow \begin{cases} -j\vec{\mathbf{k}}\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}}\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \ldots\ldots \\ \ldots\ldots\ldots \end{cases}$$

$$\frac{\partial}{\partial x} \rightarrow -jk_x \qquad \frac{\partial}{\partial y} \rightarrow -jk_y \qquad \frac{\partial}{\partial z} \rightarrow -jk_z$$

$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$$

$$\nabla\cdot\vec{\mathbf{E}} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla\cdot\vec{\mathbf{E}} = -jk_x E_x - jk_y E_y - jk_z E_z = -j\vec{\mathbf{k}}\cdot\vec{\mathbf{E}}$$

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$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases} \rightarrow \begin{cases} -j\vec{\mathbf{k}}\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}}\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ -j\vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0 \\ -j\vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

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$$\nabla\cdot\vec{\mathbf{E}} = -jk_x E_x - jk_y E_y - jk_z E_z = -j\vec{\mathbf{k}}\cdot\vec{\mathbf{E}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla \cdot \vec{\mathbf{E}} = 0 \\ \nabla \cdot \vec{\mathbf{H}} = 0 \end{cases} \rightarrow \begin{cases} -j\vec{\mathbf{k}} \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}} \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ -j\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ -j\vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

Source-free

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

Source-free

$$\rightarrow \begin{cases} -j\vec{\mathbf{k}} \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}} \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ -j\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ -j\vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

$$\begin{cases} \boxed{\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}}} \\ \boxed{\vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}}} \\ \boxed{\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{k}} \times \left(\vec{\mathbf{k}} \times \vec{\mathbf{E}}\right) = \omega\mu\boxed{\left(\vec{\mathbf{k}} \times \vec{\mathbf{H}}\right)} = \omega\mu\left(-\omega\varepsilon\vec{\mathbf{E}}\right) = \boxed{-\omega^2\mu\varepsilon\vec{\mathbf{E}}}$$

$$\boxed{\cancel{\vec{\mathbf{k}}\left(\vec{\mathbf{k}}\cdot\vec{\mathbf{E}}\right)} - \vec{\mathbf{E}}\left(\vec{\mathbf{k}}\cdot\vec{\mathbf{k}}\right)}$$

$$\boxed{-\left(\vec{\mathbf{k}}\cdot\vec{\mathbf{k}}\right)\vec{\mathbf{E}} = -\omega^2\mu\varepsilon\vec{\mathbf{E}}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}}\right) = \vec{\mathbf{B}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right) - \vec{\mathbf{C}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$$\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = \omega^2 \mu\varepsilon$$