Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

$$k=\omega\sqrt{\mu\varepsilon}$$
$$k=\beta-j\alpha$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\{E_x, H_y\}$$

$$\begin{cases}\dfrac{dE_x}{dz}=-j\omega\mu H_y\\[2mm] \dfrac{dH_y}{dz}=-j\omega\varepsilon E_x\end{cases} \qquad \frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x(z,\omega)=E_x^+(\omega)e^{-jkz}+E_x^-(\omega)e^{jkz}$$

$$\zeta H_y(z,\omega)=E_x^+(\omega)e^{-jkz}-E_x^-(\omega)e^{jkz}$$

$$\begin{cases}\varepsilon(\omega)=\varepsilon_1(\omega)-j\varepsilon_2(\omega)\\ \mu(\omega)=\mu_1(\omega)-j\mu_2(\omega)\end{cases}$$

$$k(\omega)=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega)=\beta(\omega)-j\alpha(\omega)$$

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$
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$$\{E_y, H_x\}$$
$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Fourier Domain)

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$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

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$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

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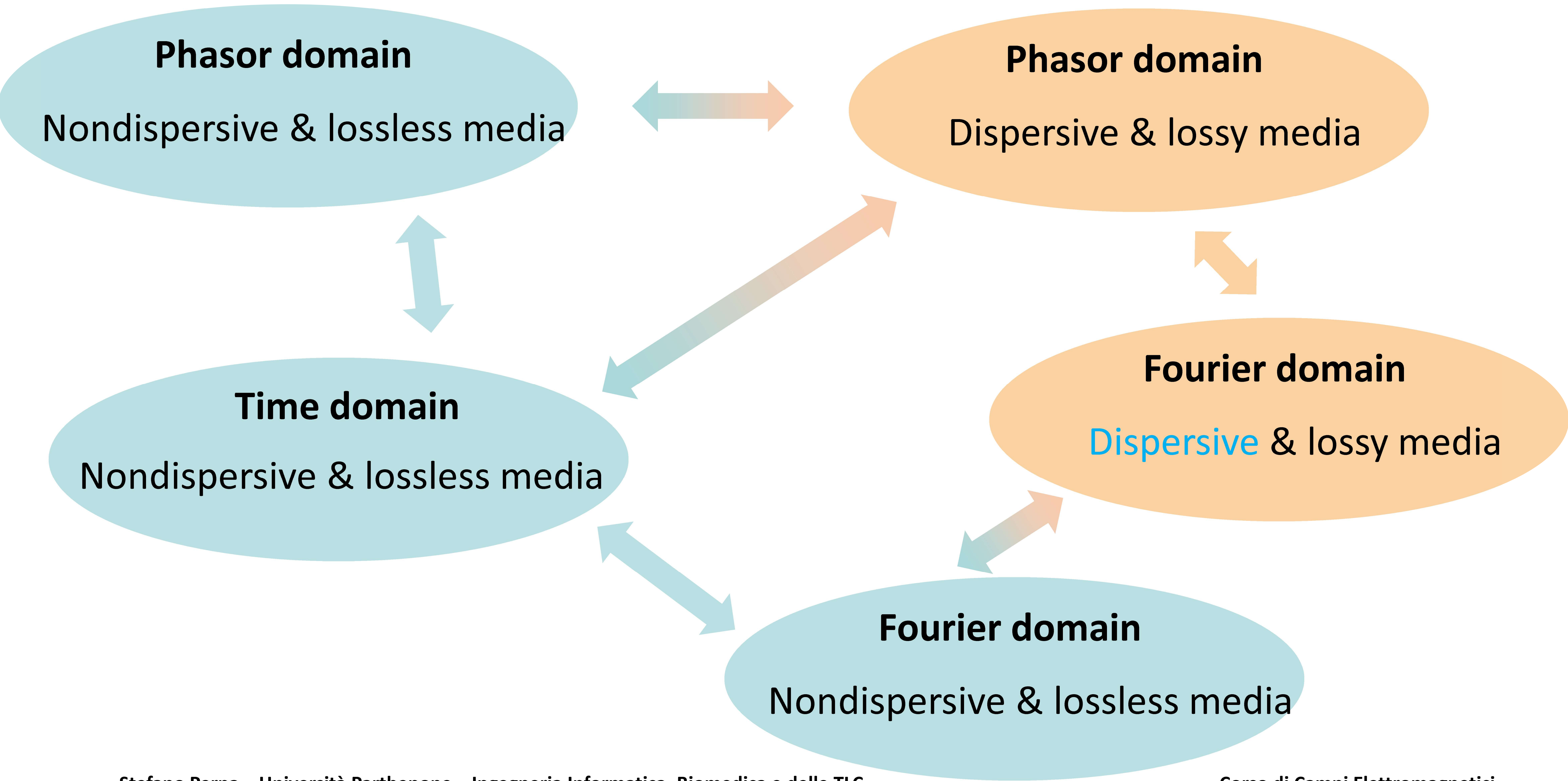
$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Rationale


Phasor domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Time domain
Nondispersive & lossless media
Fourier domain
Dispersive & lossy media
Fourier domain
Nondispersive & lossless media

Plane Waves (Fourier Domain)

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \\ \sigma : real \end{cases}$$

$$\varepsilon_{eq}(\omega) = \varepsilon(\omega)\left[1 - \frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

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$\{E_x, H_y\}$

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$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz}$$

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Fourier Domain

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$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \\ \sigma : \ real \end{cases}$$

$$\varepsilon_{eq}(\omega) = \varepsilon(\omega)\left[1 - \frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

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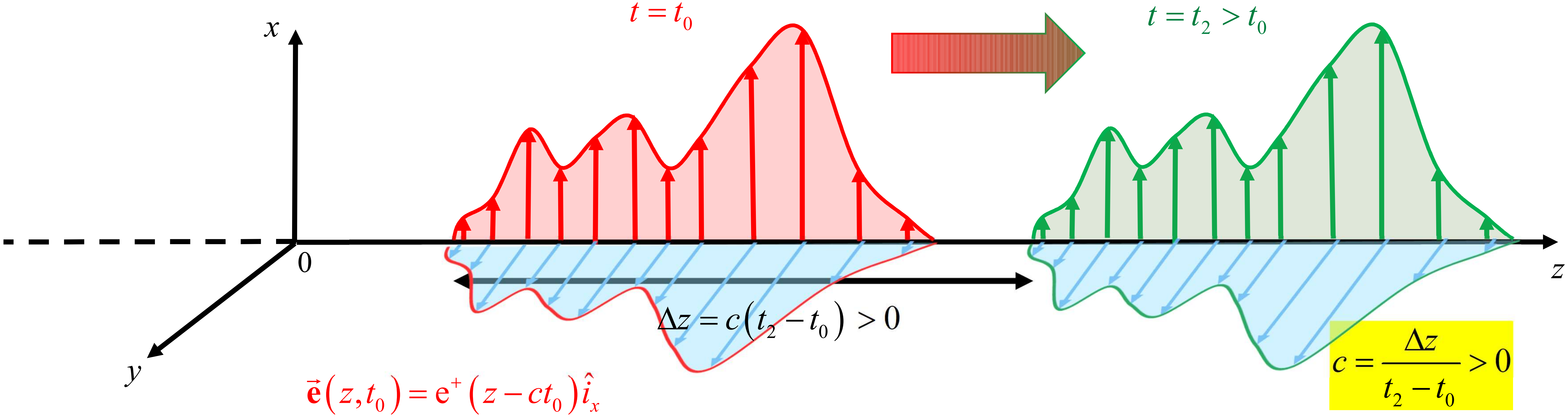
Independent each other

Plane Waves



$$\vec{\mathbf{e}}(z,t_0) = \mathrm{e}^+(z - ct_0)\,\hat{i}_x$$

$$\vec{\mathbf{e}}(z,t_2) = \mathrm{e}^+(z - ct_2)\,\hat{i}_x = \mathrm{e}^+(z - ct_0 + ct_0 - ct_2)\,\hat{i}_x = \mathrm{e}^+\left(z - ct_0 - c[t_2 - t_0]\right)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the z-axis

$\begin{cases} e^+(z - ct) \\ h^+(z - ct) \end{cases}$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves : dispersion

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega) - j\cancel{\alpha(\omega)}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta z}\cancel{e^{-\alpha z}}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta z}\cancel{e^{-\alpha z}}$$

$$\boxed{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta(\omega)z}e^{-\alpha(\omega)z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E_x^+(z,\omega)e^{j\omega t}d\omega\right\}$$

Source-free

Medium

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- $\boxed{\textbf{Lossless}}$

$$\alpha(\omega) \approx 0$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

Plane Waves : dispersion

$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta z}$$
$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta z}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta(\omega)z}e^{-\alpha(\omega)z} \approx E^+(\omega)e^{-j\beta(\omega)z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E_x^+(z,\omega)e^{j\omega t}d\omega\right\}$$

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Plane Waves : dispersion

$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$

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$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

Plane Waves : dispersion

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

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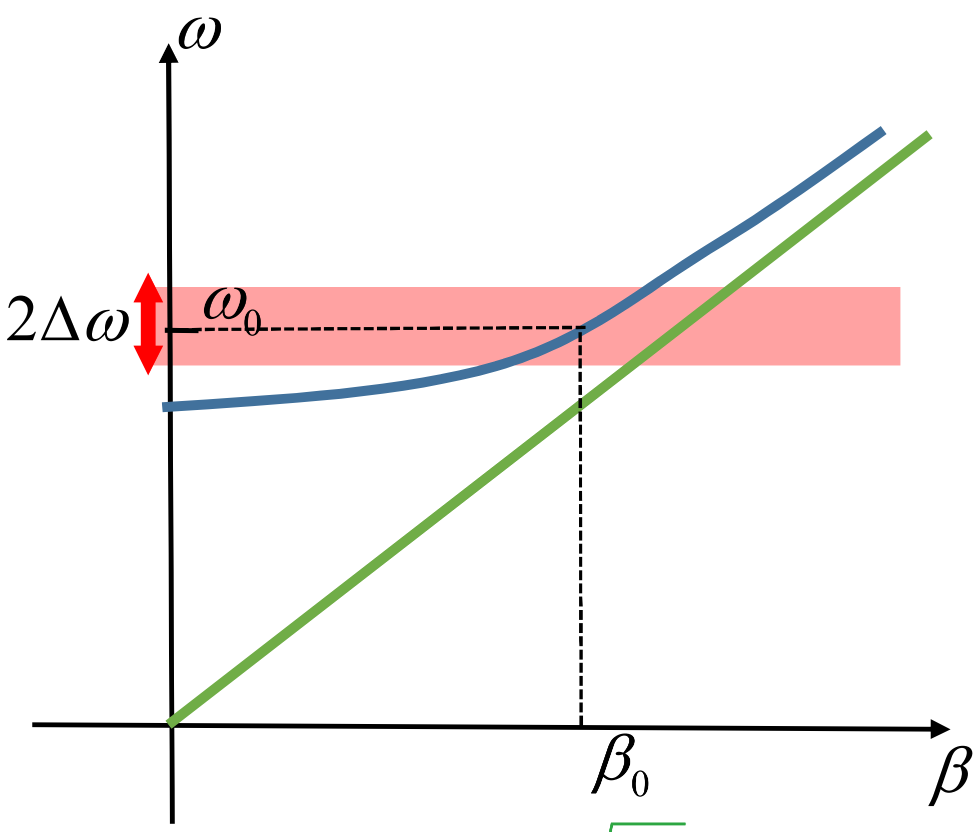

nondispersive : $\beta = \omega\sqrt{\mu\varepsilon}$

dipersive : $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2} + \cdots$$

Source-free

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$$\beta(\omega) \approx \beta_0 + \frac{(\omega-\omega_0)}{v_g}$$

$$\beta_0 = \beta(\omega_0)$$

$$v_g = \frac{1}{\beta'(\omega_0)}$$

Plane Waves : dispersion

$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\left[\beta_0+\frac{(\omega-\omega_0)}{v_g}\right]z}e^{j\omega t}d\omega$$

$$=\frac{1}{\pi}\int_{-\Delta\omega}^{\Delta\omega} E^+(\eta+\omega_0)e^{-j\left[\beta_0+\frac{\eta}{v_g}\right]z}e^{j(\eta+\omega_0)t}d\eta = \frac{1}{\pi}\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)\boxed{e^{-j\frac{\eta}{v_g}z}e^{j\eta t}}d\eta$$

$$=\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

$$\eta = \omega-\omega_0$$

$$\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}} = e^{j\omega_0\left(t-\frac{\beta_0}{\omega_0}z\right)} = \boxed{e^{j\omega_0\left(t-\frac{z}{v_p}\right)}}$$

Source-free

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$$\beta(\omega)\approx\beta_0+\frac{(\omega-\omega_0)}{v_g}$$

$$\beta_0=\beta(\omega_0)$$

$$v_g=\frac{1}{\beta'(\omega_0)}$$

$$v_p=\frac{\omega_0}{\beta_0}$$

Plane Waves : dispersion

$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$=\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

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Plane Waves : dispersion

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$$\beta_0 = \beta(\omega_0)$$

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$$v_p = \frac{\omega_0}{\beta_0}$$

$$\frac{1}{\pi}\int_0^\infty E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega = \frac{1}{\pi} e^{j\omega_0\left(t - \frac{z}{v_p}\right)} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0 + \eta) e^{j\left(t - \frac{z}{v_g}\right)\eta} d\eta$$

Plane Waves : dispersion

$$\{E_x, H_y\}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

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$$\boxed{e_x^+(z,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta\right\}}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta(\omega)z}e^{-\alpha(\omega)z} \approx E^+(\omega)e^{-j\beta(\omega)z}$$

$$\boxed{e_x^+(z,t)} = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E_x^+(z,\omega)e^{j\omega t}d\omega\right\} = \boxed{\mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega\right\}} = ?$$

$$e_x^+(z,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta\right\}$$

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Plane Waves : dispersion

$\{E_x, H_y\}$

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$$e_x^+(z=0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}\underbrace{\boxed{\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\eta t}d\eta}}_{f(t)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}f(t)\right\} = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\underbrace{\boxed{\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta}}_{f\left(t-\frac{z}{v_g}\right)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}f\left(t-\frac{z}{v_g}\right)\right\}$$

$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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Plane Waves : dispersion

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$

$$\boxed{e_x^+(z,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta\right\}}$$

$$e_x^+(z=0,t) = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t)$$

$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

Source-free

Medium

- Linear
- $\boxed{\text{Time dispersive}}$
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

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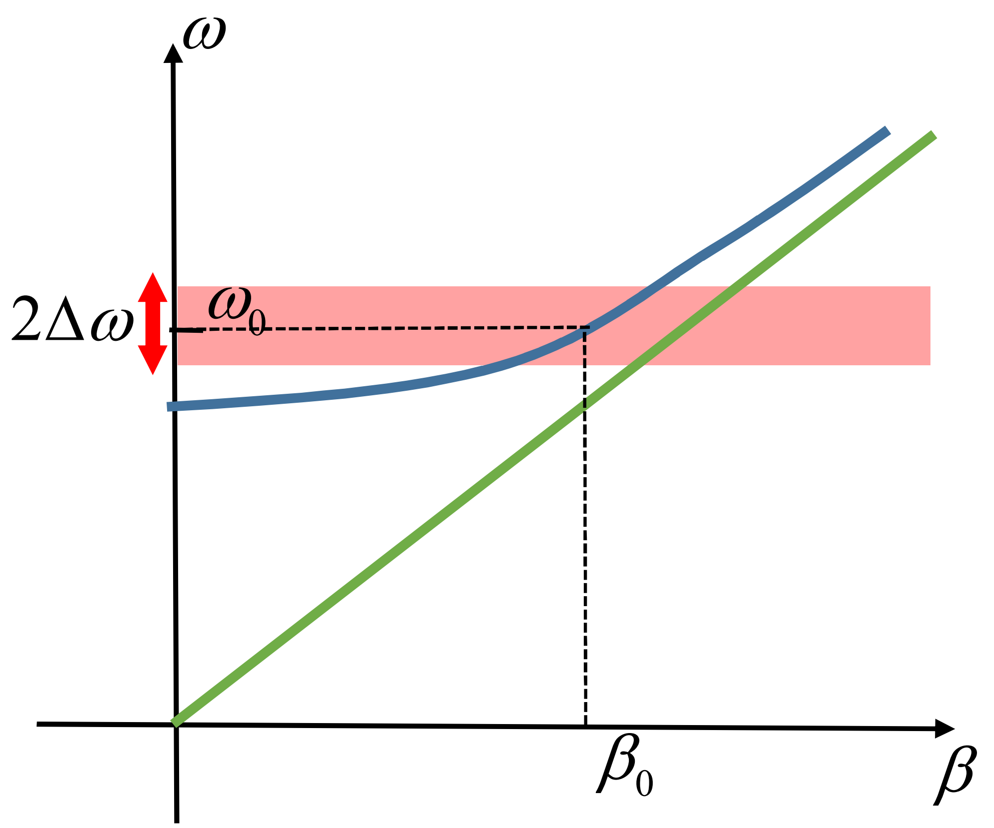

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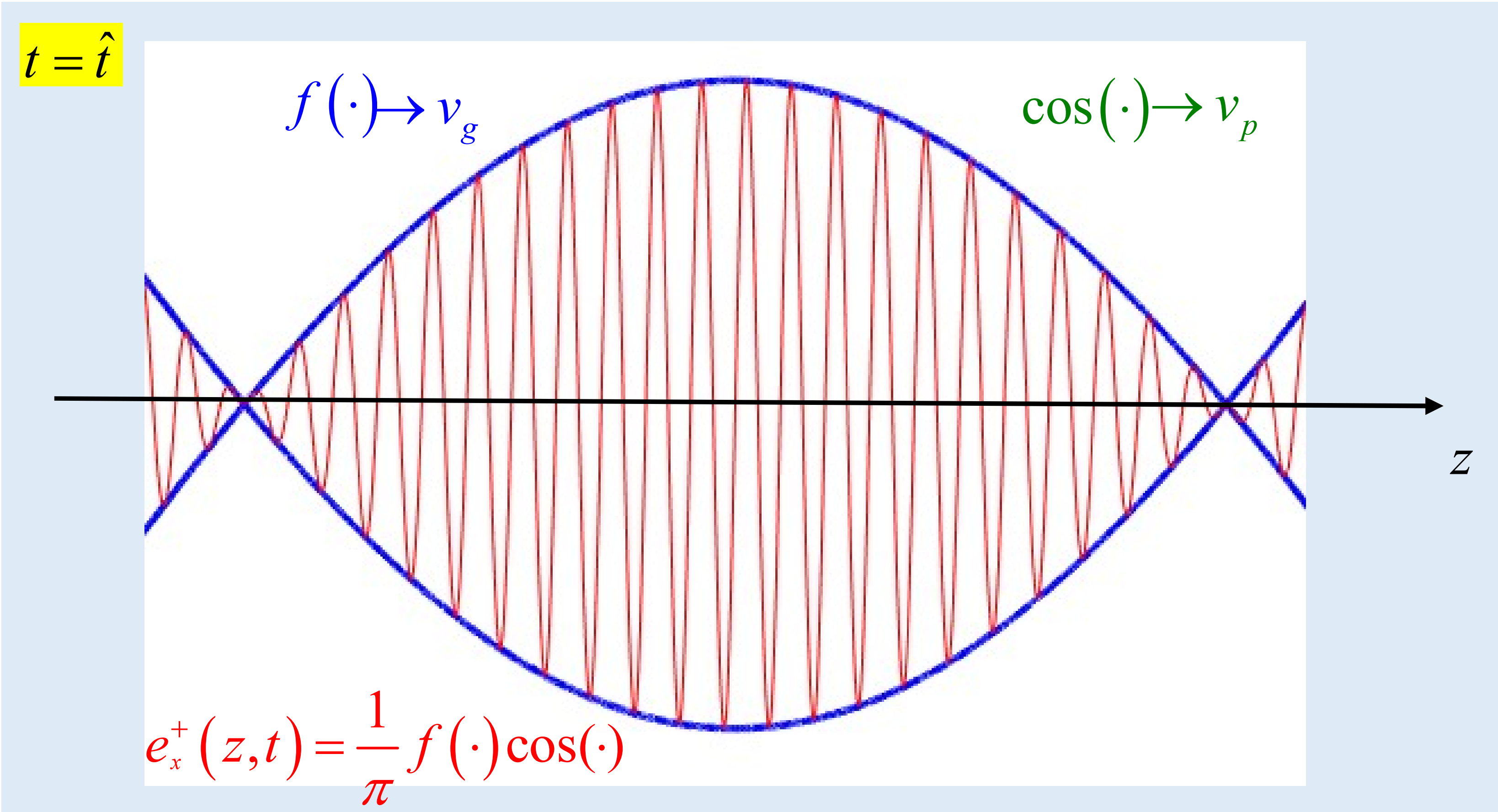
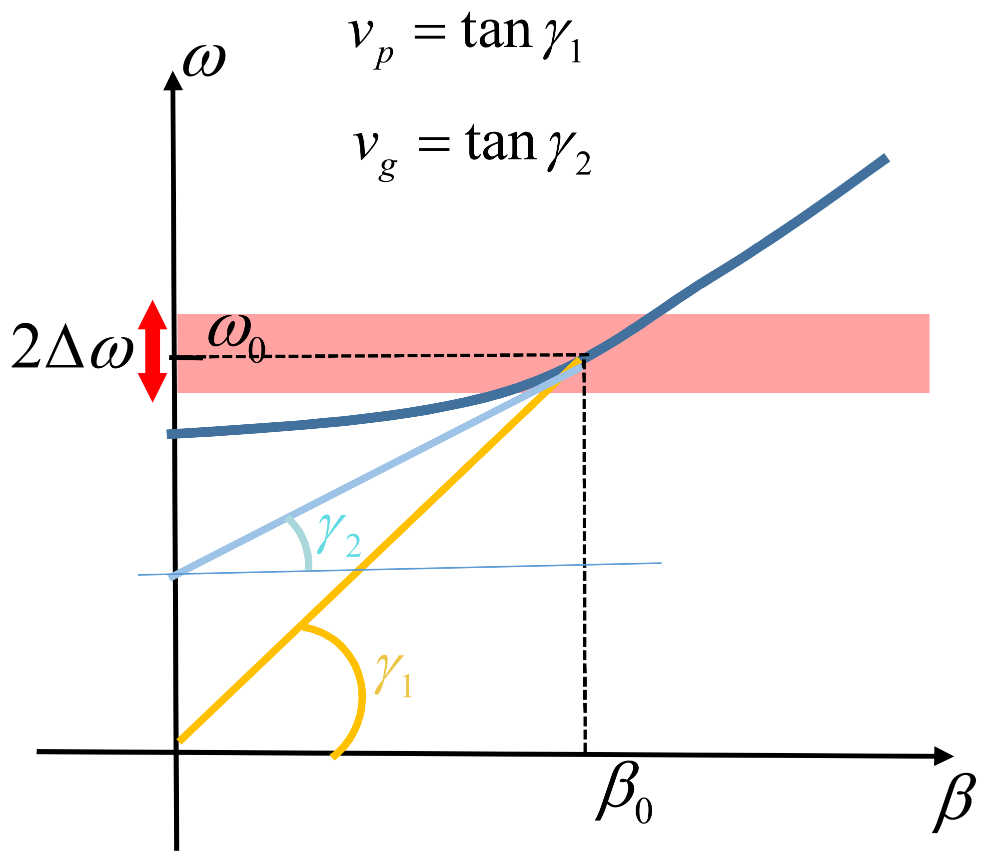


Plane Waves : dispersion



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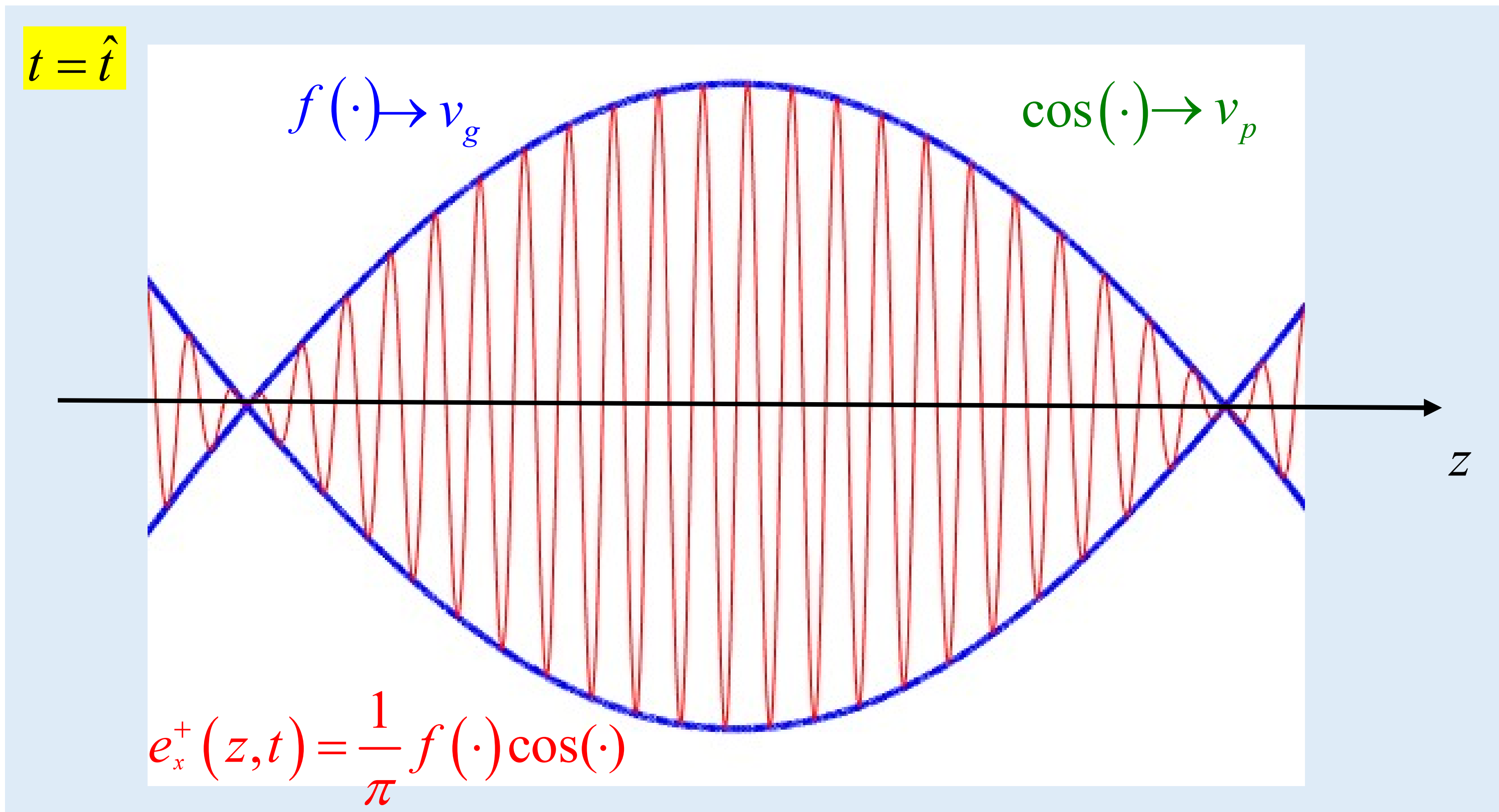

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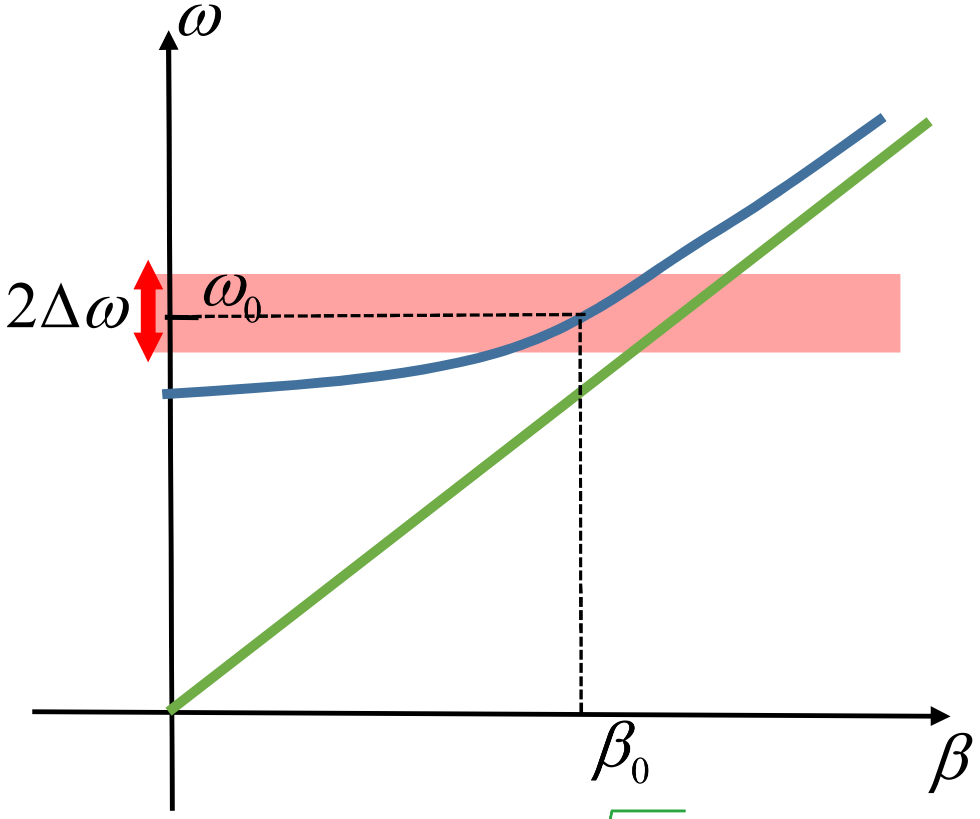

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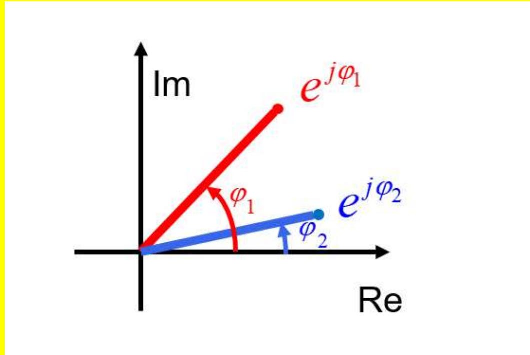
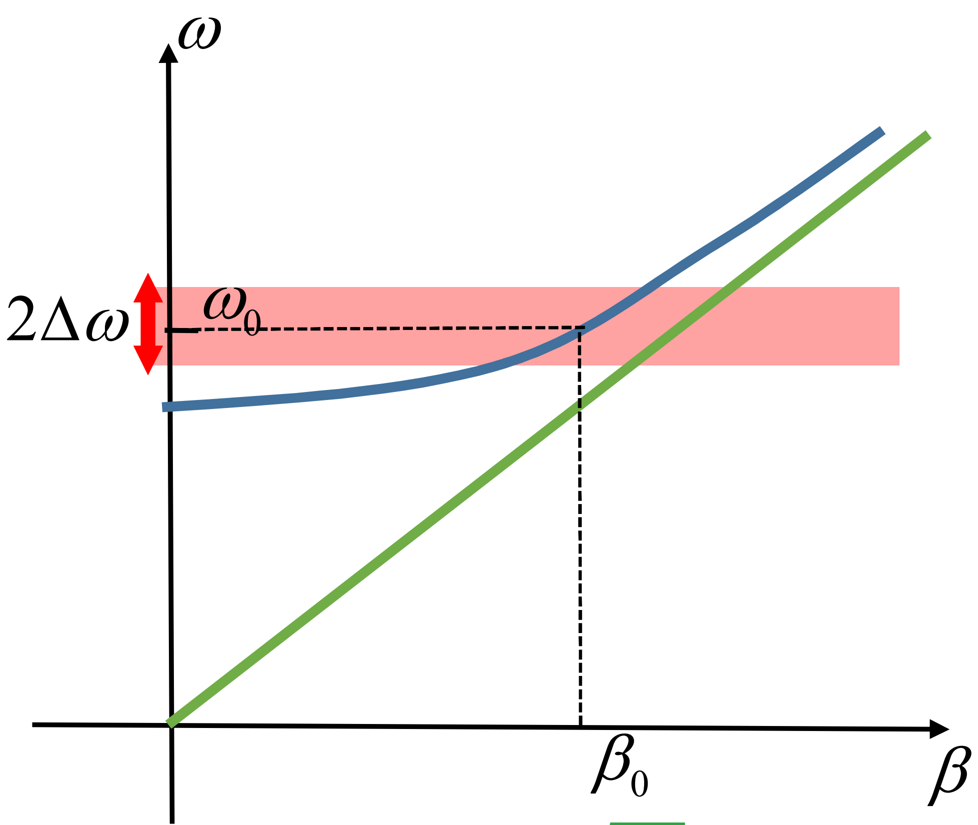
$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2}\boxed{\beta''(\omega_0)}\Delta\omega^2 z << 2\pi$$

Channel &
carrier frequency



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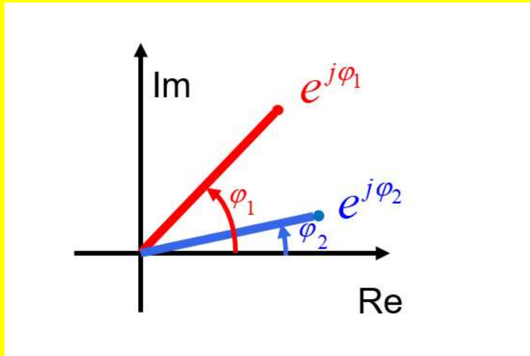
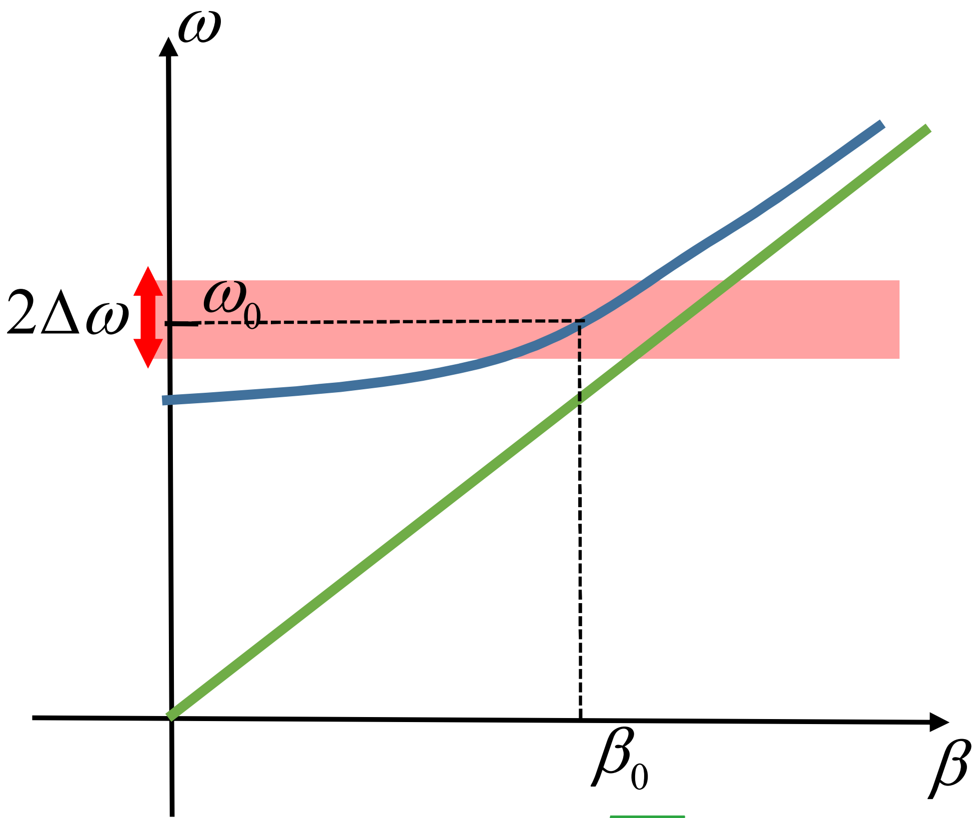


$$\frac{1}{2}\beta''(\omega_0)\boxed{\Delta\omega^2} z << 2\pi$$

Channel & carrier frequency

Bandwidth



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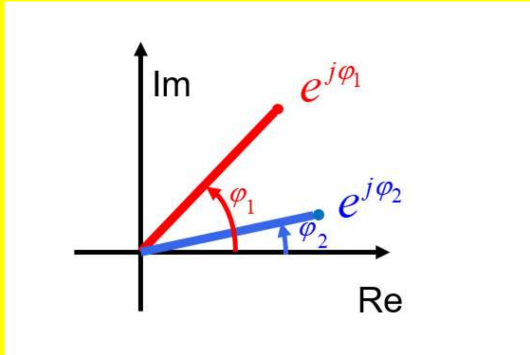

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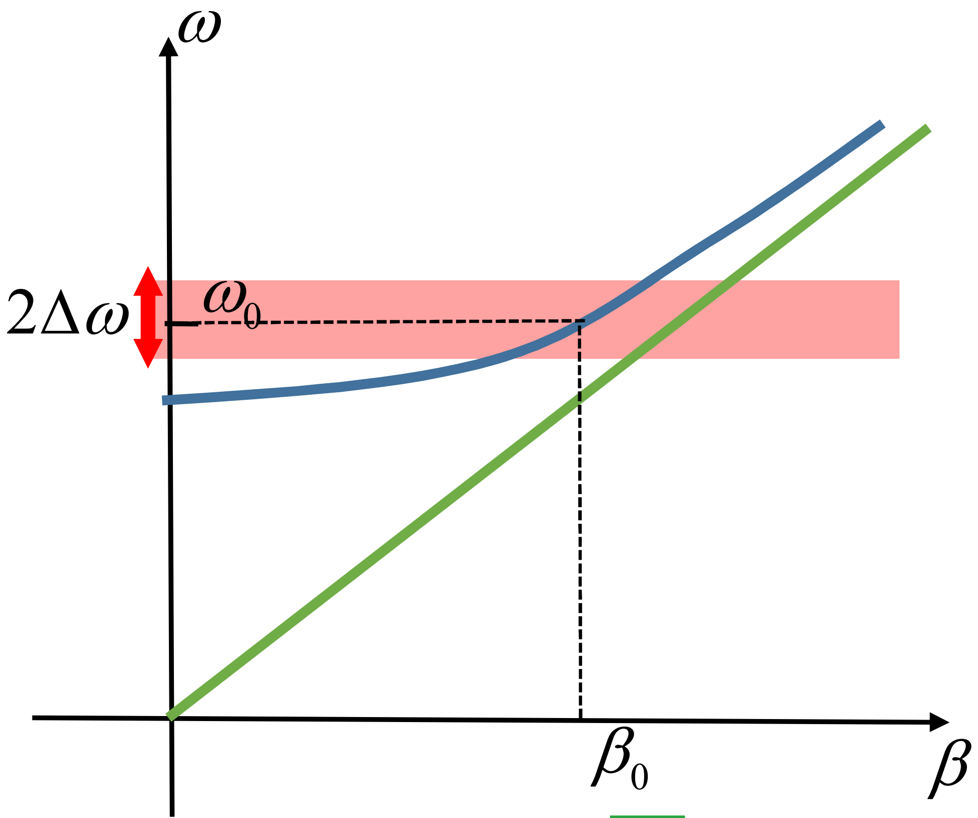
Channel & carrier frequency — Bandwidth — Distance



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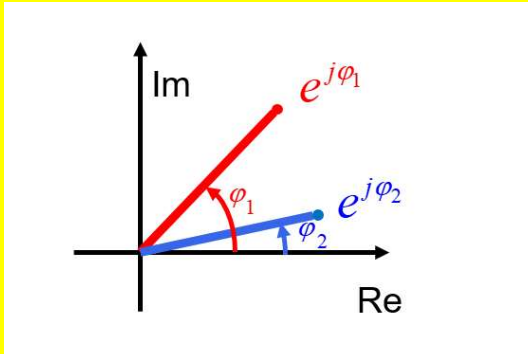

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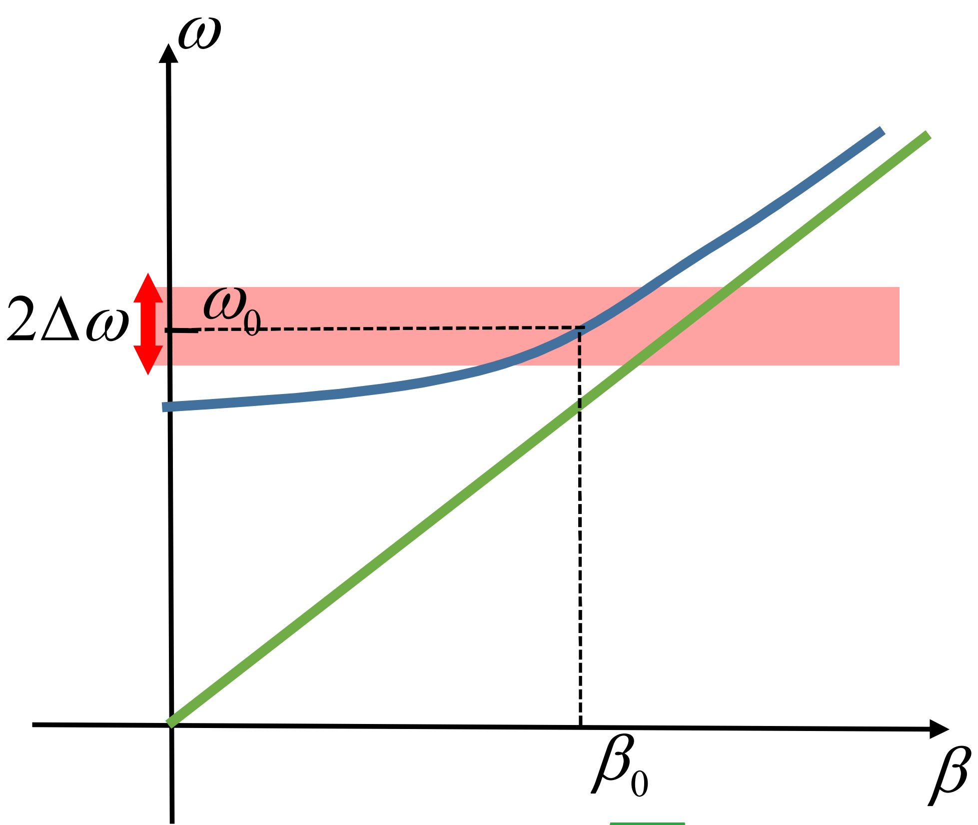
Channel & carrier frequency

Bandwidth

Distance



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