Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

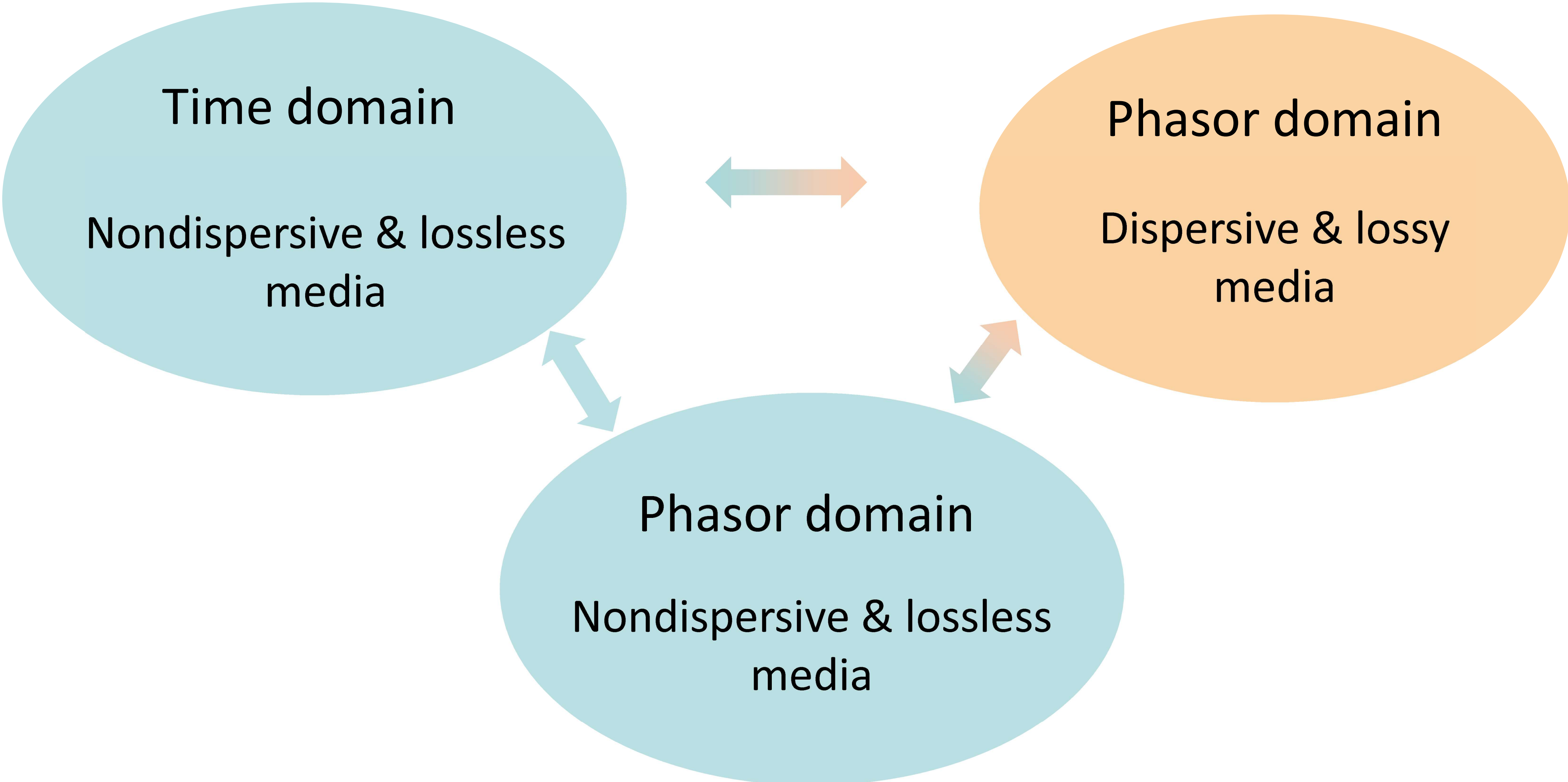
Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Phasor Domain)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\{E_y,H_x\}$ $\{E_x,H_y\}$ **Independent each other**

$\{E_x,H_y\}$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}=\beta(\omega_0)-j\alpha(\omega_0)$$

$$E_x^+(z)=E^+e^{-jkz}$$

$$\zeta H_y^+(z)=E^+e^{-jkz}$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\mathrm{Re}\left\{\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}}=\frac{1}{\mathrm{Re}\left\{\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}}=v_p(\omega_0)$$

$$E^+e^{-jkz}\quad\rightarrow\quad e^+(z,t)=e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu(\omega_0)=\mu_1(\omega_0)-j\mu_2(\omega_0)\\ \sigma:\ real\end{cases}$$

$$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$$

$$E_x^+(z)=E^+e^{-j\beta z}$$

$$\zeta H_y^+(z)=E^+e^{-j\beta z}$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu\varepsilon}}=c$$

Time nondispersive & lossless

$$\begin{cases}\varepsilon:real\\ \mu:real\\ \sigma=0\end{cases}$$

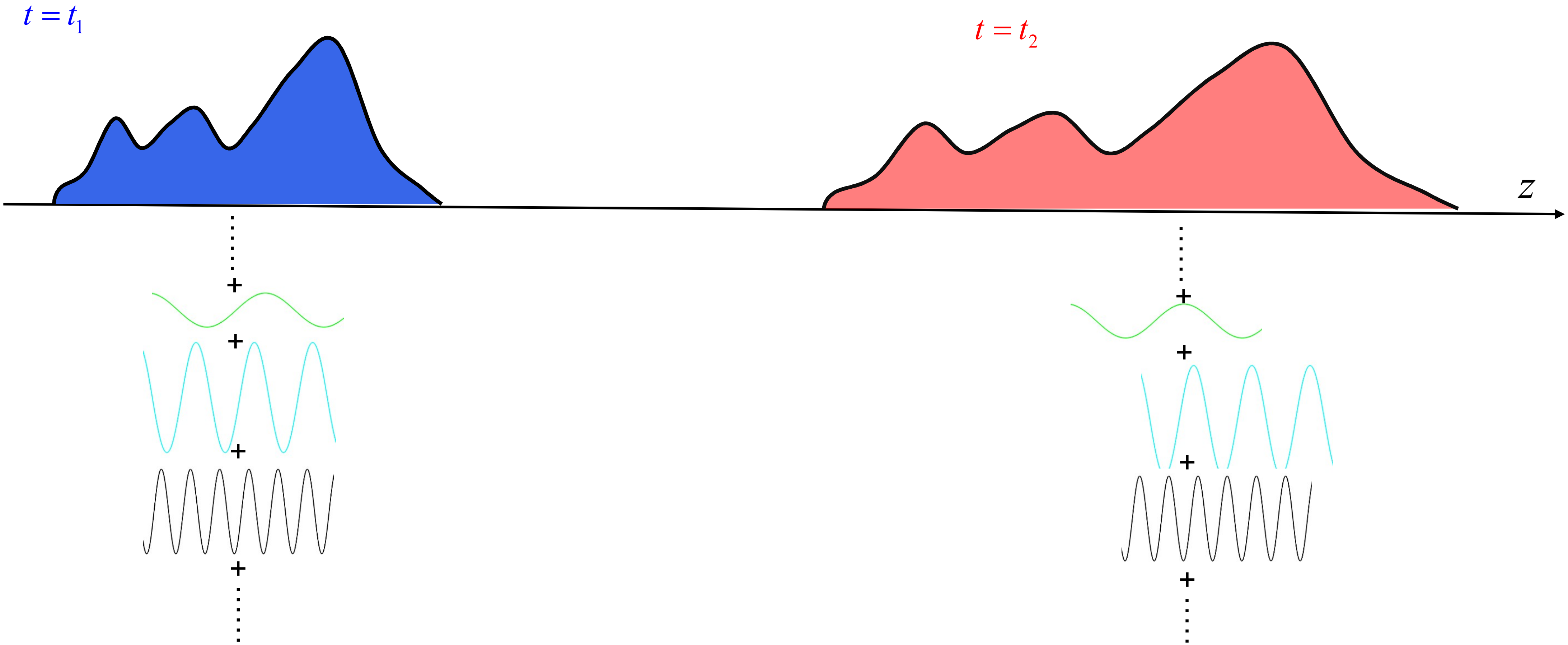
$$\boxed{c=\frac{1}{\sqrt{\mu\varepsilon}}}$$

$$E^+e^{-j\beta z}\rightarrow e_x^+(z,t)=\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)=e_x^+(z-v_pt)$$

Plane Waves

Time dispersive medium

$$\boxed{v_p = v_p(\omega_0)}$$



Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

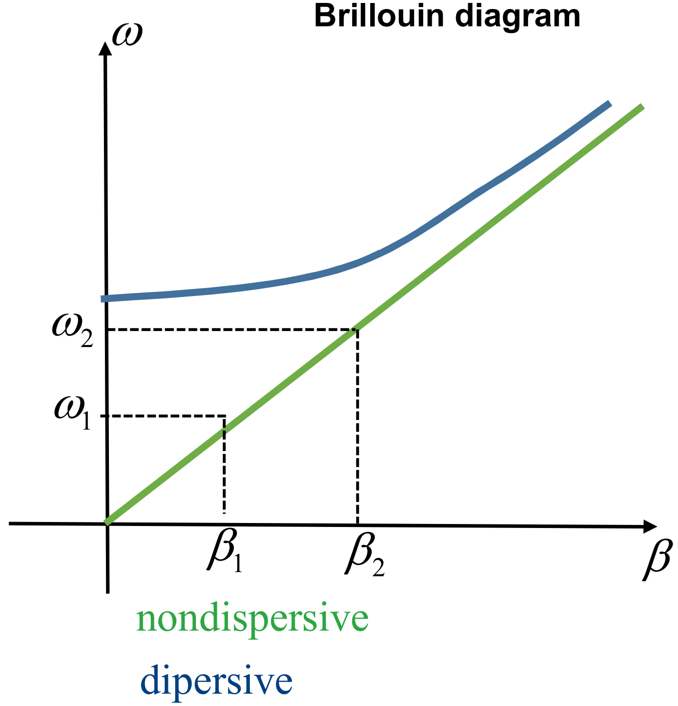
Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

Brillouin diagram



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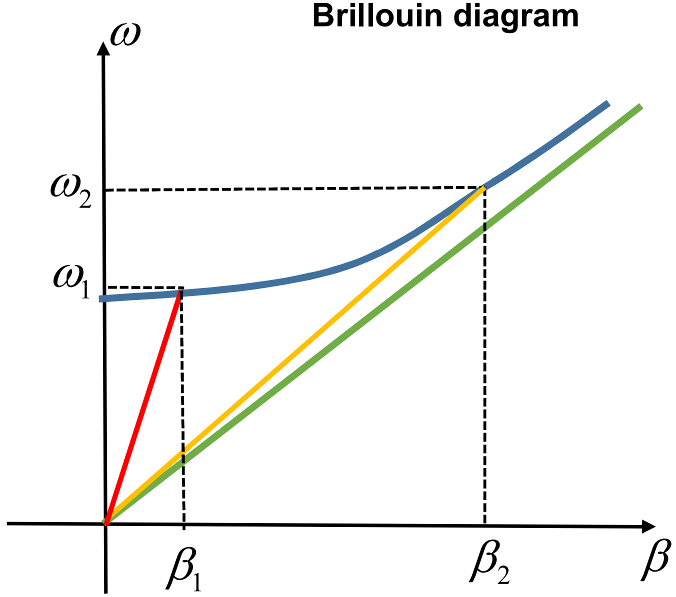

nondispersive

dipersive

$$\boxed{v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)}$$

$$\boxed{v_p = \frac{\omega_0}{\beta} = c}$$

Plane Waves (Spectral Domains)

$$\left\{E_x, H_y\right\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

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$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\left\{E_x, H_y\right\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

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$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

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$$E_z = H_z = 0$$

$$\left\{E_y, H_x\right\}$$

$$\left\{E_x, H_y\right\}$$

Independent each other

Plane Waves (Fourier Domain)

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Independent each other

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$$\{E_x, H_y\}$$

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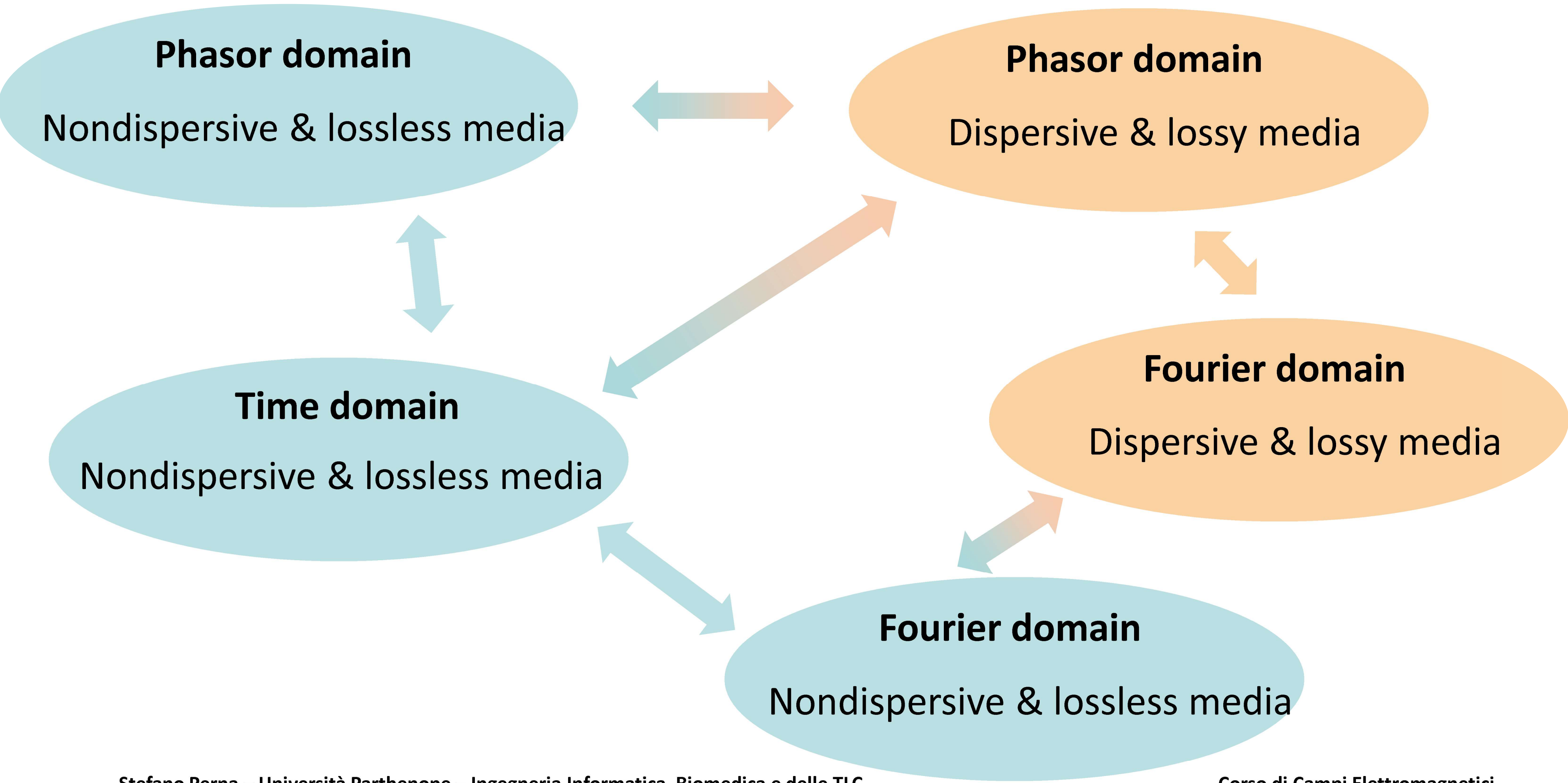
Independent each other

Rationale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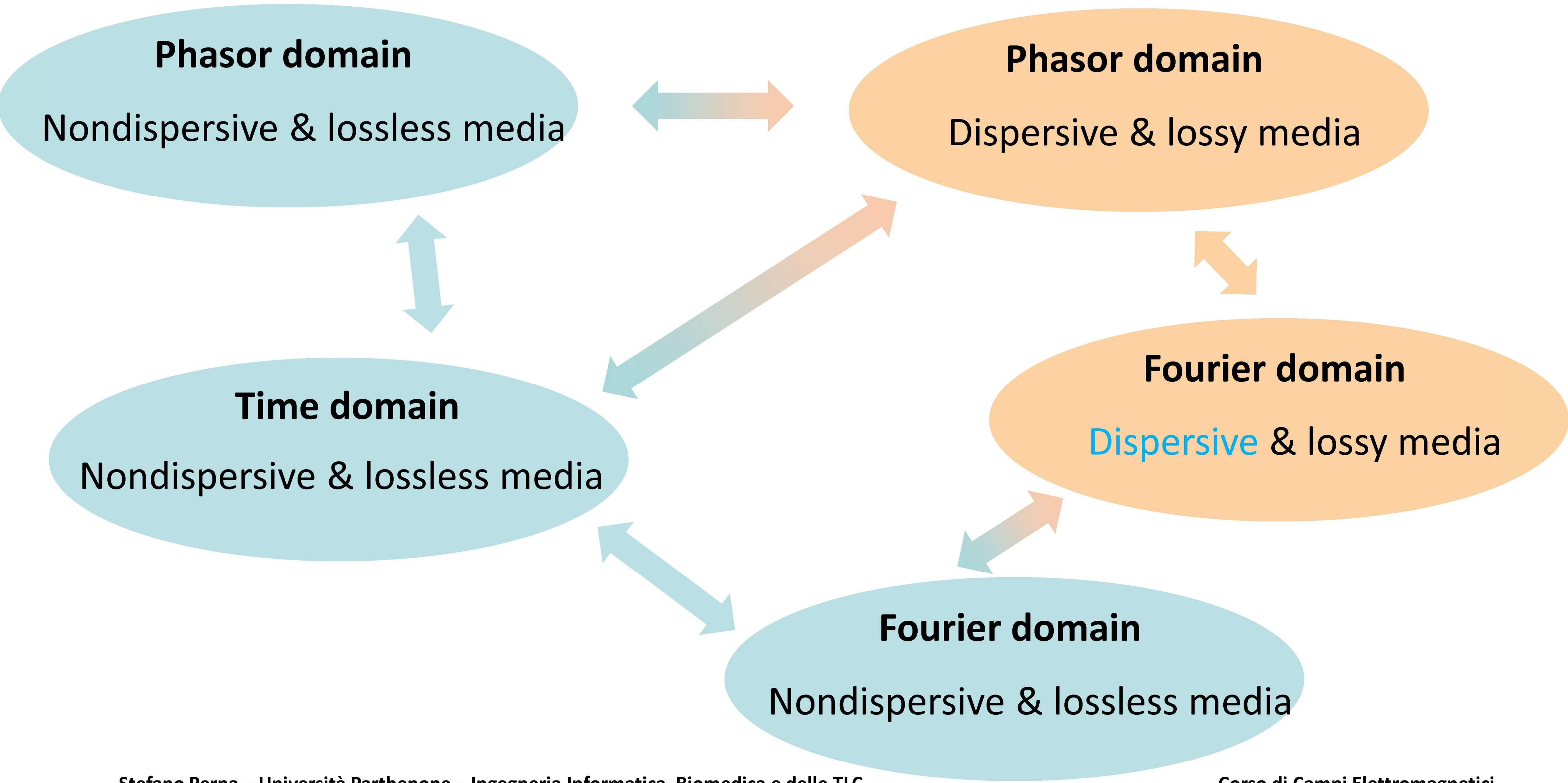
Rationale



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Plane Waves (Fourier Domain)

Time dispersive (lossy)

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$$\varepsilon_{eq}(\omega) = \varepsilon(\omega)\left[1 - \frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

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$$\Downarrow$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$
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Independent each other

Plane Waves (Fourier Domain)

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$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Fourier Domain)

Time dispersive (lossy)

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$$\{E_y, H_x\}$$
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Independent each other

Plane Waves (Fourier Domain)

$\left\{E_x, H_y\right\}$

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$$\beta = \omega\sqrt{\mu\varepsilon} \quad = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega \quad = \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega \quad = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

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$$e_x^+(z,t) =$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$\{E_y, H_x\}$
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Independent each other

Plane Waves (Fourier Domain)

$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

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$$e_x^+(z,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

<u>Progressive</u> plane wave

$$e_x^+(z=0,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega t}d\omega = f(t)$$

$$e_x^+(z>0,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega = f\left(t-\frac{z}{c}\right) = f\left[-\frac{1}{c}(z-ct)\right] = f\left[(z-ct)\right]$$

Source-free

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$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$
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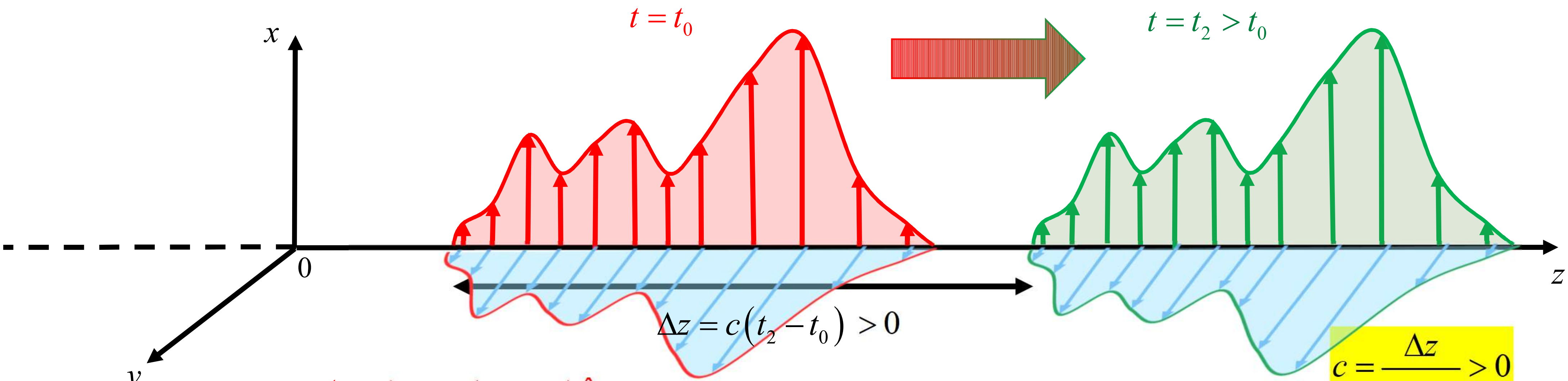
Plane Waves



$$c = \frac{\Delta z}{t_2 - t_0} > 0$$

$$\vec{\mathbf{e}}\left(z, t_0\right) = \mathrm{e}^{+}\left(z - ct_0\right)\hat{i}_x$$

$$\vec{\mathbf{e}}\left(z, t_2\right) = \mathrm{e}^{+}\left(z - ct_2\right)\hat{i}_x = \mathrm{e}^{+}\left(z - ct_0 + ct_0 - ct_2\right)\hat{i}_x = \mathrm{e}^{+}\left(z - ct_0 - c\left[t_2 - t_0\right]\right)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the z-axis

$$\begin{cases} e^{+}\left(z - ct\right) \\ h^{+}\left(z - ct\right) \end{cases}$$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (Fourier Domain)

$\left\{E_x, H_y\right\}$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

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$\{E_x, H_y\}$ **Independent each other**

Mathematical tools

$$f(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow F(\omega)$$

$$f(t)\ \text{real}\ \Rightarrow \boxed{F(\omega) = F^*(-\omega)}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \quad = \frac{1}{2\pi}\left[\boxed{\int_{-\infty}^{0} F(\omega)e^{j\omega t}d\omega} + \int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right] \quad = \frac{1}{2\pi}\left[\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega + \int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right]$$

$$\eta = -\omega$$

$$\boxed{\int_{-\infty}^{0} F(\omega)e^{j\omega t}d\omega} = -\int_{\infty}^{0} F(-\eta)e^{-j\eta t}d\eta \quad = \int_{0}^{\infty} F(-\eta)e^{-j\eta t}d\eta \quad = \int_{0}^{\infty} F^*(\eta)\left[e^{j\eta t}\right]^* d\eta = \int_{0}^{\infty}\left[F(\eta)e^{j\eta t}\right]^* d\eta = \boxed{\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega}$$

Mathematical tools

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$$= \frac{1}{2\pi}\int_{0}^{\infty} F(\omega)e^{j\omega t} + \left[F(\omega)e^{j\omega t}\right]^* d\omega = \frac{1}{2\pi}\left[\int_{0}^{\infty} 2\,\mathrm{Re}\left\{F(\omega)e^{j\omega t}\right\}d\omega\right] = \boxed{\mathrm{Re}\left\{\frac{1}{\pi}\int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right\}}$$

$$q = a + jb$$

$$q + q^* = (a + jb) + (a - jb) = 2a = 2\,\mathrm{Re}\{q\}$$

Plane Waves : dispersion

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Plane Waves : dispersion

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$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega) - j\alpha(\omega)$$

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$\alpha(\omega) \approx 0$

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$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

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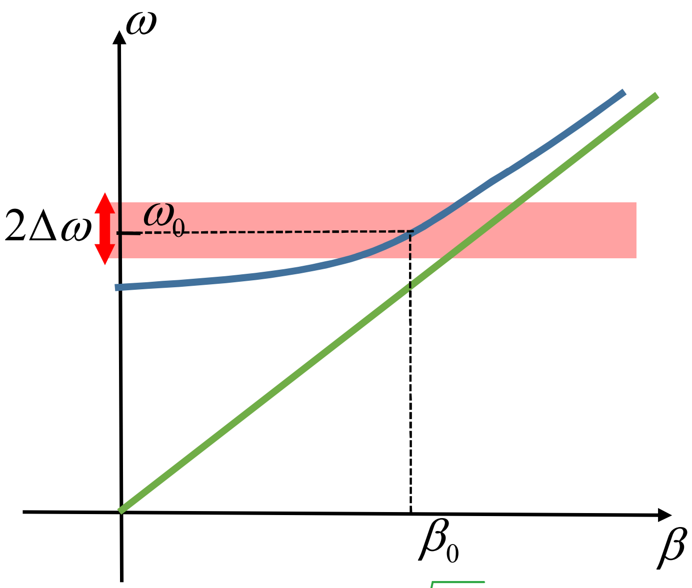

nondispersive : $\beta=\omega\sqrt{\mu\varepsilon}$

dipersive : $\beta=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$\beta(\omega)\approx\beta(\omega_0)+\beta'(\omega_0)(\omega-\omega_0)+\cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2+\cdots}$$

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Plane Waves : dispersion

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$$=\frac{1}{\pi}\int_{-\Delta\omega}^{\Delta\omega} E^+(\eta+\omega_0)e^{-j\left[\beta_0+\frac{\eta}{v_g}\right]z}e^{j(\eta+\omega_0)t}d\eta = \frac{1}{\pi}\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)\boxed{e^{-j\frac{\eta}{v_g}z}e^{j\eta t}}d\eta$$

$$=\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

$$\eta=\omega-\omega_0$$

$$\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}} = e^{j\omega_0\left(t-\frac{\beta_0}{\omega_0}z\right)} = \boxed{e^{j\omega_0\left(t-\frac{z}{v_p}\right)}}$$

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$$e_x^+(z=0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}\underbrace{\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\eta t}d\eta}_{f(t)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}f(t)\right\} = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

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$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\begin{cases}\varepsilon(\omega) = \varepsilon_1(\omega)\\ \mu(\omega) = \mu_1(\omega)\\ \sigma = 0\end{cases}$$

$$\beta(\omega) \approx \beta_0 + \frac{(\omega-\omega_0)}{v_g}$$

$$\beta_0 = \beta(\omega_0)$$

$$v_g = \frac{1}{\beta'(\omega_0)}$$

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Plane Waves : dispersion

Source-free

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$\{E_x,H_y\}$

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$$e_x^+(z=0,t)\qquad=\frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t)$$

$$=\frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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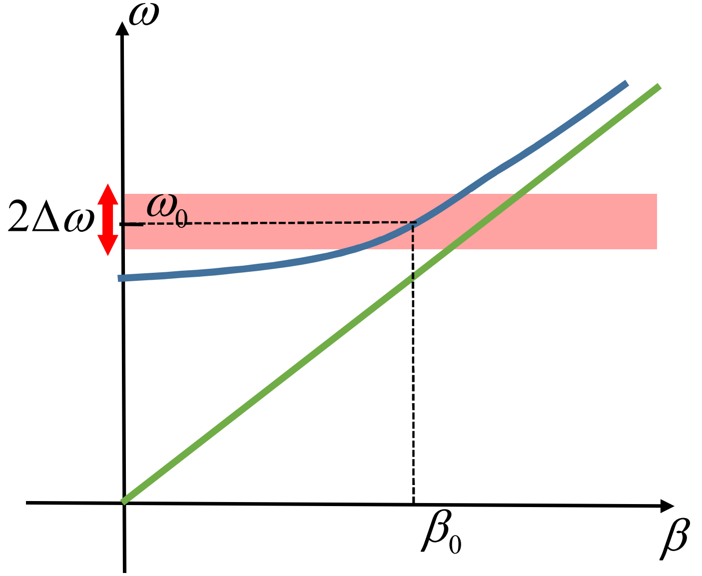
Plane Waves : dispersion



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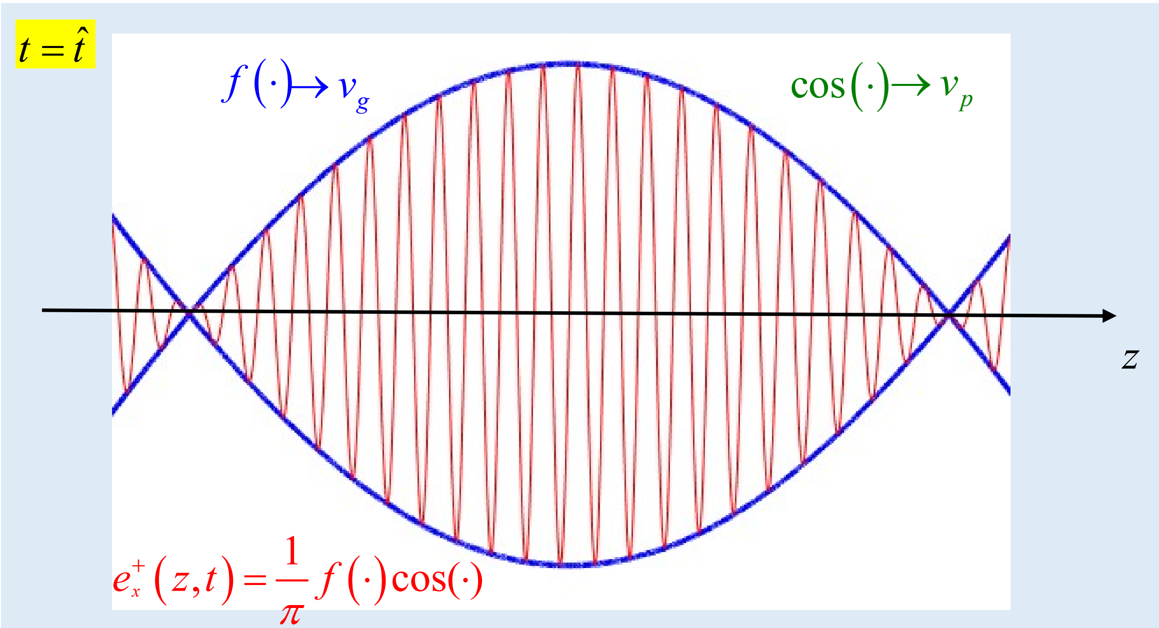
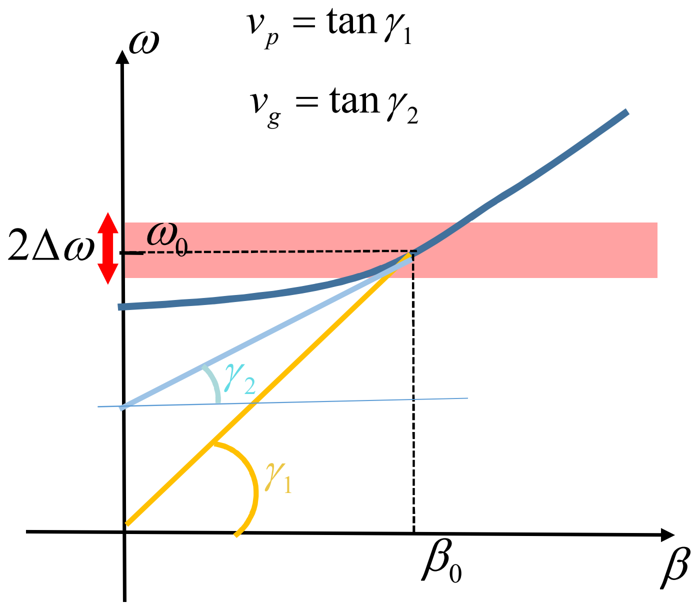


Plane Waves : dispersion



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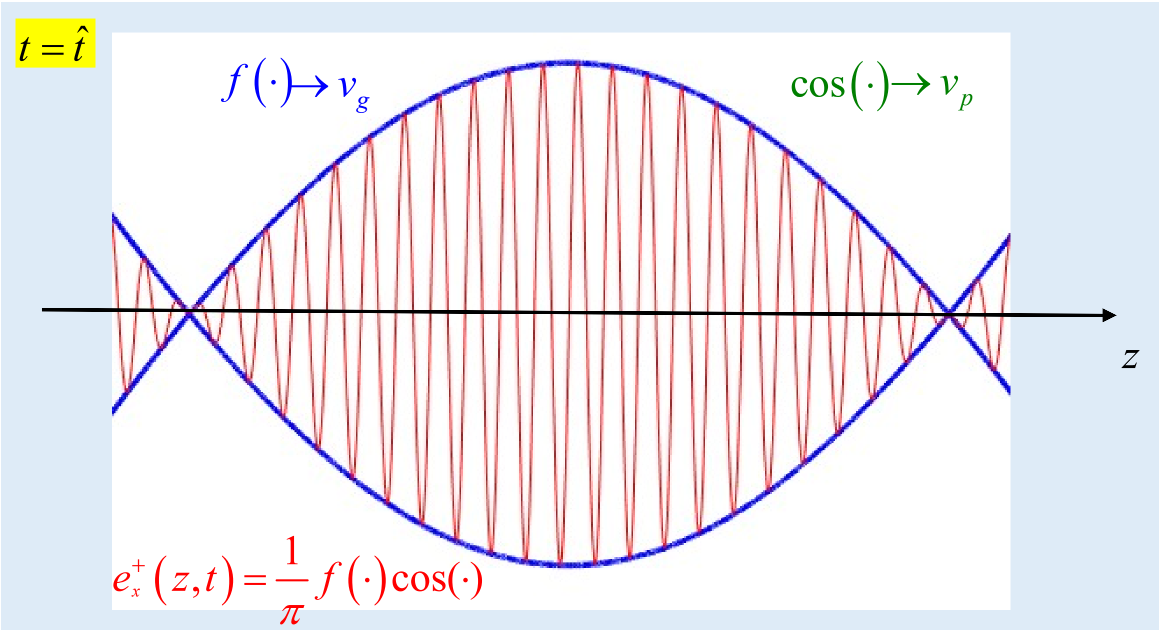

Plane Waves : dispersion

$\{E_x, H_y\}$

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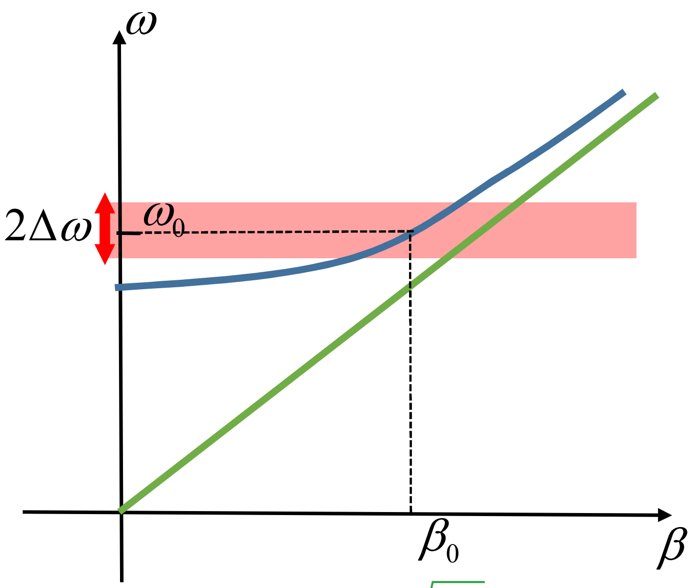

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$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2 + \cdots}$$

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Plane Waves : dispersion

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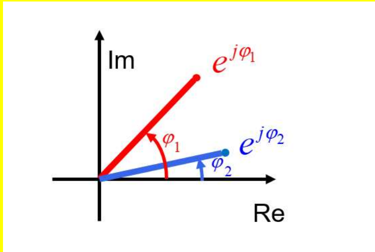
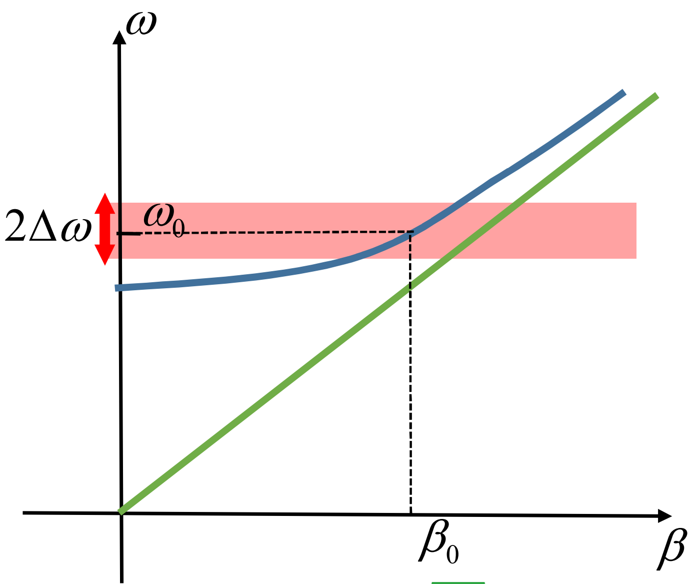
$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2}\boxed{\beta''(\omega_0)}\Delta\omega^2 z << 2\pi$$

Channel & carrier frequency



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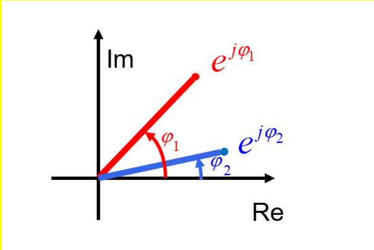

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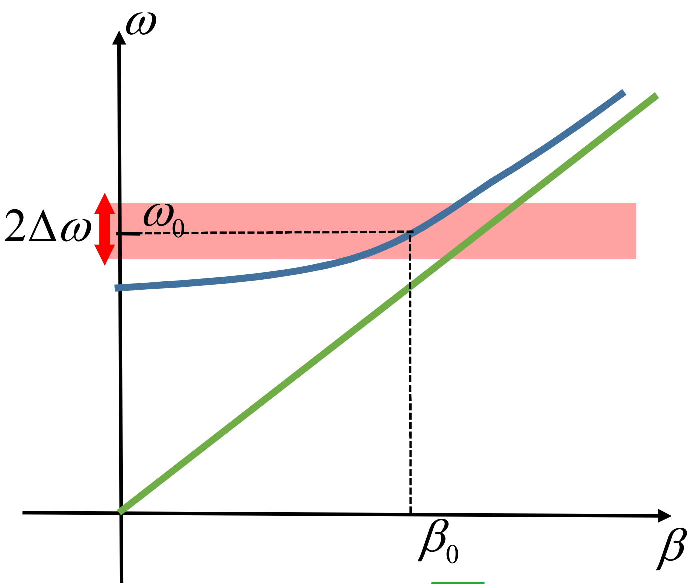
Channel & carrier frequency

Bandwidth



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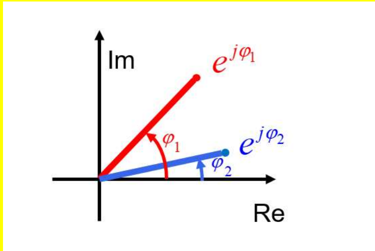

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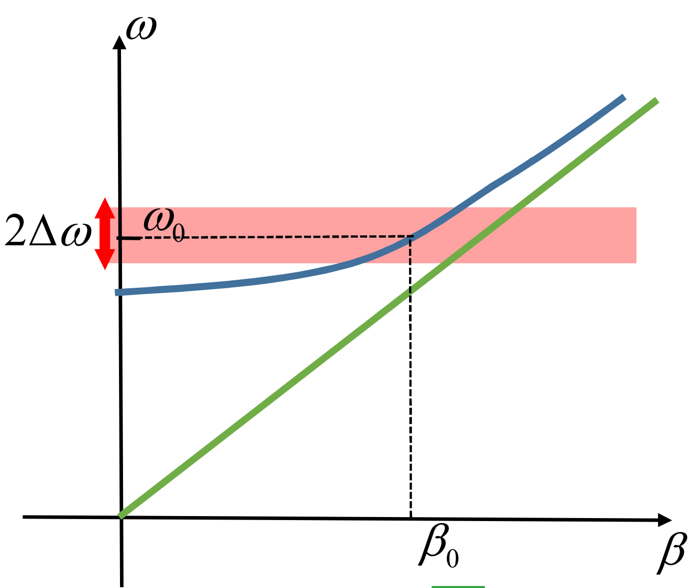
Channel & carrier frequency

Bandwidth

Distance



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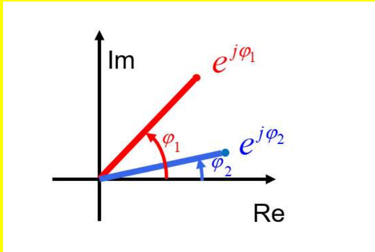

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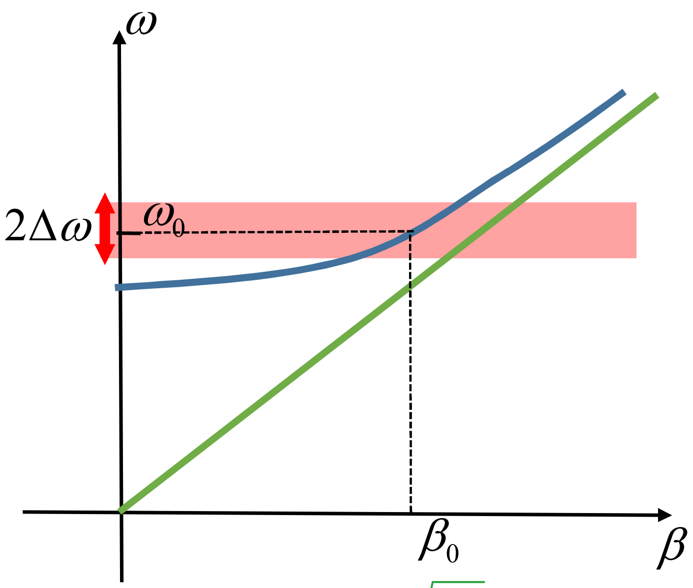

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