Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

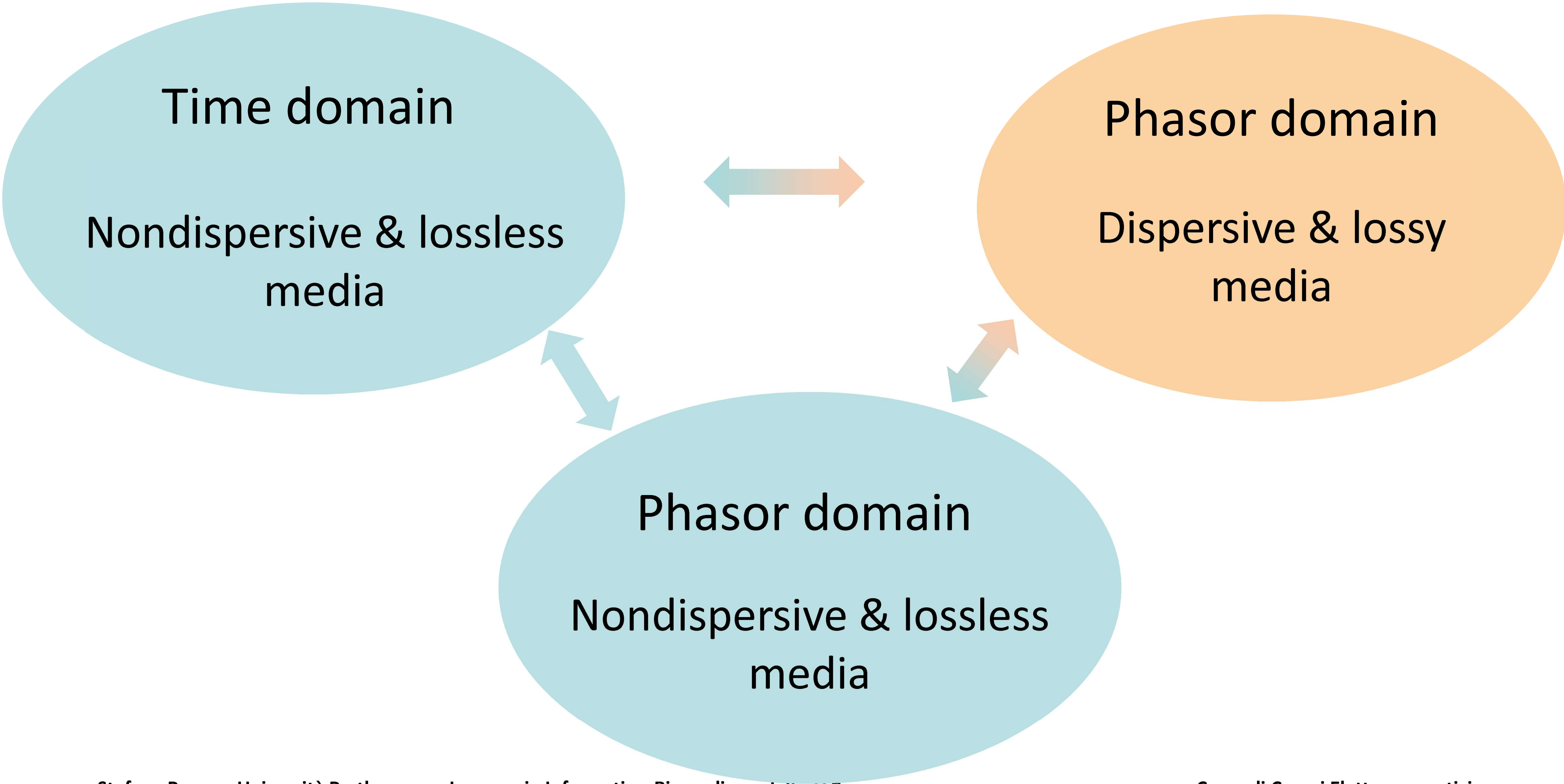
Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+| \cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

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Source-free

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- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

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$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

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$$E^+e^{-jkz}=E^+e^{-j(\beta-j\alpha)z}=E^+e^{(-j\beta z-\alpha z)}=E^+e^{-j\beta z}e^{-\alpha z}$$

$$E^+=|E^+|e^{j\varphi^+} \quad \Rightarrow E^+e^{-jkz}=|E^+|e^{j\varphi^+}e^{-j\beta z}e^{-\alpha z}$$

$$e_x^+(z,t)=\mathrm{Re}\left\{|E^+|e^{j\varphi^+}e^{-j\beta z}e^{-\alpha z}e^{j\omega_0 t}\right\}=e^{-\alpha z}|E^+|\cos(\omega_o t-\beta z+\varphi^+)$$

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- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}\left|E^+\right|\cos(\omega_o t - \beta z + \varphi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : real \end{cases}$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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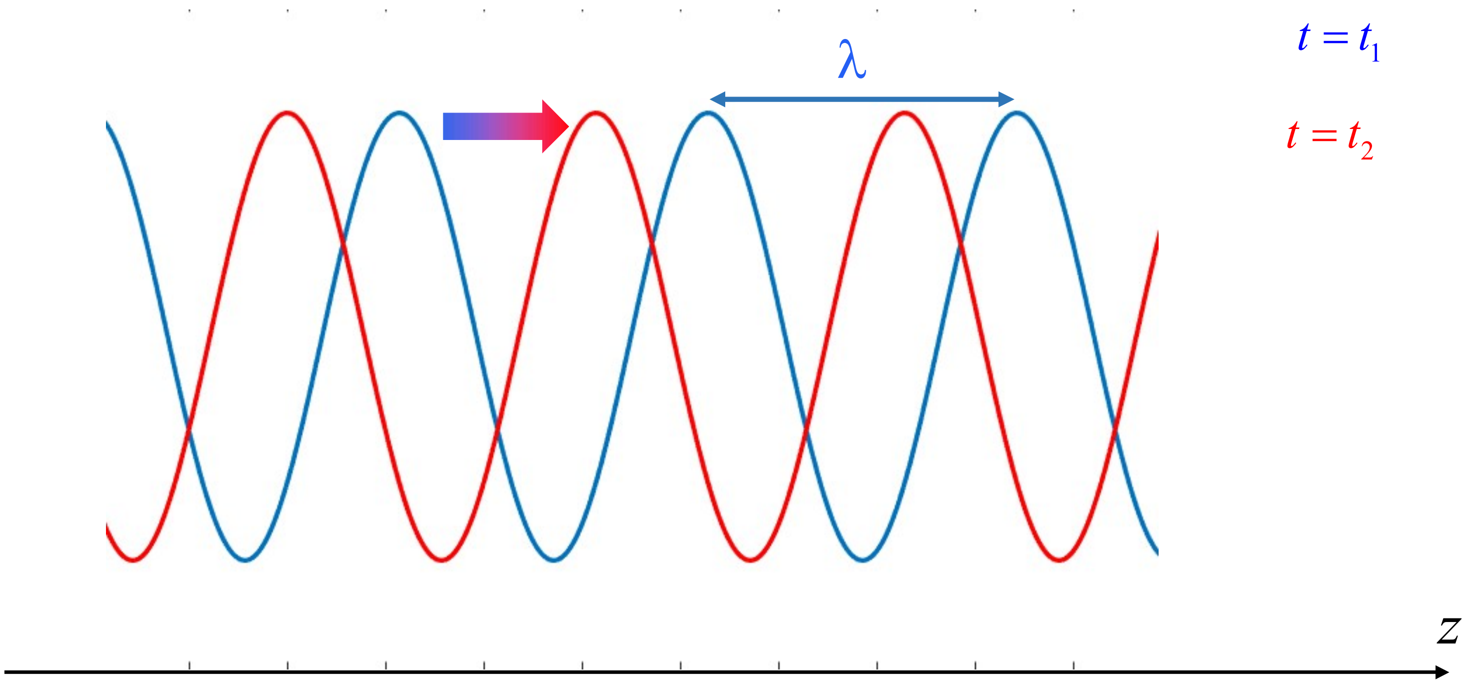
$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Plane Waves (Phasor Domain)

Time nondispersive & lossless medium

$$e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

$$v_p = \frac{\omega_0}{\beta}$$



Plane Waves (Phasor Domain)

Time dispersive & lossy medium

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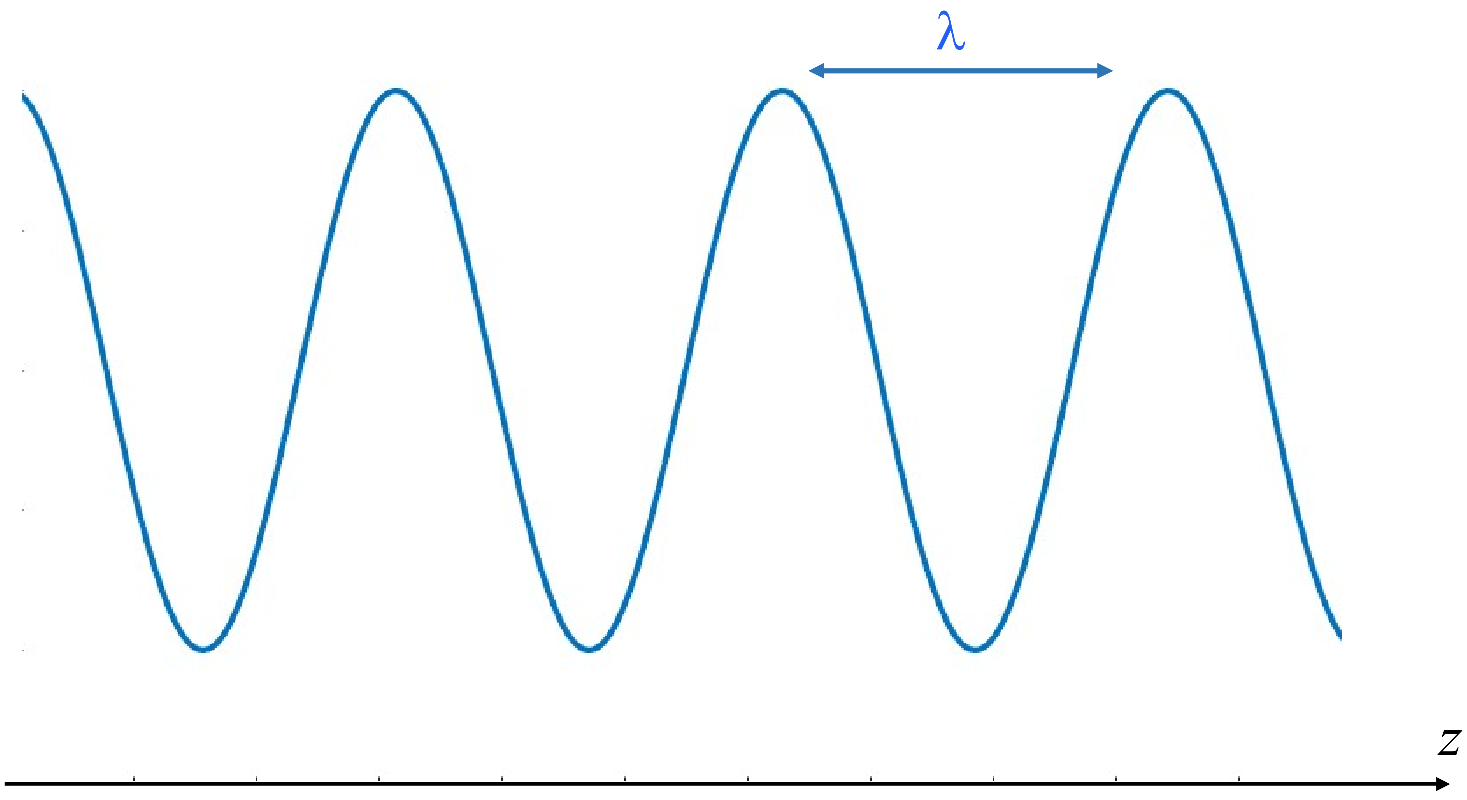

$$\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

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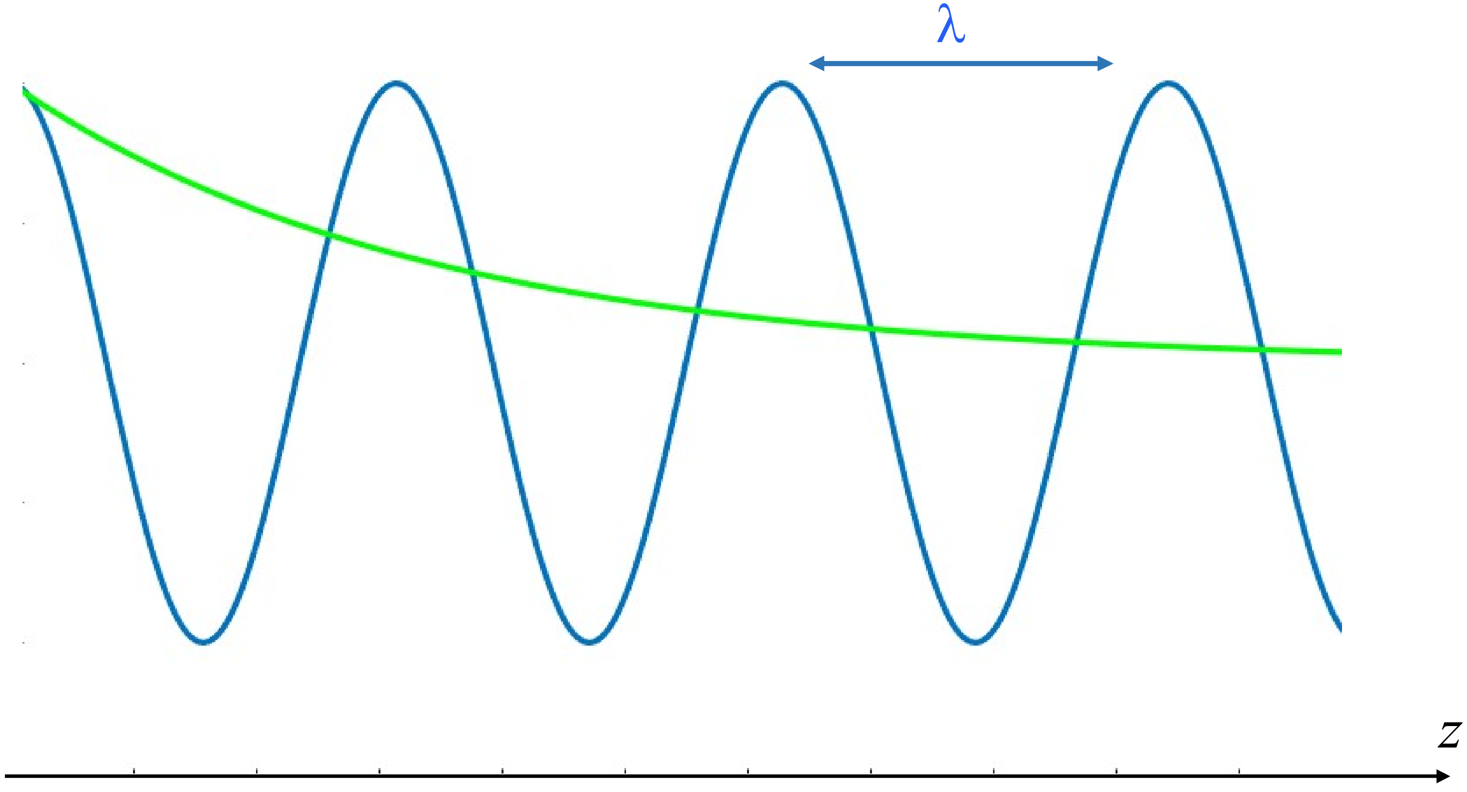

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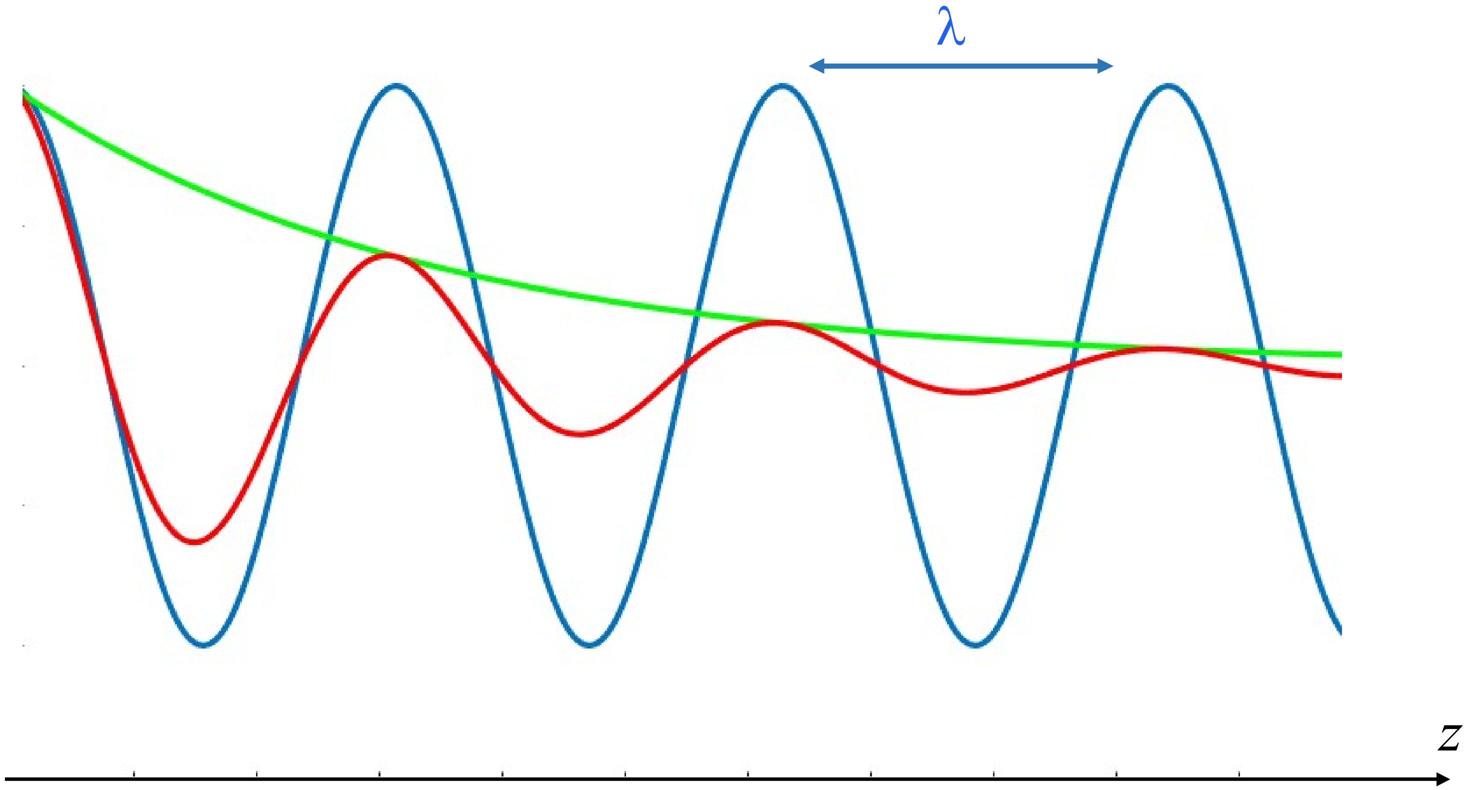

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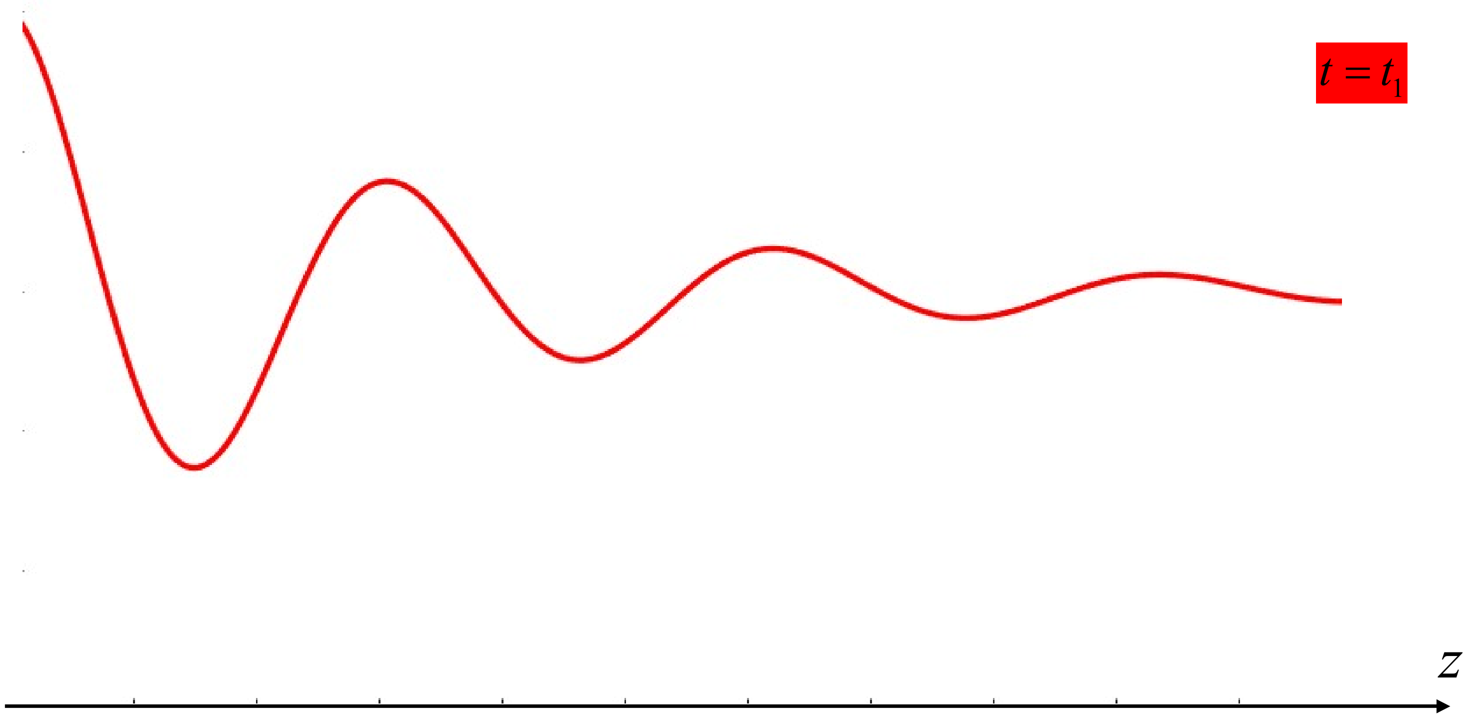

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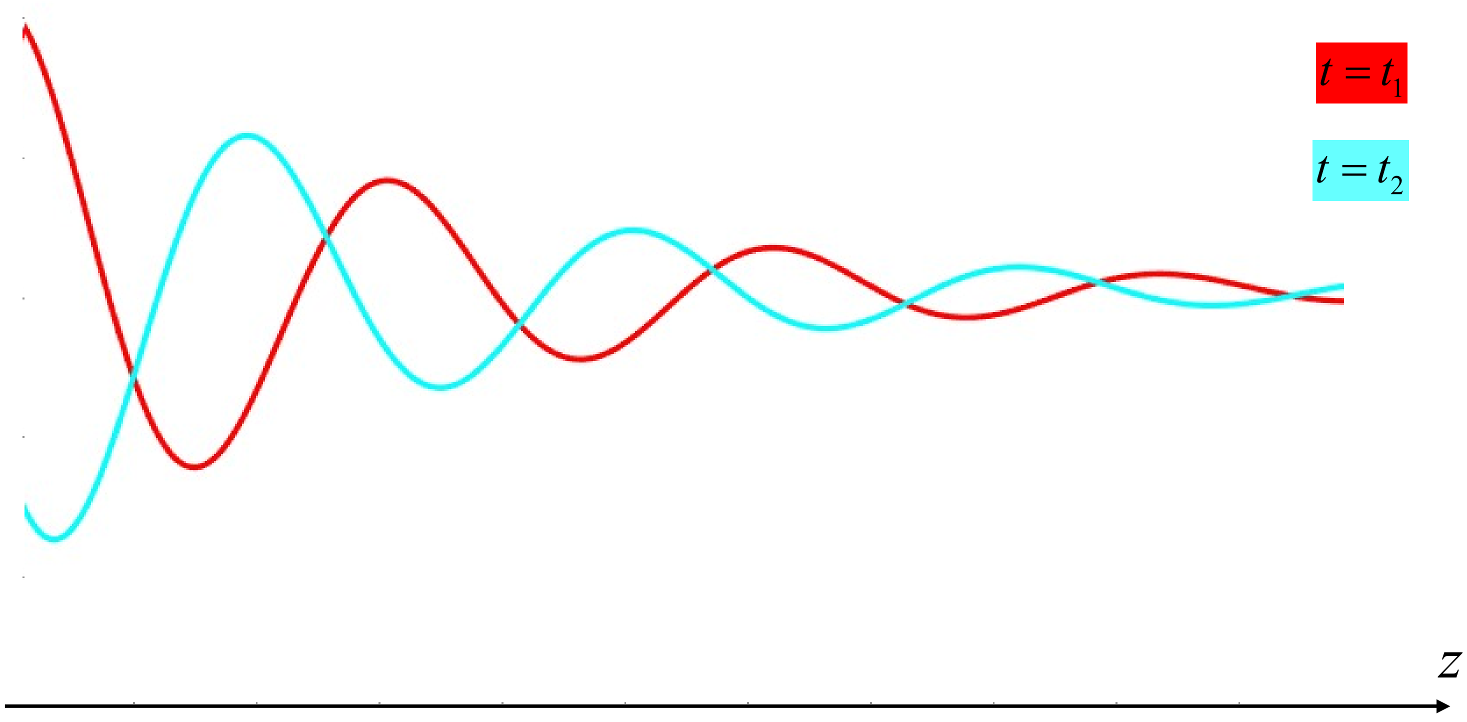

Plane Waves (Phasor Domain)

Source-free

Medium

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$\{E_x, H_y\}$

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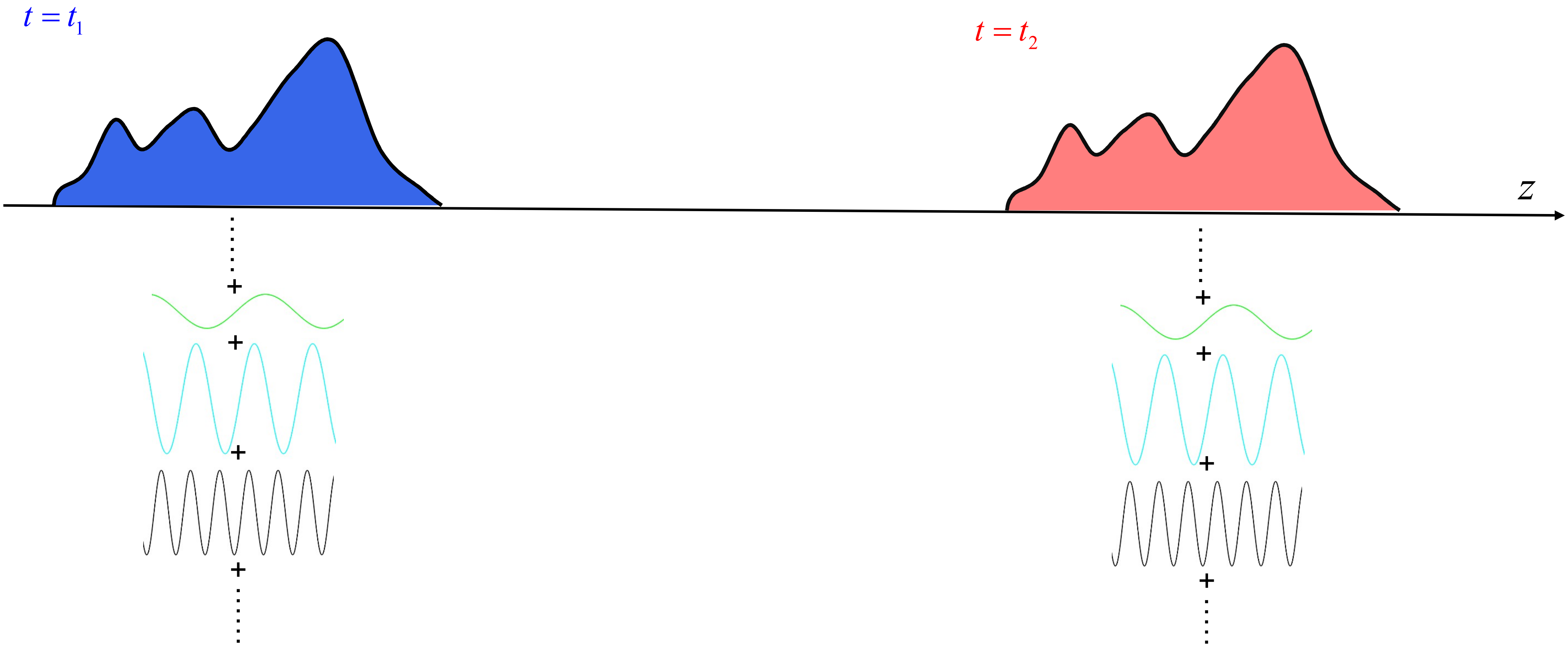
$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves

Time nondispersive & lossless medium

$v_p = c$



Plane Waves

Time dispersive medium

$$v_p = v_p(\omega_0)$$