

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Spectral domains

Plane Waves (Spectral Domains)

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z, \omega) = -j\omega \mu \vec{\mathbf{H}}(z, \omega) \\ \nabla \times \vec{\mathbf{H}}(z, \omega) = j\omega \varepsilon \vec{\mathbf{E}}(z, \omega) \end{cases}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z) \hat{i}_x + E_y(z) \hat{i}_y + E_z(z) \hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z) \hat{i}_x + H_y(z) \hat{i}_y + H_z(z) \hat{i}_z$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = E_x(z, \omega) \hat{i}_x + E_y(z, \omega) \hat{i}_y + E_z(z, \omega) \hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = H_x(z, \omega) \hat{i}_x + H_y(z, \omega) \hat{i}_y + H_z(z, \omega) \hat{i}_z$$

FD

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~- Lossless~~

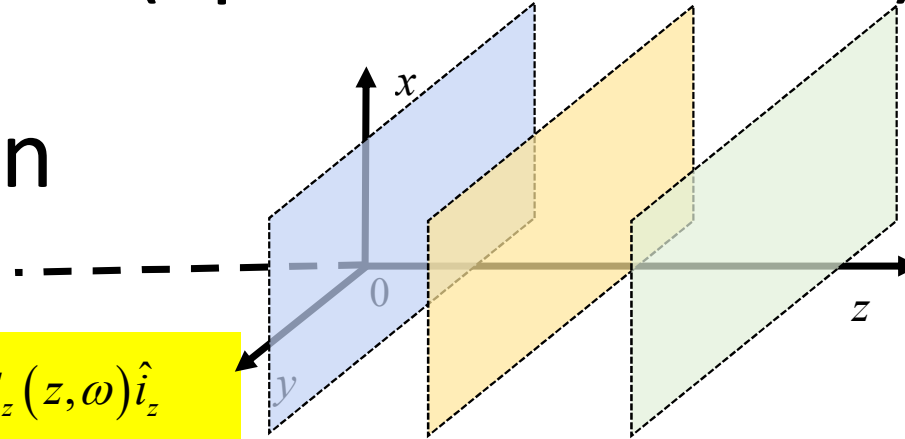
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Plane Waves (Spectral Domains)

Fourier Domain



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$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz} \right) \hat{\mathbf{i}}_x + \left(\frac{dE_x}{dz} \right) \hat{\mathbf{i}}_y$$

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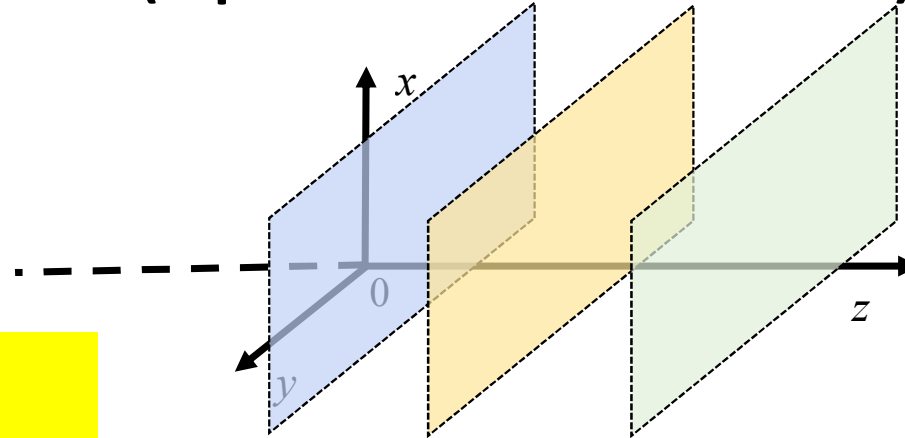
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Plane Waves (Spectral Domains)

Phasor Domain



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{\mathbf{i}}_x + E_y(z)\hat{\mathbf{i}}_y + E_z(z)\hat{\mathbf{i}}_z$$

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$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\begin{aligned} k &= \omega\sqrt{\mu\varepsilon} \\ k &= \beta - j\alpha \end{aligned}$$

$$\begin{cases} \frac{dE_y}{dz} = j\omega\mu H_x \\ \frac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \quad \{E_y, H_x\}$$

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 $\{E_x, H_y\}$ Independent each other

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$$\frac{d^2 f(z)}{dz^2} + k^2 f(z) = 0$$

$$\xi^2 + k^2 = 0 \quad \xi = \pm jk$$

$$f(z) = C_1 e^{-jkz} + C_2 e^{+jkz}$$

Plane Waves (Spectral Domains)

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$$-j\omega\mu H_y = \frac{dE_x}{dz} = -jkE_x^+ e^{-jkz} + jkE_x^- e^{jkz}$$

$$\omega\mu H_y = kE_x^+ e^{-jkz} - kE_x^- e^{jkz}$$

$$\frac{\omega\mu}{k} H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

ζ : intrinsic impedance of the medium

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

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Independent each other

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Phasor Domain

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Plane Waves (Spectral Domains)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Phasor Domain

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z, \omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z, \omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

Fourier Domain

Source-free

- Medium**
- Linear
 - **Time dispersive**
 - Space non-dispersive
 - Isotropic
 - Homogeneous (TI - SI)
 - ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ Independent each other

$\{E_x, H_y\}$

Plane Waves (Spectral Domains)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\begin{cases} k = \omega\sqrt{\mu\varepsilon} \\ k = \beta - j\alpha \end{cases}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z, \omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z, \omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

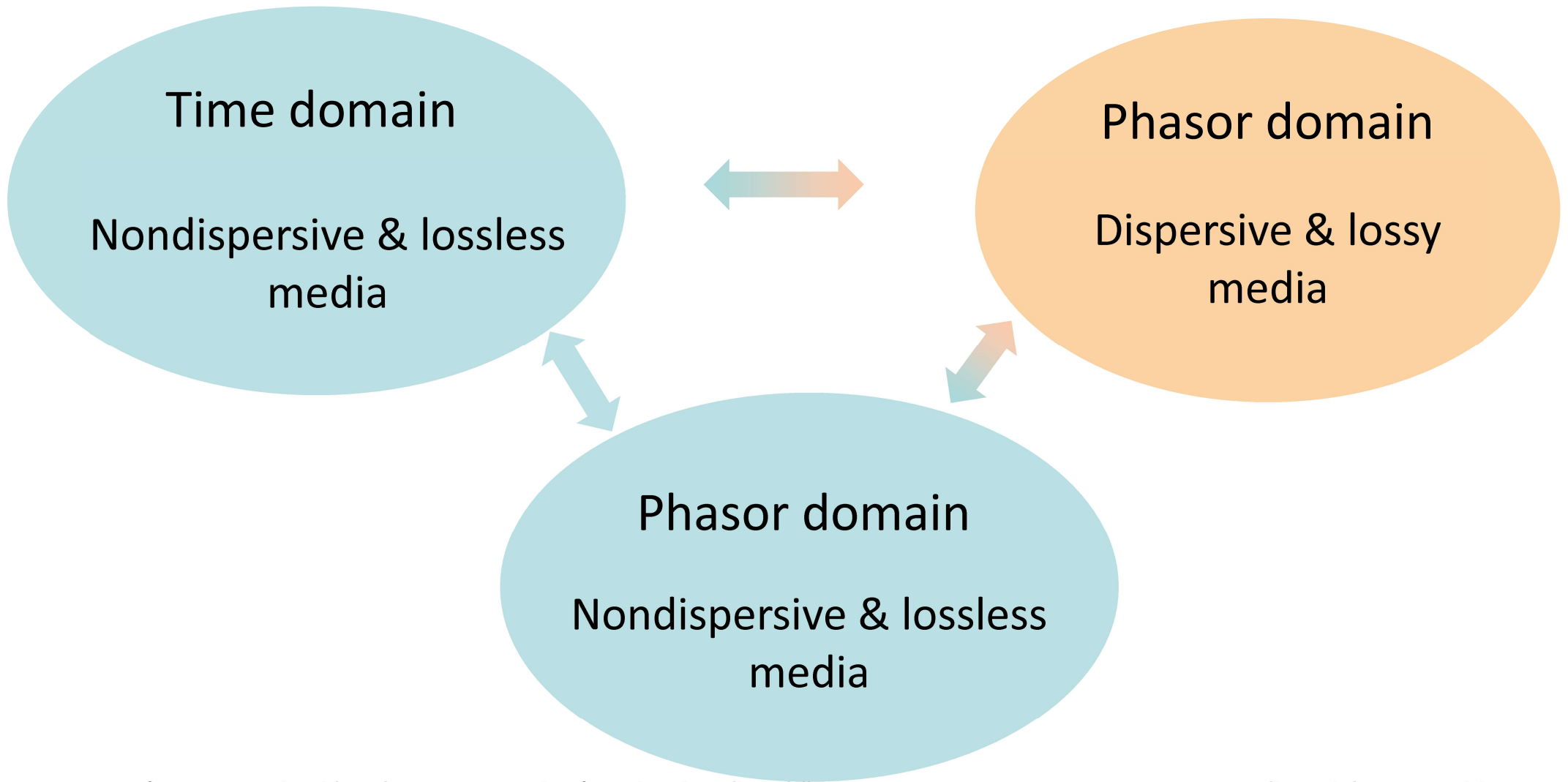
General expression of plane waves (PD)

Incidence

Plane Waves

Phasor domain

Razionale



Plane Waves (Phasor Domain)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

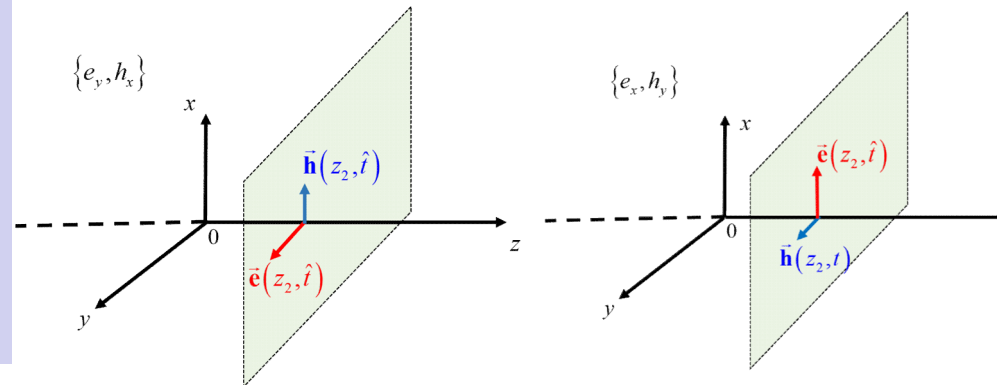
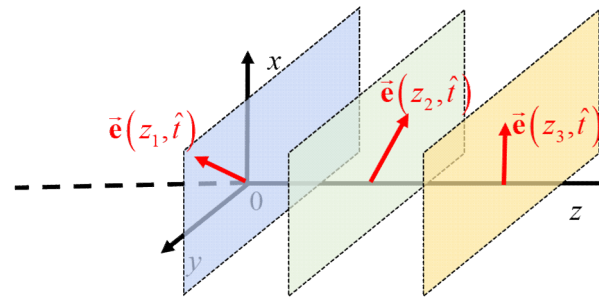
$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain



Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\begin{cases} \{E_y, H_x\} \\ \{E_x, H_y\} \end{cases} \quad \text{Independent each other}$$

Plane Waves (Phasor Domain)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

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$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

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$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

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$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Source-free

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- Linear
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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$\{E_y, H_x\}$
 $\{E_x, H_y\}$ Independent each other

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma \neq 0 \end{cases}$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma \neq 0 \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

Source-free

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$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

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$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

Source-free

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$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

- Source-free**
- Medium**
- Linear
 - **Time nondispersive**
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Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

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$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

- Source-free**
- Medium**
- Linear
 - **Time nondispersive**
 - Space non-dispersive
 - Isotropic
 - Homogeneous (TI - SI)
 - **Lossless**

$$E_x^+ = |E_x^+| e^{j\varphi^+}$$

$$E_x^+ e^{-j\beta z} = |E_x^+| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z, t) = \text{Re} \left\{ |E_x^+| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t} \right\} = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

$$= |E_x^+| \cos \left(-\beta \left[z - \frac{\omega_0}{\beta} t \right] + \varphi^+ \right) = |E_x^+| \cos \left(-\beta [z - v_p t] + \varphi^+ \right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$E_x^+ e^{-j\beta z} = |E_x^+| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z, t) = \text{Re} \left\{ |E_x^+| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t} \right\} = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

$$= |E_x^+| \cos \left(-\beta \left[z - \frac{\omega_0}{\beta} t \right] + \varphi^+ \right) = |E_x^+| \cos \left(-\beta [z - v_p t] + \varphi^+ \right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- = |E_x^-| e^{j\varphi^-}$$

$$E_x^- e^{j\beta z} = |E_x^-| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z, t) = \text{Re} \left\{ |E_x^-| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t} \right\} = |E_x^-| \cos(\omega_0 t + \beta z + \varphi^-)$$

$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
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$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E_x^-| \cos(\omega_0 t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

$$E_x^- e^{j\beta z} = |E_x^-| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z, t) = \text{Re} \left\{ |E_x^-| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t} \right\} = |E_x^-| \cos(\omega_0 t + \beta z + \varphi^-)$$

$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

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Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E_x^-| \cos(\omega_0 t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E_x^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E_x^+ e^{-j\beta z}$$

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$$\zeta H_y^-(z) = -E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

- Source-free**
- Medium**
- Linear
 - **Time nondispersive**
 - Space non-dispersive
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$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E_x^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E_x^-| \cos(\omega_0 t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

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Progressive plane wave

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Progressive plane wave

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↓

$$E_z = H_z = 0$$

$$\vec{S}(\vec{r}) = \frac{1}{2} \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})$$

$$\vec{S}^+(\vec{r}) = \frac{1}{2} \vec{E}^+(\vec{r}) \times \vec{H}^{+*}(\vec{r}) = \frac{1}{2} E_x^+(z) \hat{i}_x \times H_y^{+*}(z) \hat{i}_y = \frac{|E_x^+(z)|^2}{2\zeta} \hat{i}_z = \zeta \frac{|H_y^+(z)|^2}{2} \hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$

$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$

$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

$$\beta(\omega_0)$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

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Progressive plane wave

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$$\vec{S}(\vec{r}) = \frac{1}{2} \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})$$

$$\begin{aligned} \vec{S}^-(\vec{r}) &= \frac{1}{2} \vec{E}^-(\vec{r}) \times \vec{H}^{-*}(\vec{r}) = \frac{1}{2} E_x^-(z) \hat{i}_x \times H_y^{-*}(z) \hat{i}_y = -\frac{|E_x^-(z)|^2}{2\zeta} \hat{i}_z \\ &= -\zeta \frac{|H_y^-(z)|^2}{2} \hat{i}_z \end{aligned}$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y \quad \beta(\omega_0)$$

$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$

$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

$$\begin{cases} \{E_y, H_x\} \\ \{E_x, H_y\} \end{cases} \text{ Independent each other}$$

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Progressive plane wave

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Regressive plane wave

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Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

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$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

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Time nondispersive & lossless

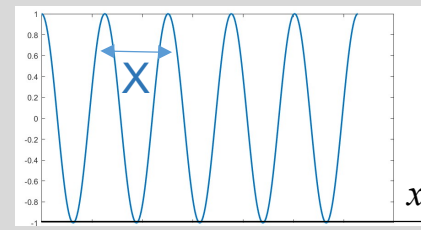
$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Memo



$$\cos(2\pi \nu x) = \cos\left(\frac{2\pi}{X} x\right)$$

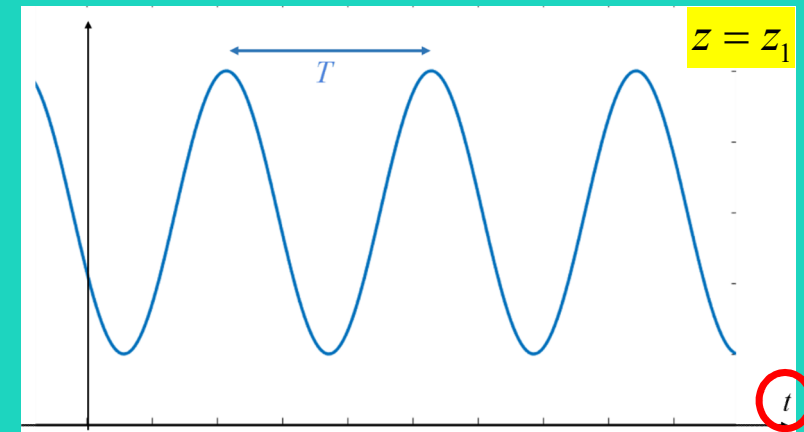
ν : frequency

$X = \frac{1}{\nu}$: period

$$e_x^+(z = z_1, t) = |E^+| \cos(\omega_0 t - \beta z_1 + \varphi^+) = |E^+| \cos\left(2\pi \left[\frac{\omega_0}{2\pi}\right] t - \beta z_1 + \varphi^+\right)$$

$$\text{frequency: } f_0 = \frac{\omega_0}{2\pi}$$

$$\text{period: } T = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$$



Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

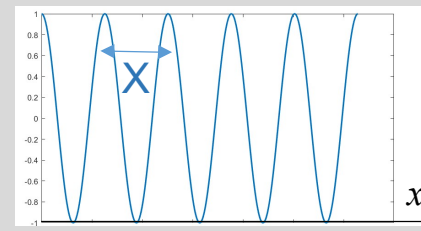
$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

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Progressive plane wave

Memo



$$\cos(2\pi \nu x) = \cos\left(\frac{2\pi}{X} x\right)$$

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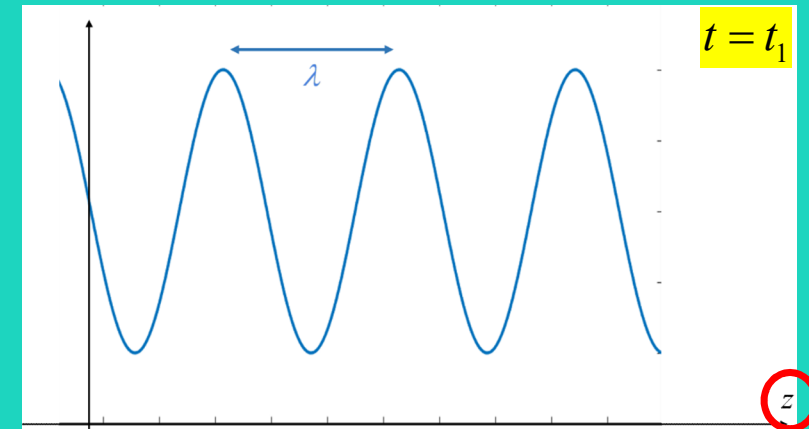
$$e_x^+(z, t = t_1) = |E^+| \cos(\omega_0 t_1 - \beta z + \varphi^+) = |E^+| \cos(-\omega_0 t_1 + \beta z - \varphi^+)$$

$$= |E^+| \cos\left(2\pi \left[\frac{\beta}{2\pi}\right] z - \omega_0 t_1 - \varphi^+\right)$$

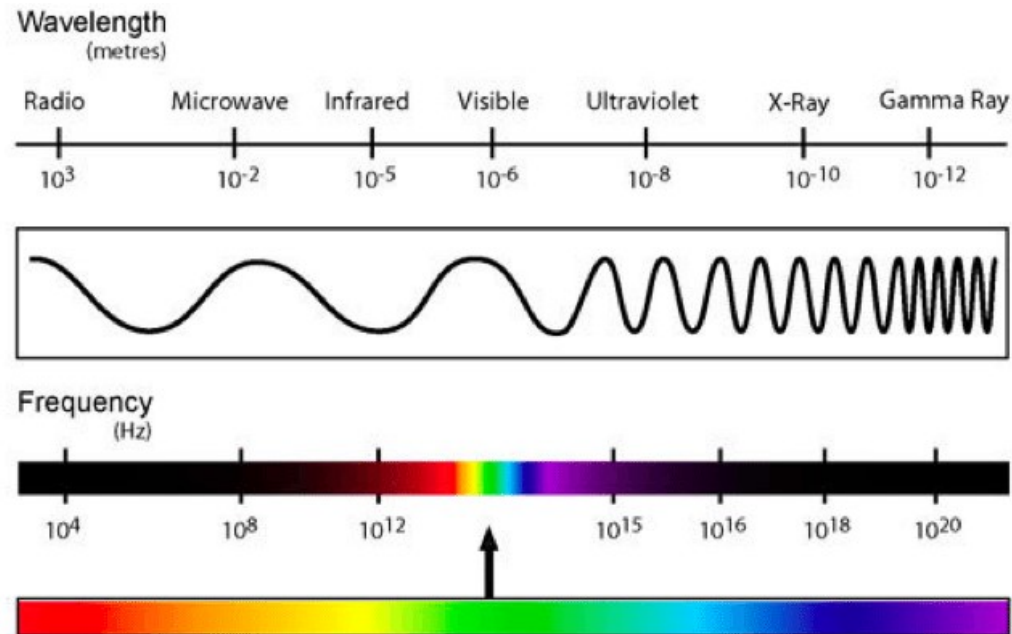
λ : wavelength

$$\text{frequency: } \nu = \frac{\beta}{2\pi}$$

$$\text{period: } \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{c}{f_0}$$



Electromagnetic spectrum



Free space (Linear, isotropic, local, homogeneous)

$$\lambda = \frac{c}{f}$$

f : frequency
 λ : wavelength
 c : light speed

Plane Waves (Phasor Domain)

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$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

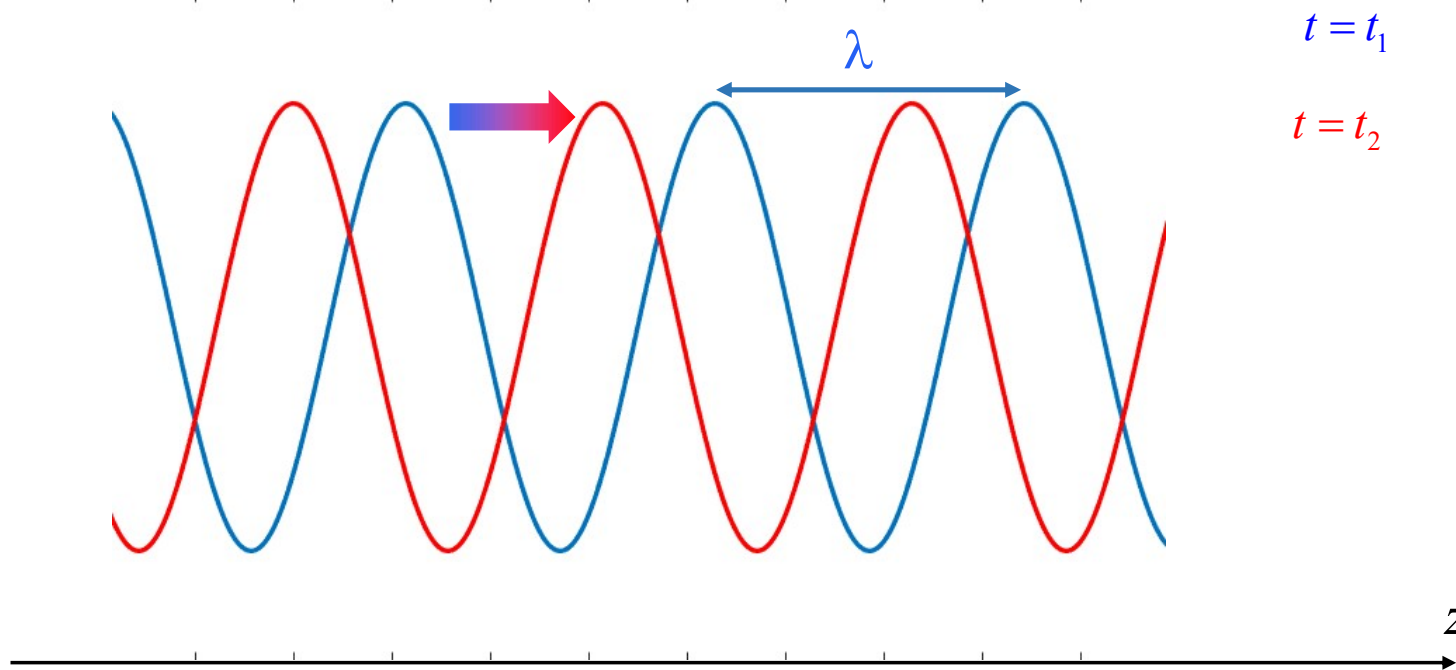
Progressive plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis

Plane Waves (Phasor Domain)

$$e_x^+(z,t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

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Plane Waves (Phasor Domain)

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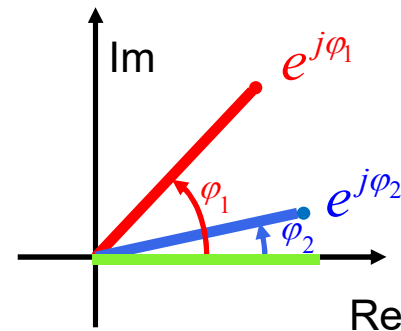
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$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$e^{-j\beta z} = e^{-j\frac{2\pi}{\lambda} z}$$

$$z \ll \lambda \Rightarrow \frac{2\pi}{\lambda} z \ll 2\pi \Rightarrow e^{-j\frac{2\pi}{\lambda} z} \approx 1$$



Plane Waves (Phasor Domain)

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$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis
- When z is small with respect to λ , the propagation effects become negligible