Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2022-2023 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

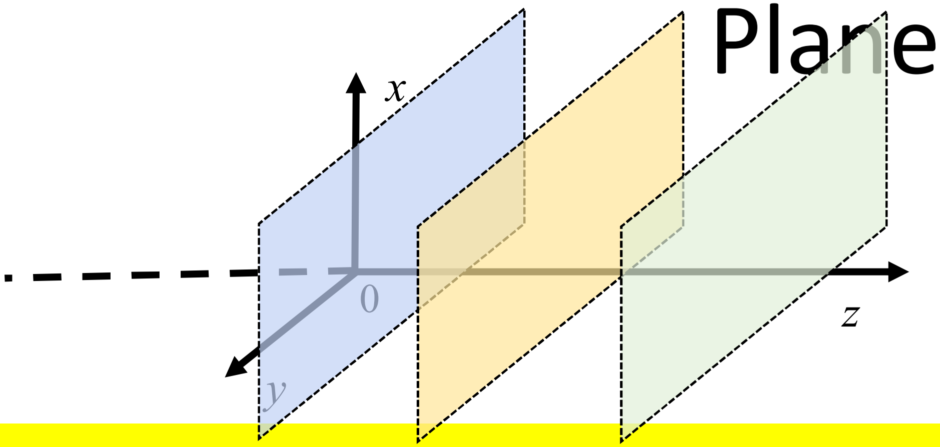
General expression of plane waves (PD)

Incidence

Plane Waves

Time domain

Plane Waves (TD)



Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=e_x(\vec{\mathbf{r}},t)\hat{i}_x+e_y(\vec{\mathbf{r}},t)\hat{i}_y+e_z(\vec{\mathbf{r}},t)\hat{i}_z=e_x(z,t)\hat{i}_x+e_y(z,t)\hat{i}_y+e_z(z,t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=h_x(\vec{\mathbf{r}},t)\hat{i}_x+h_y(\vec{\mathbf{r}},t)\hat{i}_y+h_z(\vec{\mathbf{r}},t)\hat{i}_z=h_x(z,t)\hat{i}_x+h_y(z,t)\hat{i}_y+h_z(z,t)\hat{i}_z$$

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

Time domain - Differential form

$$\begin{cases}\nabla\times\vec{\mathbf{e}}(z,t)=-\mu\dfrac{\partial\vec{\mathbf{h}}(z,t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(z,t)=\varepsilon\dfrac{\partial\vec{\mathbf{e}}(z,t)}{\partial t}\\ \varepsilon\nabla\cdot\vec{\mathbf{e}}(z,t)=0\\ \mu\nabla\cdot\vec{\mathbf{h}}(z,t)=0\end{cases}$$

$$\nabla\times\vec{\mathbf{e}}=\left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$\nabla\times\vec{\mathbf{h}}=\left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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Independent each other

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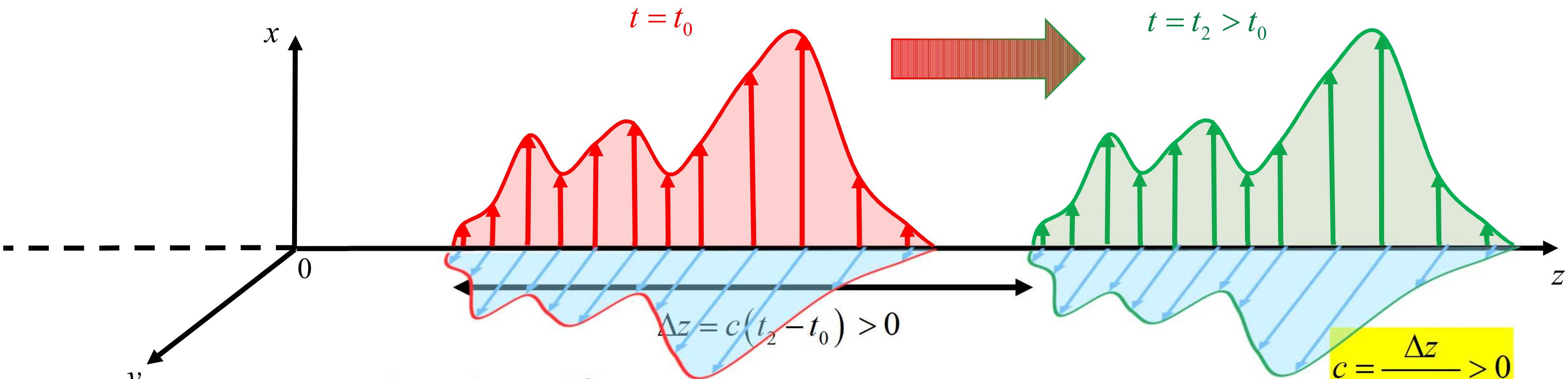

$$\vec{\mathbf{e}}\left(z,t_0\right) = \mathrm{e}^{+}\left(z - ct_0\right)\hat{i}_x$$

$$\vec{\mathbf{e}}\left(z,t_2\right) = \mathrm{e}^{+}\left(z - ct_2\right)\hat{i}_x = \mathrm{e}^{+}\left(z - ct_0 + ct_0 - ct_2\right)\hat{i}_x = \mathrm{e}^{+}\left(z - ct_0 - c\left[t_2 - t_0\right]\right)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the *z*-axis

$\begin{cases} e^{+}\left(z - ct\right) \\ h^{+}\left(z - ct\right) \end{cases}$ is referred to as electromagnetic **<u>progressive</u> plane wave**

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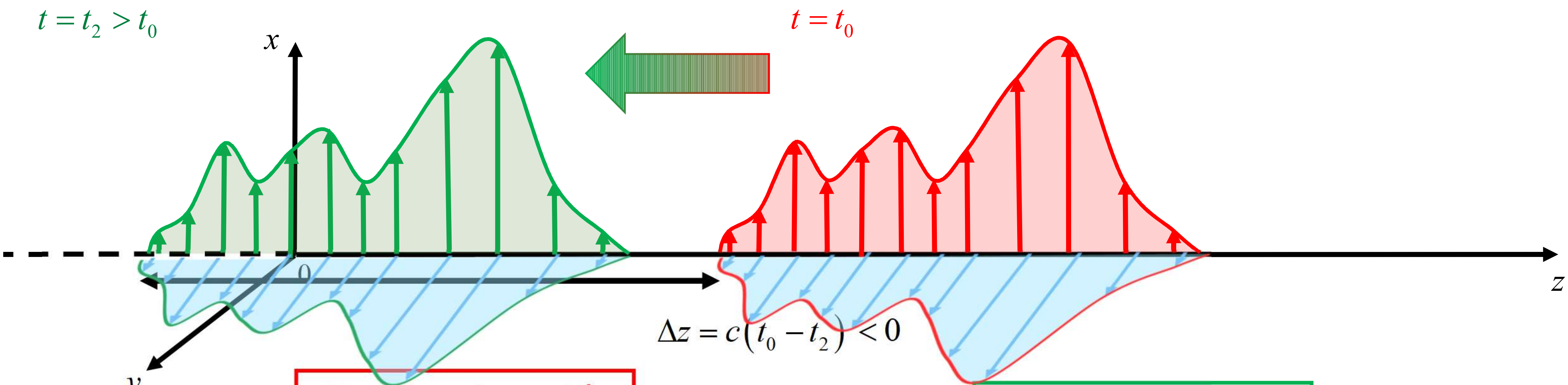

$$\vec{\mathbf{e}}(z, t_0) = \mathrm{e}^-(z + ct_0)\hat{i}_x$$

$$\vec{\mathbf{e}}(z, t_2) = \mathrm{e}^-(z + ct_2)\hat{i}_x = \mathrm{e}^-(z + ct_0 - ct_0 + ct_2)\hat{i}_x = \mathrm{e}^-\Big(z + ct_0 - \overbrace{c[t_0 - t_2]}^{\Delta z}\Big)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^-(z + ct) \\ h^-(z + ct) \end{cases}$$ is referred to as electromagnetic **<u>regressive</u> plane wave**

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$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{s}}^+ = \vec{\mathbf{e}}^+ \times \vec{\mathbf{h}}^+ = e_x^+ \hat{i}_x \times h_y^+ \hat{i}_y = e_x^+ \hat{i}_x \times \frac{e_x^+}{\zeta}\hat{i}_y = \frac{\left[e_x^+\right]^2}{\zeta}\left(\hat{i}_x \times \hat{i}_y\right) = \frac{\left[e_x^+\right]^2}{\zeta}\hat{i}_z$$

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$
$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$
$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (TD)

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$e_z(z,t)=h_z(z,t)=0$$

$\{e_y,h_x\}$, $\{e_x,h_y\}$ **Independent each other**

$$c=\frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_x,h_y\}$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\[2mm] \dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2}=0$$

$\{e_x^+,h_y^+\}$

$$\begin{cases}e_x^+(z,t)=e_x^+(z-ct)\\ \zeta h_y^+(z,t)=e_x^+(z-ct)\end{cases}$$

$\{e_x^-,h_y^-\}$

$$\begin{cases}e_x^-(z,t)=e_x^-(z+ct)\\ \zeta h_y^-(z,t)=-e_x^-(z+ct)\end{cases}$$

$$\vec{\mathbf{s}}^+=\frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^-=-\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$\{e_y,h_x\}$

$$\begin{cases}\dfrac{\partial e_y}{\partial z}=\mu\dfrac{\partial h_x}{\partial t}\\[2mm] \dfrac{\partial h_x}{\partial z}=\varepsilon\dfrac{\partial e_y}{\partial t}\end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2}=0$$

$\{e_y^+,h_x^+\}$

$$\begin{cases}e_y^+(z,t)=e_y^+(z-ct)\\ \zeta h_x^+(z,t)=-e_y^+(z-ct)\end{cases}$$

$\{e_y^-,h_x^-\}$

$$\begin{cases}e_y^-(z,t)=e_y^-(z+ct)\\ \zeta h_x^-(z,t)=e_y^-(z+ct)\end{cases}$$

$$\vec{\mathbf{s}}^+=\frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^-=-\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$\{e_x^+,h_y^+\}$ $\{e_x^-,h_y^-\}$ $\{e_y^+,h_x^+\}$ $\{e_y^-,h_x^-\}$

In all these 4 cases the Poynting vector is directed along the direction of propagation

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

Source-free

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$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

$\{e_x^+, h_y^+\}$

$$\zeta\vec{\mathbf{h}} = \zeta h_y^+ \hat{i}_y = e_x^+ \hat{i}_y$$

$$\hat{i}_p = \hat{i}_z;\ \vec{\mathbf{e}} = e_x^+ \hat{i}_x \Rightarrow \hat{i}_p \times \vec{\mathbf{e}} = \hat{i}_z \times e_x^+ \hat{i}_x = e_x^+\left(\hat{i}_z \times \hat{i}_x\right) = e_x^+ \hat{i}_y$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$
$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$
$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

Source-free

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

$\{e_x^-, h_y^-\}$

$$\zeta\vec{\mathbf{h}} = \zeta h_y^- \hat{i}_y = -e_x^- \hat{i}_y$$

$$\hat{i}_p = -\hat{i}_z\,;\ \vec{\mathbf{e}} = e_x^- \hat{i}_x \Rightarrow \hat{i}_p \times \vec{\mathbf{e}} = -\hat{i}_z \times e_x^- \hat{i}_x = -e_x^-\left(\hat{i}_z \times \hat{i}_x\right) = -e_x^- \hat{i}_y$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y \qquad \hat{i}_y = \hat{i}_z \times \hat{i}_x \qquad \hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$ $\{e_x, h_y\}$ **Independent each other**

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ points to the propagation direction

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_y^+(z-ct)\right]^2\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$e_x^+(z-ct) = \zeta h_y^+(z-ct)$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z = \zeta\left[h_y^+(z-ct)\right]^2\hat{i}_z$$

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_y^+(z-ct)\right]^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left[h_y^-(z+ct)\right]^2\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_x^+(z-ct)\right]^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left[h_x^-(z+ct)\right]^2\hat{i}_z$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0 \qquad \{e_x, h_y\}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0 \qquad \{e_y, h_x\}$$

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the Poynting vector can be written as follows:

$$\vec{\mathbf{s}} = \frac{|\vec{\mathbf{e}}|^2}{\zeta}\hat{i}_p = \zeta |\vec{\mathbf{h}}|^2 \hat{i}_p$$

where $\hat{i}_p$ **points to the propagation direction**

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_y^+(z-ct)\right]^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left[h_y^-(z+ct)\right]^2\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_x^+(z-ct)\right]^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left[h_x^-(z+ct)\right]^2\hat{i}_z$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$