Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Spectral domains

Plane Waves (Spectral Domains)

Phasor domain (PD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Source-free

Medium

- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}})} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \cancel{\rho_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \cancel{\rho_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}})} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \cancel{\rho_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \cancel{\rho_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium

- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}})} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \cancel{\rho_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}\cancel{(\vec{\mathbf{r}},\omega)} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \cancel{\rho_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}})} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \cancel{\rho_0(\vec{\mathbf{r}})} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \cancel{\rho_0(\vec{\mathbf{r}},\omega)} \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}})} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}\cancel{(\vec{\mathbf{r}},\omega)} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

PD

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

FD

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

Phasor domain (PD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}})} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \cancel{\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)} + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$

$$\begin{cases} \boldsymbol{\varepsilon} \cancel{= real} = \varepsilon_1 - j\varepsilon_2 \\ \boldsymbol{\mu} \cancel{= real} = \mu_1 - j\mu_2 \\ \boldsymbol{\sigma} \neq \mathbf{0} \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}})} \end{cases}$$

$$\vec{\mathbf{J}}(\vec{\mathbf{r}}) \rightarrow \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}})$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \cancel{\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega)} \end{cases}$$

$$\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) \rightarrow \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) + \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) + \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \left[\varepsilon - \frac{j\sigma}{\omega_0}\right] \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \left[\varepsilon - \frac{j\sigma}{\omega}\right] \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

$$\varepsilon_{eq} = \left[\varepsilon - \frac{j\sigma}{\omega}\right]$$

$$\varepsilon_{eq} = \left[\varepsilon - \frac{j\sigma}{\omega_0}\right]$$

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon_{eq} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \varepsilon_{eq} \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

$$\varepsilon_{eq} = \left[\varepsilon - \frac{j\sigma}{\omega}\right]$$

$$\varepsilon_{eq} = \left[\varepsilon - \frac{j\sigma}{\omega_0}\right]$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon_{eq} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \varepsilon_{eq} \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

$$\varepsilon_{eq} = \left[\varepsilon - \frac{j\sigma}{\omega}\right]$$

$$\varepsilon_{eq} = \left[\varepsilon - \frac{j\sigma}{\omega_0}\right]$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega \mu \vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega \varepsilon \vec{\mathbf{E}}(z,\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

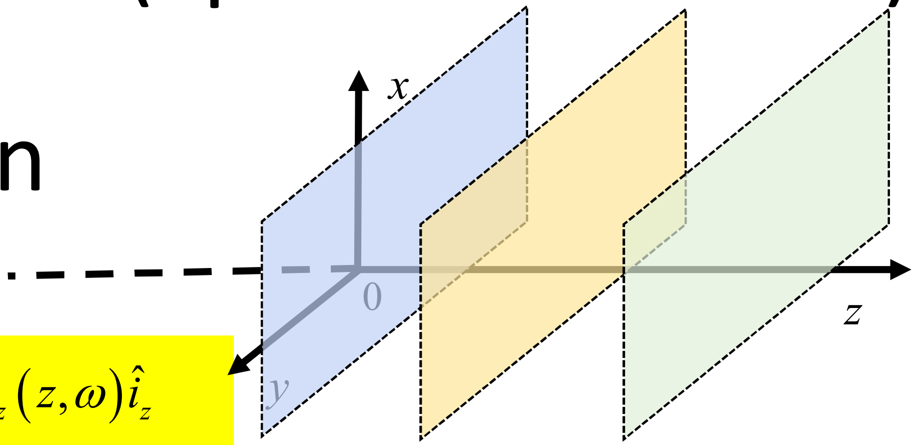
$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

Plane Waves (Spectral Domains)

Fourier Domain



$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Fourier domain (FD)

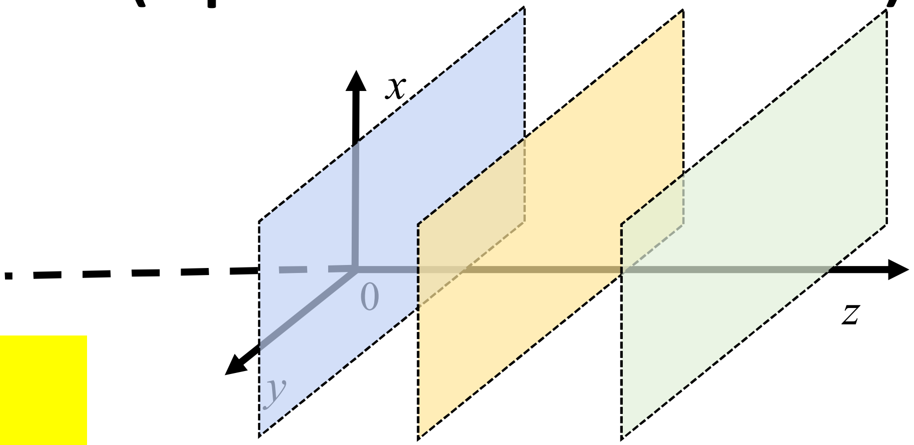
$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega\mu\vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega\varepsilon\vec{\mathbf{E}}(z,\omega) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

Phasor Domain



Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}}\\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y + \cancel{E_z\hat{i}_z}$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y + \cancel{H_z\hat{i}_z}$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$E_z = 0 \qquad H_z = 0$$

⇒

$$e_z(z,t) = 0 \qquad h_z(z,t) = 0$$

⇒ **TEM fields**

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⇓

$$E_z = H_z = 0$$

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

Source-free

Medium

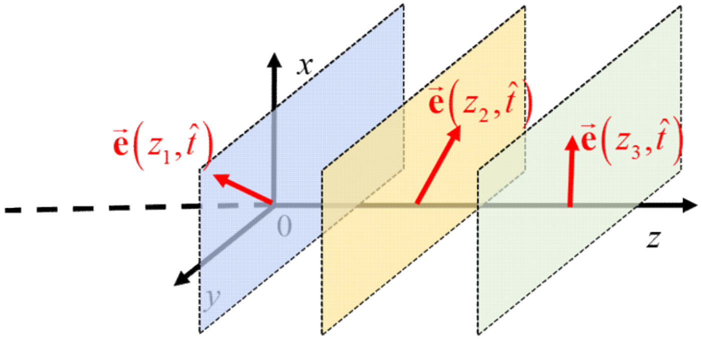
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$



Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$\{E_y, H_x\}$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \Rightarrow \frac{d^2E_x}{dz^2} = -j\omega\mu \frac{dH_y}{dz} = -\omega^2\mu\varepsilon E_x \Rightarrow \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \Rightarrow \frac{d^2E_y}{dz^2} = j\omega\mu \frac{dH_x}{dz} = -\omega^2\mu\varepsilon E_y \Rightarrow \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

k : (complex) propagation constant

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$: **Independent each other**

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other