

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

The radiation condition

$$\vec{e} \cdot \hat{i}_r = 0$$

$$\vec{h} \cdot \hat{i}_r = 0$$

$$\vec{e} - \zeta \vec{h} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and thus } \zeta \vec{h} - \hat{i}_r \times \vec{e} \sim o\left(\frac{1}{r}\right) \right)$$

as $r \rightarrow \infty$

.. on a sphere of radius r centered in the origin of the reference system, being \hat{i}_r the radial unit vector

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

$$\vec{s} = \frac{|\vec{e}|^2}{\zeta} \hat{i}_r = \zeta |\vec{h}|^2 \hat{i}_r$$

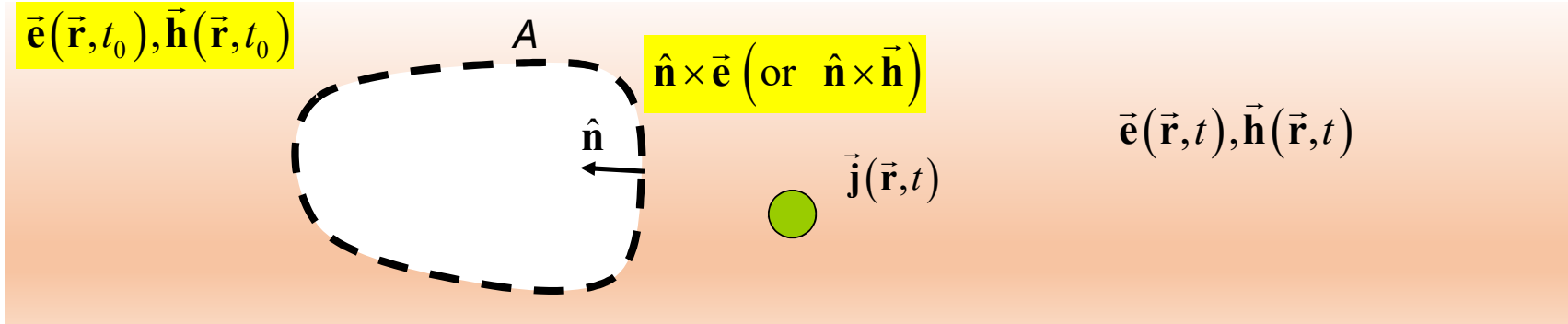


$$\vec{e} \sim O\left(\frac{1}{r}\right)$$

$$\vec{h} \sim O\left(\frac{1}{r}\right)$$

as $r \rightarrow \infty$

Uniqueness (TD-Exterior Problem)



- I Consider a source distribution $\vec{j}(\vec{r}, t)$ with its associated electromagnetic field (\vec{e}, \vec{h})
- II Consider a (smooth) surface A with an everywhere defined unit normal \hat{n}
- III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{e}(\vec{r}, t_0), \vec{h}(\vec{r}, t_0)$
- IV Consider the values of the tangential component of the electric (or magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{n} \times \vec{e}$ (or $\hat{n} \times \vec{h}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

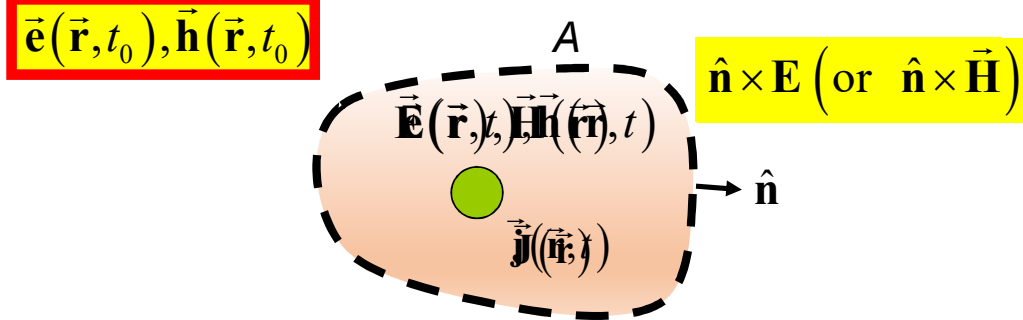
Phasor domain

Image Theory

Reciprocity

Phasor domain

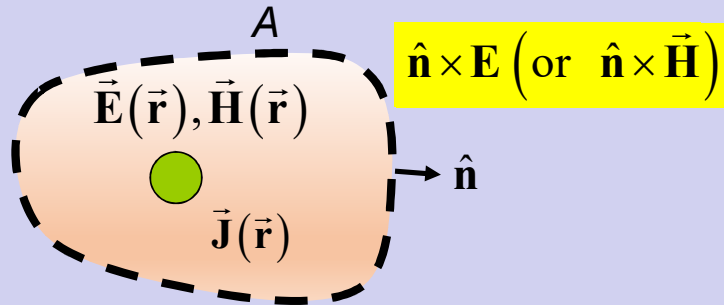
Uniqueness (PD-Interior Problem)



- I Consider a source distribution $\vec{j}(\vec{r}, t)$ with its associated electromagnetic field (\vec{e}, \vec{h})
- II Consider a (smooth) surface A with an everywhere defined unit normal \hat{n}
- III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{e}(\vec{r}, t_0), \vec{h}(\vec{r}, t_0)$
- IV Consider the values of the tangential component of the electric (or magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{n} \times \vec{e}$ (or $\hat{n} \times \vec{h}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V** bounded by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (PD-Interior Problem)



Source distribution: $\vec{J}(\vec{r})$

$$\vec{E}_1(\vec{r}), \vec{H}_1(\vec{r}) \quad \vec{E}_2(\vec{r}), \vec{H}_2(\vec{r})$$

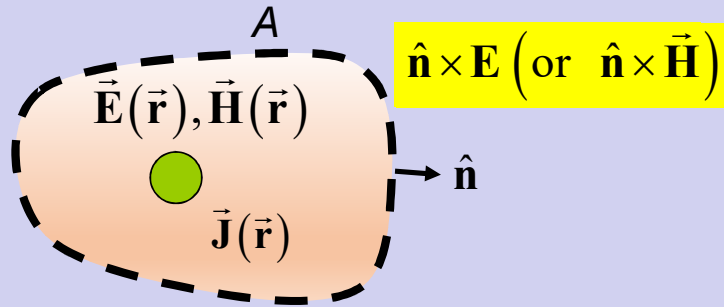
$$\hat{n} \times \vec{E}_1(\vec{r}) = \hat{n} \times \vec{E}_2(\vec{r}) \text{ on the boundary}$$

Field difference: source distribution $\vec{J}_0(\vec{r}) = 0$

$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) - \vec{E}_2(\vec{r}) \quad \vec{H}(\vec{r}) = \vec{H}_1(\vec{r}) - \vec{H}_2(\vec{r})$$

$$\hat{n} \times \vec{E}(\vec{r}) = \hat{n} \times \vec{E}_1(\vec{r}) - \hat{n} \times \vec{E}_2(\vec{r}) = 0 \text{ on the boundary}$$

Uniqueness (PD-Interior Problem)



Field difference: source distribution $\vec{J}_0(\vec{r}) = 0$

$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) - \vec{E}_2(\vec{r})$$

$$\vec{H}(\vec{r}) = \vec{H}_1(\vec{r}) - \vec{H}_2(\vec{r})$$

$$\hat{n} \times \vec{E}(\vec{r}) = \hat{n} \times \vec{E}_1(\vec{r}) - \hat{n} \times \vec{E}_2(\vec{r}) = 0 \quad \text{on the boundary}$$

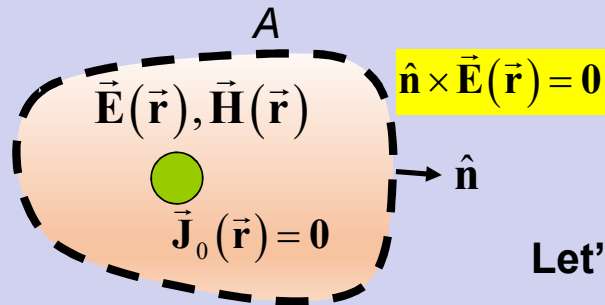
Uniqueness (PD-Interior Problem)

$$\begin{aligned}\vec{\mathbf{E}}(\vec{\mathbf{r}}) &= \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \\ \vec{\mathbf{H}}(\vec{\mathbf{r}}) &= \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})\end{aligned}$$

$$\text{Source distribution } \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0} \quad \text{on the boundary}$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-Dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{r}) = \vec{\mathbf{E}}_1(\vec{r}) - \vec{\mathbf{E}}_2(\vec{r})$$

$$\vec{\mathbf{H}}(\vec{r}) = \vec{\mathbf{H}}_1(\vec{r}) - \vec{\mathbf{H}}_2(\vec{r})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{r}) = 0$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{r}) = 0$ on the boundary

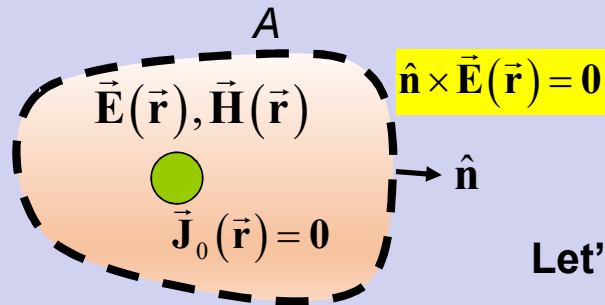
~~$$\oiint_A dA \vec{\mathbf{S}}_1(\vec{r}) \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{r})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{r})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{r})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{r}) \cdot \vec{\mathbf{J}}_0^*(\vec{r}) \right\} \right]$$~~

$$\oiint_A dA \vec{\mathbf{S}}_2(\vec{r}) \cdot \hat{\mathbf{n}} + 2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{\mathbf{H}}(\vec{r})|^2 - \frac{1}{4} \varepsilon_1 |\vec{\mathbf{E}}(\vec{r})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Im} \left\{ \vec{\mathbf{E}}(\vec{r}) \cdot \vec{\mathbf{J}}_0^*(\vec{r}) \right\} \right]$$

$$\oiint_A dA \vec{\mathbf{S}}_1(\vec{r}) \cdot \hat{\mathbf{n}} = \text{Re} \left\{ \oiint_A dA \left[\frac{1}{2} \vec{\mathbf{E}}(\vec{r}) \times \vec{\mathbf{H}}^*(\vec{r}) \right] \cdot \hat{\mathbf{n}} \right\} = \text{Re} \left\{ \oiint_A dA \left[\frac{1}{2} \hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{r}) \right] \cdot \vec{\mathbf{H}}^*(\vec{r}) \right\} = \text{Re} \left\{ \oiint_A dA \left[\frac{1}{2} \vec{\mathbf{H}}^*(\vec{r}) \times \hat{\mathbf{n}} \right] \cdot \vec{\mathbf{E}}(\vec{r}) \right\} = 0$$

$$\vec{\mathbf{A}} \cdot [\vec{\mathbf{B}} \times \vec{\mathbf{C}}] = \vec{\mathbf{C}} \cdot [\vec{\mathbf{A}} \times \vec{\mathbf{B}}] = \vec{\mathbf{B}} \cdot [\vec{\mathbf{C}} \times \vec{\mathbf{A}}]$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

Medium

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- Isotropic
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- **Time-Dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = 0$

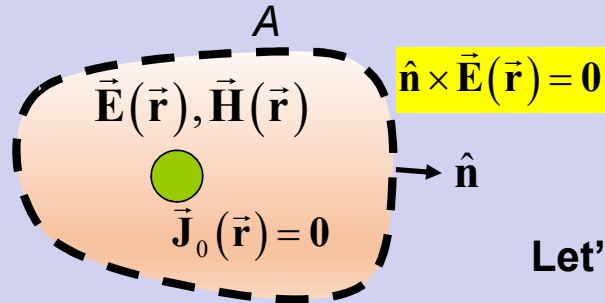
$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0$ on the boundary

~~$$\oiint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}}) \} \right]$$

$$\oiint_A dA \vec{\mathbf{S}}_2(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + 2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 - \frac{1}{4} \varepsilon_1 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}}) \} \right]$$~~

$$\oiint_A dA \vec{\mathbf{S}}_2(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} = \text{Im} \left\{ \oiint_A dA \left[\frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \right] \cdot \hat{\mathbf{n}} \right\} = \text{Im} \left\{ \oiint_A dA \left[\frac{1}{2} \hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \right] \cdot \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \right\} = 0$$

Uniqueness (PD-Interior Problem)



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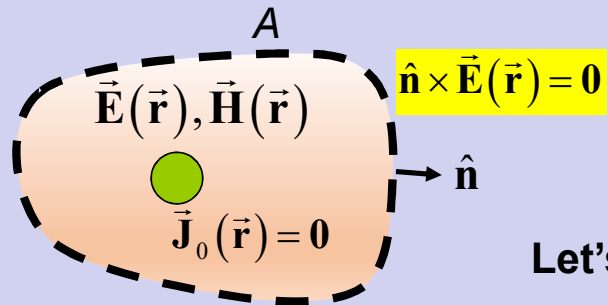
$$\text{Source distribution } \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = 0$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

~~$$\oiint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \right\} \right]$$~~

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Uniqueness (PD-Interior Problem)



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$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) - \vec{E}_2(\vec{r})$$

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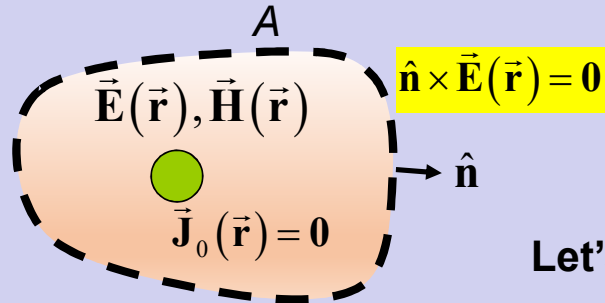
$$\text{Source distribution } \vec{J}_0(\vec{r}) = 0$$

$$\hat{n} \times \vec{E}(\vec{r}) = 0 \quad \text{on the boundary}$$

$$\iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{H}(\vec{r})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{E}(\vec{r})|^2 + \frac{1}{2} \sigma |\vec{E}(\vec{r})|^2 \right] = 0$$

$$2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{H}(\vec{r})|^2 - \frac{1}{4} \varepsilon_1 |\vec{E}(\vec{r})|^2 \right] = 0$$

Uniqueness (PD-Interior Problem)



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Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = 0$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0$ on the boundary

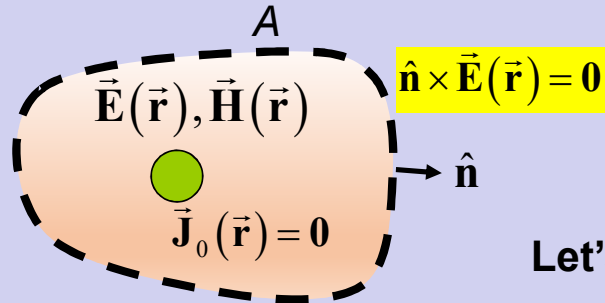
$$\iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = 0$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \quad \text{cvd}$$

$$\iiint_V dV \frac{1}{4} \mu_1 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 = \iiint_V dV \frac{1}{4} \varepsilon_1 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

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$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) - \vec{E}_2(\vec{r})$$

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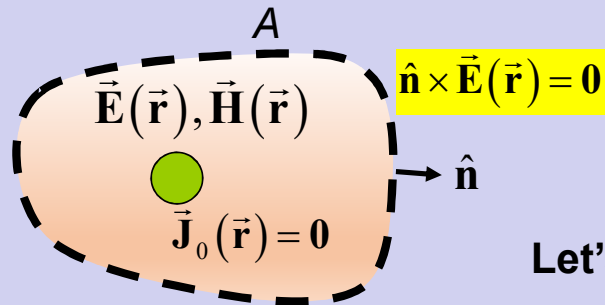
$$\text{Source distribution } \vec{J}_0(\vec{r}) = 0$$

$$\hat{n} \times \vec{E}(\vec{r}) = 0 \quad \text{on the boundary}$$

$$\iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{H}(\vec{r})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{E}(\vec{r})|^2 + \frac{1}{2} \sigma |\vec{E}(\vec{r})|^2 \right] = 0$$

$$\iiint_V dV \frac{1}{4} \mu_1 |\vec{H}(\vec{r})|^2 = \iiint_V dV \frac{1}{4} \varepsilon_1 |\vec{E}(\vec{r})|^2$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

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$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = 0$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0$ on the boundary

~~$$\iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = 0$$~~

$$\iiint_V dV \frac{1}{4} \mu_1 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 = \iiint_V dV \frac{1}{4} \varepsilon_1 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2$$

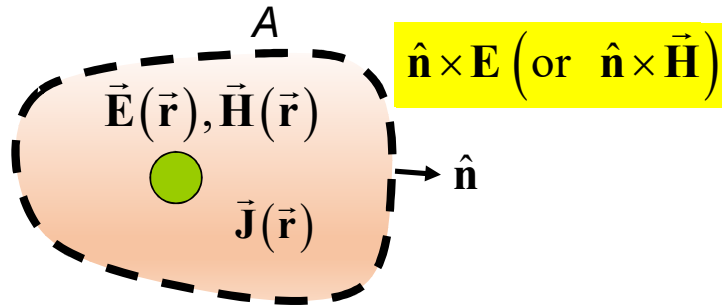
$$\begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}$$

+

No Homic losses $\sigma = 0$

Uniqueness is not ensured anymore!

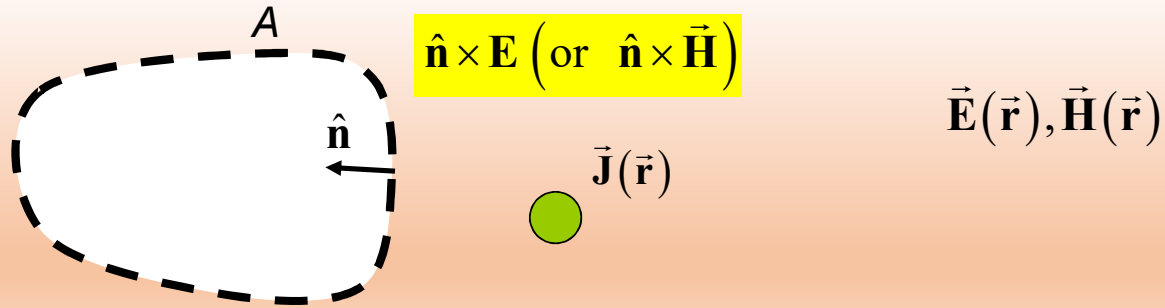
Uniqueness (PD-Interior Problem)



- I Consider a source distribution $\vec{J}(\vec{r})$ with its associated electromagnetic field $\vec{E}(\vec{r}), \vec{H}(\vec{r})$
- II Consider a (smooth) surface A with an everywhere defined unit normal \hat{n}
- IV Consider the values of the tangential component of the electric (or magnetic) field upon the surface A ; that is, consider $\hat{n} \times \vec{E}$ (or $\hat{n} \times \vec{H}$) **on the boundary**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the boundary condition** in (IV) is unique **provided that the considered medium is lossy**. **In a lossless medium**, instead, the solution is unique **but for a set of resonant solutions**.

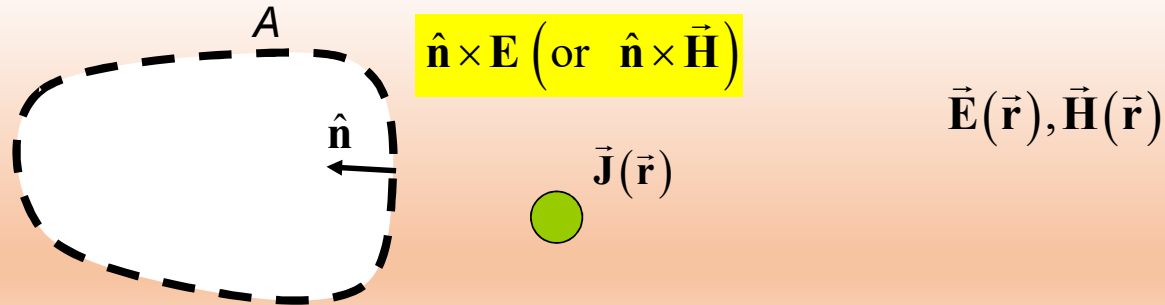
Uniqueness (PD-Exterior Problem)



- I Consider a source distribution $\vec{\mathbf{J}}(\vec{\mathbf{r}})$ with its associated electromagnetic field $\vec{\mathbf{E}}(\vec{\mathbf{r}}), \vec{\mathbf{H}}(\vec{\mathbf{r}})$
- II Consider a (smooth) surface A with an everywhere defined unit normal $\hat{\mathbf{n}}$
- IV Consider the values of the tangential component of the electric (or magnetic) field upon the surface A ; that is, consider $\hat{\mathbf{n}} \times \mathbf{E}$ (or $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$) **on the boundary**

The Uniqueness Theorem states that

Uniqueness (PD-Exterior Problem)



Source distribution: $\vec{\mathbf{J}}(\vec{\mathbf{r}})$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}), \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) \quad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}), \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

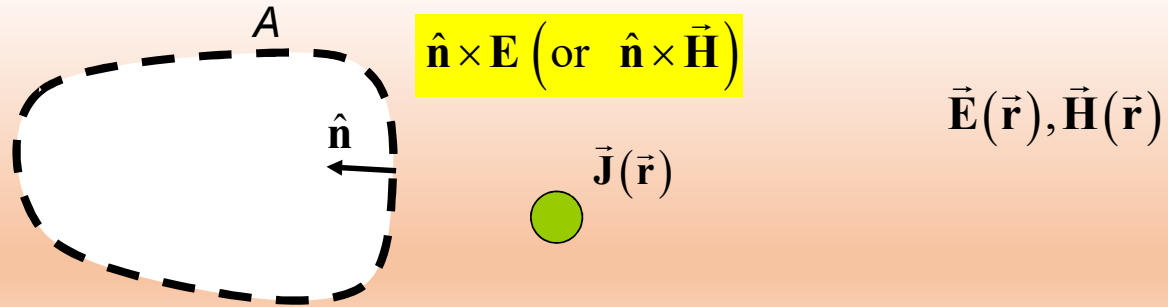
$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \text{ on the boundary}$$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = 0$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \quad \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \text{ on the boundary}$$

Uniqueness (PD-Exterior Problem)



Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = 0$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

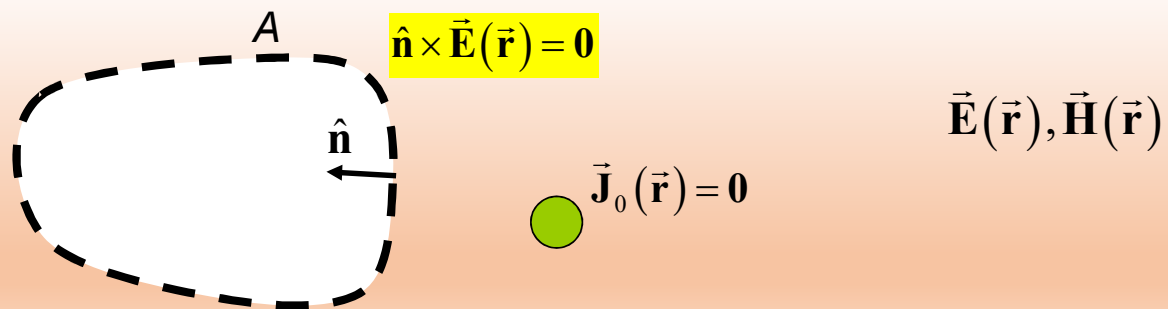
Uniqueness (PD-Exterior Problem)

$$\begin{aligned}\vec{\mathbf{E}}(\vec{\mathbf{r}}) &= \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \\ \vec{\mathbf{H}}(\vec{\mathbf{r}}) &= \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})\end{aligned}$$

$$\text{Source distribution } \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0} \quad \text{on the boundary}$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

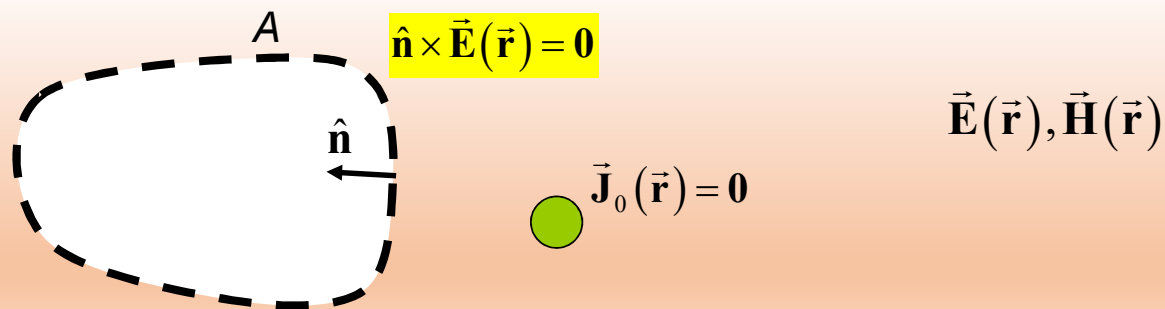
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

$$\oiint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \oiint_{A_\infty} dA_\infty \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{i}}_r + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}}) \right\} \right]$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

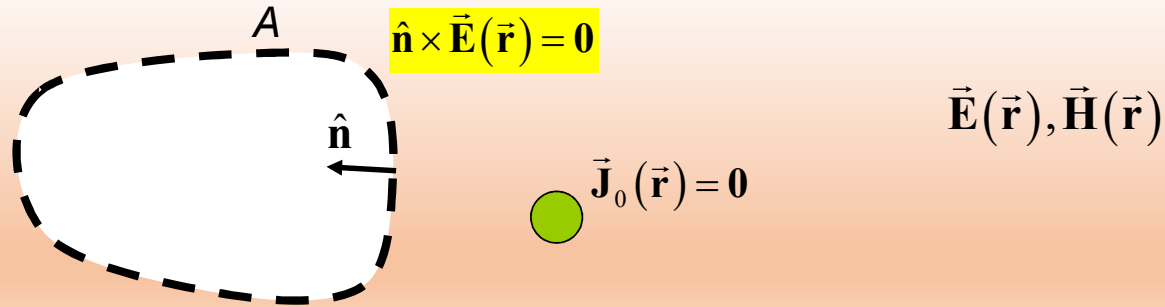
$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

~~$$\iint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iint_{A_\infty} dA_\infty \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{i}}_r + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \right\} \right]$$~~

$$\iint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} = \text{Re} \left\{ \iint_A dA \left[\frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \right] \cdot \hat{\mathbf{n}} \right\} = \text{Re} \left\{ \iint_A dA \left[\frac{1}{2} \hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \right] \cdot \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \right\} = \text{Re} \left\{ \iint_A dA \left[\frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \times \hat{\mathbf{n}} \right] \cdot \vec{\mathbf{E}}(\vec{\mathbf{r}}) \right\}$$

$$\vec{\mathbf{A}} \cdot [\vec{\mathbf{B}} \times \vec{\mathbf{C}}] = \vec{\mathbf{C}} \cdot [\vec{\mathbf{A}} \times \vec{\mathbf{B}}] = \vec{\mathbf{B}} \cdot [\vec{\mathbf{C}} \times \vec{\mathbf{A}}]$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

~~$$\iint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iint_{A_\infty} dA_\infty \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{i}}_r + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \right\} \right]$$~~

The radiation condition

$$\vec{e} \sim O\left(\frac{1}{r}\right) \quad \vec{h} \sim O\left(\frac{1}{r}\right) \quad \vec{e} - \zeta \vec{h} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and } \zeta \vec{h} - \hat{i}_r \times \vec{e} \sim o\left(\frac{1}{r}\right) \right) \quad \text{as } r \rightarrow \infty \quad \text{TD}$$
$$\hat{i}_r \cdot \vec{e} = \hat{i}_r \cdot \vec{h} = 0$$

$$\vec{E} \sim O\left(\frac{1}{r}\right) \quad \vec{H} \sim O\left(\frac{1}{r}\right) \quad \vec{E} - \zeta \vec{H} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and } \zeta \vec{H} - \hat{i}_r \times \vec{E} \sim o\left(\frac{1}{r}\right) \right) \quad \text{as } r \rightarrow \infty \quad \text{PD}$$
$$\hat{i}_r \cdot \vec{E} = \hat{i}_r \cdot \vec{H} = 0$$

$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

Note that in the PD, the quantity ζ , called **intrinsic impedance**, can be, in general, complex. At infinity, we assume that ζ is a real quantity.

The radiation condition

$$\vec{e} \sim O\left(\frac{1}{r}\right) \quad \vec{h} \sim O\left(\frac{1}{r}\right) \quad \vec{e} - \zeta \vec{h} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and } \zeta \vec{h} - \hat{i}_r \times \vec{e} \sim o\left(\frac{1}{r}\right) \right) \quad \text{as } r \rightarrow \infty \quad \text{TD}$$

$$\hat{i}_r \cdot \vec{e} = \hat{i}_r \cdot \vec{h} = 0$$

$$\vec{E} \sim O\left(\frac{1}{r}\right) \quad \vec{H} \sim O\left(\frac{1}{r}\right) \quad \vec{E} - \zeta \vec{H} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and } \zeta \vec{H} - \hat{i}_r \times \vec{E} \sim o\left(\frac{1}{r}\right) \right) \quad \text{as } r \rightarrow \infty \quad \text{PD}$$

$$\hat{i}_r \cdot \vec{E} = \hat{i}_r \cdot \vec{H} = 0$$

Note that in the PD, the quantity ζ , called **intrinsic impedance**, can be, in general, complex. At infinity, we assume that ζ is a real quantity.

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2\zeta} \vec{E} \times (\hat{i}_r \times \vec{E})^* = \frac{1}{2\zeta} \vec{E} \times (\hat{i}_r \times \vec{E}^*) = \frac{1}{2\zeta} \left[(\vec{E} \cdot \vec{E}^*) \hat{i}_r - \cancel{(\hat{i}_r \cdot \vec{E}) \vec{E}^*} \right] = \frac{|\vec{E}|^2}{2\zeta} \hat{i}_r$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = (\zeta \vec{H} \times \hat{i}_r) \times \vec{H}^* = -\vec{H}^* \times (\zeta \vec{H} \times \hat{i}_r) = -\left[\cancel{(\hat{i}_r \cdot \vec{H}^*) \zeta \vec{H}} - (\zeta \vec{H} \cdot \vec{H}^*) \hat{i}_r \right] = \frac{\zeta}{2} |\vec{H}|^2 \hat{i}_r$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$$

The radiation condition

$$\vec{e} \sim O\left(\frac{1}{r}\right) \quad \vec{h} \sim O\left(\frac{1}{r}\right) \quad \vec{e} - \zeta \vec{h} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and } \zeta \vec{h} - \hat{i}_r \times \vec{e} \sim o\left(\frac{1}{r}\right) \right) \quad \text{as } r \rightarrow \infty \quad \text{TD}$$

$$\hat{i}_r \cdot \vec{e} = \hat{i}_r \cdot \vec{h} = 0$$

$$\vec{E} \sim O\left(\frac{1}{r}\right) \quad \vec{H} \sim O\left(\frac{1}{r}\right) \quad \vec{E} - \zeta \vec{H} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \quad \left(\text{and } \zeta \vec{H} - \hat{i}_r \times \vec{E} \sim o\left(\frac{1}{r}\right) \right) \quad \text{as } r \rightarrow \infty \quad \text{PD}$$

$$\hat{i}_r \cdot \vec{E} = \hat{i}_r \cdot \vec{H} = 0$$

$$\oiint_{A_\infty} dA_\infty \vec{S}_1(\vec{r}) \cdot \hat{i}_r = \oiint_{A_\infty} dA_\infty \frac{1}{2\zeta} |\vec{E}(\vec{r})|^2 = \lim_{r \rightarrow \infty} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta r^2 \sin \vartheta \frac{|\vec{E}(\vec{r})|^2}{2\zeta}$$

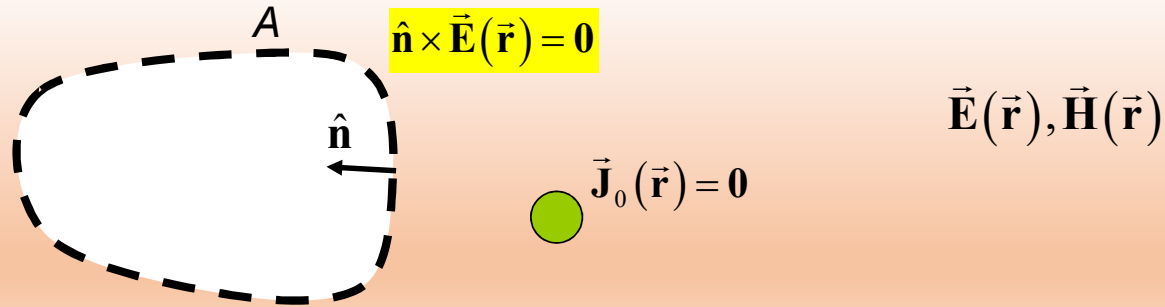
$$\left\{ \begin{array}{l} \vec{S}_1 = \frac{1}{2\zeta} |\vec{E}|^2 \hat{i}_r = \frac{\zeta}{2} |\vec{H}|^2 \hat{i}_r \\ \vec{S}_2 = \mathbf{0} \end{array} \right.$$

is a finite nonnegative quantity

A_∞ is the surface of a sphere of radius $r \rightarrow \infty$ and centered in the origin of the reference system

$$\oiint_A dA \Phi(r, \vartheta, \varphi) = \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta r^2 \sin \vartheta \Phi(r, \vartheta, \varphi)$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-Dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

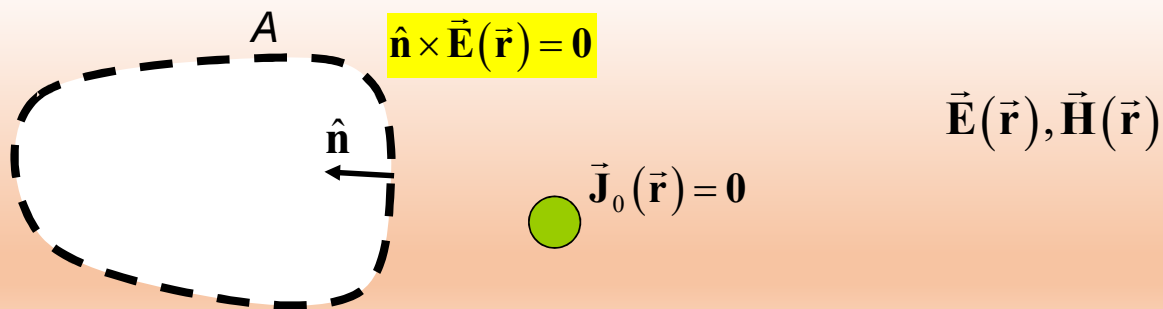
$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Radiation condition at infinity

~~$$\iint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iint_{A_\infty} dA_\infty \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{i}}_r + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \right\} \right]$$~~

$$\iint_{A_\infty} dA_\infty \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{i}}_r \geq 0$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-Dispersive**
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

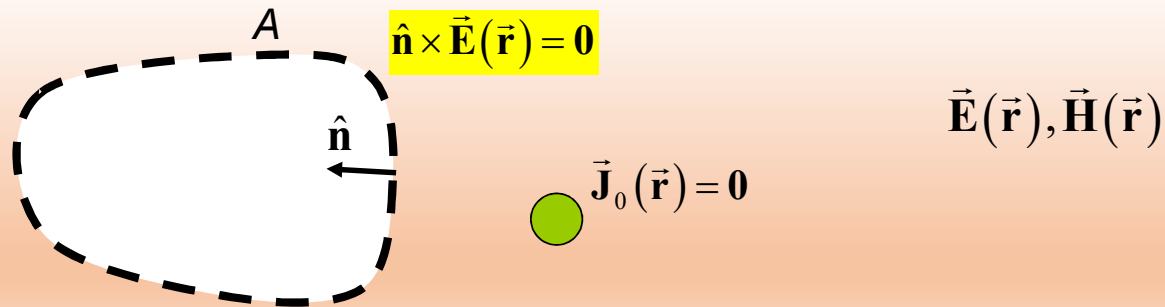
$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Radiation condition at infinity

~~$$\iint_A dA \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iint_{A_\infty} dA_\infty \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{i}}_r + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \epsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \right\} \right]$$~~

$$\iint_{A_\infty} dA_\infty \frac{1}{2\zeta} |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \epsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = 0$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- **Time-Dispersive**
- Time-invariant

$$\begin{aligned} \vec{\mathbf{E}}(\vec{\mathbf{r}}) &= \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \\ \vec{\mathbf{H}}(\vec{\mathbf{r}}) &= \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}}) \end{aligned}$$

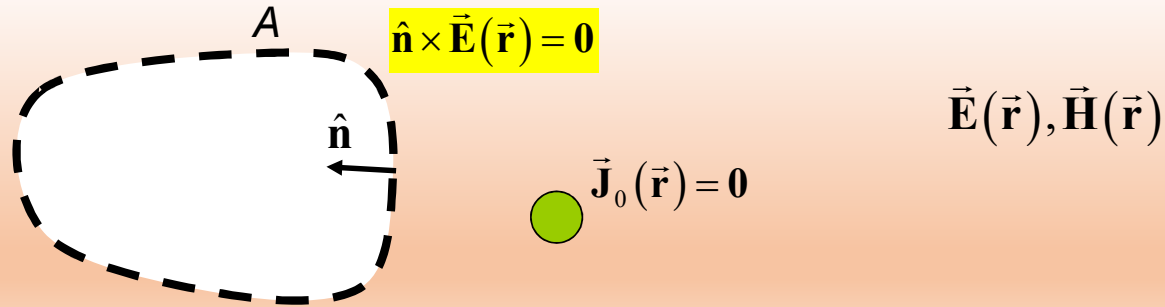
Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Radiation condition at infinity

$$\oiint_{A_\infty} dA_\infty \frac{1}{2\zeta} |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = 0 \quad \Rightarrow \quad \begin{aligned} \vec{\mathbf{E}}(\vec{\mathbf{r}}) &= \mathbf{0} \\ \vec{\mathbf{H}}(\vec{\mathbf{r}}) &= \mathbf{0} \end{aligned} \quad \text{cvd}$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- Time-Dispersive
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

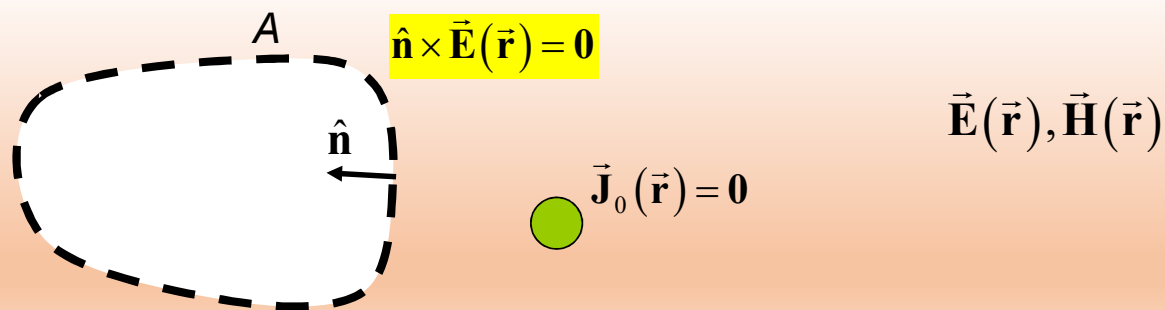
Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Radiation condition at infinity

$$\oint_{A_\infty} dA_\infty \frac{1}{2\zeta} |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = 0$$

Uniqueness (PD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- Time-Nondispersive
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Radiation condition at infinity

$$\begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}$$

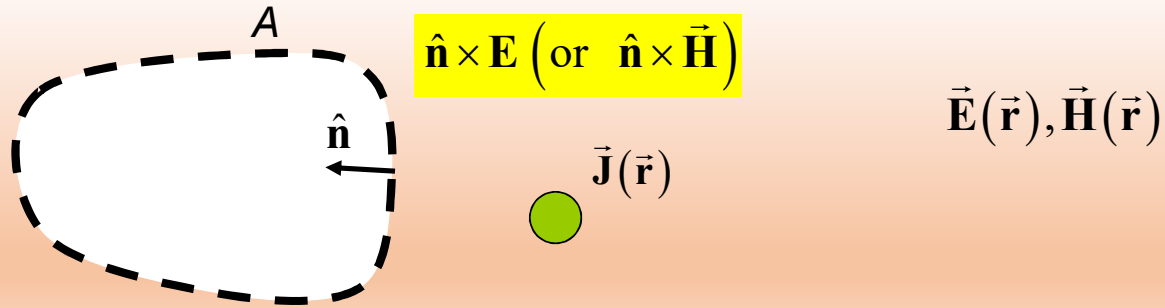
+

No Homic losses $\sigma = 0$

Uniqueness is still ensured!

$$\oint_{A_\infty} dA_\infty \frac{1}{2\zeta} |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{\mathbf{r}})|^2 \right] = 0 \quad \Rightarrow \quad \begin{matrix} \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0} \\ \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathbf{0} \end{matrix} \quad \text{cvd}$$

Uniqueness (PD-Exterior Problem)



- I Consider a source distribution $\vec{\mathbf{J}}(\vec{\mathbf{r}})$ with its associated electromagnetic field $\vec{\mathbf{E}}(\vec{\mathbf{r}}), \vec{\mathbf{H}}(\vec{\mathbf{r}})$
- II Consider a (smooth) surface A with an everywhere defined unit normal $\hat{\mathbf{n}}$
- IV Consider the values of the tangential component of the electric (or magnetic) field upon the surface A ; that is, consider $\hat{\mathbf{n}} \times \mathbf{E}$ (or $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$) **on the boundary**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the boundary condition** in (IV) **as well as the radiation condition at infinity** is unique.