Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

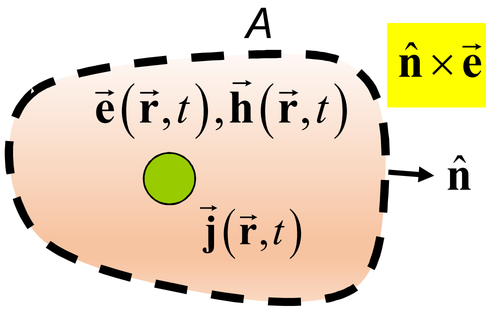
Phasor domain

Image Theory

Reciprocity

Phasor domain

Uniqueness (TD)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ (\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}})$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Interior Problem

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

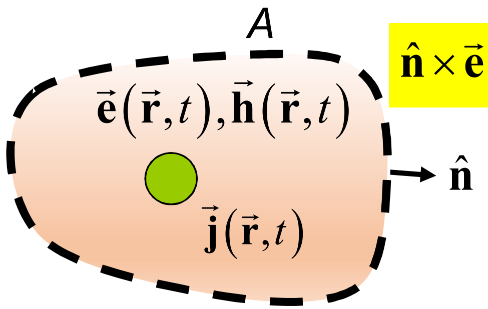
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

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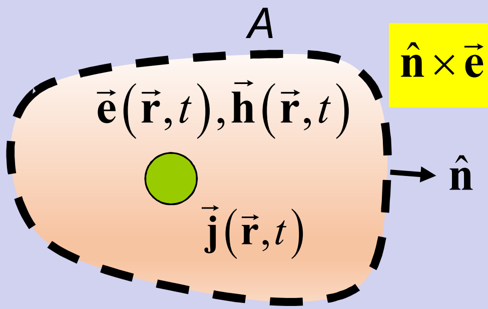

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

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Field difference: source distribution= 0

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

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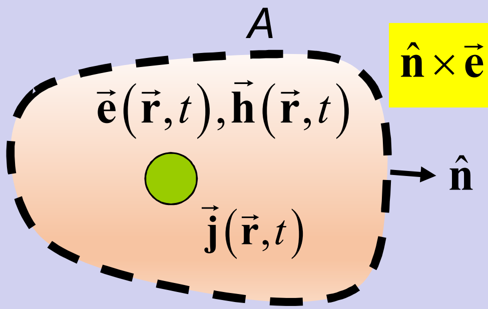

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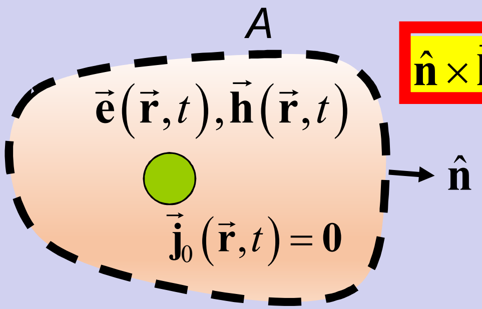
Uniqueness (TD-Interior Problem)



$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{h}}=\mathbf{0}}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
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- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

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Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

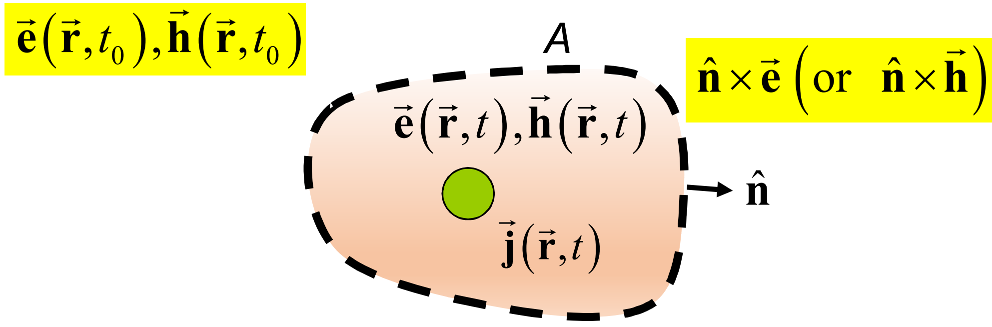
$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\quad \text{on the boundary}}$

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\oiint_A dA\boxed{\left[\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\times\hat{\mathbf{n}}\right]}\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Interior Problem)



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Mathematical tools that we will exploit today

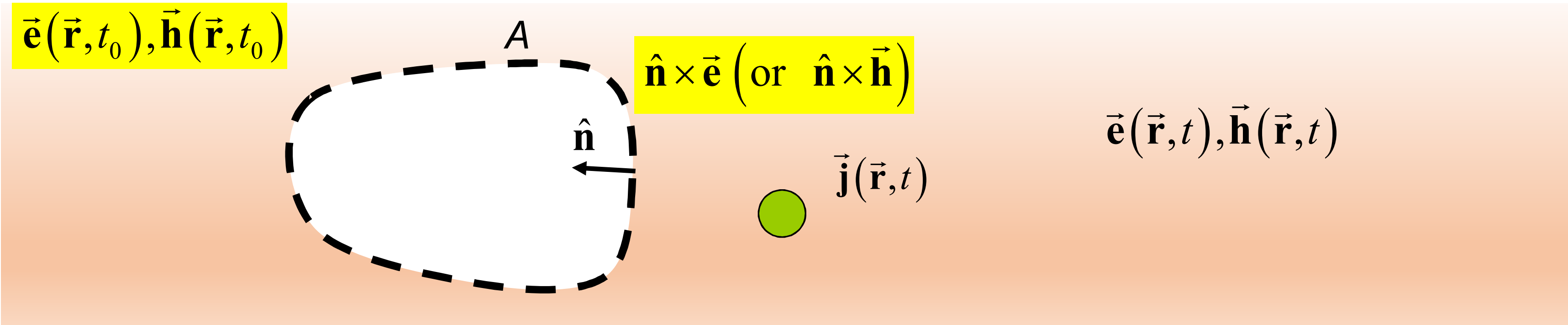
$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}} \right) = \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}} \right) \vec{\mathbf{B}} - \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \right) \vec{\mathbf{C}}$$

Let A be the surface of a sphere of radius r centered in the origin of the reference system

$$\oiint_A dA\, \Phi\left(\vec{\mathbf{r}} \right) = \oiint_A dA\, \Phi\left(r, \vartheta, \varphi \right) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta\, r^2 \sin\vartheta\, \Phi\left(r, \vartheta, \varphi \right)$$

$$dA = r^2 \sin\vartheta\, d\vartheta\, d\varphi$$

Uniqueness (TD-Exterior Problem)



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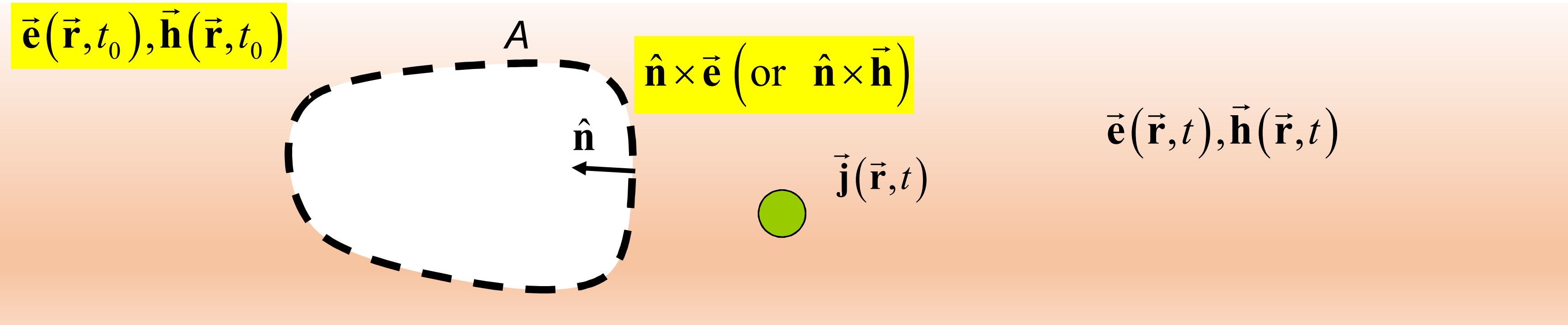

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Field difference: source distribution= 0

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

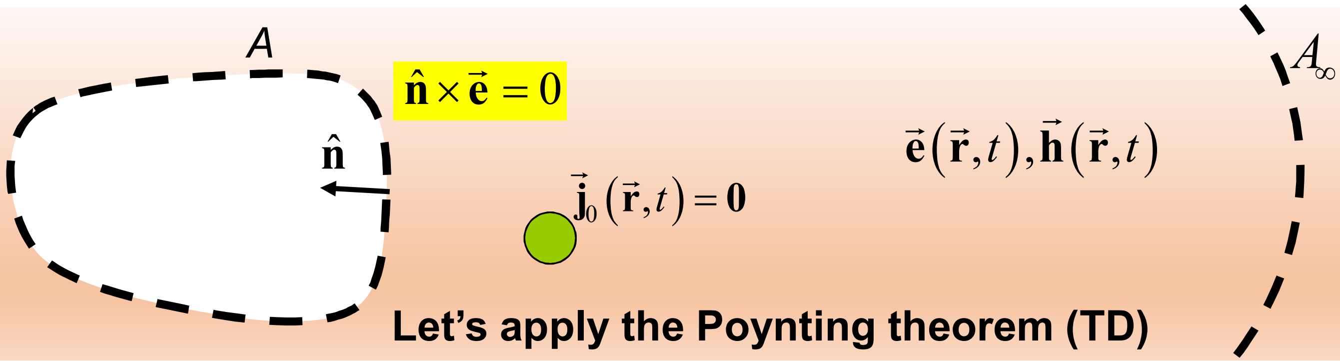
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Exterior Problem)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=0$

$\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

Let's apply the Poynting theorem (TD)

Medium

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
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Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

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$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

Uniqueness (TD-Exterior Problem)

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
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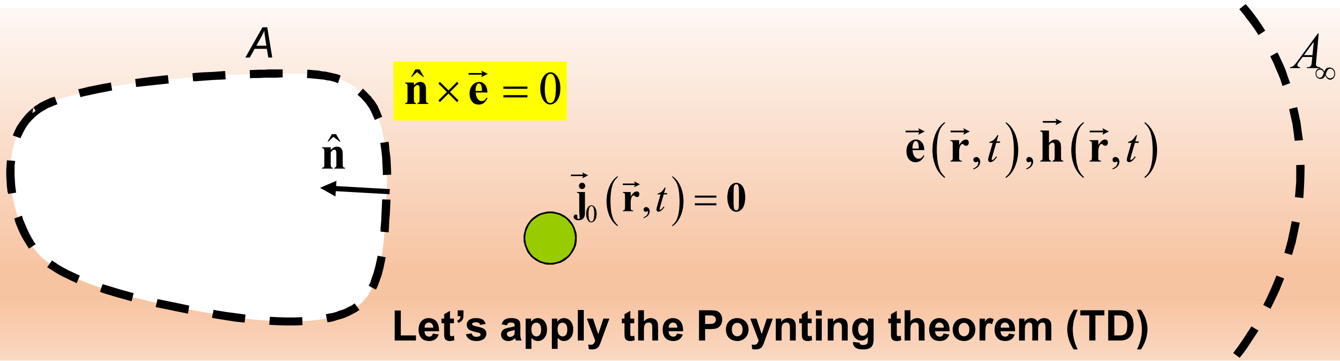

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$$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\quad\text{on the boundary}}$$

$$\cancel{\oint\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)}\cdot\hat{\mathbf{n}}+\oint\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right]+\iiint_V dV\,\sigma|\vec{\mathbf{e}}|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

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$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Exterior Problem)

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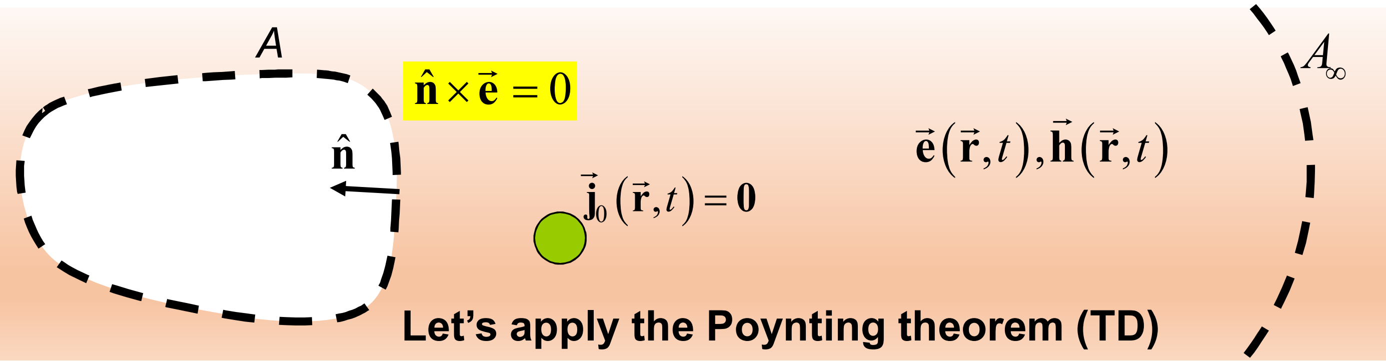

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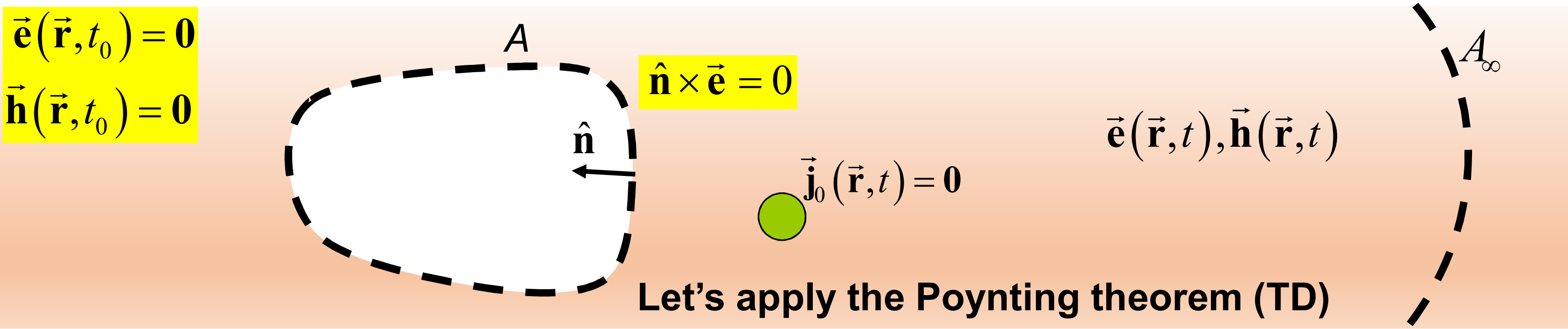
$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\oint\!\!\!\oint_{A_\infty} \cancel{dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=0$$

A_∞ is a large sphere whose radius $R>ct$, c being the speed of the light

Uniqueness (TD-Exterior Problem)



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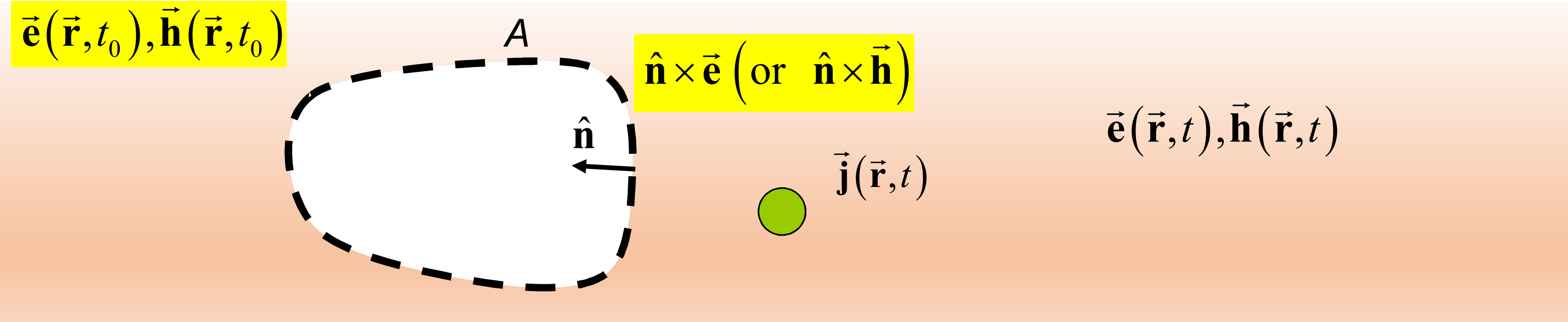
$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\oiint_{A_\infty}\cancel{dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right]+\iiint_V dV\,\sigma|\vec{\mathbf{e}}|^2=-\cancel{\iiint_V dV\,\vec{\mathbf{j}}_0}\cdot\vec{\mathbf{e}}$$

$$\Rightarrow\quad \begin{aligned}&W(t_0)=0\\&\frac{d}{dt}W(t)\le 0\\&W(t)\ge 0\end{aligned}\quad\Rightarrow\quad\begin{aligned}&\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\\&\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\end{aligned}$$

cvd

Uniqueness (TD-Exterior Problem)



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The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

The radiation condition

$$\vec{\mathbf{e}} \cdot \hat{i}_r = 0$$
$$\vec{\mathbf{h}} \cdot \hat{i}_r = 0$$
$$\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{i}_r \sim o\left(\frac{1}{r}\right)$$

$as \quad r \rightarrow \infty$

.. on a sphere of radius *r* centered in the origin of the reference system, being $\hat{i}_r$ the radial unit vector

$\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

The radiation condition

$$\vec{\mathbf{e}} \cdot \hat{i}_r = 0$$

$$\boxed{\vec{\mathbf{h}} \cdot \hat{i}_r = 0}$$

$$\boxed{\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{i}_r \sim o\left(\frac{1}{r}\right)}$$

$$\left(and\ thus\ \zeta \vec{\mathbf{h}} - \hat{i}_r \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right)$$

$as \quad r \to \infty$

.. on a sphere of radius r centered in the origin of the reference system, being $\hat{i}_r$ the radial unit vector

$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

At infinity

$$\vec{\mathbf{e}} = \zeta \vec{\mathbf{h}} \times \hat{i}_r \Rightarrow \boxed{\hat{i}_r \times \vec{\mathbf{e}}} = \hat{i}_r \times \left(\zeta \vec{\mathbf{h}} \times \hat{i}_r\right) = \left(\hat{i}_r \cdot \hat{i}_r\right)\zeta \vec{\mathbf{h}} - \cancel{\left(\hat{i}_r \cdot \zeta \vec{\mathbf{h}}\right)}\hat{i}_r \boxed{= \zeta \vec{\mathbf{h}}}$$

$$\Downarrow$$

$$\hat{i}_r \times \vec{\mathbf{e}} = \zeta \vec{\mathbf{h}}$$

$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}\right)\vec{\mathbf{B}} - \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}\right)\vec{\mathbf{C}}$$

The radiation condition

$$\boxed{\vec{\mathbf{e}} \cdot \hat{i}_r = 0}$$

$$\vec{\mathbf{h}} \cdot \hat{i}_r = 0$$

$$\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \qquad \left(\text{and thus } \boxed{\zeta \vec{\mathbf{h}} - \hat{i}_r \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right)} \right) \qquad as \quad r \to \infty$$

.. on a sphere of radius r centered in the origin of the reference system, being $\hat{i}_r$ the radial unit vector

$\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

$$\vec{\mathbf{s}} = \frac{|\vec{\mathbf{e}}|^2}{\zeta} \hat{i}_r$$

At infinity

$$\vec{\mathbf{s}} = \vec{\mathbf{e}} \times \vec{\mathbf{h}} = \frac{1}{\zeta} \vec{\mathbf{e}} \times \left(\hat{i}_r \times \vec{\mathbf{e}} \right) = \frac{1}{\zeta} \left[\left(\vec{\mathbf{e}} \cdot \vec{\mathbf{e}} \right) \hat{i}_r - \cancel{\left(\vec{\mathbf{e}} \cdot \hat{i}_r \right) \vec{\mathbf{e}}} \right] = \frac{|\vec{\mathbf{e}}|^2}{\zeta} \hat{i}_r$$

$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}} \right) = \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}} \right) \vec{\mathbf{B}} - \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \right) \vec{\mathbf{C}}$$

The radiation condition

$$\vec{\mathbf{e}} \cdot \hat{i}_r = 0$$

$$\boxed{\vec{\mathbf{h}} \cdot \hat{i}_r = 0}$$

$$\boxed{\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{i}_r \sim o\left(\frac{1}{r}\right)}$$

$$\left(and\ thus\ \zeta \vec{\mathbf{h}} - \hat{i}_r \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right)$$

$$as \quad r \to \infty$$

.. on a sphere of radius r centered in the origin of the reference system, being $\hat{i}_r$ the radial unit vector

$\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

$$\vec{\mathbf{s}} = \frac{|\vec{\mathbf{e}}|^2}{\zeta} \hat{i}_r = \zeta |\vec{\mathbf{h}}|^2 \hat{i}_r$$

At infinity

$$\vec{\mathbf{s}} = \vec{\mathbf{e}} \times \vec{\mathbf{h}} = \left(\zeta \vec{\mathbf{h}} \times \hat{i}_r\right) \times \vec{\mathbf{h}} = -\vec{\mathbf{h}} \times \left(\zeta \vec{\mathbf{h}} \times \hat{i}_r\right) = -\left[\cancel{\left(\vec{\mathbf{h}} \cdot \hat{i}_r\right)} \zeta \vec{\mathbf{h}} - \left(\vec{\mathbf{h}} \cdot \zeta \vec{\mathbf{h}}\right) \hat{i}_r\right] = \zeta |\vec{\mathbf{h}}|^2 \hat{i}_r$$

$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}\right) \vec{\mathbf{B}} - \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}\right) \vec{\mathbf{C}}$$

The radiation condition

$$\vec{\mathbf{e}} \cdot \hat{i}_r = 0$$

$$\vec{\mathbf{h}} \cdot \hat{i}_r = 0$$

$$\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \qquad \left(and\ thus\ \zeta \vec{\mathbf{h}} - \hat{i}_r \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right)$$

$as \quad r \to \infty$

.. on a sphere of radius *r* centered in the origin of the reference system, being $\hat{i}_r$ the radial unit vector

$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, **which is assumed homogeneous, isotropic, nondispersive and lossless at infinity**

$$\vec{\mathbf{s}} = \frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta} \hat{i}_r = \zeta \left|\vec{\mathbf{h}}\right|^2 \hat{i}_r$$

⬇

$$\vec{\mathbf{e}} \sim O\left(\frac{1}{r}\right) \qquad \vec{\mathbf{h}} \sim O\left(\frac{1}{r}\right) \qquad as \quad r \to \infty$$

The radiation condition

$$\hat{i}_r \cdot \vec{\mathbf{e}} = \hat{i}_r \cdot \vec{\mathbf{h}} = 0 \qquad \vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{i}_r \sim o\left(\frac{1}{r}\right) \qquad \left(and\ \zeta \vec{\mathbf{h}} - \hat{i}_r \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right)$$

$$\boxed{\vec{\mathbf{e}} \sim O\left(\frac{1}{r}\right)} \quad \boxed{\vec{\mathbf{h}} \sim O\left(\frac{1}{r}\right)} \qquad \vec{\mathbf{s}} = \frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta} \hat{i}_r = \zeta \left|\vec{\mathbf{h}}\right|^2 \hat{i}_r$$

TD

$as \quad r \to \infty$

.. on a sphere of radius r centered in the origin of the reference system, being $\hat{i}_r$ the radial unit vector

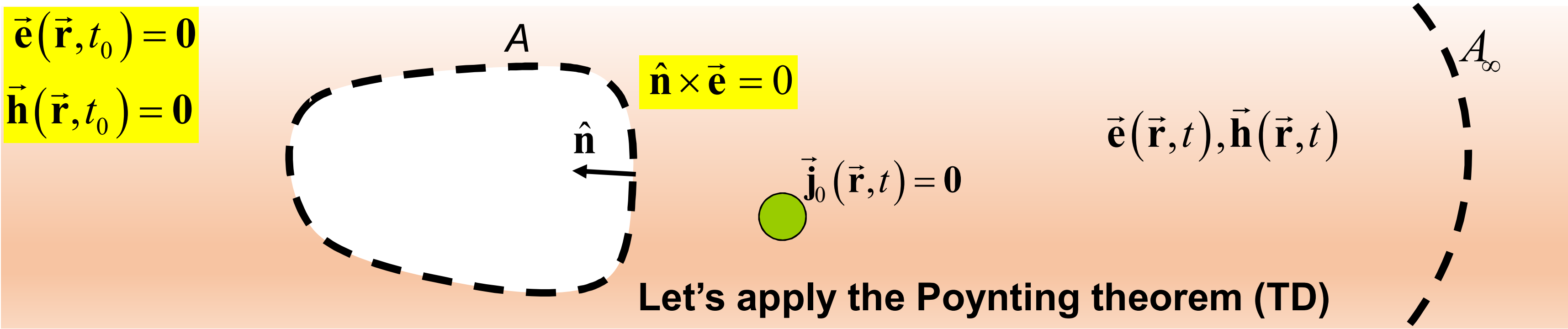
$$\oiint_{A_\infty} dA_\infty \, \vec{\mathbf{s}} \cdot \hat{i}_r = \oiint_{A_\infty} dA_\infty \frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta} = \lim_{r \to \infty} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \, r^2 \sin\vartheta \, \frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta}$$ is a finite nonnegative quantity

$$\oiint_{A_\infty} dA_\infty \, \vec{\mathbf{s}} \cdot \hat{i}_r = \oiint_{A_\infty} dA_\infty \, \zeta \left|\vec{\mathbf{h}}\right|^2 = \lim_{r \to \infty} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \, r^2 \sin\vartheta \, \zeta \left|\vec{\mathbf{h}}\right|^2$$ is a finite nonnegative quantity

A_∞ is the surface of a sphere of radius $r \to \infty$ and centered in the origin of the reference system

$$\oiint_A dA \, \Phi(r,\vartheta,\varphi) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \, r^2 \sin\vartheta \, \Phi(r,\vartheta,\varphi)$$

Uniqueness (TD-Exterior Problem)



Medium
- Linear
- Isotropic
- Space-Nondispersive
- Time-Nondispersive
- Time-invariant

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

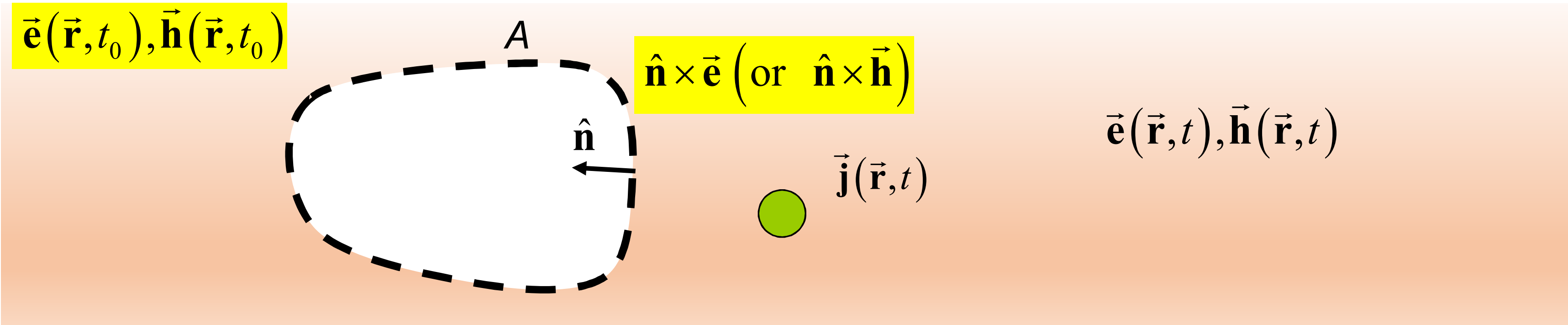
$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)}\cdot\hat{\mathbf{n}}+\oint\!\!\!\oint_{A_\infty}\cancel{dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{i}_r}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=\cancel{-\iiint_V dV\,\vec{\mathbf{j}}_0}\cdot\vec{\mathbf{e}}$$

$$\Rightarrow\quad \begin{array}{l} W(t_0)=0 \\ \dfrac{d}{dt}W(t)\le 0 \\ W(t)\ge 0 \end{array}\quad\Rightarrow\quad \begin{array}{l}\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\end{array}$$

cvd

Uniqueness (TD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.