

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Poynting theorem (TD)

$$\vec{s}(\vec{r}, t) = \vec{e}(\vec{r}, t) \times \vec{h}(\vec{r}, t)$$

Poynting vector

$$[\vec{s}]: \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{s}(\vec{r}, t) + \frac{\partial}{\partial t} w(\vec{r}, t) + p_j(\vec{r}, t) = p_0(\vec{r}, t)$$

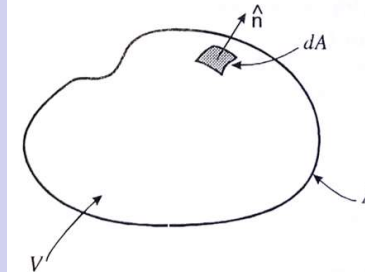
$$\oiint_A dA \vec{s}(\vec{r}, t) \cdot \hat{n} + \frac{d}{dt} \iiint_V dV w(\vec{r}, t) + \iiint_V dV p_j(\vec{r}, t) = \iiint_V dV p_0(\vec{r}, t)$$

Electromagnetic
power flux

$$P_S(t) + \frac{d}{dt} W(t) + P_j(t) = P_0(t)$$

Hypotheses on the medium

- Linear
- Local (TND & SND)
- Isotropic
- Time-invariant



$$w(\vec{r}, t) = \frac{1}{2} \mu |\vec{h}|^2 + \frac{1}{2} \varepsilon |\vec{e}|^2 \quad \text{Energy density of the e.m. field} \quad \Rightarrow \quad \iiint_V dV w(\vec{r}, t) = W(t) \quad \text{Energy of the e.m. field}$$

$$p_j(\vec{r}, t) = \sigma |\vec{e}|^2 \quad \text{Power density dissipated in the conducting medium} \quad \Rightarrow \quad \iiint_V dV p_j(\vec{r}, t) = P_j(t) \quad \text{Power dissipated in the conducting medium}$$

$$p_0(\vec{r}, t) = -\vec{j}_0 \cdot \vec{e} \quad \text{Power density delivered by the sources to the field} \quad \Rightarrow \quad \iiint_V dV p_0(\vec{r}, t) = P(t) \quad \text{Power delivered by the sources to the field}$$

Mathematical tools that we will exploit today

$$\nabla \cdot [\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}})] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}})]$$

$$\iiint_V dV \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\begin{aligned} \nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) &= \frac{1}{2} \nabla \cdot [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})] = \frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}})] - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})] = \\ &= \frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}})] - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}})]^* = \\ &= \frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot [-j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}})] - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot [j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})]^* \\ &= -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}}) \end{aligned}$$

$$\nabla \cdot [\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}})] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}})]$$

Phasor domain

$$\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}})$$

$$\nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

Hypotheses on the medium (TD)

- Linear
- Isotropic
- Time-invariant
- Local (TND & SND)

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \\ \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \sigma \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \end{cases}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

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- Linear
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$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma: \text{real} \end{cases}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
- Space-Nondispersive

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon: real \\ \mu: real \\ \sigma: real \end{cases}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

Hypotheses on the medium (PD)

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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\begin{aligned} \nabla \cdot \vec{\mathbf{S}} &= -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0 \\ &= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 \end{aligned}$$

$$-j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} = -j\omega_0 \vec{\mathbf{H}}^* \cdot [(\mu_1 - j\mu_2) \vec{\mathbf{H}}] = -j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \omega_0 \mu_2 |\vec{\mathbf{H}}|^2$$

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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\begin{aligned} \nabla \cdot \vec{\mathbf{S}} &= -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}} - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0 \\ &= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 \end{aligned}$$

$$j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* = j\omega_0 \vec{\mathbf{E}} \cdot [(\varepsilon_1 - j\varepsilon_2) \vec{\mathbf{E}}]^* = j\omega_0 \vec{\mathbf{E}} \cdot [(\varepsilon_1 + j\varepsilon_2) \vec{\mathbf{E}}^*] = j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2$$

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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0$$

$$= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0$$

$$-\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* = -\vec{\mathbf{E}} \cdot \sigma \vec{\mathbf{E}}^* = -\sigma |\vec{\mathbf{E}}|^2$$

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Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}} - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

$$= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = \nabla \cdot \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\nabla \cdot \vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 = -\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 = -\frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Im} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\}$$

Hypotheses on the medium (PD)

- Linear
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$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = \nabla \cdot \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j \nabla \cdot \vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 = -\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 = -\frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

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$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

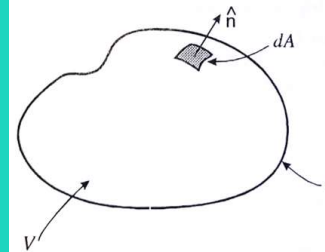
Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$



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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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$$\iiint_V dV \nabla \cdot \vec{\mathbf{S}}_1 + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \} \right]$$

$$\oiint_A dA \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \} \right]$$

$$\iiint_V dV \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

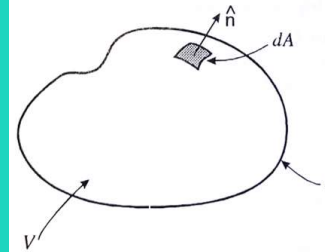
Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$



Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
- Space-Nondispersive

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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$$\oiint_A dA \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \} \right]$$

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TD

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

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TD

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

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Poynting theorem

...MEMO : phasors and time averages

$$\begin{aligned} \dot{\mathbf{f}}_1(\vec{r}, t) &\longrightarrow \dot{\mathbf{F}}_1(\vec{r}) \\ \dot{\mathbf{f}}_2(\vec{r}, t) &\longrightarrow \dot{\mathbf{F}}_2(\vec{r}) \end{aligned}$$

$$\begin{aligned} \vec{\mathbf{S}}_1(\vec{r}) &= \text{Re}\{\vec{\mathbf{S}}(\vec{r})\} = \text{Re}\left\{\frac{1}{2}\vec{\mathbf{E}}(\vec{r})\times\vec{\mathbf{H}}^*(\vec{r})\right\} = \frac{1}{2}\text{Re}\{\vec{\mathbf{E}}(\vec{r})\times\vec{\mathbf{H}}^*(\vec{r})\} \\ &= \langle\vec{\mathbf{e}}(\vec{r}, t)\times\vec{\mathbf{h}}(\vec{r}, t)\rangle = \langle\vec{\mathbf{s}}(\vec{r}, t)\rangle \end{aligned}$$

$$\langle\vec{\mathbf{f}}_1(\vec{r}, t)\cdot\vec{\mathbf{f}}_2(\vec{r}, t)\rangle = \frac{1}{T}\int_0^T\vec{\mathbf{f}}_1(\vec{r}, t)\cdot\vec{\mathbf{f}}_2(\vec{r}, t)dt = \frac{1}{2}\text{Re}\{\vec{\mathbf{F}}_1(\vec{r})\cdot\vec{\mathbf{F}}_2^*(\vec{r})\}$$

$$\langle\vec{\mathbf{f}}_1(\vec{r}, t)\times\vec{\mathbf{f}}_2(\vec{r}, t)\rangle = \frac{1}{T}\int_0^T\vec{\mathbf{f}}_1(\vec{r}, t)\times\vec{\mathbf{f}}_2(\vec{r}, t)dt = \frac{1}{2}\text{Re}\{\vec{\mathbf{F}}_1(\vec{r})\times\vec{\mathbf{F}}_2^*(\vec{r})\}$$

$$\oiint_A dA \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = \iiint_V dV \left[-\frac{1}{2} \text{Re}\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\} \right]$$

Time averaged power flux associated to the e.m. field

$$\oiint_A dA \vec{\mathbf{s}}(\vec{r}, t) \cdot \hat{\mathbf{n}} + \frac{d}{dt} \iiint_V dV \left[\frac{1}{2} \mu |\vec{\mathbf{h}}|^2 + \frac{1}{2} \varepsilon |\vec{\mathbf{e}}|^2 \right] + \iiint_V dV \sigma |\vec{\mathbf{e}}|^2 = - \iiint_V dV \vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

TD

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

$$\begin{cases} \vec{\mathbf{D}}(\vec{r}) = \varepsilon(\vec{r})\vec{\mathbf{E}}(\vec{r}) \\ \vec{\mathbf{B}}(\vec{r}) = \mu(\vec{r})\vec{\mathbf{H}}(\vec{r}) \\ \vec{\mathbf{J}}(\vec{r}) = \sigma\vec{\mathbf{E}}(\vec{r}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : \text{real} \end{cases}$$

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$$\text{Re} \left\{ -\frac{1}{2} \vec{\mathbf{E}}(\vec{r}) \cdot \vec{\mathbf{J}}_0^*(\vec{r}) \right\} = \langle -\dot{\mathbf{j}}_0(\vec{r}, t) \cdot \vec{\mathbf{e}}(\vec{r}, t) \rangle$$

$$\langle \vec{\mathbf{f}}_1(\vec{r}, t) \cdot \vec{\mathbf{f}}_2(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{r}, t) \cdot \vec{\mathbf{f}}_2(\vec{r}, t) dt = \frac{1}{2} \text{Re} \{ \dot{\mathbf{F}}_1(\vec{r}) \cdot \dot{\mathbf{F}}_2^*(\vec{r}) \}$$

$$\langle \vec{\mathbf{f}}_1(\vec{r}, t) \times \vec{\mathbf{f}}_2(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{r}, t) \times \vec{\mathbf{f}}_2(\vec{r}, t) dt = -\frac{1}{2} \text{Re} \{ \dot{\mathbf{F}}_1(\vec{r}) \times \dot{\mathbf{F}}_2^*(\vec{r}) \}$$

$$\oiint_A dA \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \} \right]$$

Time averaged power flux associated to the e.m. field

Time averaged power delivered by the sources to the field

$$\begin{aligned} \vec{\mathbf{D}}(\vec{r}) &= \varepsilon(\vec{r}) \vec{\mathbf{E}}(\vec{r}) \\ \vec{\mathbf{B}}(\vec{r}) &= \mu(\vec{r}) \vec{\mathbf{H}}(\vec{r}) \\ \vec{\mathbf{J}}(\vec{r}) &= \sigma \vec{\mathbf{E}}(\vec{r}) \end{aligned}$$

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TD

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

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$$\begin{aligned} \frac{1}{2} \sigma |\vec{\mathbf{E}}(\vec{r})|^2 &= \frac{1}{2} \text{Re} \left\{ \sigma |\vec{\mathbf{E}}(\vec{r})|^2 \right\} = \frac{1}{2} \text{Re} \left\{ \sigma \vec{\mathbf{E}}(\vec{r}) \cdot \vec{\mathbf{E}}^*(\vec{r}) \right\} = \langle \sigma \vec{\mathbf{e}}(\vec{r}, t) \cdot \vec{\mathbf{e}}(\vec{r}, t) \rangle \\ &= \langle \sigma |\vec{\mathbf{e}}(\vec{r}, t)|^2 \rangle \end{aligned}$$

$$\langle \vec{\mathbf{f}}_1(\vec{r}, t) \cdot \vec{\mathbf{f}}_2(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{r}, t) \cdot \vec{\mathbf{f}}_2(\vec{r}, t) dt = \frac{1}{2} \text{Re} \left\{ \vec{\mathbf{F}}_1(\vec{r}) \cdot \vec{\mathbf{F}}_2^*(\vec{r}) \right\}$$

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$$\oiint_A dA \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\} \right]$$

Time averaged power flux associated to the e.m. field

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

$$\oiint_A dA \vec{\mathbf{s}}(\vec{r}, t) \cdot \hat{\mathbf{n}} + \frac{d}{dt} \iiint_V dV \left[\frac{1}{2} \mu |\vec{\mathbf{h}}|^2 + \frac{1}{2} \varepsilon |\vec{\mathbf{e}}|^2 \right] + \iiint_V dV \sigma |\vec{\mathbf{e}}|^2 = - \iiint_V dV \vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}} \quad \text{TD}$$

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

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$$\vec{S}(\vec{r}) = \vec{S}_1(\vec{r}) + j\vec{S}_2(\vec{r})$$

Where is the energy?

Remember that the energy is a state function

$$\int_{t_1}^{t_2} dt \frac{\partial}{\partial t} w(t) = w(t_2) - w(t_1)$$

$$\oiint_A dA \vec{S}_1 \cdot \hat{n} + \iiint_V dV \frac{1}{2} \sigma |\vec{E}|^2 = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{E} \cdot \vec{J}_0^* \} \right]$$

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TD

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$$\oiint_A dA \vec{S}_1 \cdot \hat{n} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{H}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{E}|^2 - \frac{1}{2} \sigma |\vec{E}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \{ \vec{E} \cdot \vec{J}_0^* \} \right]$$

Time averaged power flux associated to the e.m. field

LOSSES
(ε_2) electric losses
(μ_2) magnetic losses

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

$$\begin{cases} \vec{D}(\vec{r}) = \varepsilon(\vec{r}) \vec{E}(\vec{r}) \\ \vec{B}(\vec{r}) = \mu(\vec{r}) \vec{H}(\vec{r}) \\ \vec{J}(\vec{r}) = \sigma \vec{E}(\vec{r}) \end{cases}$$

$$\varepsilon_2 > 0; \mu_2 > 0; \sigma > 0$$

Dispersion and losses are related each other: a (time) dispersive medium presents losses

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : \text{real} \end{cases}$$

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\oiint_A dA \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\} \right]$$

$$\oiint_A dA \vec{\mathbf{S}}_2 \cdot \hat{\mathbf{n}} + 2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{4} \varepsilon_1 |\vec{\mathbf{E}}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Im} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\} \right]$$

Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- **Time-Dispersive**
- **Space-Nondispersive**

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

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$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

...MEMO : phasors and time averages

$$\vec{f}_1(\vec{r}, t) \longrightarrow \vec{F}_1(\vec{r})$$

$$\vec{f}_2(\vec{r}, t) \longrightarrow \vec{F}_2(\vec{r})$$

$$\langle \vec{f}_1(\vec{r}, t) \cdot \vec{f}_2(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{f}_1(\vec{r}, t) \cdot \vec{f}_2(\vec{r}, t) dt = \frac{1}{2} \text{Re} \{ \vec{F}_1(\vec{r}) \cdot \vec{F}_2^*(\vec{r}) \}$$

$$\langle \vec{f}_1(\vec{r}, t) \times \vec{f}_2(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{f}_1(\vec{r}, t) \times \vec{f}_2(\vec{r}, t) dt = \frac{1}{2} \text{Re} \{ \vec{F}_1(\vec{r}) \times \vec{F}_2^*(\vec{r}) \}$$

nting theorem (PD)

nting vector $[\vec{S}] : \frac{Watt}{m^2}$

$$\oiint_A dA \vec{S}_2 \cdot \hat{n} + 2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{H}|^2 - \frac{1}{4} \varepsilon_1 |\vec{E}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Im} \{ \vec{E} \cdot \vec{J}_0^* \} \right]$$

$$= \left\langle \frac{1}{2} \mu |\vec{h}(\vec{r}, t)|^2 \right\rangle = \frac{1}{2} \mu \left\langle |\vec{h}(\vec{r}, t)|^2 \right\rangle = \frac{1}{2} \mu \text{Re} \left\{ \frac{1}{2} \vec{H}(\vec{r}) \cdot \vec{H}(\vec{r})^* \right\} = \frac{1}{2} \mu \text{Re} \left\{ \frac{1}{2} |\vec{H}(\vec{r})|^2 \right\}$$

$$= \frac{1}{2} \mu \frac{1}{2} |\vec{H}(\vec{r})|^2 = \frac{1}{4} \mu |\vec{H}(\vec{r})|^2$$

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$$\begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}$$

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$$\oiint_A dA \vec{S}_2 \cdot \hat{n} + 2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{H}|^2 - \frac{1}{4} \varepsilon_1 |\vec{E}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Im} \{ \vec{E} \cdot \vec{J}_0^* \} \right]$$

$$\frac{1}{4} \mu |\vec{H}(\vec{r})|^2 = \left\langle \frac{1}{2} \mu |\vec{h}(\vec{r}, t)|^2 \right\rangle$$

$$\frac{1}{4} \varepsilon |\vec{E}(\vec{r})|^2 = \left\langle \frac{1}{2} \varepsilon |\vec{e}(\vec{r}, t)|^2 \right\rangle$$

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$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : \text{real} \end{cases}$$

$$\begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}$$

Poynting theorem (PD)

$$\vec{S}(\vec{r}) = \frac{1}{2} [\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})]$$

Poynting vector

$$[\vec{S}] : \frac{Watt}{m^2}$$

$$\vec{S}(\vec{r}) = \vec{S}_1(\vec{r}) + j\vec{S}_2(\vec{r})$$

$$\oint_A dA \vec{S}_2 \cdot \hat{n} + 2\omega_0 \iiint_V dV \left[\frac{1}{4} \mu_1 |\vec{H}|^2 - \frac{1}{4} \varepsilon_1 |\vec{E}|^2 \right] = \iiint_V dV \left[-\frac{1}{2} \text{Im} \left\{ \vec{E} \cdot \vec{J}_0^* \right\} \right]$$

Time averaged magnetic energy density

Time averaged electric energy density

$$\frac{1}{4} \mu |\vec{H}(\vec{r})|^2 = \left\langle \frac{1}{2} \mu |\vec{h}(\vec{r}, t)|^2 \right\rangle$$

$$\frac{1}{4} \varepsilon |\vec{E}(\vec{r})|^2 = \left\langle \frac{1}{2} \varepsilon |\vec{e}(\vec{r}, t)|^2 \right\rangle$$

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$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : \text{real} \end{cases} \quad \begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}$$