Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2023-2024 - Laurea “Triennale” – Secondo semestre - Secondo anno

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Constitutive relationships

Time domain - Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

Even assuming knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t); \rho_0(\vec{\mathbf{r}},t)$

Number of independent scalar equations: 7

Number of unknown scalar quantities: 16

Maxwell equations involve a number of unknowns larger than the number of equations!

The additional missing equations are provided by the **constitutive relationships**, which describe interaction of fields and matter from a macroscopic point of view

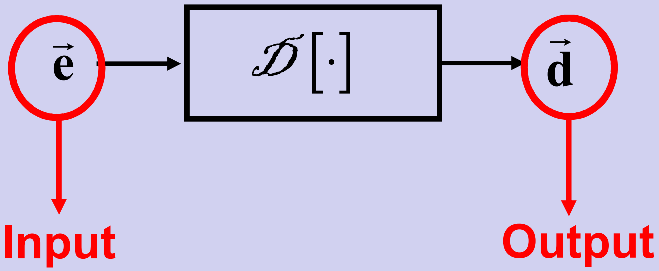
Constitutive relationships

Linear media that are NOT bianisotropic

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right]$$



Constitutive relationships

Linear media

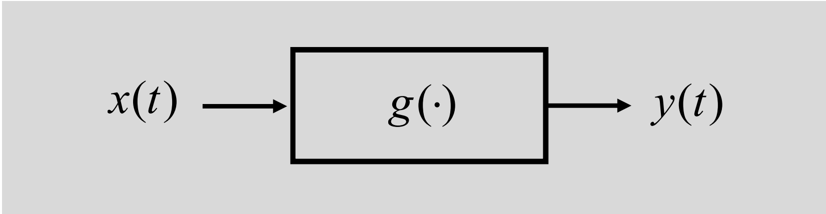
Class	Property	Effect on the I-O relation
Time-dispersive		
Time-nondispersive		
Time-variant		
Time-invariant		

Constitutive relationships

Linear media

Class	Property	Effect on the I-O relation
Space-dispersive		
Space-nondispersive		
Space-variant		
Space-invariant		

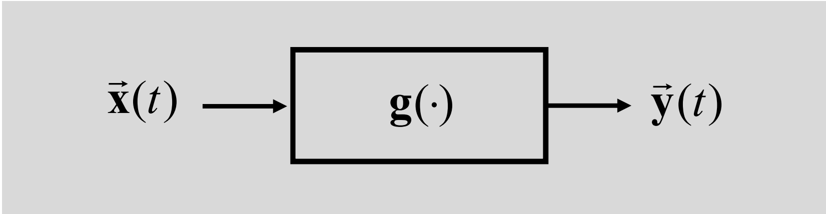
Memo: linear systems



$x(t)$ and $y(t)$ are scalar

	Time-dispersive (TD)	**Time-nondispersive (TND)**
Time-variant (TV)	$y(t)=\int dt'\, g(t,t')\, x(t')$	$y(t)=\tilde{g}(t)\, x(t)$
Time-invariant (TI)	$y(t)=\int dt'\, g(t-t')\, x(t')$	$y(t)=\tilde{g}\, x(t)$

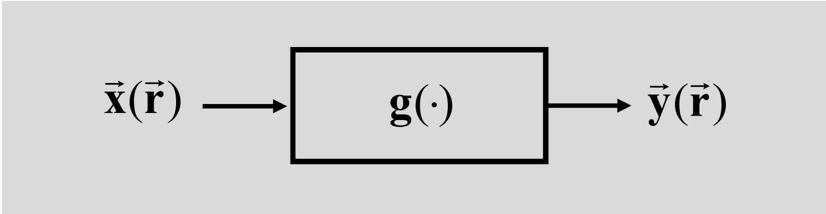
Linear systems



$\vec{\mathbf{x}}(t)$ and $\vec{\mathbf{y}}(t)$ are vectors

	Time-dispersive (TD)	**Time-nondispersive (TND)**
Time-variant (TV)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t,t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}(t)\cdot\vec{\mathbf{x}}(t)$
Time-invariant (TI)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t-t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(t)$

Linear systems



$\vec{\mathbf{x}}(\vec{\mathbf{r}})$ and $\vec{\mathbf{y}}(\vec{\mathbf{r}})$ are vectors

	Space-dispersive (SD)	**Space-nondispersive (SND)**
Space-variant (SV)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}},\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$
Space-invariant (SI)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$

Linear and Anisotropic Media

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t,t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}(t)\cdot\vec{\mathbf{x}}(t)$
Time-invariant (TI)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t-t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(t)$

	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}},\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$
Space-invariant (SI)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$

MEMO

Linear media

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \boldsymbol{\varepsilon}(\vec{\mathbf{r}},t) \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)$: 3x3 matrix

- **Local (nondispersive) media**
- **Anisotropic media**

Class	**Property**	**Effect on the I-O relation**
Isotropic	A **rotation** of the input implies **the same rotation** of the output	$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)$ becomes scalar. It is not a matrix anymore!

Linear and Isotropic Media

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{\mathbf{y}}(t)=\int dt'\, g\left(t,t'\right)\vec{\mathbf{x}}\left(t'\right)$	$\vec{\mathbf{y}}(t)=\tilde{\mathrm{g}}\left(t\right)\vec{\mathbf{x}}\left(t\right)$
Time-invariant (TI)	$\vec{\mathbf{y}}(t)=\int dt'\, \mathrm{g}\left(t-t'\right)\vec{\mathbf{x}}\left(t'\right)$	$\vec{\mathbf{y}}(t)=\tilde{\mathrm{g}}\,\vec{\mathbf{x}}\left(t\right)$

	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\, \mathrm{g}\left(\vec{\mathbf{r}},\vec{\mathbf{r}}'\right)\vec{\mathbf{x}}\left(\vec{\mathbf{r}}'\right)$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathrm{g}}\left(\vec{\mathbf{r}}\right)\vec{\mathbf{x}}\left(\vec{\mathbf{r}}\right)$
Space-invariant (SI)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\, \mathrm{g}\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right)\vec{\mathbf{x}}\left(\vec{\mathbf{r}}'\right)$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathrm{g}}\,\vec{\mathbf{x}}\left(\vec{\mathbf{r}}\right)$

Linear and Anisotropic Media

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t,t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}(t)\cdot\vec{\mathbf{x}}(t)$
Time-invariant (TI)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t-t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(t)$

	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}},\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$
Space-invariant (SI)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$

Constitutive relationships

Linear & Anisotropic & Dispersive media & Space-variant & Time-variant

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, \mathbf{g}_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\cdot\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right] \qquad \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, \mathbf{g}_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\cdot\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, \mathbf{g}_j\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\cdot\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Linear & Isotropic & Dispersive media & Space-variant & Time-variant

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right] \qquad \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, g_j\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Constitutive relationships

In the following, just for the sake of simplicity, we will consider isotropic media

Time: dispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$

Time: nondispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Dispersive (SD) Time-Nondispersive (TND)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$

Time: dispersive
Space: nondispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Dispersive (TD)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$

Dispersive (time & space) vs. nondispersive (time & space)

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Nondispersive (TND)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}},t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}}\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$

Permittivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$

$$[\varepsilon]=?$$

$$[\varepsilon]=\frac{Coulomb}{m^2}\frac{m}{Volt}=\frac{Coulomb}{Volt}\frac{1}{m}=\frac{Farad}{m}$$

$$\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]=\frac{Volt}{m} \qquad \left[\vec{\mathbf{d}}(\vec{\mathbf{r}},t)\right]=\frac{Coulomb}{m^2}$$

$$C=\frac{q}{\Delta \mathrm{v}} \qquad Farad=\frac{Coulomb}{Volt}$$

Permeability

	SND+TND: Local
SV-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}},t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

$$[\mu]=?$$

$$[\mu]=\frac{Weber}{m^2}\frac{m}{Ampere}=\frac{Weber}{Ampere}\frac{1}{m}=\frac{Henry}{m}$$

$$\left[\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\right]=\frac{Weber}{m^2} \qquad \left[\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\frac{Ampere}{m}$$

$$L=\frac{\Phi_{\vec{\mathbf{b}}}}{i} \qquad Henry=\frac{Weber}{Ampere}$$

Conductivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$

$$[\sigma]=?$$

$$[\sigma]=\frac{Ampere}{m^2}\frac{m}{Volt}=\frac{Ampere}{Volt}\frac{1}{m}=\frac{1}{\Omega\, m}=\frac{Siemens}{m}$$

$$\left[\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\right]=\frac{Ampere}{m^2} \qquad \left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]=\frac{Volt}{m}$$

$$\Delta \mathrm{v}=R\,i \qquad \Omega=\frac{Volt}{Ampere}=\frac{1}{Siemens}$$

Exercise n.1

Linear+ Isotropic + Space-Variant (SV) + Time-Variant (TV)

Space-dispersive (SD) +Time-dispersive (TD) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\left[g_d\right]=\frac{Farad}{s\cdot m^4}$
Space-dispersive (SD) +Time-nondispersive (TND) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$	$\left[g_d\right]=\frac{Farad}{m^4}$
Space-nondispersive (SND) +Time-dispersive (TD) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\left[g_d\right]=\frac{Farad}{s\cdot m}$
Space-nondispersive (SND) +Time-nondispersive (TND) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}},t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$	$\left[\varepsilon\right]=\frac{Farad}{m}$

Exercise n.2

Linear+ Isotropic + Space-Variant (SV) + Time-Invariant (TI)

Space-Dispersive (SD) +Time-Dispersive (TD)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\left[g_b\right]=\frac{Henry}{s\cdot m^4}$$

Space-Dispersive (SD) +Time-Nondispersive (TND)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t\right)$$

$$\left[g_b\right]=\frac{Henry}{m^4}$$

Space-Nondispersive (SND) +Time-Dispersive (TD)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int dt'\, g_b\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t'\right)$$

$$\left[g_b\right]=\frac{Henry}{s\cdot m}$$

Space-Nondispersive (SND) +Time-Nondispersive (TND)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu\left(\vec{\mathbf{r}}\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t\right)$$

$$\left[\mu\right]=\frac{Henry}{m}$$

Conductors

$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Metal	Conductivity σ [siemens/m]
Silver, 99.98% pure	6.14×10^7
Copper, annealed	5.80×10^7
Copper, hard drawn	5.65×10^7
Gold, pure drawn	4.10×10^7
Aluminum, commercial hard drawn	3.54×10^7
Magnesium	2.17×10^7
Tungsten	1.81×10^7
Zinc	1.74×10^7
Nickel	1.28×10^7
Iron, 99.98% pure	1.00×10^7
Steel	$1.00\text{–}0.5 \times 10^7$
Lead	0.48×10^7
Mercury	0.10×10^7

From G.Franceschetti, ‘Electromagnetics, Theory, Techniques, and Engineering paradigms’, Plenum Press

Perfect Electric Conductors (PEC)

$$\sigma \to \infty \quad \Rightarrow \quad \vec{\mathbf{e}} = 0 \quad \Rightarrow \quad \vec{\mathbf{h}} = 0$$

Vacuum

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon_0 \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu_0 \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\sigma = 0$$

$$\mu_0 = 4\pi \times 10^{-7} \quad Henry\,/\,m$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \quad Farad\,/\,m$$