



MASTER IN ENTREPRENEURSHIP  
INNOVATION MANAGEMENT  
IN COLLABORATION WITH **MIT SLOAN**

IN COLLABORATION WITH  
**MIT MANAGEMENT**  
SLOAN SCHOOL



UNIVERSITÀ DEGLI STUDI DI NAPOLI  
**PARTHENOPE**

OPERATIONS

# Advanced and sustainable manufacturing strategies

Decision Making and strategies

Antonella PETRILLO  
University of Napoli «Parthenope»

# Lesson mapping

## Aim of the Workshop

Is to offer a quick and intuitive understanding of the the Analytic Hierarchy Process (AHP) that is a mathematically simple methodology in the field of multi-criteria decision-making in operations research (OR).

## One point lesson:

MCDM overview

Understanding the AHP

- Understanding How to Structure an AHP Model
- Building AHP Models Using Super Decisions
- Building Sensitivity in AHP models

Changing from AHP to ANP thinking





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# Research Activities

University of Pittsburgh  
Prof. Thomas SAATY



**KATZ**  
UNIVERSITY OF PITTSBURGH  
JOSEPH M. KATZ  
GRADUATE SCHOOL  
OF BUSINESS



 University of Pittsburgh

**PITT BUSINESS** | Joseph M. Katz  
Graduate School of Business



# Research Activities

## AHP Academy



Particularly, the **AHP Academy** will promote events and offer tools for the diffusion of information regarding the field of decision making, such as:

- 📌 **international lectures;**
- 📌 **publications on Decision Making/AHP;**
- 📌 **training courses;**
- 📌 **scholarships.**



### Honorary President

Thomas L. Saaty (born 1926 in Mosul, Iraq) is an American mathematician. He teaches in the Joseph M. Katz Graduate School of Business. He is the inventor, architect, and primary theoretician of the Analytic Hierarchy Process, a decision-making framework used for large-scale, multiparty, multi-criteria decision analysis, and of the Analytic Network Process, its generalization to decisions with dependence and feedback.

## AHP Academy

The AHP Academy promotes the diffusion of the culture and methodologies of Decision Making, with particular reference to those based on **Analytic Hierarchy Process**. The aim of the association is to support the development of studies, researches and applications within the Decision Making and the AHP, and to create a place to share experiences and results of the researches on decision making among researchers, experts, public and private institutions of around the whole world.

### THE AIMS

The goals of AHP Academy are:

- 📌 Promote the spread of a culture of methodologies of Decision Making in the world, working for the sharing of experience and knowledge of among the members.
- 📌 Facilitate the exchange of experience and knowledge between the parties concerned with issues of Decision Making, including the identification of areas of interest and the prevailing development of partnerships,
- 📌 Promote a more effective dialogue between research and business, encouraging and promoting joint initiatives, support the university in identifying training needs and research priorities for the sector.
- 📌 Evolve as a center of expertise and collaborate with national and international associations involved in the standardization and certification of methods, criteria and tools for decision making, taking into account the quality system.



# Research Activities

Home

The Event

Call for Papers

Register

Program

Past Conferences About AHP Contact



**ISAHP 2020**  
WEB CONFERENCE

**DECEMBER 3 - DECEMBER 6, 2020 / WEB CONFERENCE**

International Symposium on the Analytic Hierarchy Process

## International Symposium on the Analytic Hierarchy Process

*AHP/ANP: The Next Generation*

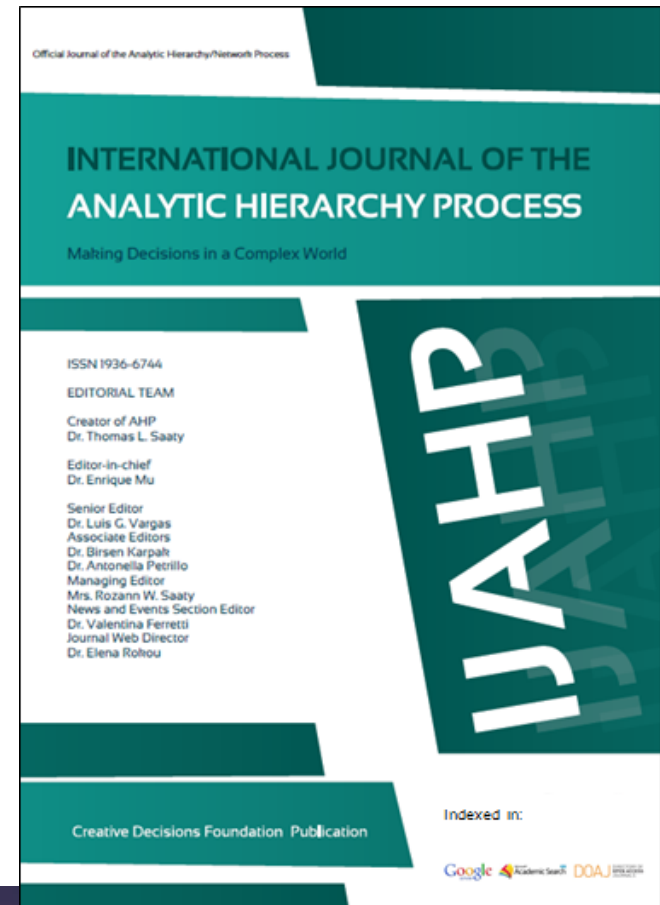
	Year	Location		Year	Location		Year	Location
1	1988	Tianjin, China	8	2005	Honolulu, United States	15	2018	Hong Kong, HK
2	1991	Pittsburgh, PA, U.S.A.	9	2007	Viña del Mar, Chile	16	2020	Virtual
3	1994	Washington, D.C., U.S.A.	10	2009	Pittsburgh, PA, U.S.A.			
4	1996	Vancouver, Canada	11	2011	Sorrento, Italy			
5	1999	Kobe, Japan	12	2013	Kuala Lumpur, Malaysia			
6	2001	Berne, Switzerland	13	2014	Washington, United States			
7	2003	Nusa Dua, Indonesia	14	2016	London, UK			

# Research Activities

## International Journal of the Analytic Hierarchy Process

IJAHP is a scholarly journal that publishes papers about research and applications of the Analytic Hierarchy Process(AHP) and Analytic Network Process(ANP).

The journal encourages research papers in both theory and applications. Empirical investigations, comparisons and exemplary real-world applications in diverse areas are particularly welcome.



# Research Activities

## Some Publications

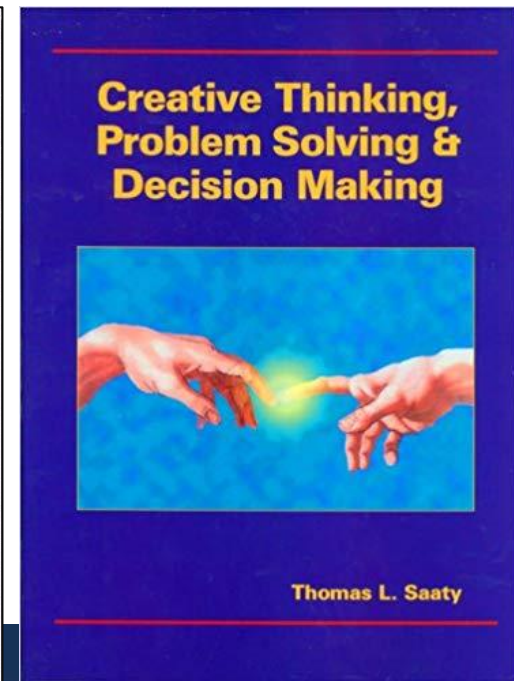
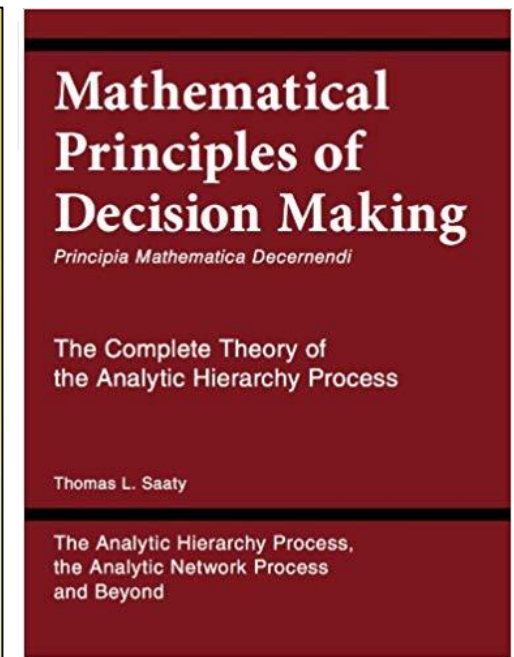
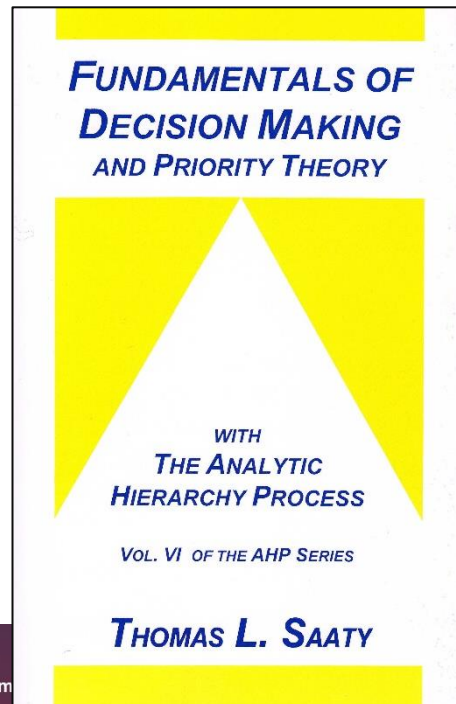
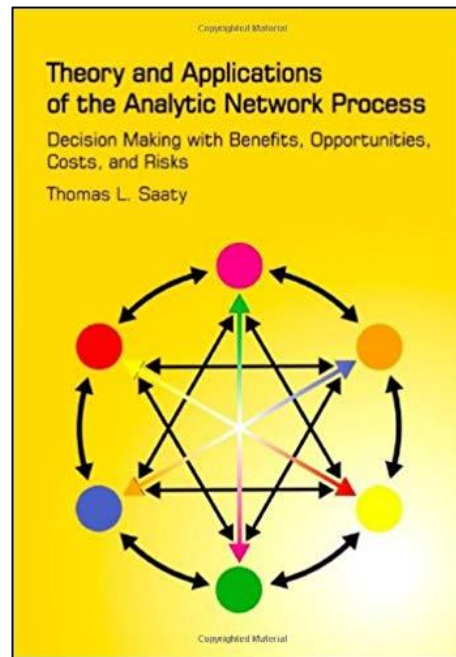
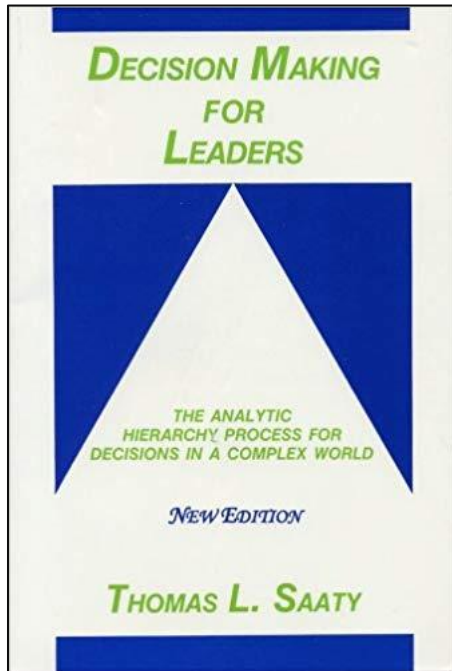
Since the *landmark* publication of “**Decision making for Leaders**” by Prof. Thomas Saaty in **1980**, there have been **several books** on the topic.

Some of them deal with the theory of the Analytic Hierarchy Process (AHP) and others discuss its applications.



# Research Activities

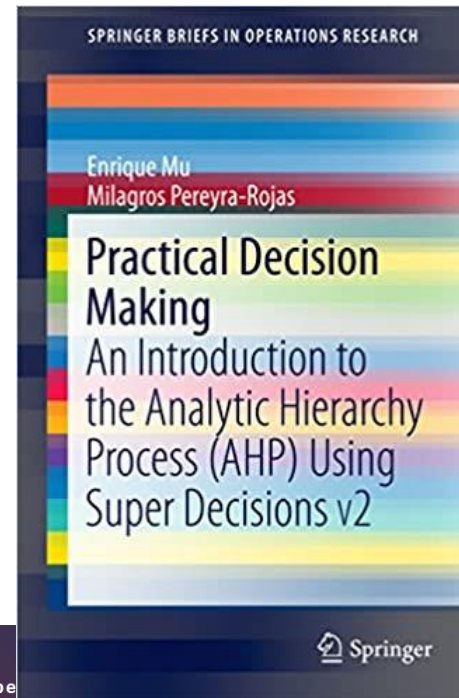
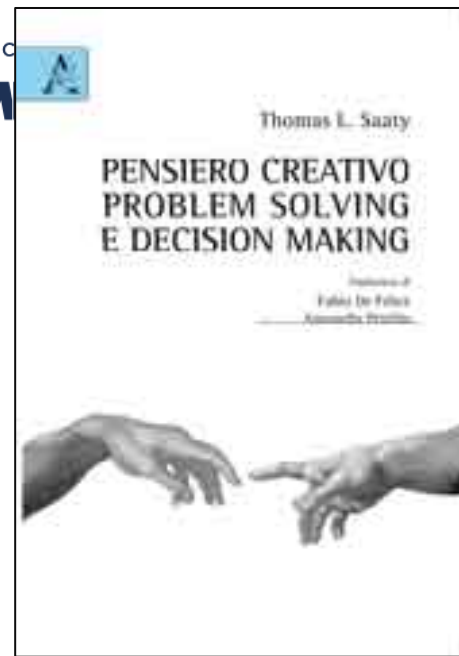
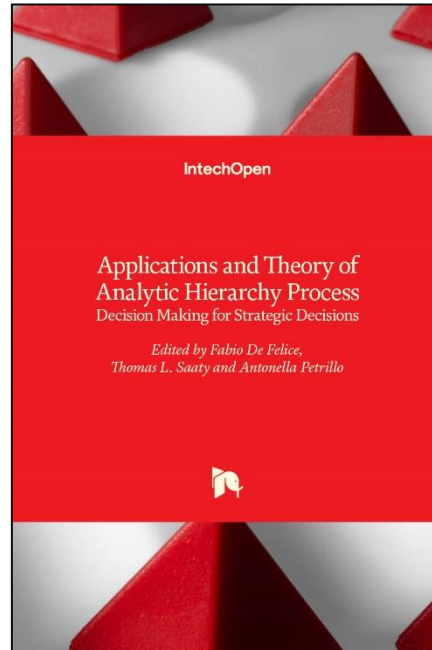
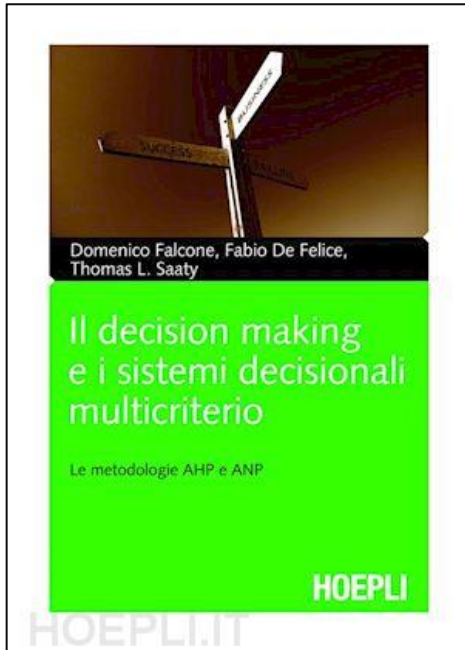
## Some Publications





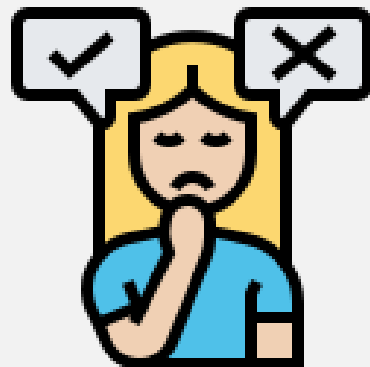
# Research Activities

## Some Publications



# Decision making

## MCDM overview



# What is decision making?

# Decision Making

Decision making today is a **science**.

Every day we have **hard decisions** to make.

The survival of the business depends on making the **right decision**.



# Decision Making

*Our lives are the **sum** of our **decisions**,  
whether in **business** or in **personal**  
spheres.*

*Often, **when** we decide is as important as **what** we decide.*

*To be a person is to be a decision maker.*

*Thomas Saaty*

**There are different  
kind of decisions!**

# Decision Making

## Different Kinds of Decisions

1. **Instantaneous** like what **restaurant to eat at** and what kind of rice cereal to buy.



2. **Personal** but allowing a little time like which job to choose and what **house to buy** or car to drive.



3. **Long term decisions** and any decisions that involve **planning** and **resource allocation** and more significantly group decision making.



# TED Ideas worth spreading



1145 results

## How to make hard choices

Chang's research focuses on decision-making and the human condition. Ruth Chang asks why some choices are so hard and what that means for the human condition.

[https://www.ted.com/speakers/ruth\\_chang](https://www.ted.com/speakers/ruth_chang)

TED Speaker



## Ruth Chang

Philosopher

[ruthchang.net](http://ruthchang.net) [Read: "Resolving to Create a New You"](#)  
[Book: Making Comparisons Count](#)

8,465,118  
Views



# Decision Making

Think of a hard choice you'll face in the near future

## Which career should I pursue?

it might be between two careers



## Should I break up -- or get married?!

or even between two people to marry



## Where should I live?

or even between two cities to live



# Decision Making

What is the difference between

**Simple** choices!?!? .. **Hard** choice



$$1 + 1 = 2$$



# Decision Making

Simple choices!?!? ..Simple decisions



**Choice 1**



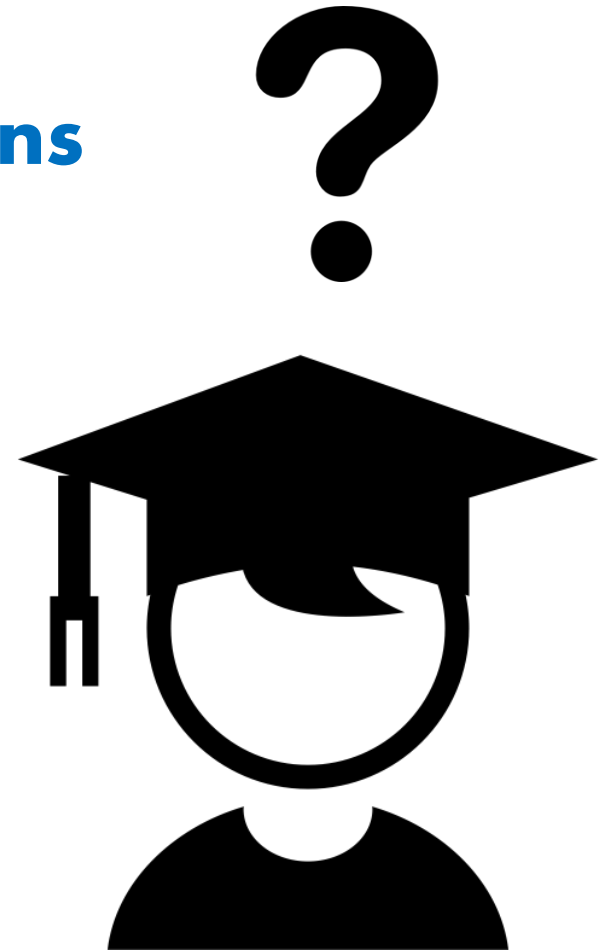
**Choice 2**

# Decision Making

## Hard choices!?!?... Hard decisions



**Choice 1**



**Choice 2**

# Decision Making

*Big decisions like these can be agonizingly difficult.*

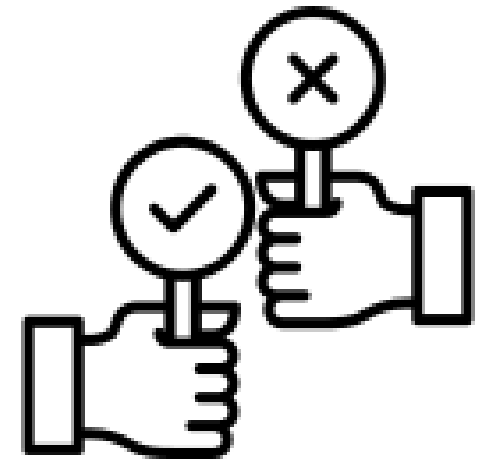
*But that's because we think about them the **wrong way!***



Hard choices!?!?... Hard decisions

Simple choices!?!? ..Simple decisions

# Why is a choice simple? or hard?



# Decision Making

*... Hard Choices ... How to make it!!!*

Hard Choices are hard because there is no best option.

In an easy choice one alternative is better than the other.

In **hard choice** one alternative is better in some ways, the other alternative is better in other ways and neither is better than the other overall.

**The alternatives are equally good!**

**The point is...**

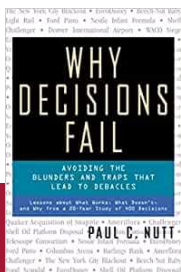
**It is important to decide  
....and decide well**



# Decision Making

## Importance of decision making process

- At least 50% of decisions should not be successful.
- 33% of decisions are never implemented .
- 50% of the decisions implemented is left after 2 years.
- 66% of decisions are based on methods used to failure.
- The decisions that use a high level of participation are successful in 80% of cases, but this occurs only 20% of the time.
- In practice, any error is unavoidable decision.

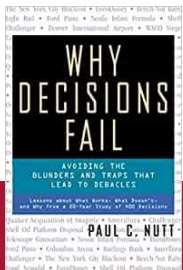


Source: Why Decisions Fail - Author Paul Nut - Publisher; Berret & Koehler 2002

# Decision Making

## Importance of decision making process

- 11 Million meetings in the U.S. per day
- Most professionals attend a total of 61.8 meetings per month
- Research indicates that over 50 percent of this meeting time is **wasted**
- Professionals **lose 31 hours per month** in unproductive meetings, or approximately four work days.



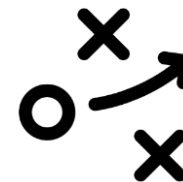
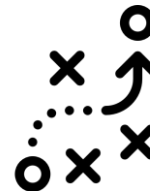
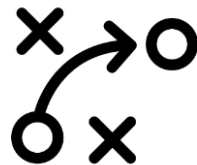
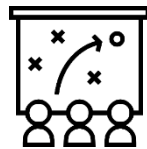
Source: Why Decisions Fail - Author Paul Nut - Publisher; Berret & Koehler 2002

# Decision Making

*Decision making is **difficult enough...***

...It is necessary to develop strategies and **measures** to manage these risks!

*Of course*.....The **success parameters** for any project **are** on time completion, within specific **budget** and with **requisite performance** (technical requirement).



# Decision Making

To make a decision in complex systems

# Multi-Criteria Decision Making Methods

overview

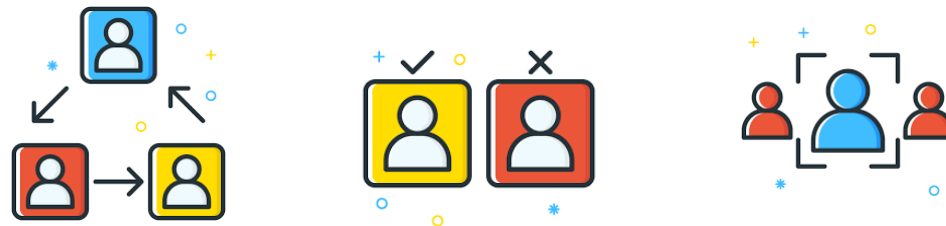
# Decision Making

## Multi-Criteria Decision Making Methods

Human beings are required to make decisions at individual and collective levels.

Initially, the decision-making process was studied as **rational process** of analyzing a problem and seeking solutions.

However, in recent years it has become clear that **human beings are far from making decisions in a rational way**, either as individual or as part of a group.



# Decision Making

## Multi-Criteria Decision Making Methods

**Psychological studies** have found cognitive anomalies or biases experienced by human beings when making decisions.

Some experiments have shown that individuals are easy victims of a series of cognitive biases

such as the **Phenomenon of framing**

“Changing the way a decision is framed – e.g., as a win or loss – makes individuals change their opinions”

Such as the **Phenomenon of anchoring**

“The individual’s decision is influenced by what piece of information is shown first”

# Decision Making

## Multi-Criteria Decision Making Methods

### Phenomenon of framing

“Changing the way a decision is framed – e.g., as a win or loss – makes individuals change their opinions”

For example, if two **investment projects** are presented to a group of people, one where there is the probability of losing 20% of the investment and another in which there is 80% chance of making a profit; people prefer to invest in the second project, although both have the same risk (20% probability of losing and 80% winning).



# Decision Making

## Multi-Criteria Decision Making Methods

### Phenomenon of anchoring

“The individual’s decision is influenced by what piece of information is shown first”

In other studies, it has been found that if a group of individuals is asked to estimate the following product:

$$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9$$

and another group composed of individuals of similar age, education...are asked to estimate the product

$$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

The first group estimate systematically lower results than the second group. This is because people are influenced by the first number shown!



# Decision Making

## Multi-Criteria Decision Making Methods

These cognitive biases and increasing complexity of modern problems make it extremely important to adopt a

**METHODOLOGY** for making simple and effective decisions.

It is essential to minimize cognitive biased and obtain a **group participation's synergy**.



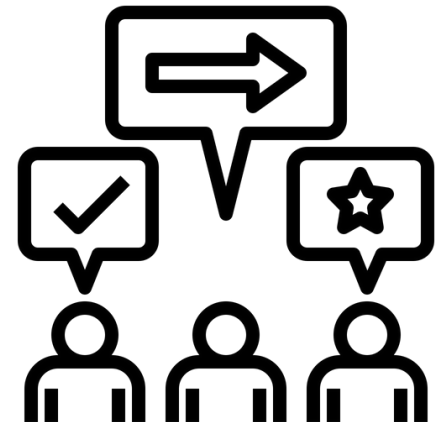
# Decision Making

## Multi-Criteria Decision Making Methods

Multi Criteria Decision Making (MCDM) provides **strong decision** making in domains where selection of best alternative is highly complex.

MCDM methods have evolved to accommodate **various types of applications**.

**Dozens of methods** have been developed, with even small variations to existing methods causing the creation of new branches of research.

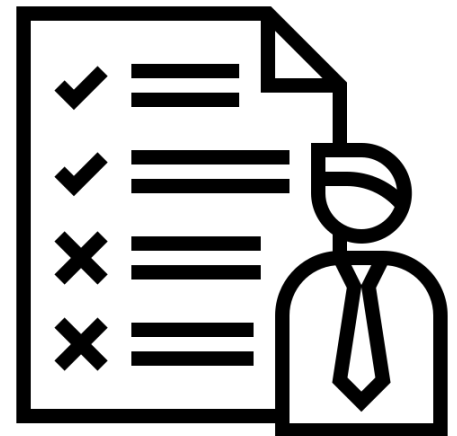


# Decision Making

## Multi-Criteria Decision Making Methods

In our day today life, so many decisions are being made from various criteria's, so the decision can be made by providing weights to different **criteria's** and all the weights are obtain from **expert groups**. It is important to determine the structure of the problem and explicitly evaluate multi criteria.

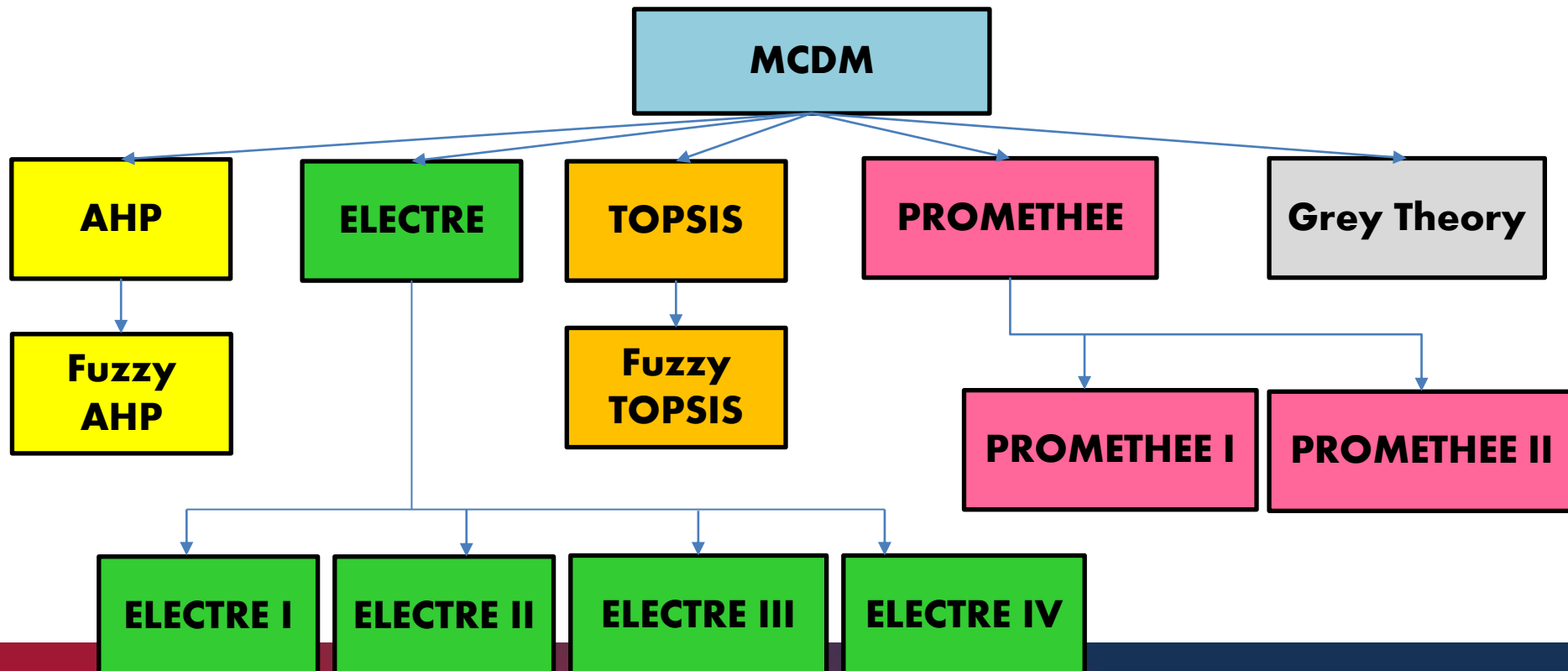
For example, in **building a nuclear power plant**, certain decisions are taken based on different criteria. There are not only very complex issues involving multi criteria, some criteria may have effect toward some problem, but over all to have an optimum solution, all the alternatives must have common criteria which clearly lead to more informed and better decisions.



# Decision Making

To make a decision in complex systems

## Multi-Criteria Decision Making Methods



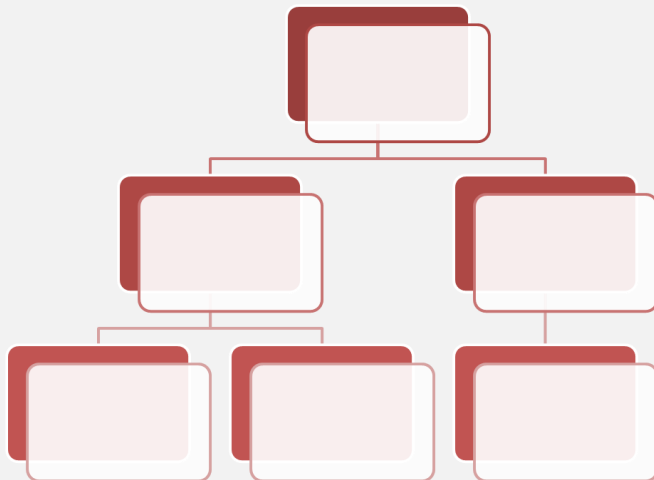
# MCDM methods with its merits and demerits

Sl. No	MCDM Methods	Description	Advantages	Disadvantages
1.	Analytic hierarchy process (AHP)	It also includes pair wise comparison of different alternatives for different criterion.	<ol style="list-style-type: none"> <li>1. Flexible, intuitive and checks inconsistencies</li> <li>2. Since problem is constructed into a hierarchical structure, the importance of each element becomes clear.</li> <li>3. No bias in decision making</li> </ol>	<ol style="list-style-type: none"> <li>1. Irregularities in ranking</li> <li>2. Additive aggregation is used. So important information may be lost.</li> <li>3. More number of pair wise comparisons are needed</li> </ol>
2	Analytic Network Process(ANP)	AHP builds the decision problem from arrangement of different goals, criteria and alternatives and pair wise comparison of the criteria to obtain the best alternative	<ol style="list-style-type: none"> <li>1. Independence among elements is not required.</li> <li>2. Prediction is accurate because priorities are improved by feedback.</li> </ol>	<ol style="list-style-type: none"> <li>1. Time consuming</li> <li>2. Uncertainty – not supported</li> <li>3. Hard to convince decision making</li> </ol>
3.	Data envelopment analysis (DAE)	DAE is a method where it is used to find the efficiency of combination of multi inputs and multi outputs of the problem.	<ol style="list-style-type: none"> <li>1. Multiple inputs and outputs can be handled.</li> <li>2. Relation between inputs and outputs are not necessary.</li> <li>3. Comparisons are directly against peers</li> <li>4. Inputs and outputs can have very different units</li> </ol>	<ol style="list-style-type: none"> <li>1. Measurement error can cause significant problems</li> <li>2. Absolute efficiency cannot be measured.</li> <li>3. Statistical tests are not applicable.</li> <li>4. Large problems can be demanding.</li> </ol>
4.	Aggregated Indices Randomization method (AIRM)	This method solves the complex problem where uncertainty occurs which has incomplete information for the problem to be solved.	<ol style="list-style-type: none"> <li>1. Non-numeric, non-exact and non-complete expert information can be used to solve multi criteria decision making problems.</li> <li>2. Transparent mathematical foundation assures exactness and reliability of results.</li> </ol>	It aims only at complex objects multi-criteria estimation under uncertainty.
5.	Weighted Product model(WPM)	Alternatives are being compared with the other by the weights and ratio of one for each criterion.	<ol style="list-style-type: none"> <li>1. Can remove any unit of measure.</li> <li>2. Relative values are used rather than actual ones.</li> </ol>	No solution with equal weight of DMs ones
6.	Weighted Sum Model (WSM)	It is used for evaluating a number of alternatives in accordance to the different criteria which are expressed in the same unit.	Strong in a single dimensional problems	Difficulty emerges on multi-dimensional problems
7.	Goal Programming	Goal programming is a division where it has more than one objective which conflicts with each other, and by arranging the goals or target have to be achieved by minimizing the irrelevant information.	<ol style="list-style-type: none"> <li>1. Handles large numbers of variables, constraints and objectives.</li> <li>2. Simplicity and ease of use</li> </ol>	<ol style="list-style-type: none"> <li>1. Setting of appropriate weights.</li> <li>2. Solutions are not pair to efficient.</li> </ol>
8.	ELECTRE	It is used to select the best choice with maximum advantage and least conflict in the function of various criteria	Outranking is used	Time consuming
9.	Grey analysis	This methods deal with all incomplete data and to overcome the deficiencies of other methods.	Perfect information has a unique solution	Does not provide optimal solution.



# Understanding the Analytic Hierarchy Process

## Overview



# Analytic hierarchy process

The analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed by Thomas L. Saaty in the 1970s.

It represents the most accurate approach for quantifying the weights of criteria. Individual experts' experiences are utilized to estimate the relative magnitudes of factors through pair-wise comparisons.

**In AHP, the decision problem is decomposed into a **hierarchy** of more easily comprehended sub-problems.**

Experts estimate the relative magnitudes of factors through **pairwise comparisons**.

# Analytic hierarchy process

AHP has been widely discussed and used since its official appearance. From its origins in the academia and in the government is recognized as essential tool of modern managers and leaders.

If you try to investigate SCOPUS, the largest abstract and citation database of peer-reviewed literature...you will understand the **phenomenon of AHP**....

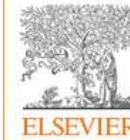
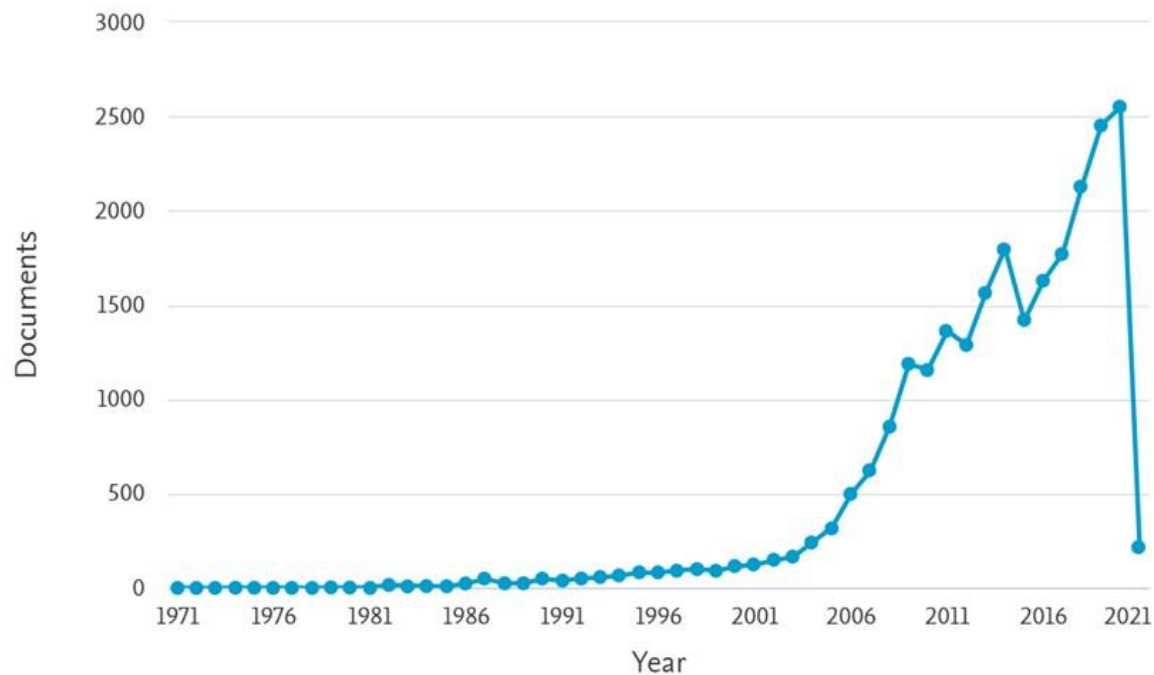


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Documents by year



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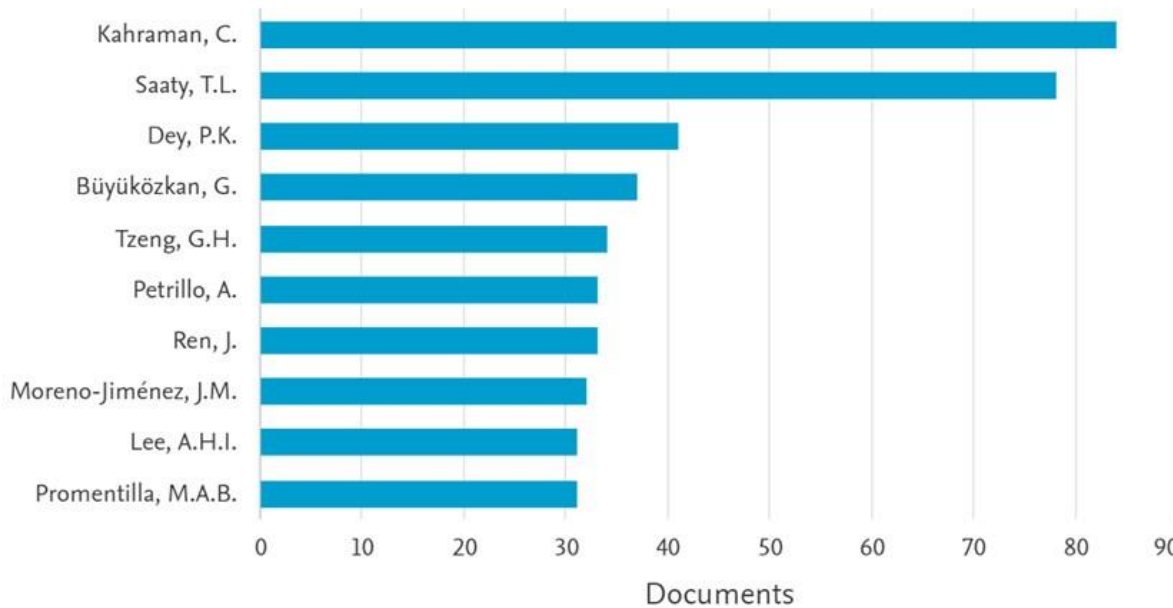
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# Documents by authors

## Documents by author

Compare the document counts for up to 15 authors.



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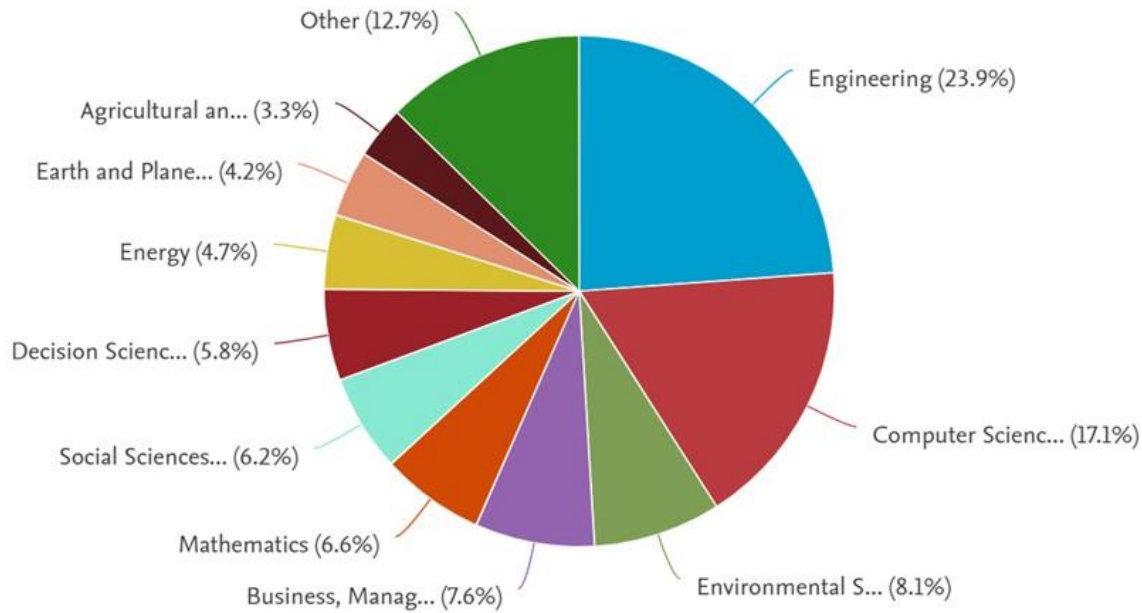
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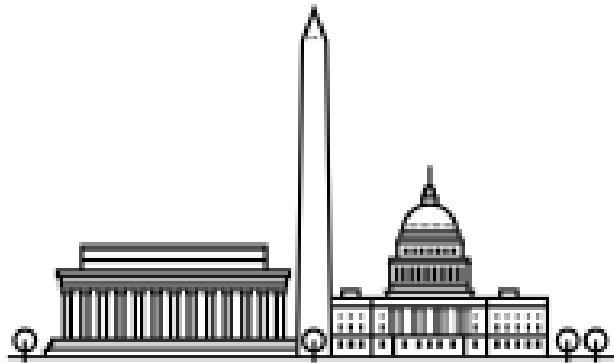


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# Who uses AHP?

# Analytic Hierarchy Process

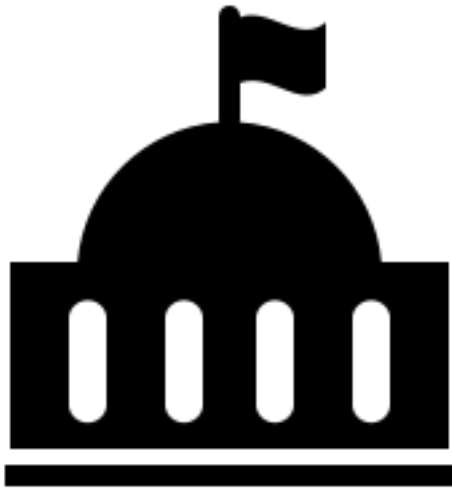
John & Daniel Saaty



WASHINGTON DC



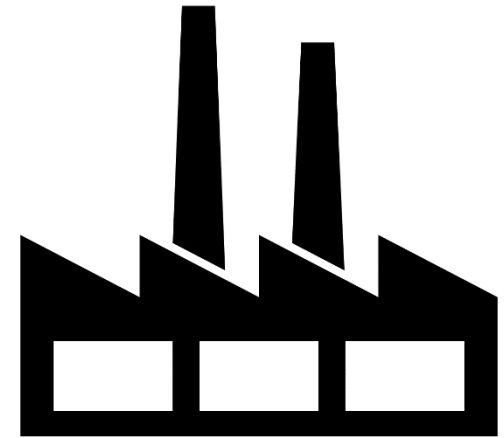
# Analytic Hierarchy Process



**Federal**



**State & Local**



**Private Sector**

# Analytic Hierarchy Process



## THE COMPANY

AIR FORCE A8-XP is the strategic planning division of the Air Force. It focuses on orchestrating their annual integration effort to **prioritize and allocate resources in their 30-year plan.**

## THE PROBLEM

Their current process was not flexible enough to handle on-the-fly adjustments while still accounting for the long-term payout of the programs.

## THE SOLUTION

The development of AHP model specifically related to decisions and longer-term, strategic planning choices. This framework made it easy to manipulate and update data, which helped them look at resource decisions across multiple time periods, both mid-term and long-term.

# Analytic Hierarchy Process



## THE ORGANIZATION

**Arizona Department of Transportation (ADOT).** ADOT strategically prioritizes the investment strategy for over 160 projects in a typical Statewide Transportation Improvement Plan (STIP) cycle. The cycle usually lasts for 4 to 5 years and are accountable for around \$1.2 billion of transportation funding, which is comprised of 7 different funding sources.

## THE PROBLEM

ADOT needed to incorporate project performance into their planning process and provide a system-wide perspective during their planning decision process.

## THE SOLUTION

The development of AHP model to improve their performance measures in place. This helped enable them to spend their budget with a direct correlation to expected performance and answer questions of what extra funding would yield.



# Analytic Hierarchy Process

## THE ORGANIZATION

**Italian Ministry of Agricultural Policies** is responsible for the elaboration and coordination of agricultural, forestry, agri-food policies as well as for fishing at national, European and international level, representing Italy in the European Union for the matters of competence.

## THE PROBLEM

Identification of a “quality” model for Italian racecourse for the distribution of economic resources. Prioritizes the resources allocation strategy.

## THE SOLUTION

The development of AHP model helped to define key factors to improve Italian racecourse performance. This helped them to allocate better their resources and to spend better their public budget.

# Analytic Hierarchy Process



## THE ORGANIZATION

MBDA is a world leader in missile systems offering a comprehensive international product range incorporating today's most advanced innovations.

## THE PROBLEM

Train managers in decision making. For senior executives, managers for building high-performing teams and key decision makers.

## THE SOLUTION

The “Decision-Making School” deals with planning and implementing top level training seminars for MBDA executives on various aspects of the theory of rational decisions.

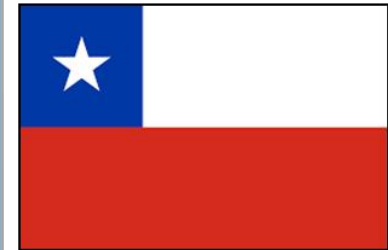
# Analytic Hierarchy Process

## IMPLEMENTATION OF AHP METHODOLOGY INTO NATIONAL DEFENSE EVALUATION & QUALIFICATION PROJECTS



Subsecretaría de  
Defensa

**JULIO BAEZA VON BOHLEN**



# In which kind of decision we apply AHP?

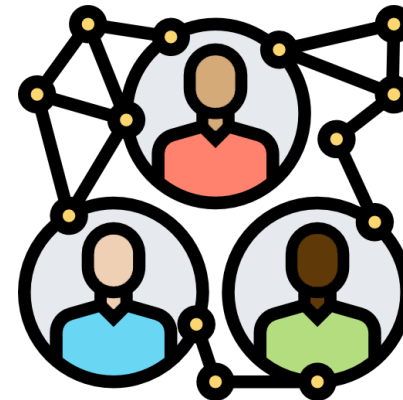
# Analytic Hierarchy Process

## Most Decision Problems are Multicriteria

- Maximize profits
- Satisfy customer demands
- Maximize employee satisfaction
- Satisfy shareholders
- Minimize costs of production
- Satisfy government regulations
- Minimize taxes
- Maximize bonuses

# Analytic Hierarchy Process

Perhaps the biggest advantage of AHP is that allows the inclusion of **intangibles** such as experience, subjective preference and intuition in a logical and structured way!



# Could you list some examples of tangible and intangible factors?



# Analytic Hierarchy Process

In our life we need to prioritize both **tangible** and **intangible** criteria:

In most decisions, **intangibles** such as:

- political factors and
- social factors

take precedence over **tangibles** such as:

- economic factors and
- technical factors



# Analytic Hierarchy Process

**You don't need to know everything to  
get to the answer.**



# Analytic Hierarchy Process

It is **not** the **precision** of **measurement** on a particular factor that determines the validity of a decision, **but** the **importance** we attach to the factors involved.

**How do** we **assign importance** to all the factors and synthesize this diverse information to make the best decision?

# Analytic Hierarchy Process

**AHP allows** to assign a **weight of importance** to each factors.

**AHP allows** to measure **intagibles elements** through expert's judgment.

AHP choose the “**best**” among several alternatives.

Differently from common optimization methods AHP uses derived measurements or subjective.

**Subjectivity  $\neq$  Arbitrariness**

# Analytic Hierarchy Process

The increasing complexity of modern problems make it **extremely important to adopt a methodology** for making easy to use and understand.

The **ANALYTIC HIERARCHY PROCESS** meets these requirements.



# Analytic Hierarchy Process

**Analytic:** Decompose the problem into its elementary components.

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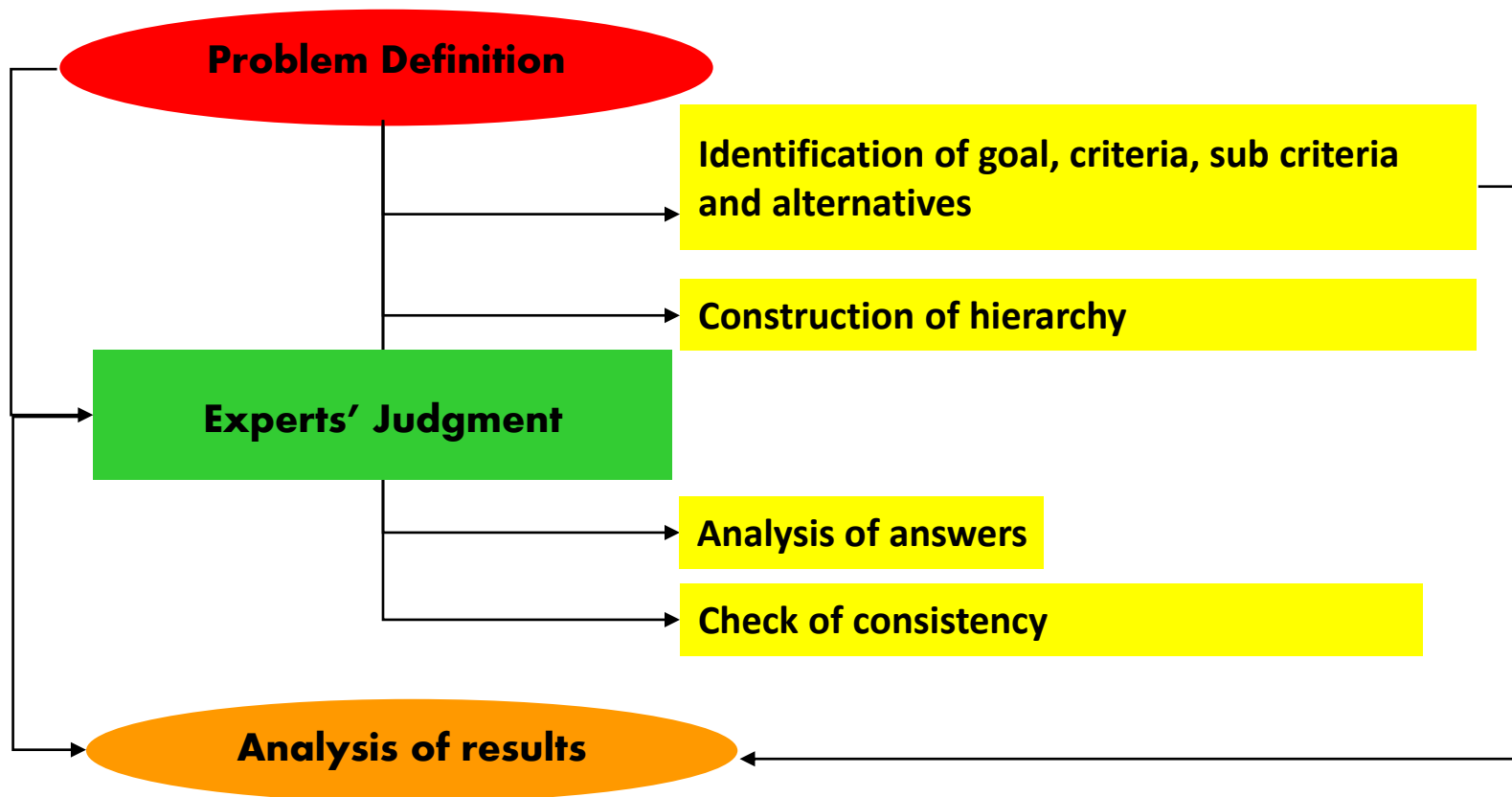
**Hierarchy:** Design the decision problem in a hierarchical or network defining the goal, criteria and the sub-criteria.

---

**Process:** Process the data and evaluations in order to achieve the final result

# Analytic Hierarchy Process

## AHP Logic Diagram



# Analytic Hierarchy Process

## Main Phases of AHP

**Phase#1.** Definition of the Hierarchy

**Phase#2.** Pairwise comparison

**Phase#3.** Consistency Index (CI) calculation

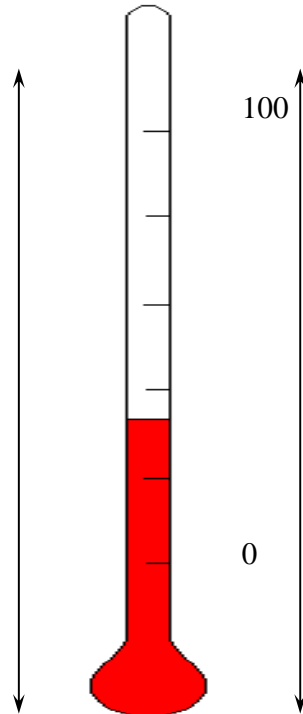
# Analytic Hierarchy Process

## Measurement scales

Good for  
preserving food

Bad for  
preserving food

Good for  
preserving food



Temperature

Bad for  
comfort

Good for  
comfort

Bad for  
comfort





# Analytic Hierarchy Process

## Measurement scales

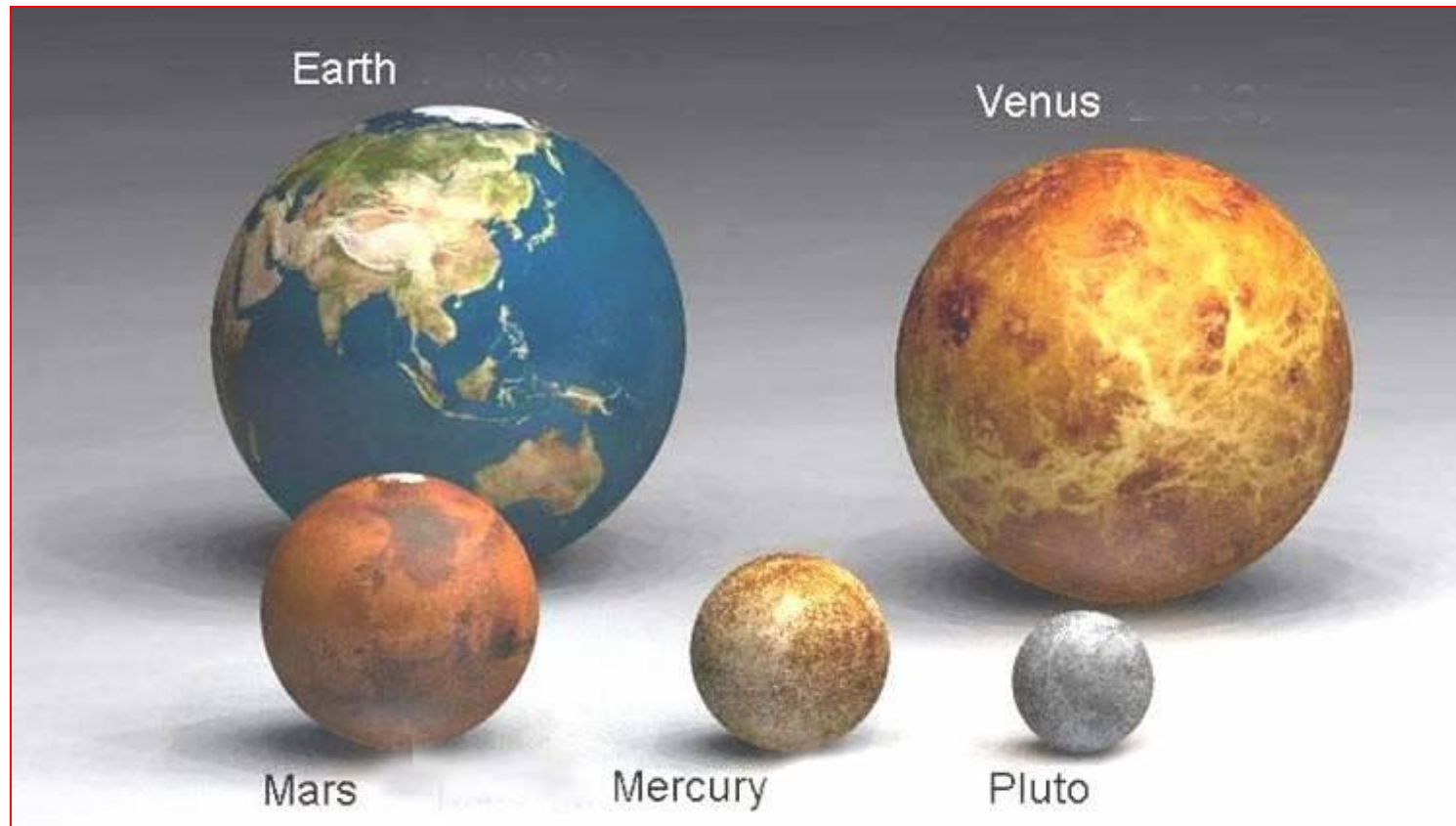
Obviously, it is important to compare homogenous elements with each other!

Let's do some examples!



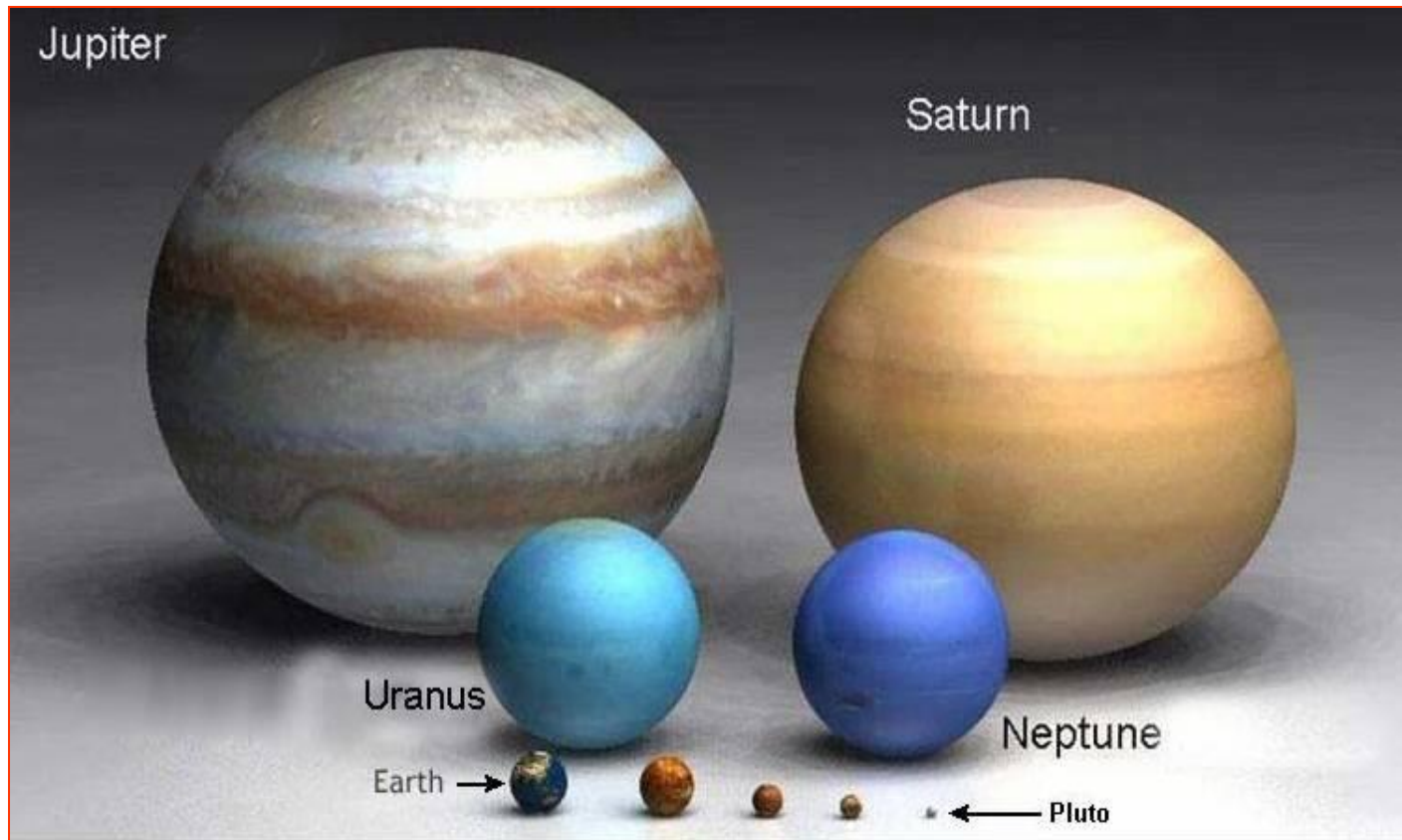
# Analytic Hierarchy Process

## Measurement scales



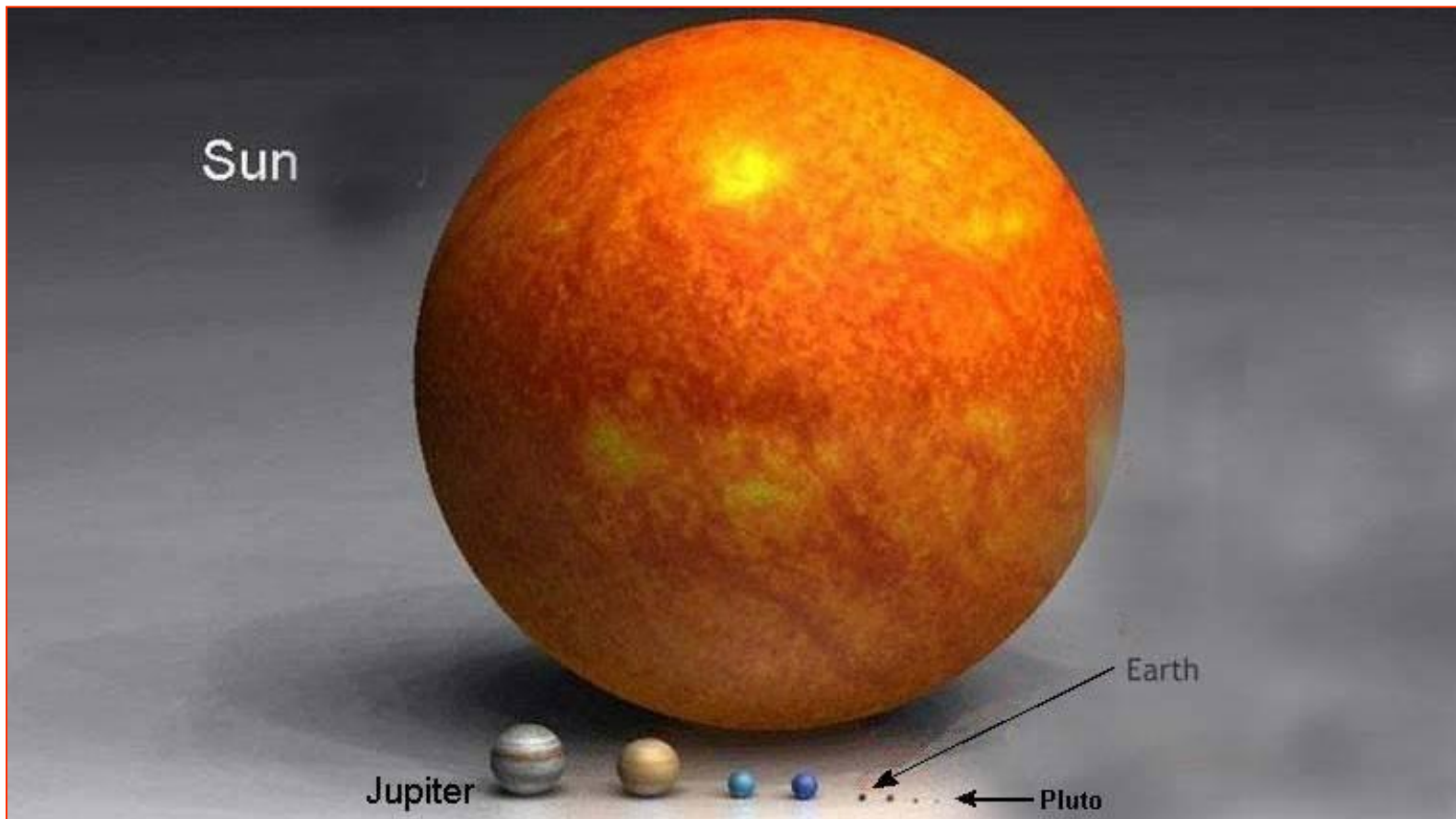
# Analytic Hierarchy Process

## Measurement scales



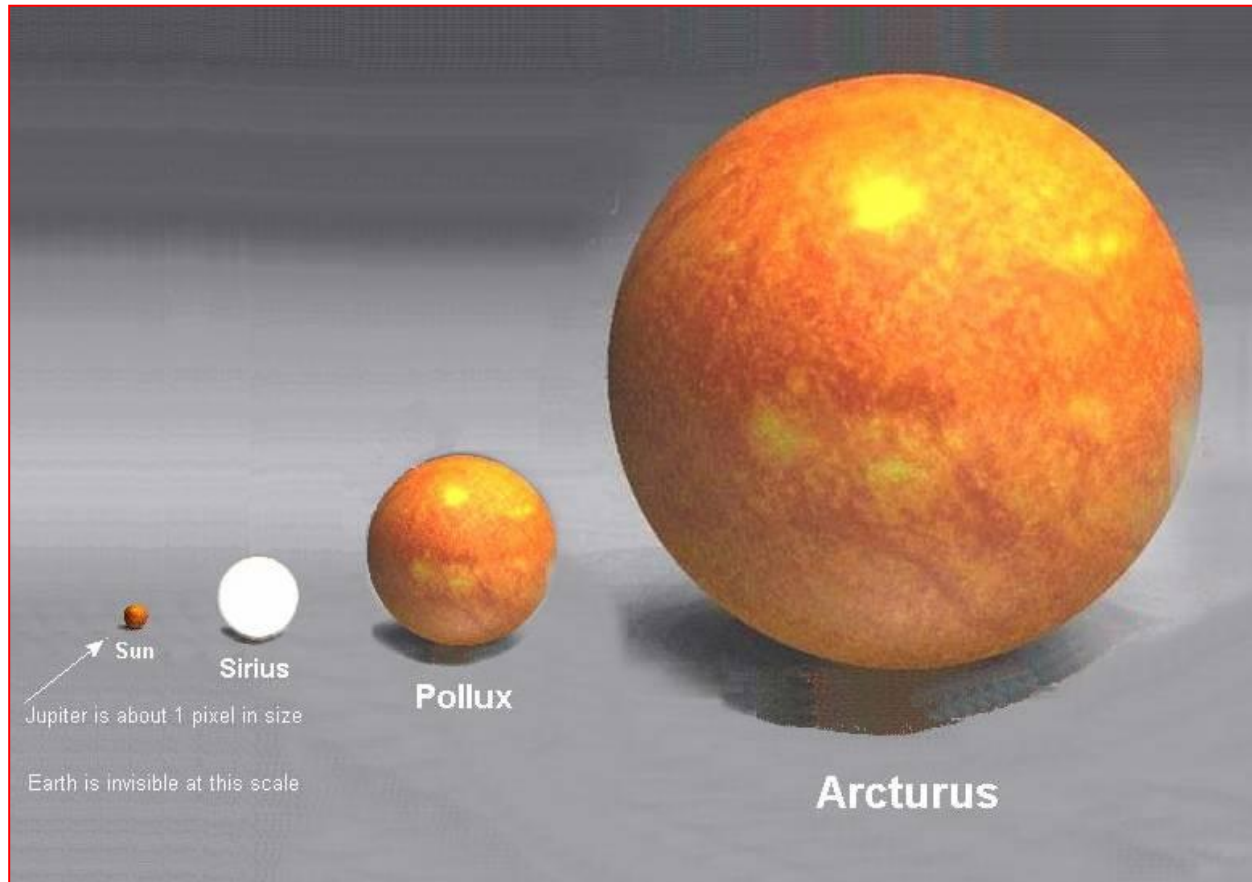
# Analytic Hierarchy Process

## Measurement scales



# Analytic Hierarchy Process

## Measurement scales



# Analytic Hierarchy Process

## Measurement scales



# Analytic Hierarchy Process

## Saaty's Scale

Intensity of Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance is demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed

# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

Prof. Nokata (1995) classified knowledge into **explicit** and **tacit**.

**EXPLICIT KNOWLEDGE** can be explained, coded and easily transmitted from one person to the other.

The information contained in **encyclopedias** and **textbooks** are **good examples of explicit knowledge**.

The most common forms of explicit knowledge are manuals, documents, procedures, and how-to videos.





# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

Nokata (1995) classified knowledge into **explicit** and **tacit**.

**TACIT KNOWLEDGE (or implicit)** is the kind of knowledge that is difficult to transfer to another person.

Tacit knowledge can be defined as skills, ideas and experiences that people have but are not codified.



One of the most convincing examples of tacit knowledge is **facial recognition**. *We know a person's face, and can recognize it among a thousand, indeed a million. Yet we usually cannot tell how we recognize a face we know, so most of this cannot be put into words.* When you see a face, you are not conscious about your knowledge of the individual features (eye, nose, mouth), but you see and recognize the face as a whole

# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

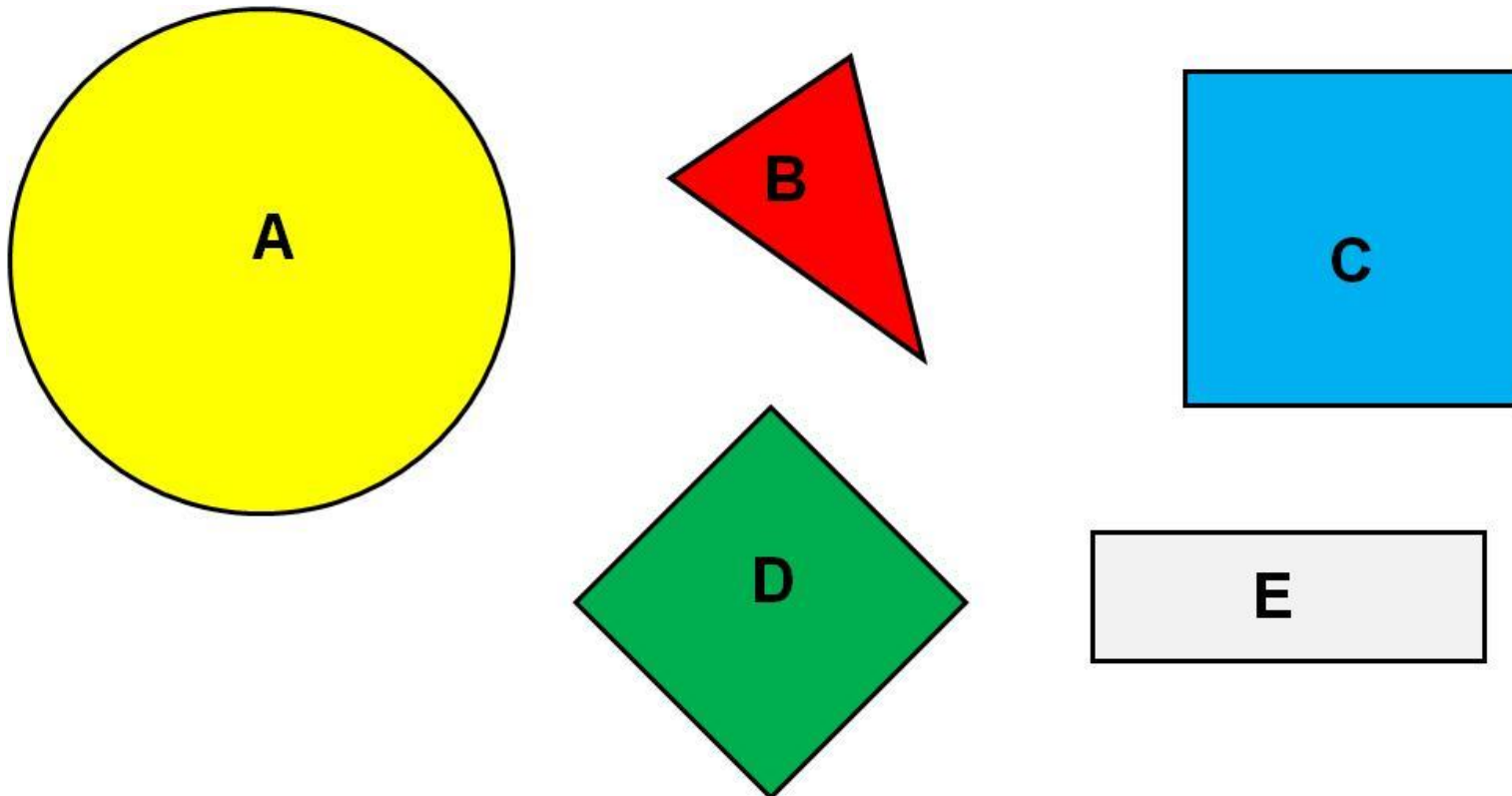
Several validation studies have been conducted over the years to illustrate the effectiveness of the **AHP to facilitate** the *eliciting* of *tacit knowledge* in the decision-making process.

In one **classical study** (Saaty, 2008) a group of participants were shown the some **geometric figures** and were requested to rank the geometric figures in terms of **AREA SIZE** and to estimate the **relative AREAS OF EACH FIGURES**.



# Analytic Hierarchy Process

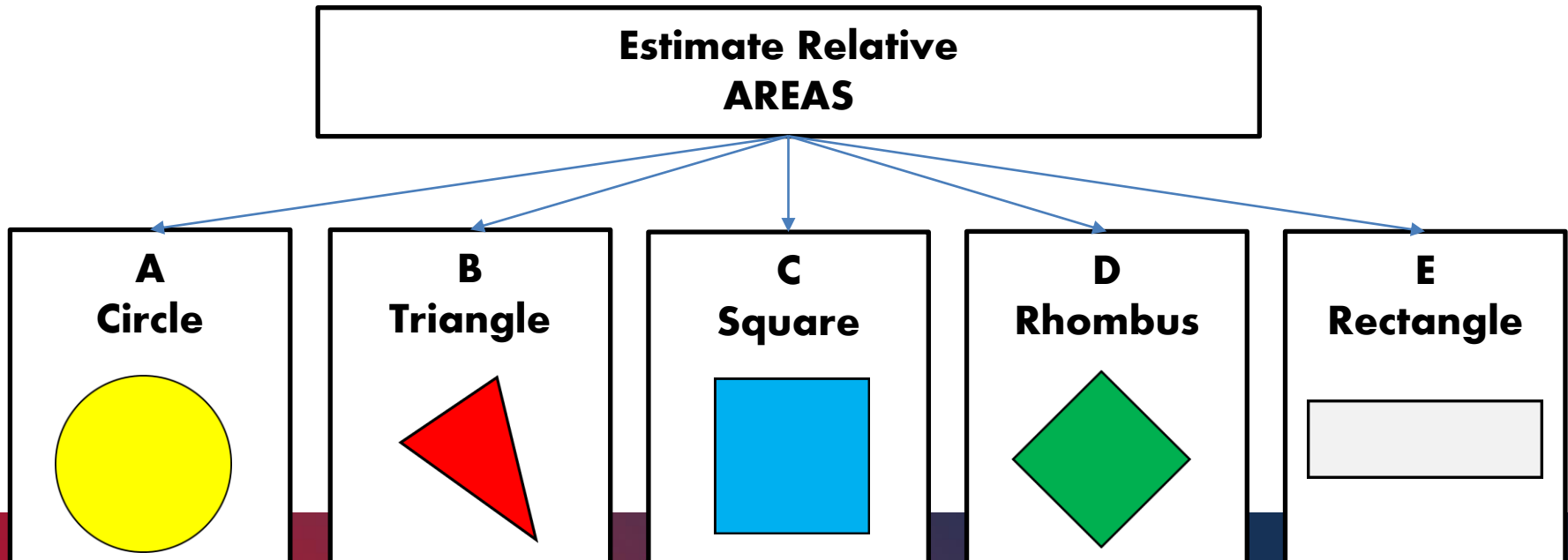
## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale



# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

From an *AHP point of view*, such task can be conceptualized as a *hierarchical* decision-making task comprese of a decision goal and the alternative to choose from (i.e. the different geometric figures).



# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

In **technical terms**, the AHP method consists of **pairwise comparisons** of the areas of the geometric figures followed by a calculation of the final priorities.

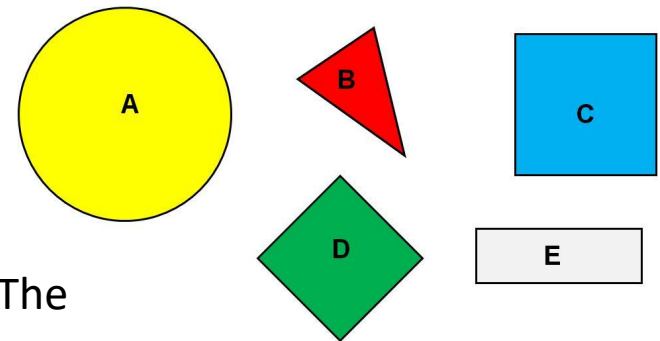
### Assumptions:

The **Circle (A)** is the largest figure in the cluster

The **Triangle (B)** is the smallest figure in the cluster

It is clear that the Circle is the biggest figures in the cluster. The most important.

**Question:** How much the Circle is bigger than the Triangle?

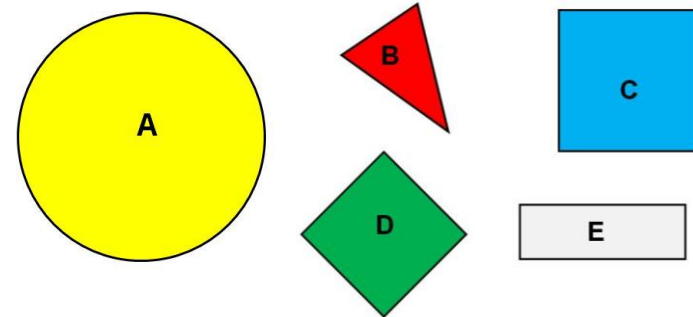


	A	B	C	D	E
A	1	9			

# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

Similarly, I compare the circle with all the other geometric figures I get that...



	A	B	C	D	E
A	1	9	3	4	5

# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

Completed pairwise comparison matrix for the geometric figures

	A	B	C	D	E	Priorities (Weight)
A	1	9	2	4	5	0,480
B	1/9	1	1/5	1/3	1/2	0,049
C	1/2	5	1	2	3	0,250
D	1/4	3	1/2	1	2	0,138
E	1/5	2	1/3	1/2	1	0,085

The matrix is symmetrical, reciprocal and consistent

# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

When the final priorities are compared against the actual relative sizes, we can see that the differences are minimal!

### Completed pairwise comparison matrix for the geometric figures

	A	B	C	D	E	Priorities (Weight)	Relative size
A	1	9	2	4	5	0,480	0,470
B	1/9	1	1/5	1/3	1/2	0,049	0,050
C	1/2	5	1	2	3	0,250	0,240
D	1/4	3	1/2	1	2	0,138	0,150
E	1/5	2	1/3	1/2	1	0,085	0,100

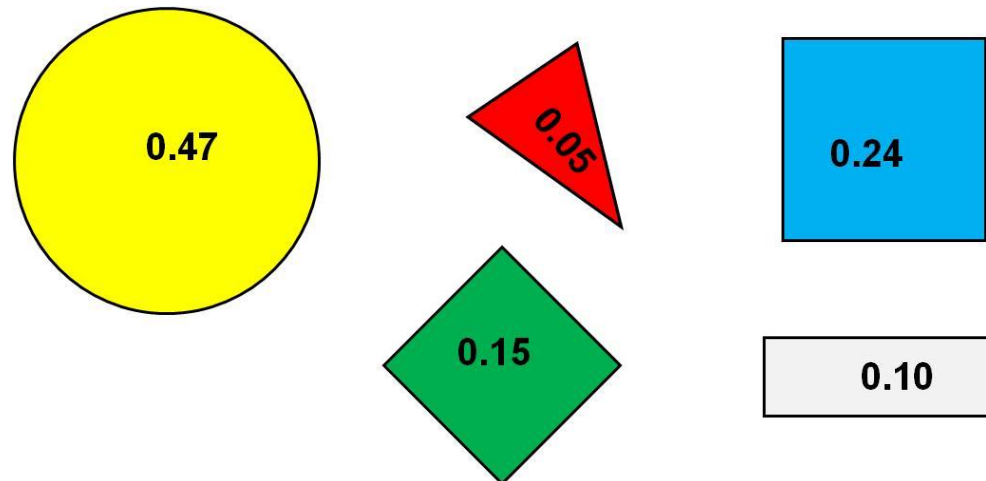


# Analytic Hierarchy Process

## Use of AHP to Elicit Tacit Knowledge and Saaty's Scale

In other words, **if we went to calculate the area of the figures with the mathematical rules we will get the following results!**

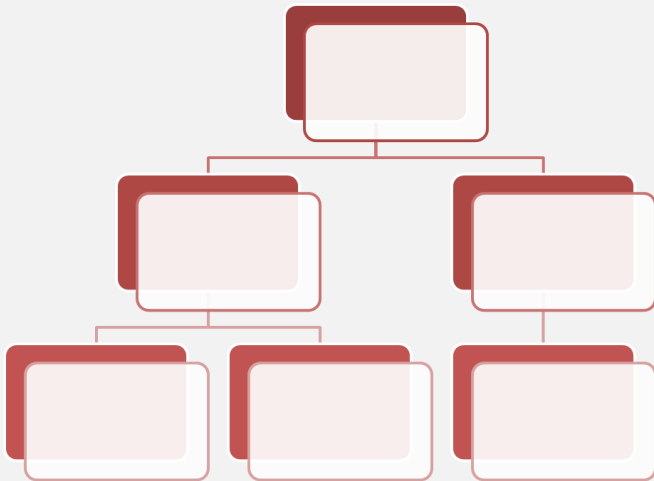
The **purpose** of the example is to show that if the judgments are carefully expressed, the results are very reliable.



# Understanding the Analytic Hierarchy Process

Basic

AHP simple model



# AHP Model: Case Study 1

To explain **how use** AHP we will use a **simple example**.

Our **GOAL** is to buy a **new car**.

Our purchase is based on different **CRITERIA** such as **cost**, **comfort**, and **safety**.

We could evaluate several **ALTERNATIVES** but let us assume that we have only two: **CAR 1** and **CAR 2**.

.....let's see.... (at glance):

# AHP Model: Case Study 1

To analyze the decision of buying a car using the AHP we should follow the **next 6 steps:**

- **Step#1: Develop a model for the decision:** Break down the decision into a hierarchy of goals, criteria, and alternatives.
- **Step#2: Derive priorities (weights) for the criteria:** The importance of criteria are compared pairwise with respect of the desired goal to derive their weights. We then check the consistency of judgments; that is, a review of the judgments is done in order to ensure a reasonable level of consistency in terms of proportionality and transitivity.
- **Step#3: Derive the local priorities (preferences) for the alternatives:** Derive priorities for the alternatives with respect to each criterion. Check and adjust the consistency, if necessary.
- **Step#4: Derive the Overall Priorities (Model Synthesis):** All alternative priorities obtained are combined as a weighted sum – to take into account the weight of each criterion – to establish the overall priorities of the alternatives. The alternative with the highest overall priority constitutes the best choice.
- **Step#5: Perform Sensitivity Analysis:** A study of how changes in the weights of the criteria could effect the result of done to understand the rationale behind the obtained results.
- **Step#6: Making a Final Decision:** Based on the synthesis results ad sensitivity analysis, a decision can be made.

# AHP Model: Case Study 1

## GOAL

## Buying a new car

## ALTERNATIVES

## CAR1 and CAR2



**CAR 1**



**CAR 2**

# AHP Model: Case Study 1

## CRITERIA: Safety, Cost, Comfort

**SAFETY**



**COST**



**COMFORT**



**Experts**

# AHP Model: Case Study 1

## Step#1: Develop a model for the decision

The first step in an AHP analysis is to build a **HIERARCHY** for the decision. This is also called **decision modelling** and it simply consists of building a hierarchy to analyze the decision.

The **advantages** of the hierarchy decomposition are clear.

By structuring the problem in this way it is possible to better understand the decision to be achieved, the criteria to be used and the alternatives to be evaluated.

This, step is crucial and this is where, in more complex problems, it is desirable to request the **participation of experts** to ensure that all criteria and possible alternatives have been considered.

# AHP Model: Case Study 1

Step#1: Develop a model for the decision

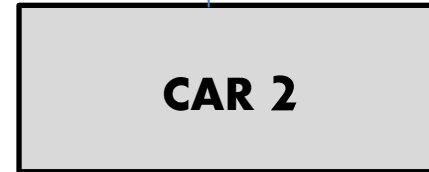
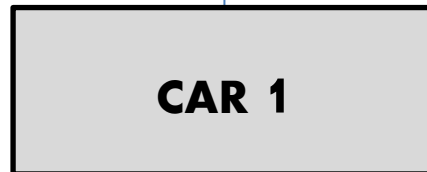
Level 1: **GOAL**



Level 2: **CRITERIA**



Level 3: **ALTERNATIVES**





# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

It is clear that **when buying** a car **not all criteria are equally important** in a given time.

### For example,

- a **student** may give more importance to the **cost factor** rather than to comfort and safety;
- while a **parent** may give more importance to the **safety factor** rather than to the others.

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

Clearly, the importance or weight of each criterion will be different.

Because of this, we first are required to derive by pairwise comparisons the **relative priority** of each criterion with respect to each of the others using a numerical scale of comparison developed by Prof. Saaty, the so-called **semantic scale of Saaty's**.

# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

## Sematic scale of Saaty's

Numeric value	Verbal judgment
1	Equal importance
2, 3	Moderate importance of one over another
4, 5	Strong or essential importance
6, 7	Very strong or demonstrated importance
8, 9	Extreme importance

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

To perform the pairwise comparison you need to create a **comparison matrix** of the criteria involved in the decision.

Buying a car	COST	COMFORT	SAFETY
COST			
COMFORT			
SAFETY			

**Cells in comparison matrices** will have a value from the numeric scale to reflect **our relative preference** in each of the compared pairs.

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

For example, if we consider that the **cost** is *very strongly more important* than the **comfort** factor, the cost-comfort factor comparison cell will contain the value 7.

Buying a car	COST	COMFORT	SAFETY
COST		7	
COMFORT			
SAFETY			

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

Of course, the opposite comparison, the importance of comfort relative to the importance of cost, will yield the reciprocal of this value (comfort/cost =  $1/7$ ).

Buying a car	COST	COMFORT	SAFETY
COST		7	
COMFORT	1/7		
SAFETY			

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

If we consider that the **cost** is *moderately more important* than safety, we will enter 3 in the cost-safety cell and the safety-cost cell will contain the reciprocal.

Buying a car	COST	COMFORT	SAFETY
COST		7	3
COMFORT	1/7		
SAFETY	1/3		

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

Finally, if we feel that **safety** is *moderately more important* than **comfort**, the safety-comfort cell will contain the value 3 and the comfort-safety cell, will have the reciprocal  $1/3$ .

Buying a car	COST	COMFORT	SAFETY
COST		7	3
COMFORT	$1/7$		<b><math>1/3</math></b>
SAFETY	$1/3$	<b>3</b>	



# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

**Note** that in comparison matrix when the importance of a criterion is compared with itself the input value is 1.

Pairwise comparison matrix with intensity judgment

Buying a car	COST	COMFORT	SAFETY
COST	<b>1</b>	7	3
COMFORT	1/7	<b>1</b>	1/3
SAFETY	1/3	3	<b>1</b>

# AHP Model: Case Study 1

## Step#2: Derive priorities (weights) for the criteria

**At this stage** you can see on of the great advantages of the AHP:

- Its natural simplicity;
- Regardless of how many factors are involved in making the decision, the AHP method requires to compare a pair of elements at any time;
- It allows the inclusion of **tangible variables** (e.g., cost) as well **intangible ones** (e.g., comfort) as criteria in the decision.

# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

To calculate the priorities... weights for each criterion

Buying a car	COST	COMFORT	SAFETY	Product
COST	<b>1.000</b>	7.000	3.000	<b>21.00</b>
COMFORT	0.143	<b>1.000</b>	0.333	<b>0.048</b>
SAFETY	0.333	3	<b>1.000</b>	<b>1.000</b>

# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

To calculate the priorities... weights for each criterion

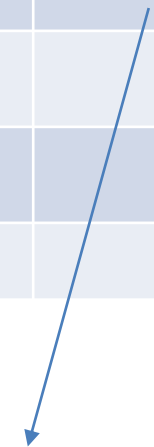
Buying a car	COST	COMFORT	SAFETY	Root3
COST	<b>1.000</b>	7.000	3.000	<b>2.758</b>
COMFORT	0.143	<b>1.000</b>	0.333	<b>0.362</b>
SAFETY	0.333	3	<b>1.000</b>	<b>1.000</b>

# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

To calculate the priorities... weights for each criterion

Buying a car	COST	COMFORT	SAFETY	Root3	Normalization
COST	<b>1.000</b>	7.000	3.000	<b>2.758</b>	<b>0.669</b>
COMFORT	0.143	<b>1.000</b>	0.333	<b>0.362</b>	<b>0.087</b>
SAFETY	0.333	3	<b>1.000</b>	<b>1.000</b>	<b>0.242</b>
				<b>4.121</b>	<b>1</b>

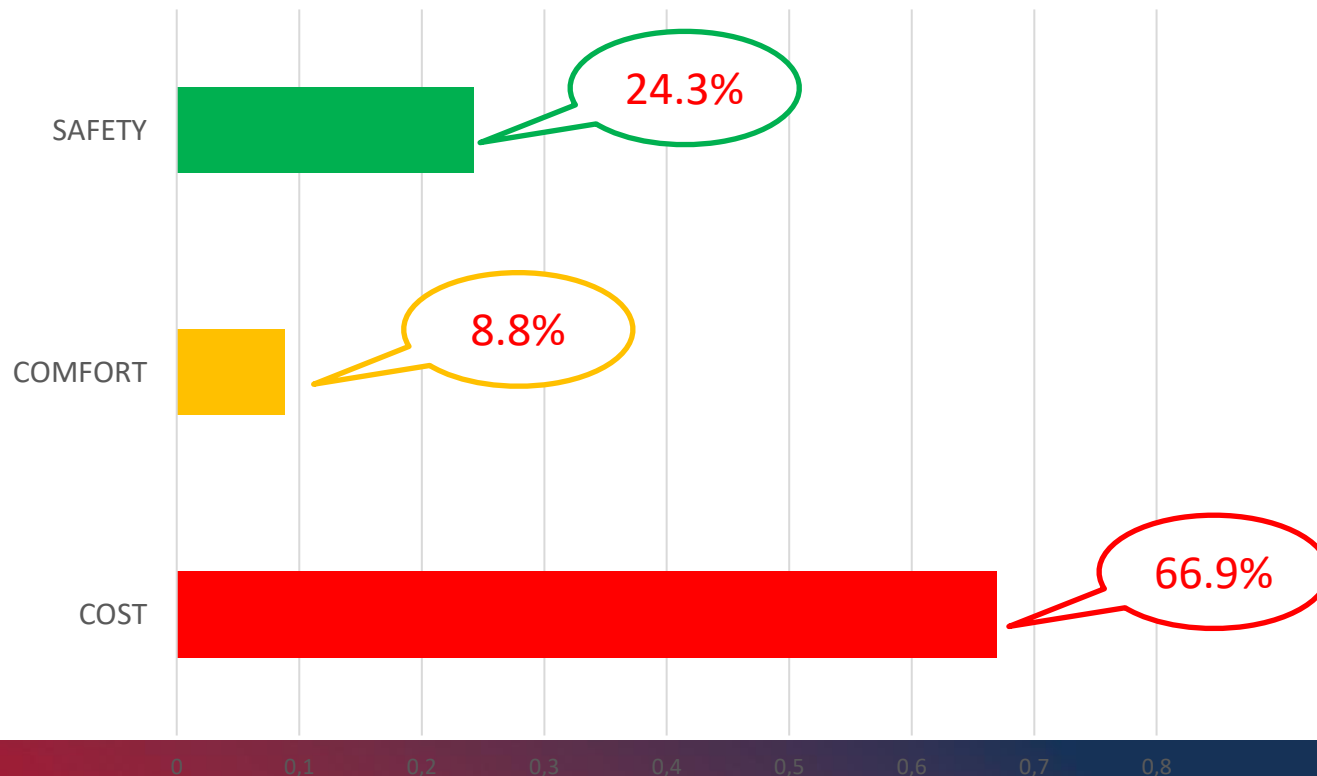


$$2,75/4,12=0,669$$

# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

The ... **weights** for each criterion are:



# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

## Consistency

Once judgments have been entered, it is necessary to **check that they are consistent.**

Since the numeric values are derived from **subjective preferences** of individuals, it is possible to avoid some inconsistency in the final matrix of judgments.

Because the world of experience is vast and we deal with it in pieces according to whatever goals concern us at the time, **our judgments can never be perfectly precise.**

# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

## Consistency

The question is

**How much inconsistency is acceptable?**

For this purpose, AHP calculates the  
**Consistency Index (CI) of the matrix**

$$CI = (\lambda_{\max} - n) / (n-1) < 10\%$$

Where n is the number of compared elements (in our example n = 3)



# AHP Model: Case Study 1

Step#2: Derive priorities (weights) for the criteria

## Consistency

Buying a car	COST	COMFORT	SAFETY	Root3	Normaliz ation	Coeff	$\lambda_{max}$ Eigenvalue
COST	<b>1.000</b>	7.000	3.000	<b>2.758</b>	<b>0.669</b>	<b>1</b>	<b>0.988</b>
COMFORT	0.143	<b>1.000</b>	0.333	<b>0.362</b>	<b>0.087</b>	<b>0.1313</b>	<b>0.967</b>
SAFETY	0.333	3	<b>1.000</b>	<b>1.000</b>	<b>0.242</b>	<b>0.362</b>	<b>1.051</b>
<b>Sum</b>	<b>1.476</b>	<b>11</b>	<b>4.333</b>	<b>4.121</b>	<b>1</b>		<b>3.007</b>

For example:

For COST  $(2.75) * (1.476) / \text{tot } (4.12) = 0,988$  (Eigenvalue)

$$CI = (3.007 - 3) / (3-1) = 0.004 < 0.10$$

Since the value is less than 0.10, we can assume that our judgments matrix is **reasonable consistent**.

# AHP Model: Case Study 1

## Step#3: Derive the local priorities (preferences) for the alternatives

Our third step consists of deriving the **relative priorities** (preferences) **of the alternatives with respect of each criterion.**

In our case are cost, comfort, and safety.

In our example we have only **2 alternatives** CAR1 and CAR 2 and we have **3 criteria.**

This means that there will be **3 comparison matrices** corresponding to the following three comparisons:

- With respect of the cost criterion: Compare CAR 1 with CAR 2
- With respect of the comfort criterion: Compare CAR 1 with CAR 2
- With respect of the safety criterion: Compare CAR 1 with CAR 2

# AHP Model: Case Study 1

Step#3: Derive the local priorities (preferences) for the alternatives

With respect of the cost criterion which alternative is preferable?  
CAR 1 or CAR 2?

Let us assume that we prefer **very strongly** the CAR 1 over the CAR 2

COST	CAR 1	CAR 2
CAR 1	1	7
CAR 2	1/7	1



C.I. = 0



Priority:

- 0.875 for CAR 1 = 87.5%
- 0.125 for CAR 2 = 12.5%

# AHP Model: Case Study 1

Step#3: Derive the local priorities (preferences) for the alternatives

With respect of the comfort criterion which alternative is preferable? CAR 1 or CAR 2?

Let us assume that we prefer **strongly** the CAR 2 over the CAR 1

COMFORT	CAR 1	CAR 2
CAR 1	1	1/5
CAR 2	5	1



C.I. = 0



Priority:

0.833 for CAR 2 = 83,3%

0.167 for CAR 1 = 16,7%

# AHP Model: Case Study 1

Step#3: Derive the local priorities (preferences) for the alternatives

With respect of the safety criterion which alternative is preferable? CAR 1 or CAR 2?

Let us assume that we prefer **extremely** the CAR 2 over the CAR 1

COMFORT	CAR 1	CAR 2
CAR 1	1	1/9
CAR 2	9	1

▶ C.I. = 0



Priority:

0.90 for CAR 2 = 90%

0.10 for CAR 1 = 10%

# AHP Model: Case Study 1

## Step#3: Derive the local priorities (preferences) for the alternatives

We can **summarize** the results indicating that:

1. if our only criterion were **cost**, CAR 1 would be our best option (priority 0.875);
2. if our only criterion were **comfort** our best option would be the CAR 2 (priority 0.833);
3. if our sole purchase criteria were **safety** our best option would be the CAR 2 (priority 0.90)



COST = 87,5% for CAR 1



COMFORT = 83,3% for CAR 2



SAFETY = 90% for CAR 2

# AHP Model: Case Study 1

## Step#4: Derive the Overall Priorities (Model Syntesis)

	COST	COMFORT	SAFETY	Overall priority
<i>Criteria weights</i>	0.669	0.088	0.243	
CAR 1	0.875	0.167	0.100	<b>0.146</b>
CAR 2	0.125	0.833	0.900	<b>0.853</b>

CAR 1 = 14.6%

CAR 2 = 85.3%



# AHP Model: Case Study 1

## Step#5: Perform Sensitivity Analysis

It is useful to perform a "**what-if**" analysis to see how the final results would have a change if the **weights** of the **criteria** would have been **different**.

Sensitivity analysis allows us to understand **how robust** is our **original decision**.

To perform a sensitivity analysis it is necessary to **make changes to the weights** of the criterion and see how the change the overall priority.



# AHP Model: Case Study 1

## Step#5: Perform Sensitivity Analysis

### Scenario 1: all criteria same weight

	COST	COMFORT	SAFETY	Overall priority
<i>Criteria weights</i>	<b>0.333</b>	<b>0.333</b>	<b>0.333</b>	
CAR 1	0.875	0.167	0.100	<b>0.130</b>
CAR 2	0.125	0.833	0.900	<b>0.869</b>



**CAR 2 = 86.9%**



# AHP Model: Case Study 1

## Step#5: Perform Sensitivity Analysis

### Scenario 2: cost weight leading

	COST	COMFORT	SAFETY	Overall priority
<i>Criteria weights</i>	<i>0.500</i>	<i>0.250</i>	<i>0.250</i>	
CAR 1	0.875	0.167	0.100	<b>0.129</b>
CAR 2	0.125	0.833	0.900	<b>0.435</b>

▶ **CAR 2 = 43.5%**

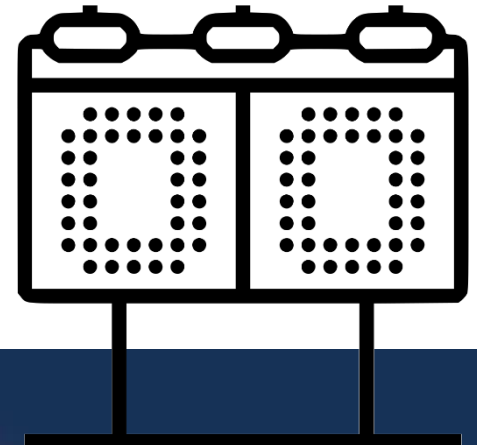


# AHP Model: Case Study 1

## Step#6: Making a final decision

The model is **rather robust** since CAR 2 is the best choice even when changing scenarios!

We can analyze different possible scenarios of interest to understand in which cases the best original choice is no longer so.



# AHP Model: Case Study 1

## Remarks

It is important to note that the results should be interpreted as a **blueprint of preference** and alternatives based on the level of importance obtained for the different criteria taking into consideration our comparative judgments.

In other words, the AHP methodology allows us to determine which **alternative is the most consistent** with our criteria and the level of importance that we give them.



# How to use excel



# Pairwise comparison

## Example

All 1s are located on the diagonal.

In fact it is evident that in the comparison with itself (A with A) there is parity, that is, according to the Saaty scale, value 1.

In the comparison A with B, A was preferred to B by attributing the value 4; automatically when comparing B with A, B took  $\frac{1}{4}$ . And so on.

The condition ( $a_{ij} = 1 / a_{ji}$ ), known as the relationship of reciprocity, arises from the need to guarantee the symmetry of the judgments of importance. In fact, if, for example, it is believed that A is worth twice B ( $A = 2B$ ), it necessarily follows that B is worth half ( $1/2$ ) of A ( $B = \frac{1}{2} A$ ).

		j			
		A	B	C	D
i	A	1	4	3	7
	B	1/4	1	1	2
	C	1/3	1	1	2
	D	1/7	1/2	1/2	1

# Pairwise comparison

## Example

It has been scientifically demonstrated that in the case of perfect consistency of judgments, the matrix of comparisons that is formed has particular properties: it is **symmetrical, reciprocal and consistent**.

The matrix has a single eigenvalue, called the **maximum eigenvalue**, equal to the order  $n$  of the matrix and that the elements of the corresponding eigenvector.

# Pairwise comparison

## Example: weight definition

For each row is calculated what is called "weight", given by the multiplication of the values present on that row and the nth root is calculated on this product. The weights (Xi) derive from the calculation of the geometric mean: that is, from the product of the values of the row and this result placed under the nth root. For example, in the first line we make the product  $1 \times 4 \times 3 \times 7 = 84$  and we calculate the **fourth root** (since the number of factors is 4) of **84 which is 3.027**; proceed in the same way for all the other lines.

Weight Normalization Coefficient Eigenvalue

					N. elem.	Pesi Xi	Normalizzazione Pesi	Coeff.	Auto valore	
i	A	1	4	3	7	4	3,027	0,581	1,00	1,004
	B	1/4	1	1	2		0,841	0,162	0,278	1,050
	C	1/3	1	1	2		0,904	0,174	0,298	0,955
	D	1/7	1/2	1/2	1		0,435	0,083	0,144	1,002
Totale		1,726	6,50	5,50	12,00					
Yj										
Totale somma somma Xi							5,207	1,000		4,010



# Pairwise comparison

## Example: Normalization

The weights are then normalized; that is, given that in the example their sum is **5.207** and this must be brought to one, all weights are reduced in proportion (e.g.  $3.027 / 5.207 = 0.581$  and so on).

This leads to the values of the "weight normalization" column.

Weight Normalization Coefficient Eigenvalue

		A	B	C	D	N. elem.	Pesi $X_i$	Normalizzazione Pesi	Coeff.	Auto valore	
i	A	1	4	3	7	4	3,027	0,581	1,00	1,004	
	B	1/4	1	1	2		0,841	0,162	0,278	1,050	
	C	1/3	1	1	2		0,904	0,174	0,298	0,955	
	D	1/7	1/2	1/2	1		0,435	0,083	0,144	1,002	
Totale		1,726	6,50	5,50	12,00						
Yj											
Totale somma somma $X_i$							5,207	1,000			4,010

# Pairwise comparison

## Example: Coefficients

We then proceed, on the basis of these values, to calculate the coefficient so that whoever has obtained the highest weight will have the coefficient 1 and the others in proportion; then the formula  $P_i / P_{max}$  is applied.

Since the maximum value in our case is 0.581, element A will take the coefficient 1 and the others in proportion.

Weight Normalization Coeff. Eigenvalue

		A	B	C	D	N. elem.	Pesi $X_i$	Normalizzazione Pesi	Coeff.	Auto valore	
i	A	1	4	3	7	4	3,027	0,581	1,00	1,004	
	B	1/4	1	1	2		0,841	0,162	0,278	1,050	
	C	1/3	1	1	2		0,904	0,174	0,298	0,955	
	D	1/7	1/2	1/2	1		0,435	0,083	0,144	1,002	
Totale		1,726	6,50	5,50	12,00						
Yj											
Totale somma somma $X_i$							5,207	1,000			4,010

# Pairwise comparison

## Example: Consistency

First, the eigenvalue for each row must be calculated; to calculate the eigenvalue, the ratio between the product of  $X_i$  of each row multiplied by the total  $Y_j$  of the relative column and the sum of  $X_i$  is performed.

In formula.  $X_i * \text{total } Y_j / \text{total } X_i$ .

For A you will have.  $X_i (3.027) * \text{tot } Y_j (1.726) / \text{tot } X_i (5.207) = 1.004$  (eigenvalue)

Weight Normalization      Coeff.    Eigenvalue

		A	B	C	D	N. -- elem.	Pesi $X_i$	Normalizzazione Pesi	Coeff.	Auto valore	
i	A	1	4	3	7	4	3,027	0,581	1,00	1,004	
	B	1/4	1	1	2		0,841	0,162	0,278	1,050	
	C	1/3	1	1	2		0,904	0,174	0,298	0,955	
	D	1/7	1/2	1/2	1		0,435	0,083	0,144	1,002	
Totale $Y_j$		1,726	6,50	5,50	12,00						
Totale somma somma $X_i$							5,207	1,000			4,010

# Pairwise comparison

## Example: Consistency

**The sum of the self values and the main eigenvalue (4.010) also known as the maximum self value.**

If the evaluations were expressed in a logical way, without contradictions and uncertainties, the maximum eigenvalue would be **equal to 4** (order  $n$  of the matrix). But in practice this does not always happen, precisely because the evaluations are almost never perfectly consistent (coherent).

**Even in our example the evaluations are not perfectly consistent since the maximum eigenvalue is 4.010 instead of 4.00.**

However, the evaluations do not need to be perfectly consistent; precisely because we are in the field of subjectivity it is possible to allow a certain degree of inconsistency, as long as this is not exaggerated.

# Pairwise comparison

## Example: Consistency index calculation

The maximum eigenvalue provides a measure of consistency of the estimate of the vector of relative weights, as it is directly related to the degree of coherence of the decision maker.

At this point there remains the problem of establishing whether the weights obtainable from reflect the judgments of those who made the comparisons.

The AHP method defines the following consistency index (CI) which allows to measure the overall difference between these two sets of values:

$$CI = (\lambda_{max} - n) / (n-1)$$

# Pairwise comparison

## Example: Consistency index calculation

In the case of perfect consistency CI is equal to zero:

- when the matrix  $A$  is perfectly consistent, the main eigenvalue  $\lambda_{\max}$  is in fact equal to  $n$ ;
- as the inconsistency increases, the value of CI increases.

Taking up the previous example analyzed we have:

- principal eigenvalue 4.010
- $n = 4$ ,
- $n-1 = 3$

Thus, the calculation of the consistency index leads to the following value:  
 $(4.010-4) / 3 = 0.003$ .

# AHP Model: Case Study 2

## AHP Model:

## Buying a car

## Using Superdecision



By **Creative Decisions** Foundation

4922 Ellsworth Avenue  
Pittsburgh, PA 15213  
Phone: 412-621-6546  
Fax: 412-681-4510



The **Super Decisions** is decision support software **that implements the AHP and ANP.**

The **Analytic Hierarchical Process (AHP)** and the **Analytic Network Process (ANP)** make it possible to include **intangibles** in decision making.

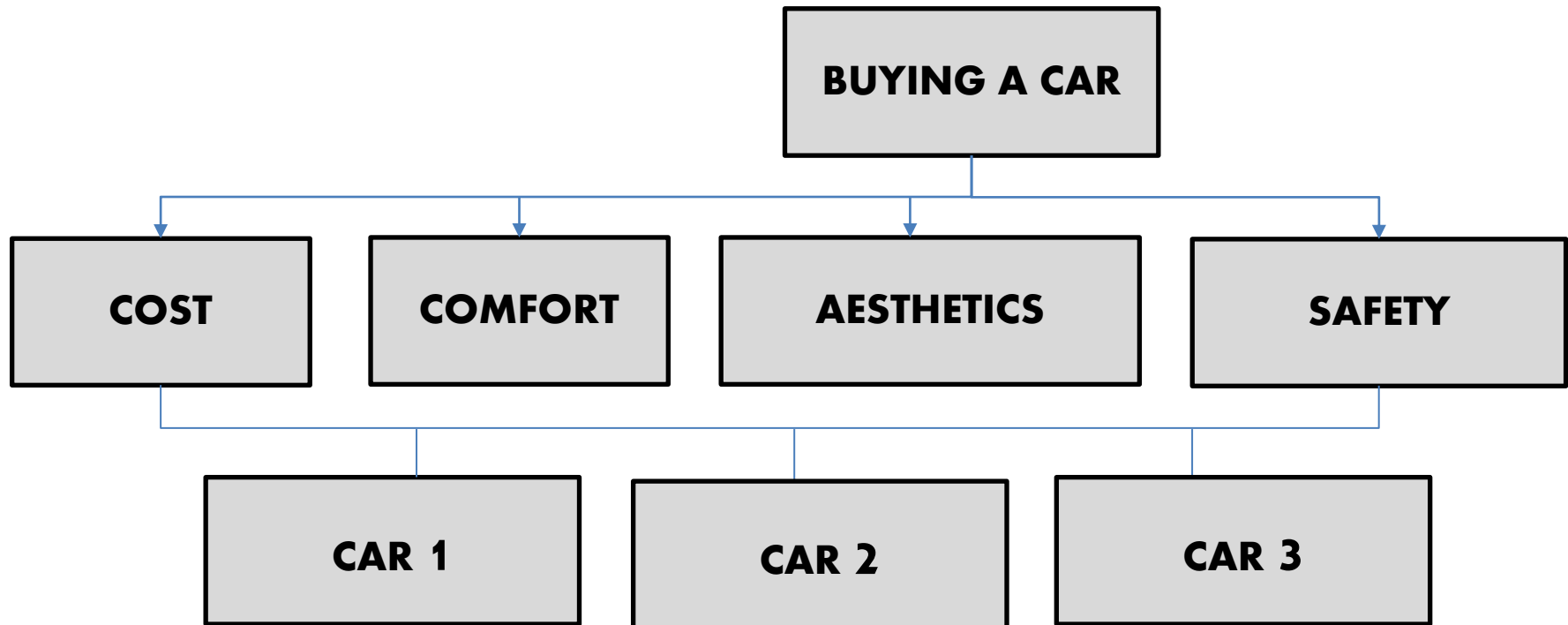
*AHP/ANP are the most powerful synthesis methodologies for combining judgment and data to effectively rank options and predict outcomes.*



# Case Study 2



# 1. Developing a Model



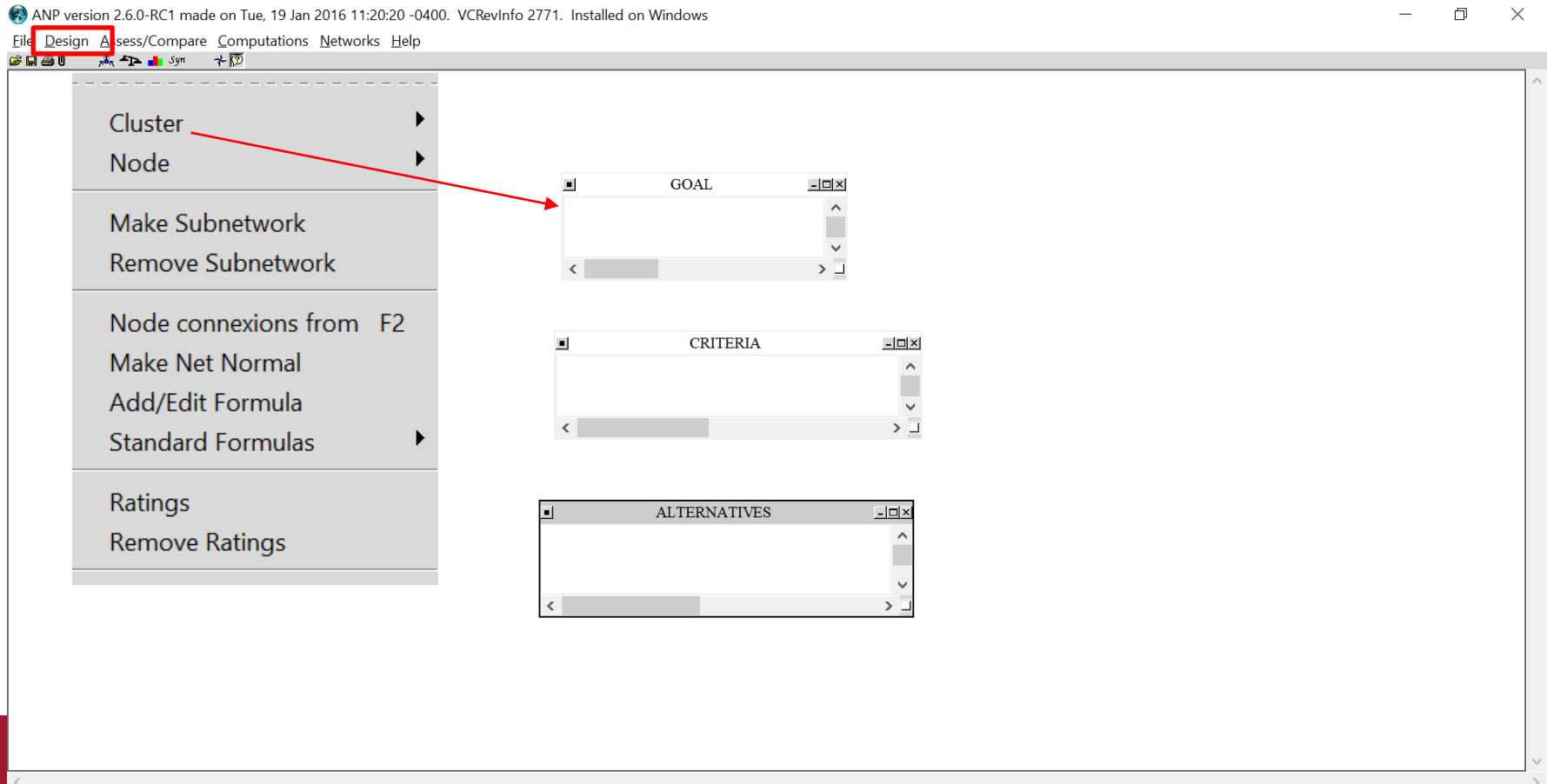
# 1. Developing a Model

	<b>AESTHETIC/ Prestige</b>	<b>COMFORT</b>	<b>COST</b>	<b>SAFETY</b>
	Excellent	Medium	22.500,00 EUR	Medium
	Good	Excellent	26.700,00 EUR	Excellent
	Medium	Good	28.200,00 EUR	Good

# 1. Developing a Model

Select Design>Cluster>New to create cluster

Then Enter cluster name



ANP version 2.6.0-RC1 made on Tue, 19 Jan 2016 11:20:20 -0400. VCRvInfo 2771. Installed on Windows

File Design Assess/Compare Computations Networks Help

Cluster  
Node  
Make Subnetwork  
Remove Subnetwork  
Node connexions from F2  
Make Net Normal  
Add/Edit Formula  
Standard Formulas  
Ratings  
Remove Ratings

GOAL

CRITERIA

ALTERNATIVES

The screenshot shows the ANP software interface. The 'Design' menu is highlighted with a red box, and a red arrow points from the 'Cluster' option to the 'GOAL' table. The 'GOAL' table is empty. The 'CRITERIA' table is also empty. The 'ALTERNATIVES' table is empty. The interface includes a menu bar with 'File', 'Design', 'Assess/Compare', 'Computations', 'Networks', and 'Help'. The title bar indicates the software version and installation details.

# 1. Developing a Model

Select Design>Node>New to create node  
Then Enter node name

ANP version 2.6.0-RC1 made on Tue, 19 Jan 2016 11:20:20 -0400. VCRevInfo 2771. Installed on Windows

File Design Assess/Compare Computations Networks Help

The screenshot shows the ANP software interface. On the left, a menu is open with 'Design' highlighted. Under 'Design', the 'Node' option is selected, and a red arrow points from it to a 'Buying a car' node in the 'GOAL' window. The 'GOAL' window contains a single node labeled 'Buying a car'. Below it, the 'CRITERIA' window contains four nodes: 'COST', 'COMFORT', 'SAFETY', and 'AESTHETICS'. At the bottom, the 'ALTERNATIVES' window contains three nodes: 'CAR 1', 'CAR 2', and 'CAR 3'. The software title bar indicates it is installed on Windows.

# 1. Developing a Model

Next step is to connect the node. Select from the menu option «do connexion»



ANP version 2.6.0-RC1 made on Tue, 19 Jan 2016 11:20:20 -0400. VCRevInfo 2771. Installed on Windows

File Design Assess/Compare Computations Networks Help

The screenshot displays the ANP software interface with three main windows:

- Node Sel...:** A dialog box titled "Node Sel..." showing "Node connexions from 'Buying a car'". The list includes AESTHETICS, Buying a car, CAR 1, CAR 2, CAR 3, COMFORT, COST, and SAFETY. The "COMFORT" and "SAFETY" nodes are highlighted in blue. "Okay" and "Cancel" buttons are at the bottom.
- GOAL:** A window titled "GOAL" containing a single node labeled "Buying a car".
- CRITERIA:** A window titled "CRITERIA" containing four nodes: "COST", "COMFORT", "SAFETY", and "AESTHETICS".
- ALTERNATIVES:** A window titled "ALTERNATIVES" containing three nodes: "CAR 1", "CAR 2", and "CAR 3".

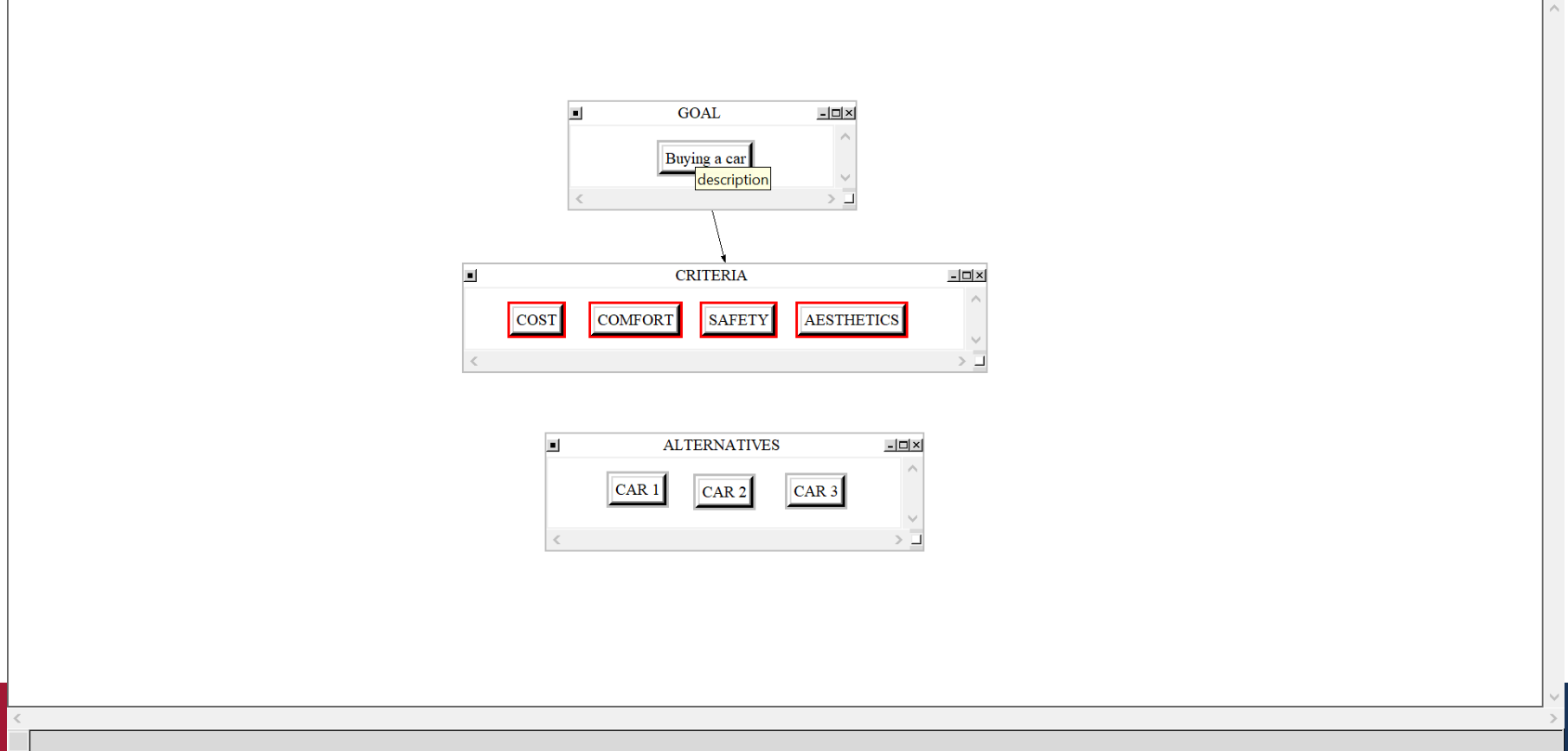
# 1. Developing a Model

Note that to see the connections made, it is required to press the button 

ANP version 2.6.0-RC1 made on Tue, 19 Jan 2016 11:20:20 -0400. VCRevInfo 2771. Installed on Windows

File Design Assess/Compare Computations Networks Help





# 1. Developing a Model

ANP version 2.6.0-RC1 made on Tue, 19 Jan 2016 11:20:20 -0400. VCRinfo 2771. Installed on Windows

File Design Assess/Compare Computations Networks Help

The screenshot displays the ANP software interface with a hierarchical model for 'Buying a car'. The model consists of three levels:

- GOAL:** A single node labeled 'Buying a car'.
- CRITERIA:** Four nodes labeled 'COST', 'COMFORT', 'SAFETY', and 'AESTHETICS'. A 'description' field is visible below these nodes.
- ALTERNATIVES:** Three nodes labeled 'CAR 1', 'CAR 2', and 'CAR 3', each enclosed in a red box.

Arrows indicate the flow of influence from the GOAL node to the CRITERIA nodes, and from the CRITERIA nodes to the ALTERNATIVES nodes. A dialog box titled 'Node connexions from "COST"' is open on the left, showing a list of nodes connected to the 'COST' criterion. The list includes 'AESTHETICS', 'Buying a car', 'CAR 1', 'CAR 2', 'CAR 3', 'COMFORT', 'COST', and 'SAFETY'. The 'CAR 1', 'CAR 2', and 'CAR 3' nodes are highlighted in blue. The dialog box has 'Okay' and 'Cancel' buttons at the bottom.

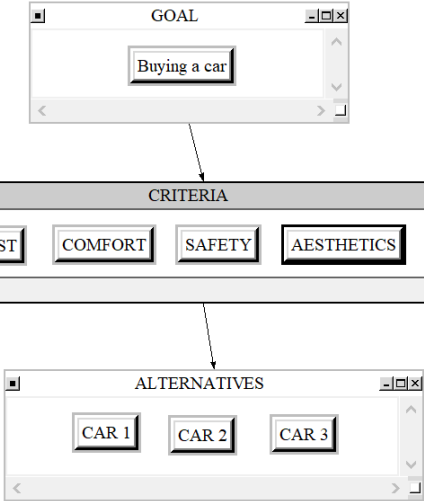


## 2. Deriving Priorities (weights) for Criteria

To compare the criteria press the "balance" button 

ANP version 2.6.0-RC1 made on Tue, 19 Jan 2016 11:20:20 -0400. VCRevInfo 2771. Installed on Windows

File Design Assess/Compare Computations Networks Help



The screenshot displays the ANP software interface with a hierarchical structure for a goal. The hierarchy is as follows:

- GOAL:** Buying a car
- CRITERIA:** COST, COMFORT, SAFETY, AESTHETICS
- ALTERNATIVES:** CAR 1, CAR 2, CAR 3

Arrows indicate the flow from the goal to the criteria, and from the criteria to the alternatives.

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## 2. Deriving Priorities (weights) for Criteria

Questionnaire mode for comparison of criteria with respect to the “buying a Car” node.

Comparisons for Super Decisions Main Window: Unnamed file 0

1. Choose
2. Node comparisons with respect to Buying a car

Node	Cluster	Graphical	Verbal	Matrix	Questionnaire	Direct
Choose Node		Comparisons wrt "Buying a car" node in "CRITERIA" cluster				
Buying a car		AESTHETICS is ?????? more important than COMFORT				
Cluster: GOAL						
Choose Cluster						
CRITERIA						
Restore						

1. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	COMFORT
2. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	COST
3. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY
4. COMFORT	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	COST
5. COMFORT	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY
6. COST	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY

## 2. Deriving Priorities (weights) for Criteria

Comparison of criteria with respect to the “buying a Car” node.

Comparisons for Super Decisions Main Window: Unnamed file 0

1. Choose	2. Node comparisons with respect to Buying a car	3. Results																																																																																																																																											
Node Cluster <b>Choose Node</b> <span style="float: right;">▶◀</span> Buying a car <hr/> Cluster: GOAL <hr/> <b>Choose Cluster</b> <span style="float: right;">▶◀</span> CRITERIA <hr/> Restore	Graphical Verbal Matrix Questionnaire Direct Comparisons wrt "Buying a car" node in "CRITERIA" cluster <b>SAFETY is very strongly more important than COMFORT</b> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>1. AESTHETICS</td> <td>&gt;=9.5</td> <td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td> <td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> <td>&gt;=9.5</td> <td>No comp.</td> <td>COMFORT</td> </tr> <tr> <td>2. AESTHETICS</td> <td>&gt;=9.5</td> <td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td> <td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> <td>&gt;=9.5</td> <td>No comp.</td> <td>COST</td> </tr> <tr> <td>3. AESTHETICS</td> <td>&gt;=9.5</td> <td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td> <td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> <td>&gt;=9.5</td> <td>No comp.</td> <td>SAFETY</td> </tr> <tr> <td>4. COMFORT</td> <td>&gt;=9.5</td> <td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td> <td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> <td>&gt;=9.5</td> <td>No comp.</td> <td>COST</td> </tr> <tr> <td>5. COMFORT</td> <td>&gt;=9.5</td> <td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td> <td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> <td>&gt;=9.5</td> <td>No comp.</td> <td>SAFETY</td> </tr> <tr> <td>6. COST</td> <td>&gt;=9.5</td> <td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td> <td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> <td>&gt;=9.5</td> <td>No comp.</td> <td>SAFETY</td> </tr> </table>	1. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	COMFORT	2. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	COST	3. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY	4. COMFORT	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	COST	5. COMFORT	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY	6. COST	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY	<div style="text-align: right;"> <input type="button" value="Normal"/> <input type="button" value="Hybrid"/> </div> <p style="text-align: center;">Inconsistency: 0.08704</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 60%;">AESTHETICS</td> <td style="width: 30%;"><div style="width: 33.96%; background-color: #007bff; height: 10px;"></div></td> <td style="width: 10%; text-align: right;">0.03960</td> </tr> <tr> <td>COMFORT</td> <td><div style="width: 10.61%; background-color: #007bff; height: 10px;"></div></td> <td style="text-align: right;">0.10612</td> </tr> <tr> <td>COST</td> <td><div style="width: 40.20%; background-color: #007bff; height: 10px;"></div></td> <td style="text-align: right;">0.40207</td> </tr> <tr> <td>SAFETY</td> <td><div style="width: 45.22%; background-color: #007bff; height: 10px;"></div></td> <td style="text-align: right;">0.45221</td> </tr> </table> <div style="text-align: center; margin-top: 10px;"> <input type="checkbox"/> Completed <span style="float: right;">▶◀</span>  <input type="checkbox"/> Comparison <span style="float: right;">▶◀</span>  <input type="button" value="Copy to clipboard"/> </div>	AESTHETICS	<div style="width: 33.96%; background-color: #007bff; height: 10px;"></div>	0.03960	COMFORT	<div style="width: 10.61%; background-color: #007bff; height: 10px;"></div>	0.10612	COST	<div style="width: 40.20%; background-color: #007bff; height: 10px;"></div>	0.40207	SAFETY	<div style="width: 45.22%; background-color: #007bff; height: 10px;"></div>	0.45221
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6. COST	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY																																																																																																																								
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## 2. Deriving Priorities (weights) for Criteria

Graphic comparison of criteria with respect to the “buying a Car” node.

Comparisons for Super Decisions Main Window: Unnamed file 0

### 1. Choose

Node Cluster

**Choose Node** ◀ ▶

Buying a car ▾

Cluster: GOAL

**Choose Cluster** ◀ ▶

CRITERIA ▾

Restore

### 2. Node comparisons with respect to Buying a car

Graphical Verbal Matrix Questionnaire Direct

COMFORT

SAFETY

Help for graphical mode.

1. Click and drag the circle to adjust the judgment.
2. Click the "No comparison" button to set the judgment to zero.
3. Use Tab/Enter to move between judgments or use the navigation buttons on the right.
4. Type a number to vote.
5. Hit - or / to invert.

No comparison

### 3. Results

Normal ▾ Hybrid ▾

Inconsistency: 0.08704

AESTHETICS		0.03960
COMFORT		0.10612
COST		0.40207
SAFETY		0.45221

Completed ◀ ▶

Comparison ◀ ▶

Copy to clipboard

## 2. Deriving Priorities (weights) for Criteria

Verbal comparison of criteria with respect to the “buying a Car” node.

Comparisons for Super Decisions Main Window: Unnamed file 0

**1. Choose**

Node Cluster

Choose Node

Buying a car

Cluster: GOAL

Choose Cluster

CRITERIA

Restore

**2. Node comparisons with respect to Buying a car**

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "Buying a car" node in "CRITERIA" cluster

**SAFETY is very strongly more important than COMFORT**

Extreme

Very strong

Strongly

Moderately

Equal

Help for verbal mode.

1. Click and drag to adjust the judgment.
2. Click the "Invert comparison" button to invert.
3. Use Tab/Enter to move between judgments or use the navigation buttons on the right.
4. Click below equals to give a zero judgment.
5. Type a number to vote.
6. Hit - or / to invert.

**3. Results**

Normal Hybrid

Inconsistency: 0.08704

AESTHETICS	<div style="width: 3.5%;"></div>	0.03960
COMFORT	<div style="width: 10.6%;"></div>	0.10612
COST	<div style="width: 40.2%;"></div>	0.40207
SAFETY	<div style="width: 45.2%;"></div>	0.45221

Completed Comparison

Copy to clipboard

## 2. Deriving Priorities (weights) for Criteria Consistency

- Click on the **Inconsistency** button (at top left corner of matrix)
- Choose Basic **Inconsistency Report**; the first cell
- Left-click on either the Current or **Best Value** cell to return to the matrix and input a new value . You can use the suggested value to improve the final CI.

Comparisons for Super Decisions Main Window: Unnamed file 0

1. Choose	2. Node comparisons with respect to Buying a car	3. Results																								
Node Cluster <b>Choose Node</b> Buying a car Cluster: GOAL <b>Choose Cluster</b> CRITERIA Restore	Graphical Verbal Matrix Questionnaire Direct Comparisons wrt "Buying a car" node in "CRITERIA" cluster <b>COMFORT is 5 times more important than AESTHETICS</b> Inconsistency COMFORT COST SAFETY <table border="1"> <tr> <td>AESTHETIC</td> <td>↑ 5</td> <td>↑ 8</td> <td>↑ 8</td> </tr> <tr> <td>COMFORT</td> <td></td> <td>↑ 5</td> <td>↑ 7</td> </tr> <tr> <td>COST</td> <td></td> <td></td> <td>← 1</td> </tr> </table> Copy to clipboard	AESTHETIC	↑ 5	↑ 8	↑ 8	COMFORT		↑ 5	↑ 7	COST			← 1	Normal  Hybrid Inconsistency: 0.08704 <table border="1"> <tr> <td>AESTHETICS</td> <td></td> <td>0.03960</td> </tr> <tr> <td>COMFORT</td> <td></td> <td>0.10612</td> </tr> <tr> <td>COST</td> <td></td> <td>0.40207</td> </tr> <tr> <td>SAFETY</td> <td></td> <td>0.45221</td> </tr> </table> Completed Comparison Copy to clipboard	AESTHETICS		0.03960	COMFORT		0.10612	COST		0.40207	SAFETY		0.45221
AESTHETIC	↑ 5	↑ 8	↑ 8																							
COMFORT		↑ 5	↑ 7																							
COST			← 1																							
AESTHETICS		0.03960																								
COMFORT		0.10612																								
COST		0.40207																								
SAFETY		0.45221																								

# 3. Deriving Local Priorities (preferences) for the Alternatives

## AESTHETIC Criteria

Comparisons for Super Decisions Main Window: Unnamed file 0

**1. Choose**

Node Cluster

Choose Node

AESTHETICS

Cluster: CRITERIA

Choose Cluster

ALTERNATIVES

Restore

**2. Node comparisons with respect to AESTHETICS**

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "AESTHETICS" node in "ALTERNATIVES" cluster

CAR 2 is moderately more important than CAR 3

1. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 2
2. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3
3. CAR 2	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3

**3. Results**

Normal Hybrid

Inconsistency: 0.06239

CAR 1	<div style="width: 73.064%;"></div>	0.73064
CAR 2	<div style="width: 18.839%;"></div>	0.18839
CAR 3	<div style="width: 8.096%;"></div>	0.08096

Completed Comparison

Copy to clipboard



# 3. Deriving Local Priorities (preferences) for the Alternatives

## COMFORT Criteria

Comparisons for Super Decisions Main Window: Unnamed file 0

1. Choose	2. Node comparisons with respect to COMFORT	3. Results									
Node Cluster Choose Node <b>COMFORT</b> Cluster: CRITERIA Choose Cluster <b>ALTERNATIVES</b> Restore	Graphical Verbal Matrix Questionnaire Direct Comparisons wrt "COMFORT" node in "ALTERNATIVES" cluster <b>CAR 2 is moderately more important than CAR 3</b> 1. CAR 1 >=9.5 9 8 7 6 5 4 3 2 2 3 4 5 6 7 8 9 >=9.5 No comp. CAR 2 2. CAR 1 >=9.5 9 8 7 6 5 4 3 2 2 3 4 5 6 7 8 9 >=9.5 No comp. CAR 3 3. CAR 2 >=9.5 9 8 7 6 5 4 3 2 2 3 4 5 6 7 8 9 >=9.5 No comp. CAR 3	Normal Hybrid Inconsistency: 0.03703 <table border="1"> <tr> <td>CAR 1</td> <td><div style="width: 10%; background-color: blue;"></div></td> <td>0.10473</td> </tr> <tr> <td>CAR 2</td> <td><div style="width: 63.699%; background-color: blue;"></div></td> <td>0.63699</td> </tr> <tr> <td>CAR 3</td> <td><div style="width: 25.282%; background-color: blue;"></div></td> <td>0.25282</td> </tr> </table> Completed Comparison Copy to clipboard	CAR 1	<div style="width: 10%; background-color: blue;"></div>	0.10473	CAR 2	<div style="width: 63.699%; background-color: blue;"></div>	0.63699	CAR 3	<div style="width: 25.282%; background-color: blue;"></div>	0.25282
CAR 1	<div style="width: 10%; background-color: blue;"></div>	0.10473									
CAR 2	<div style="width: 63.699%; background-color: blue;"></div>	0.63699									
CAR 3	<div style="width: 25.282%; background-color: blue;"></div>	0.25282									





# 3. Deriving Local Priorities (preferences) for the Alternatives

## COST Criteria

Comparisons for Super Decisions Main Window: Unnamed file 0

**1. Choose**

Node Cluster

Choose Node

COST

Cluster: CRITERIA

Choose Cluster

ALTERNATIVES

Restore

**2. Node comparisons with respect to COST**

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "COST" node in "ALTERNATIVES" cluster

CAR 2 is moderately more important than CAR 3

1. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 2
2. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3
3. CAR 2	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3

**3. Results**

Normal Hybrid

Inconsistency: 0.09040

CAR 1	<div style="width: 71.724%;"></div>	0.71724
CAR 2	<div style="width: 19.469%;"></div>	0.19469
CAR 3	<div style="width: 8.808%;"></div>	0.08808

Completed Comparison

Copy to clipboard



# 3. Deriving Local Priorities (preferences) for the Alternatives

## SAFETY Criteria

Comparisons for Super Decisions Main Window: Unnamed file 0

**1. Choose**

Node Cluster

Choose Node

SAFETY

Cluster: CRITERIA

Choose Cluster

ALTERNATIVES

Restore

**2. Node comparisons with respect to SAFETY**

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "SAFETY" node in "ALTERNATIVES" cluster

CAR 2 is moderately more important than CAR 3

1. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 2
2. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3
3. CAR 2	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3

**3. Results**

Normal Hybrid

Inconsistency: 0.05156

CAR 1	<div style="width: 15%; background-color: blue;"></div>	0.15706
CAR 2	<div style="width: 75%; background-color: blue;"></div>	0.59363
CAR 3	<div style="width: 15%; background-color: blue;"></div>	0.24931

Completed Comparison

Copy to clipboard



# 4. Deriving Overall Priorities (Model Synthesis)

## Final RESULTS

**CAR2:  
42,1%**

New synthesis for: Super Decisions Main Window: Unnamed file 0

Here are the overall synthesized priorities for the alternative CAR2 you synthesized from the network Super Decisions Main Window: Unnamed file 0

Name	Graphic	Ideals	Normals	Raw
CAR 1		0.947042	0.399477	0.199723
CAR 2		1.000000	0.421784	0.210892
CAR 3		0.423842	0.178770	0.089385

The Raw column gives the priorities from the limiting supermatrix (which also appear in the Limiting column above),

Normals column shows the final preferences, in standardized form.

Ideals column is obtained by dividing each value in the Normals column by highest value of said column

# 5. Sensitivity Analysis

## Scenario 1: all criteria same weight

Comparisons for Super Decisions Main Window: AHP\_3 CARS.sdmod

1. Choose
2. Node comparisons with respect to Buying a car

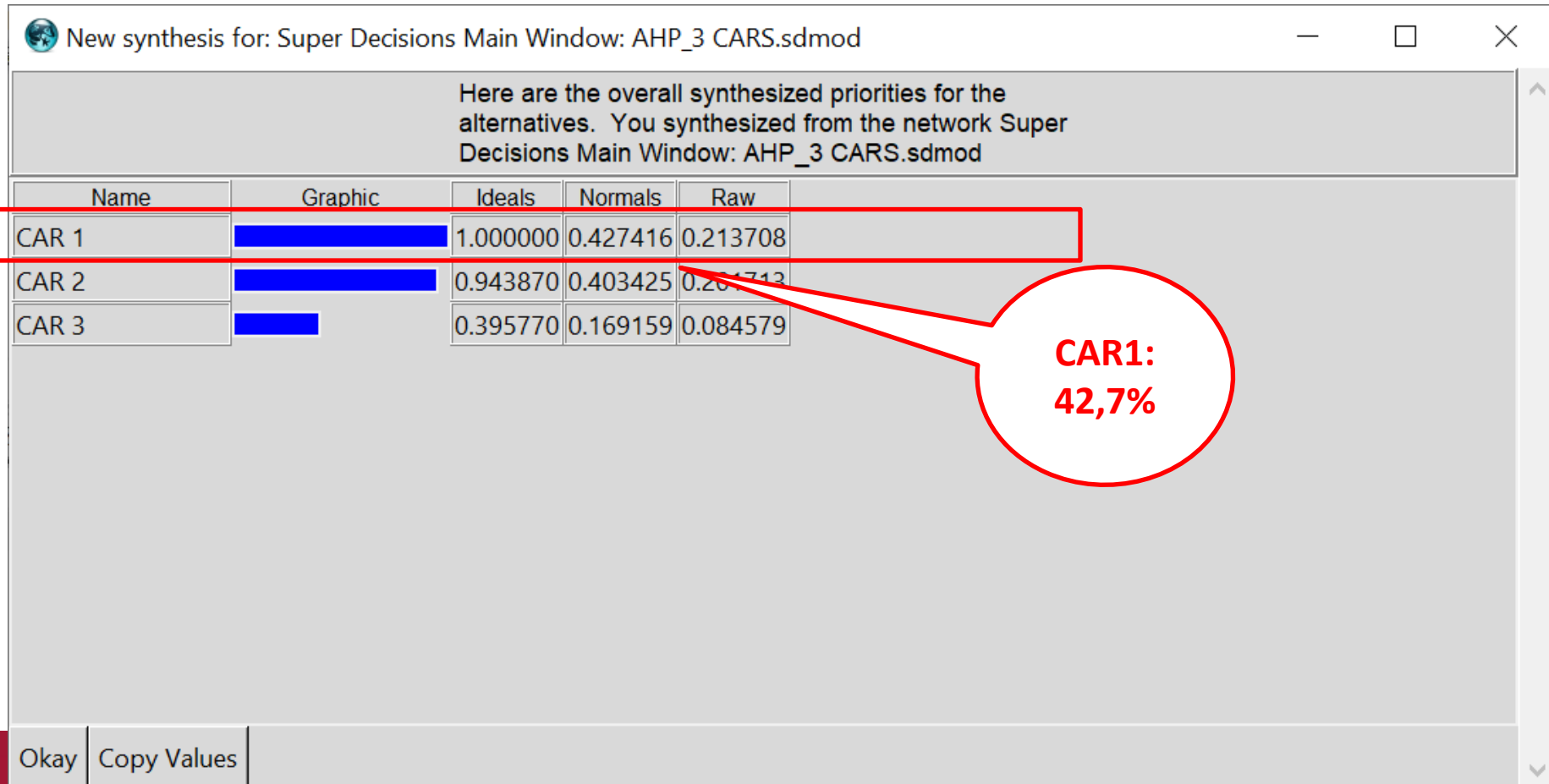
Node	Cluster	Graphical	Verbal	Matrix	Questionnaire	Direct								
<b>Choose Node</b> <span style="float: right;">◀▶</span> Buying a car <span style="float: right;">▾</span>		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;">AESTHETICS</td> <td style="width: 50%;">0.25</td> </tr> <tr> <td>COMFORT</td> <td>0.25</td> </tr> <tr> <td>COST</td> <td>0.25</td> </tr> <tr> <td>SAFETY</td> <td>0.25</td> </tr> </table>					AESTHETICS	0.25	COMFORT	0.25	COST	0.25	SAFETY	0.25
AESTHETICS	0.25													
COMFORT	0.25													
COST	0.25													
SAFETY	0.25													
<b>Choose Cluster</b> <span style="float: right;">◀▶</span> CRITERIA <span style="float: right;">▾</span>														
Restore		<input type="checkbox"/> Invert												

This is the direct data input area.  
Type in new direct data here, and/or  
Click the invert box invert priorities for this  
direct data.

NOTE: Any changes made in direct data take  
effect immediately and overwrite  
pre-existing data inputted in the  
other modes.

# 5. Sensitivity Analysis

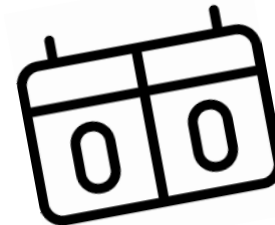
## Scenario 1: all criteria same weight



## 6. Final Decision

If all criteria have the same weight the best choice becomes CAR 1, but just a little. It means that model is quite robust.

We can analyze different possible scenarios of interest to understand in which cases the best original choice is no longer so.

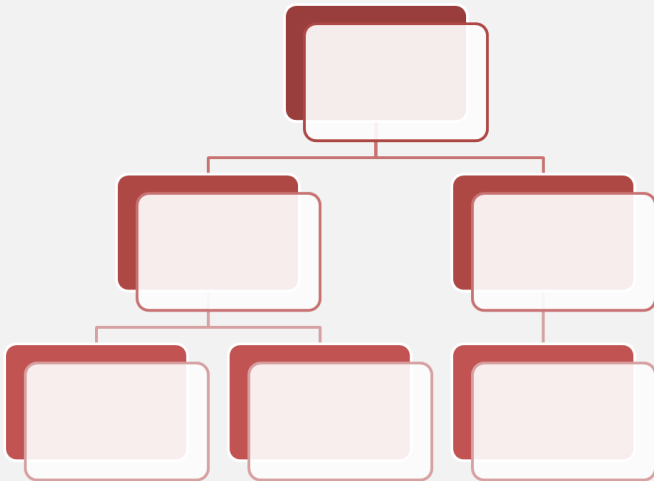


# Understanding the Analytic Hierarchy Process

## Process

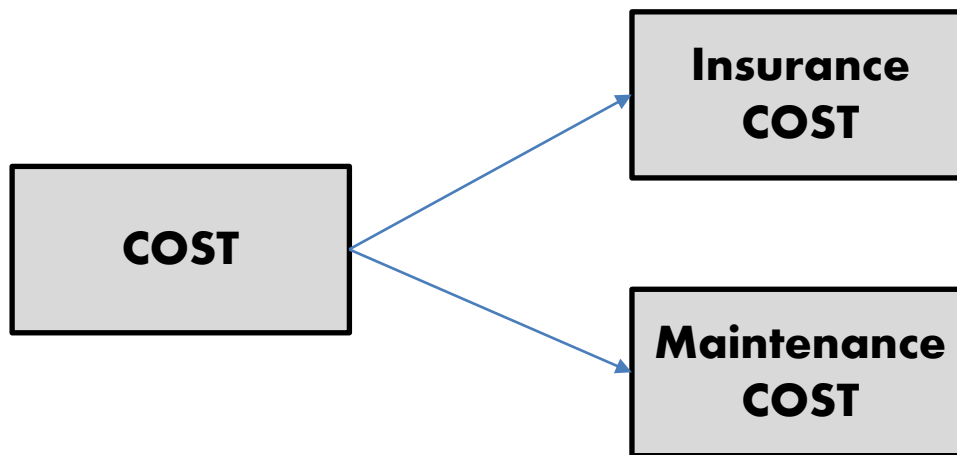
## Intermediate

AHP model with sub-criteria



# AHP model with sub-criteria

## How to modify the model?





# AHP model with sub-criteria

## How to modify the model?

Super Decisions Main Window: AHP\_3 CARS with subcriteria.sdmod

File Design Assess/Compare Computations Networks Help

The screenshot displays the Super Decisions software interface for an AHP model titled "AHP\_3 CARS with subcriteria.sdmod". The model structure is as follows:

- GOAL:** Buying a car
- CRITERIA:** COST, COMFORT, SAFETY, AESTHETICS
- ALTERNATIVES:** CAR 1, CAR 2, CAR 3
- COST sub-criteria:** insurance cost, maintenance cost

Red annotations highlight the sub-criteria for the COST criterion, indicating a focus on how to modify the model.

# How to modify the model?

	AESTHETIC/ Prestige	COMFORT	COST		SAFETY
			Insurance	Mainten.	
	Excellent	Medium	22.500,00 EUR		Medium
			500,00	300,00	
	Good	Excellent	26.700,00 EUR		Excellent
			550,00	350,00	
	Medium	Good	28.200,00 EUR		Good
			500,00	400,00	

# How to modify the model?

## 1 new matrix for insurance cost

Comparisons for Super Decisions Main Window: AHP\_3 CARS with subcriteria.sdmod

**1. Choose**

Node Cluster

**Choose Node** <>

insurance cost

Cluster: COST sub-criter~

**Choose Cluster** <>

ALTERNATIVES

Restore

**2. Node comparisons with respect to insurance cost**

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "insurance cost" node in "ALTERNATIVES" cluster

**CAR 3 is moderately to strongly more important than CAR 2**

1. CAR 1	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 2	
2. CAR 1	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3
3. CAR 2	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3	

**3. Results**

Normal Hybrid

Inconsistency: 0.00000

CAR 1	0.44444
CAR 2	0.11111
CAR 3	0.44444

Completed Comparison

Copy to clipboard

# How to modify the model?

## 1 new matrix for maintenance cost

Comparisons for Super Decisions Main Window: AHP\_3 CARS with subcriteria.sdmod

### 1. Choose

Node Cluster

**Choose Node**

maintenance co~

Cluster: COST sub-criter~

**Choose Cluster**

ALTERNATIVES

Restore

### 2. Node comparisons with respect to maintenance cost

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "maintenance cost" node in "ALTERNATIVES" cluster

**CAR 1 is moderately to strongly more important than CAR 3**

1. CAR 1	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 2
2. CAR 1	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3
3. CAR 2	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	CAR 3

### 3. Results

Normal Hybrid

Inconsistency: 0.07069

CAR 1	0.61441
CAR 2	0.26837
CAR 3	0.11722

Completed Comparison

Copy to clipboard

# How to modify the model?

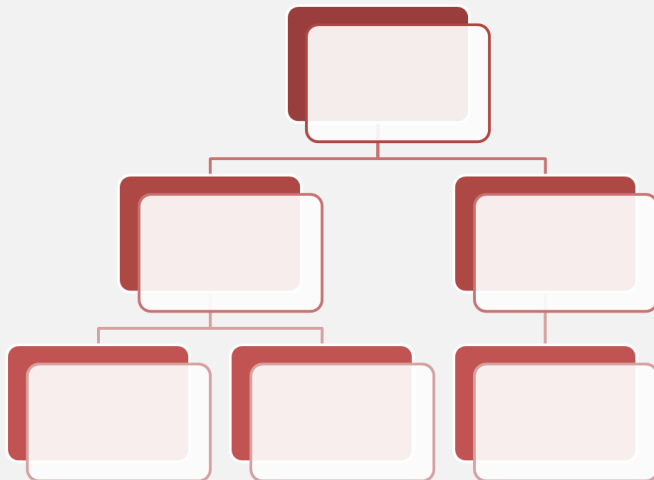
In **summary**, the procedure to insert sub-criteria to a specific criterion (e.g., cost) consist of:

- Create **sub-criteria cluster** for the specific criterion;
- Create the **sub-criteria nodes**;
- **Connect the criterion node to the alternatives**;
- Compare pairwise the sub-criteria to obtain the relative sub-criteria weights;
- Compare the alternatives with respect to these sub-criteria.

# Understanding the Analytic Hierarchy Process

## Intermediate

### AHP Absolute Model (or Rating Model)



# Absolute model or called Rating Model

Sometimes there is a **large number of alternatives** to consider. For example, in the case of **evaluating employees** for promotion, it would not be unusual to have to evaluate 30 or more.

This would make a **pairwise comparison very difficult** due to the excessive number of required comparisons.

A similar situation occurs when you are constantly adding or removing alternatives.

**A pairwise comparison requires a repetitive comparative process.**

**This process is tedious!**

# Absolute model or called Rating Model

To resolve these two situations **ratings model or absolute models** have been developed by Prof. Saaty.

In this approach, criteria priority is still derived by pairwise comparison.

A **rating scale** is specifically developed for each of the criteria and the alternatives are evaluated, independently of each other, using these scales.



# Absolute model or called Rating Model

In an Absolute model a **hierarchy is developed** in the usual way down to the level of criteria or sub-criteria.

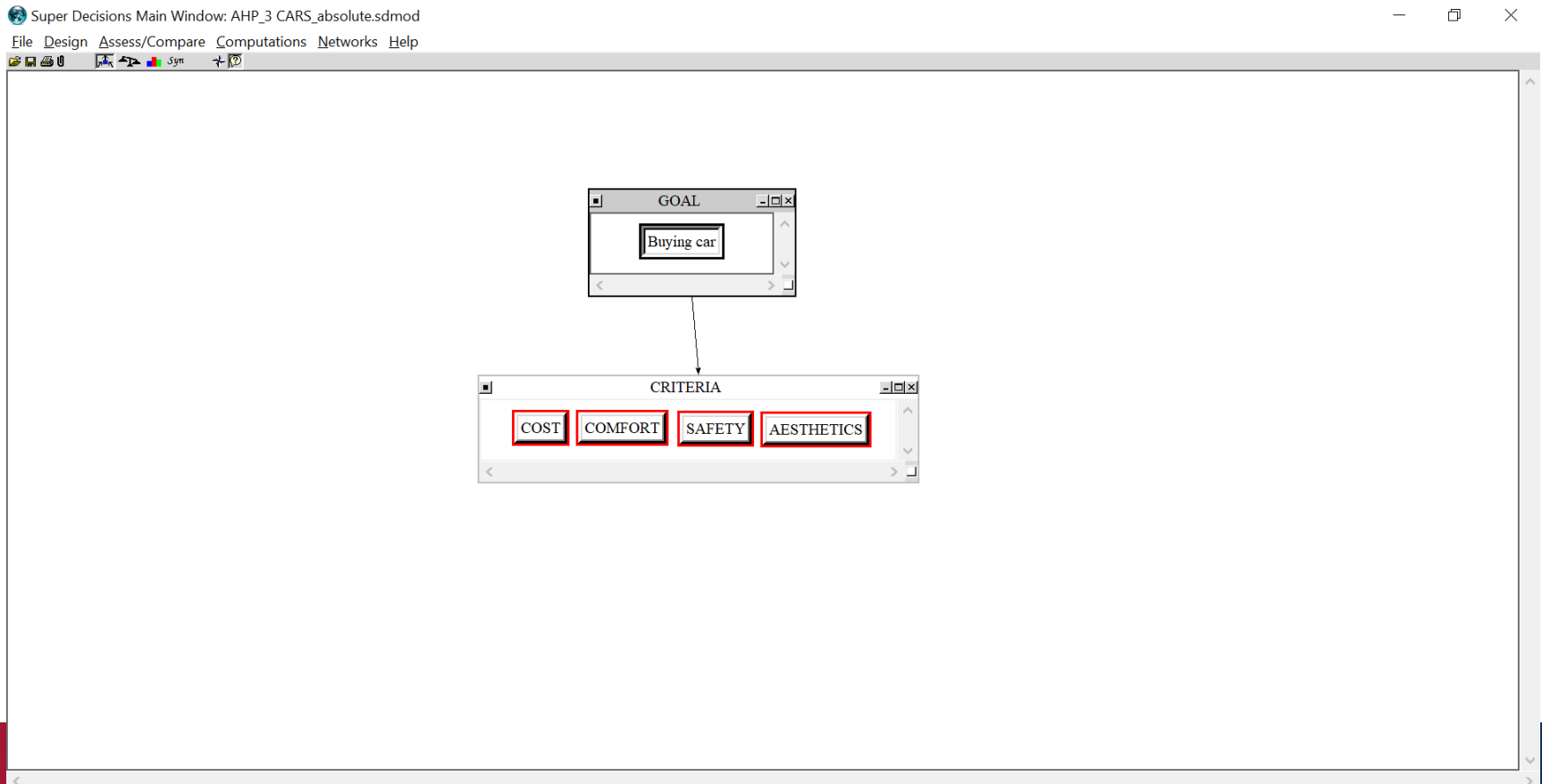
The criteria or sub-criteria are further subdivided into a **level for intensities.**

**Let us see how it works!**



# How to build the model?

In Absolute model the hierarchy is formed only by the GOAL and the CRITERIA



# How to build the model?

Let us assume that the judgments for the criteria comparisons have been entered.

Comparisons for Super Decisions Main Window: AHP\_3 CARS\_absolute.sdmod

### 1. Choose

Node Cluster

Choose Node ◀▶

Buying car

Cluster: GOAL

Choose Cluster ◀▶

CRITERIA

Restore

### 2. Node comparisons with respect to Buying car

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "Buying car" node in "CRITERIA" cluster

**COST is equally as important as SAFETY**

1. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	COMFORT	
2. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	COST	
3. AESTHETICS	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY	
4. COMFORT	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	COST	
5. COMFORT	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY	
6. COST	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	SAFETY

### 3. Results

Normal ▾ Hybrid ▾

Inconsistency: 0.08704

AESTHETICS	0.03960
COMFORT	0.10612
COST	0.40207
SAFETY	0.45221

Completed ▶ Comparison ?

Copy to clipboard

# How to build the model?

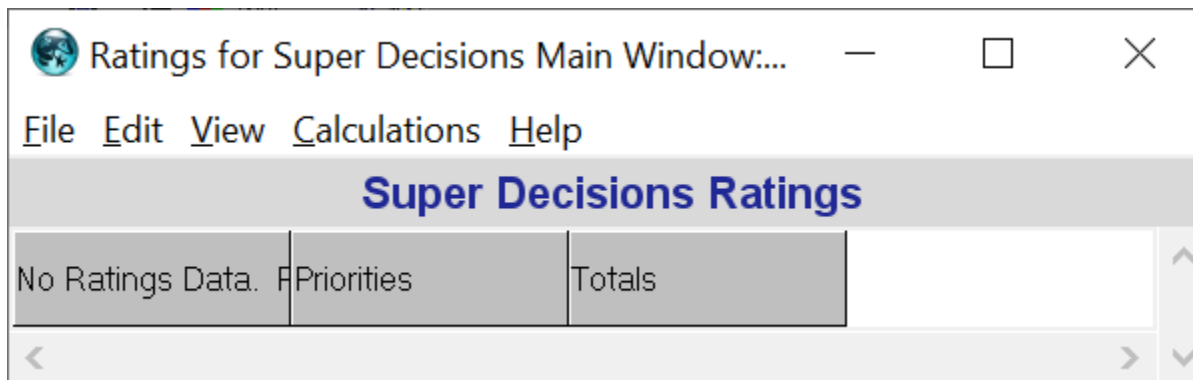
In ratings models, the **evaluation of the alternatives** (car 1, car2 and car3) is **NOT** done via pairwise comparison but by **rating them** with respect to each criterion separately.

**Thus**, instead of comparing the alternative pairwise we need to **create a ratings scale** for each criterion and the alternatives will be scored against each criterion accordingly.

# How to build the model?

First, we need to create a **ratings model**.

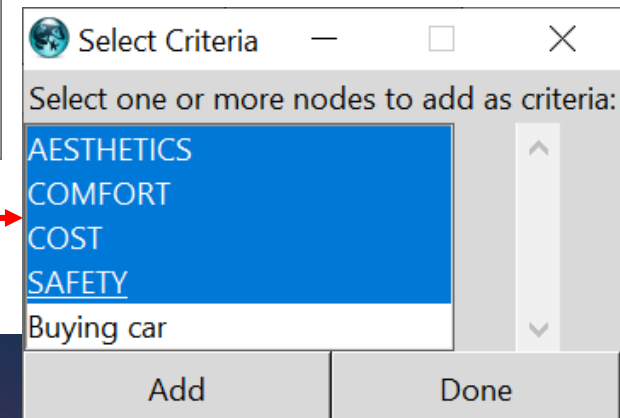
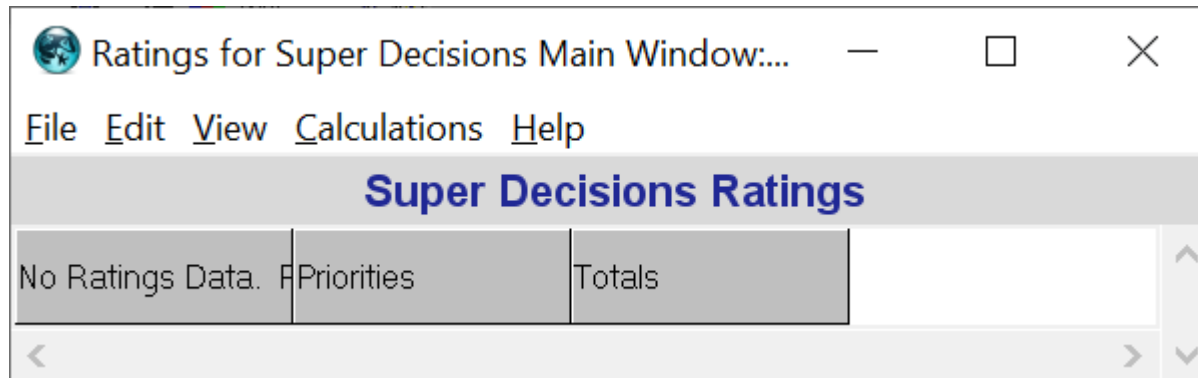
To create a ratings model using **Super Decisions**, select from the main windows, the option **Assess/Compare** followed by ratings and we will get the screen in Figure.



# How to build the model?

On this screen select the *edit* option followed by *criteria*. Press the *New* button and a new window named *Select Criteria* will appear.

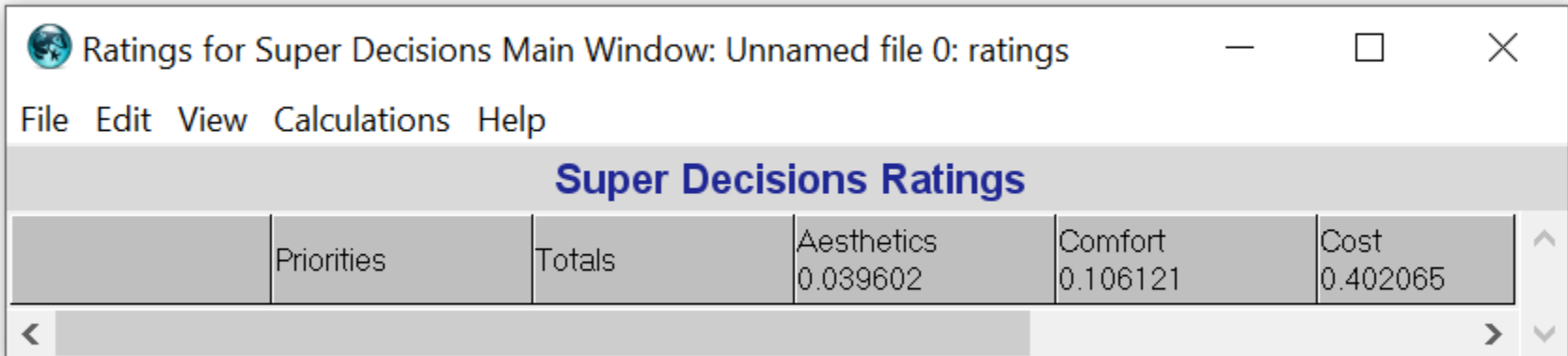
In this new window, select cost, comfort, aesthetics, and safety criteria.



*Select Criteria*

# How to build the model?

Click on the button **Add** followed by **Done** to build the header of the ratings model matrix shown in Figure.



The screenshot shows a software window titled "Ratings for Super Decisions Main Window: Unnamed file 0: ratings". The window contains a menu bar with "File", "Edit", "View", "Calculations", and "Help". Below the menu bar is a header "Super Decisions Ratings". The main content is a table with the following data:

	Priorities	Totals	Aesthetics	Comfort	Cost
			0.039602	0.106121	0.402065

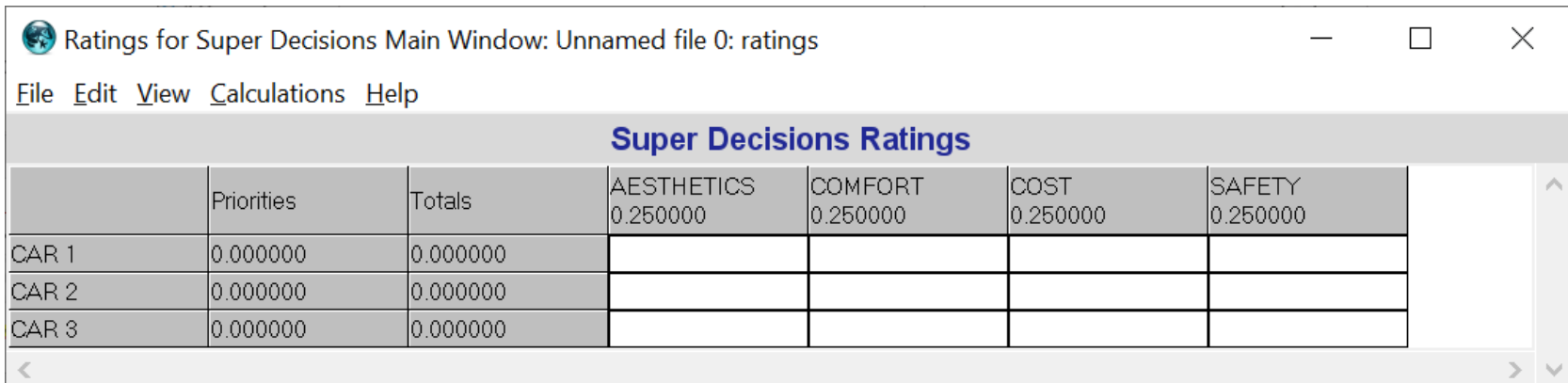
Note that the weight of the criteria are automatically assigned by the software based on the comparison of criteria previously made.

# How to build the model?

Now we need to add the **alternatives**.

For this we select ***Edit/Alternatives/New*** and proceed to enter the name of the first alternative.

This process is repeated as many times as necessary.



The screenshot shows a software window titled "Ratings for Super Decisions Main Window: Unnamed file 0: ratings". The window contains a menu bar with "File", "Edit", "View", "Calculations", and "Help". Below the menu bar is a table titled "Super Decisions Ratings". The table has seven columns: "Priorities", "Totals", "AESTHETICS", "COMFORT", "COST", and "SAFETY". Each of the last four columns has a value of 0.250000. The table has three rows labeled "CAR 1", "CAR 2", and "CAR 3". All values in the "Priorities" and "Totals" columns are 0.000000.

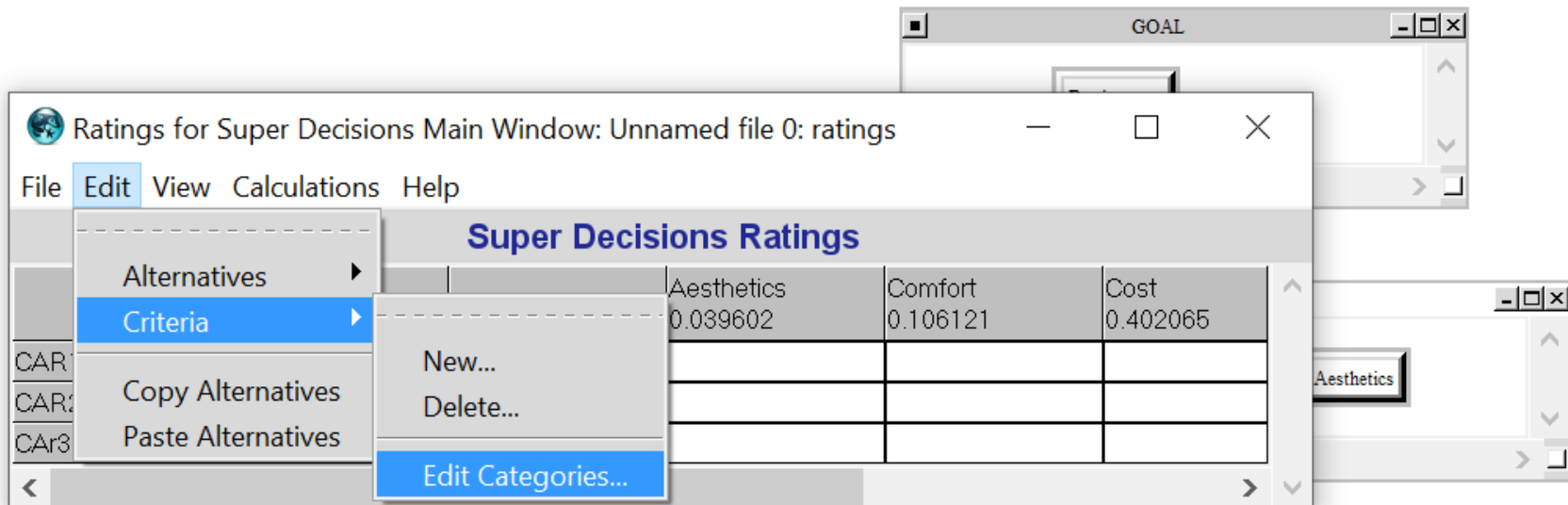
	Priorities	Totals	AESTHETICS 0.250000	COMFORT 0.250000	COST 0.250000	SAFETY 0.250000
CAR 1	0.000000	0.000000				
CAR 2	0.000000	0.000000				
CAR 3	0.000000	0.000000				



# How to build the model?

Now you must create a **rating scale** for each criterion.

For this select ***Edit/Criteria/Edit Categories***



The screenshot displays the 'Ratings for Super Decisions' main window. The 'Edit' menu is open, and the 'Criteria' option is selected, which has opened a sub-menu with 'Edit Categories...' highlighted. In the background, a 'GOAL' window is visible, and a table of criteria ratings is shown.

	Aesthetics	Comfort	Cost
	0.039602	0.106121	0.402065

# How to build the model?

and select *Comfort... Aesthetic....Cost...Safety...*and click **OK**.

The screenshot displays the 'Ratings for Super Decisions' main window and a 'Criteria' dialog box. The main window shows a table with columns for Priorities, Totals, Aesthetics, Comfort, and Cost. The 'Criteria' dialog box is open, showing a list of criteria to be edited, with 'Aesthetics' selected.

	Priorities	Totals	Aesthetics 0.039602	Comfort 0.106121	Cost 0.402065
CAR1	0.000000	0.000000			
CAR2	0.000000	0.000000			
CAR3	0.000000	0.000000			

Criteria dialog box content:

Please select a criteria whose categories you want to edit:

- Aesthetics**
- Comfort
- Cost
- Safety

Buttons: OK, Cancel

# How to build the model?

Add the ratings/comparisons

The screenshot shows a software interface for building a decision model. It includes several windows:

- GOAL**: A window containing the goal "Buying a car".
- CRITERIA**: A window containing criteria "Cost", "Comfort", "Safety", and "Aesthetics".
- Category Dialog**: A dialog box titled "Categor..." with a text input field containing "Above Average" and "OK" and "Cancel" buttons.
- Context Menu**: A menu with options: "Move Up", "Move Down", "New", "Rename", "Remove", and "Comparisons".
- Ratings for Super Decisions Main Window**: A window displaying a table of ratings.

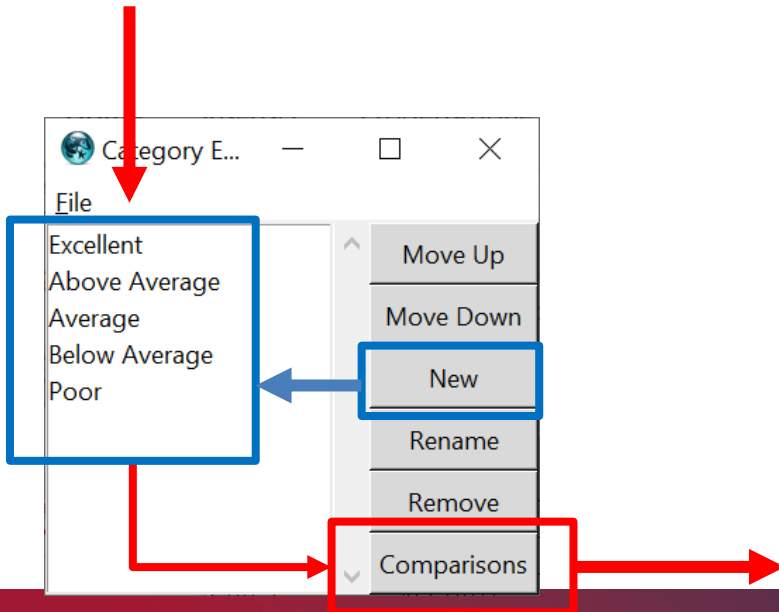
The "Ratings for Super Decisions Main Window" displays the following table:

Super Decisions Ratings					
	Priorities	Totals	Aesthetics	Comfort	Cost
CAR1	0.000000	0.000000	0.039602	0.106121	0.402065
CAR2	0.000000	0.000000			
CAR3	0.000000	0.000000			

# How to build the model?

Now we need to give a score to each category. To do that, press the button **Comparisons** in the **Category Editor** window.

By entering these weights in the window we conclude with the weighting of the categories.



Comparisons wrt "Criteria Compares for COMFO..."

File Computations Misc Help

Graphical Verbal Matrix Questionnaire Direct

Comparisons wrt "Criteria Compares for COMFORT" in Categories.

1.	Excellent	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Above Average	
2.	Excellent	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Average	
3.	Excellent	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Below Average	
4.	Excellent	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Poor	
5.	Above Average	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Average	
6.	Above Average	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Below Average	
7.	Above Average	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Poor	
8.	Average	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Below Average	
9.	Average	>=9.5	9	8	7	6	5	4	3	2	2	3	4	5	6	7	8	9	>=9.5	No comp.	Poor	
10.	Below Average	>=9.5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	>=9.5	No comp.	Poor

# How to build the model?

Now you must evaluate alternatives using the ratings model

GOAL

Ratings for Super Decisions Main Window: Unnamed file 0: ratings

File Edit View Calculations Help

### Super Decisions Ratings

	Priorities	Totals	Aesthetics 0.039602	Comfort 0.106121	Cost 0.402065	Safety 0.452212
CAR1	0.000000	0.000000				
CAR2	0.000000	0.000000				
CAR3	0.000000	0.000000				

- Excellent
- Above Average
- Average
- Below Average
- Poor
- Numeric Value
- No Value
- Cancel

# How to build the model?

It is necessary repeat the process of all criterion and to evaluate alternatives using the ratings model.

Figure shows the final matrix.

Ratings for Super Decisions Main Window: AHP\_3 CARS\_absolute.sdmod: ratings

File Edit View Calculations Help

**Super Decisions Ratings**

	Priorities	Totals	AESTHETICS 0.039602	COMFORT 0.106121	COST 0.402065	SAFETY 0.452212
CAR 1	0.322476	0.593840	Excellent	Average	Excellent	Average
CAR 2	0.439653	0.809621	Above Average	Excellent	Above Average	Excellent
CAR 3	0.237871	0.438040	Average	Above Average	Average	Above Average

# How to build the model?

Rating scale values for comfort.

Ratings for Super Decisions Main Window: AHP\_3 CARS\_absolute.sdmod: ratings

File Edit View Calculations Help

### Super Decisions Ratings

	Priorities	Totals	AESTHETICS 0.039602	COMFORT 0.106121	COST 0.402065	SAFETY 0.452212
CAR 1	0			Average	Excellent	Average
CAR 2	0			Excellent	Above Average	Excellent
CAR 3	0			Above Average	Average	Above Average

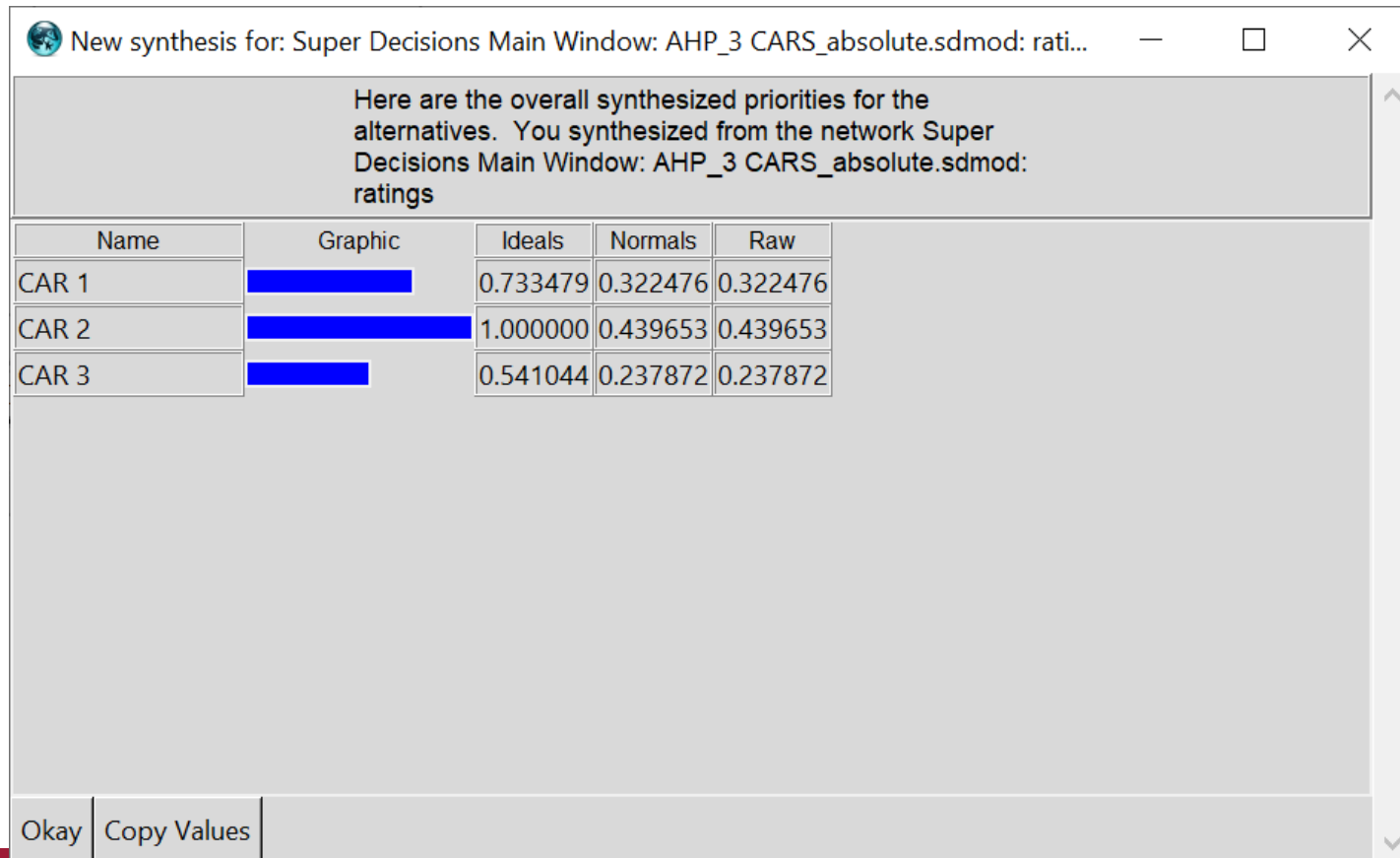
Column Priorities f... ✕

Priorities for columns of ratings system.

AESTHETICS	<div style="width: 3.96%;"></div>	0.039602
COMFORT	<div style="width: 10.61%;"></div>	0.106121
COST	<div style="width: 40.21%;"></div>	0.402065
SAFETY	<div style="width: 45.22%;"></div>	0.452212

# How to build the model?

## Final Results





# The AHP is the Method of Prioritization

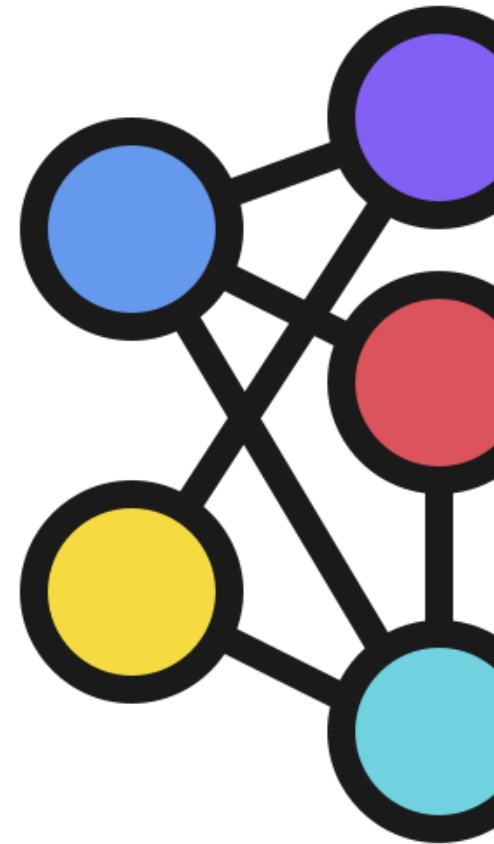
1. AHP captures priorities from paired comparison judgments of the elements of the decision with respect to each of their parent criteria.
2. Paired comparison judgments can be arranged in a matrix.
3. Priorities are derived from the matrix as its principal eigenvector, which defines a ratio scale.
4. Thus, the eigenvector is an intrinsic concept of a correct prioritization process. It also allows for the measurement of inconsistency in judgment.
5. Priorities derived this way satisfy the property of a ratio scale just like pounds and yards do.



## WHY IS AHP EASY TO USE?

AHP does not take for granted the measurements on scales but asks that **scale values be interpreted** according to the objectives of the problem.

It relies on elaborate hierarchic structures to represent decision problems and **is able to handle** problems of risk, conflict, and prediction.



# WHY THE AHP IS POWERFUL IN CORPORATE PLANNING

1. Breaks down criteria into manage-able components.
2. Leads a group into making a specific decision for consensus or tradeoff.
3. Provides opportunity to examine disagreements and stimulate discussion and opinion.



## Practical tips

### What is the best kind of decision problem for AHP?

AHP can be used in a wide number of decision-making problems. It is traditionally used in selection, prioritization, and forecasting.

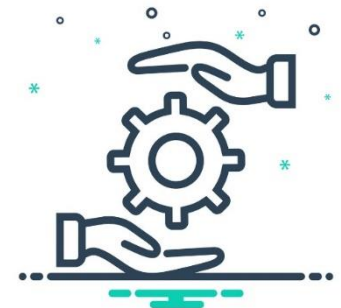
### How many criteria are needed for the AHP hierarchy?

Saaty's scale intensity, as well as AHP as a whole, is based on the findings from cognitive science that suggest that a person's working memory capacity is in the order of  $7 \pm 2$ ;

that is between 5 and 9 elements.

This suggests that 5-9 criteria should be the ideal.

If you have more than that you may consider grouping some of them into an overall criterion and creating sub-criteria.



## Practical tips

### How many levels should an AHP hierarchy have?

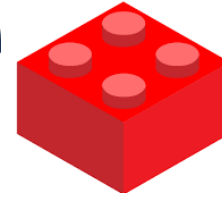
The same rationale from the previous question can be applied here. While, there is not a limit to the number of levels in a hierarchy you may want keep it within the  $7 \pm 2$ , if possible.

### What is the potential limitations of using AHP?

Based on our experience in the use of AHP, the following limitations have been found:

- a) The comparison process may be long if the decision is complex;
- b) The comparison judgment may be unreliable if the participants are not fully engaged in the process;
- c) The decision-making transparency may be counter-productive for managers who are interested in manipulating the results.

# Lessons Learned

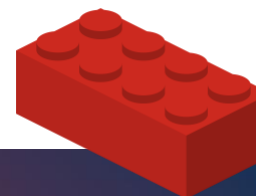
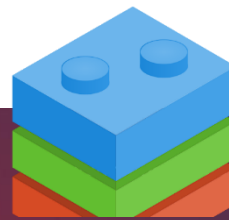
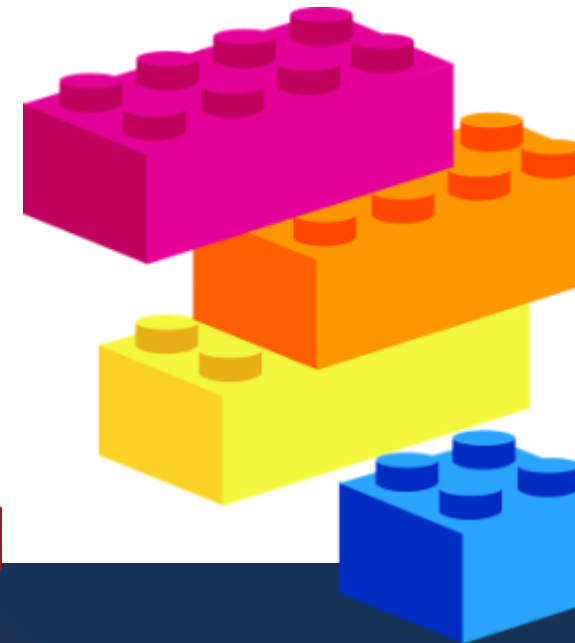


## AHP can be used in different decision settings.

AHP is rather similar to working with LEGO blocks.

The number of different blocks is rather limited and relatively easy to grasp; however, the **possibilities** of what can be done with them is **rather unlimited**.

Similarly, using a relative limited set of concepts: hierarchical modeling, pairwise comparison, consistency, sensitivity; it is possible to **address a very broad number of decision-making problems and situations**.



# Scientific & Technical References

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