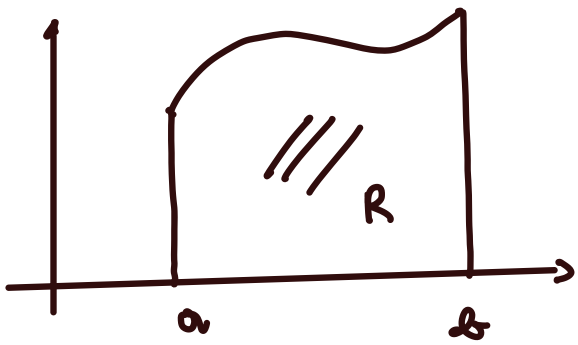
Lezioni del 15 e 19 Dicembre 2023

$f \geq 0$, $f : [a, b] \longrightarrow \mathbb{R}$

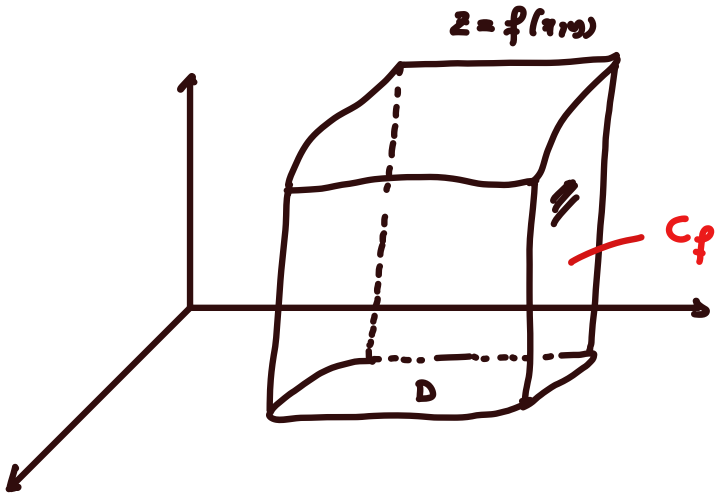
$$\int_a^b f(x)\,dx = \text{Area}(R)$$



$f(x,y)$ $\quad f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ continua

$f \geq 0$ $\quad \iint_D f(x,y)\,dx\,dy$??

$$C_f = \{ (x,y,z) \in \mathbb{R}^3 : (x,y) \in D, \quad 0 \leq z \leq f(x,y) \}$$

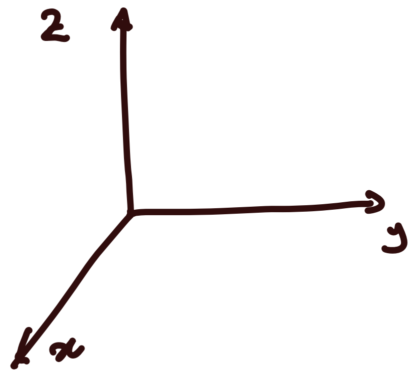


CILINDROIDE RELATIVO AD $f(x,y)$

$$\iint_D f(x,y)\,dx\,dy = \mathrm{VOL}(C_f)$$

Integrali tripli

$$\iiint_E f(x,y,z)\,dx\,dy\,dz$$



Def. (Dominio normale rispetto a xy)

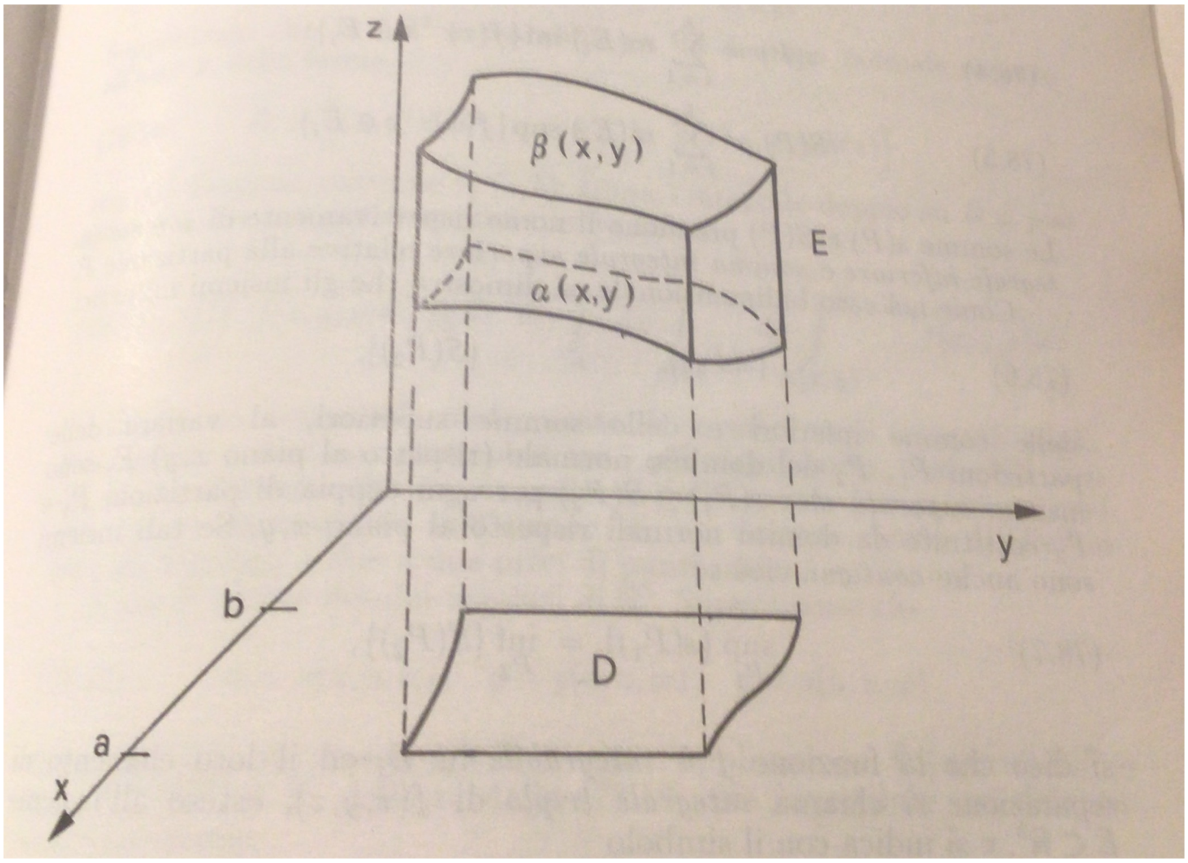
$E \subseteq \mathbb{R}^3$

$$E = \{(x,y,z) \in \mathbb{R}^3 : (x,y) \in D,\ \alpha(x,y) \leq z \leq \beta(x,y)\}$$

$D \subseteq \mathbb{R}^2$ dominio regolare

$\alpha(x,y) \leq \beta(x,y) \quad \forall (x,y) \in D$

funzioni continue



$$\mathrm{Vol}(E) = \iint_D \left[\beta(x,y) - \alpha(x,y)\right]$$

$E \subseteq \mathbb{R}^3$ normale rispetto al piano yz : $D \subseteq \mathbb{R}^2_{yz}$

$$E = \{ (x,y,z) \in \mathbb{R}^3 : (y,z) \in D \,, \; \alpha(y,z) \le x \le \beta(y,z) \}$$

$$\mathrm{VOL}(E) = \iint_D \left[\beta(y,z) - \alpha(y,z)\right] dy\,dz$$

$E \subseteq \mathbb{R}^3$ normale rispetto al piano xz:

$$E = \{ (x,y,z) \in \mathbb{R}^3 : (x,z) \in D, \ \alpha(x,z) \leq y \leq \beta(x,z) \}$$

$$\mathrm{VOL}(E) = \iint_D [\, \beta(x,z) - \alpha(x,z) \,] \, dx dz$$

$f(x,y,z)$, $\quad f: E \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$ limitata

E dominio normale

$$\iiint_E f(x,y,z) \, dx dy dz$$

f continua su $E \implies f$ è integrabile su E.

Formule di Riduzione

$f(x,y,z)$, $f: E \longrightarrow \mathbb{R}$ continua

Allora: su

$$E = \{ (x,y,z) \in \mathbb{R}^3 : (x,y) \in D, \ \alpha(x,y) \leq z \leq \beta(x,y) \}$$

$$\iiint_E f(x,y,z)\,dx\,dy\,dz = \iint_D dx\,dy \int_{\alpha(x,y)}^{\beta(x,y)} f(x,y,z)\,dz$$

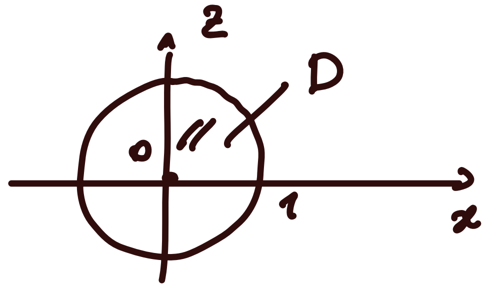
$$E = \{ (x,y,z) \in \mathbb{R}^3 : (y,z) \in D, \; \alpha(y,z) \le x \le \beta(y,z) \}$$

$$\iiint_E f(x,y,z)\,dx\,dy\,dz = \iint_D dy\,dz \int_{\alpha(y,z)}^{\beta(y,z)} f(x,y,z)\,dx$$

$$E = \{ (x,y,z) \in \mathbb{R}^3 : (x,z) \in D, \; \alpha(x,z) \le y \le \beta(x,z) \}$$

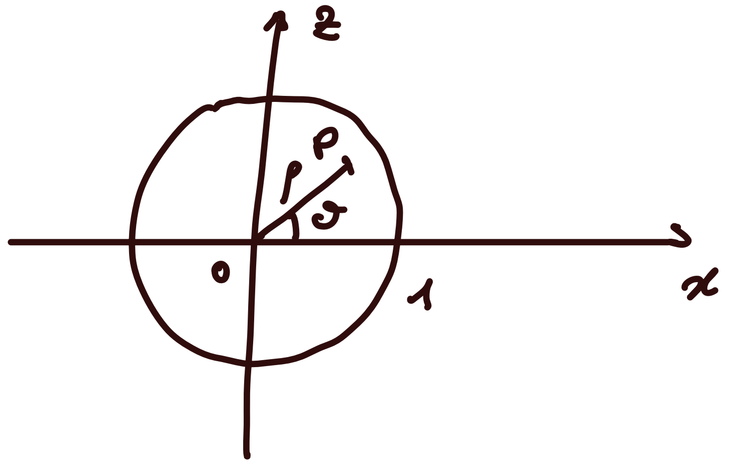
$$\iiint_E f(x,y,z)\,dx\,dy\,dz = \iint_D dx\,dz \int_{\alpha(x,z)}^{\beta(x,z)} f(x,y,z)\,dy$$

1) $\displaystyle\iiint_E x\,dx\,dy\,dz$ $\qquad E = \{ (x,y,z) : z^2 + x^2 \le 1, \; \underbrace{0}_{\alpha} \le y \le \underbrace{1-x-z}_{\beta} \}$



normale rispetto ad xz

$$= \iint_D dx\,dz \int_0^{1-x-z} x\,dy = \iint_D x(1-x-z)\,dx\,dz$$



$$\begin{cases} x = \rho\cos\theta \\ z = \rho\sin\theta \end{cases}$$

$$T = \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le \rho \le 1 \end{cases}$$

$$= \iint_T \rho\cos\theta\,(1-\rho\cos\theta-\rho\sin\theta)\,\rho\,d\rho\,d\theta$$

$$= \iint_T \rho^2\cos\theta\,d\rho\,d\theta - \iint_T \rho^3\cos^2\theta\,d\theta - \iint_T \rho^3\sin\theta\cos\theta\,d\rho\,d\theta$$

$$= \int_0^1 d\rho \int_0^{2\pi} \rho^2 \overbrace{\cos\theta}^{=0}\,d\rho\,d\theta$$

$$- \int_0^1 d\rho \int_0^{2\pi} \rho^3\cos^2\theta\,d\theta \qquad \int \cos^2\theta\,d\theta$$

$$- \int_0^1 d\rho \underbrace{\int_0^{2\pi}}_{=0} \rho^3 \underbrace{(\sin\theta\cos\theta)}\,d\theta$$

$$\int \sin\theta\cos\theta\,d\theta = \frac{\sin^2\theta}{2} + c$$

$$\int \cos^2\theta \, d\theta = \int \cos\theta \cos\theta \, d\theta = \text{(P. PARTI)} =$$

$$= \sin\theta\cos\theta + \int \sin^2\theta \, d\theta =$$

$$= \sin\theta\cos\theta + \theta - \int \cos^2\theta \, d\theta$$

$$\int \cos^2\theta \, d\theta = \frac{\sin\theta\cos\theta + \theta}{2} + c$$

$$\iiint \dots = -\frac{1}{4}\left(\frac{\sin\theta\cos\theta + \theta}{2}\right)_0^{2\pi} = -\frac{\pi}{4}$$

2) Calcolare il volume di

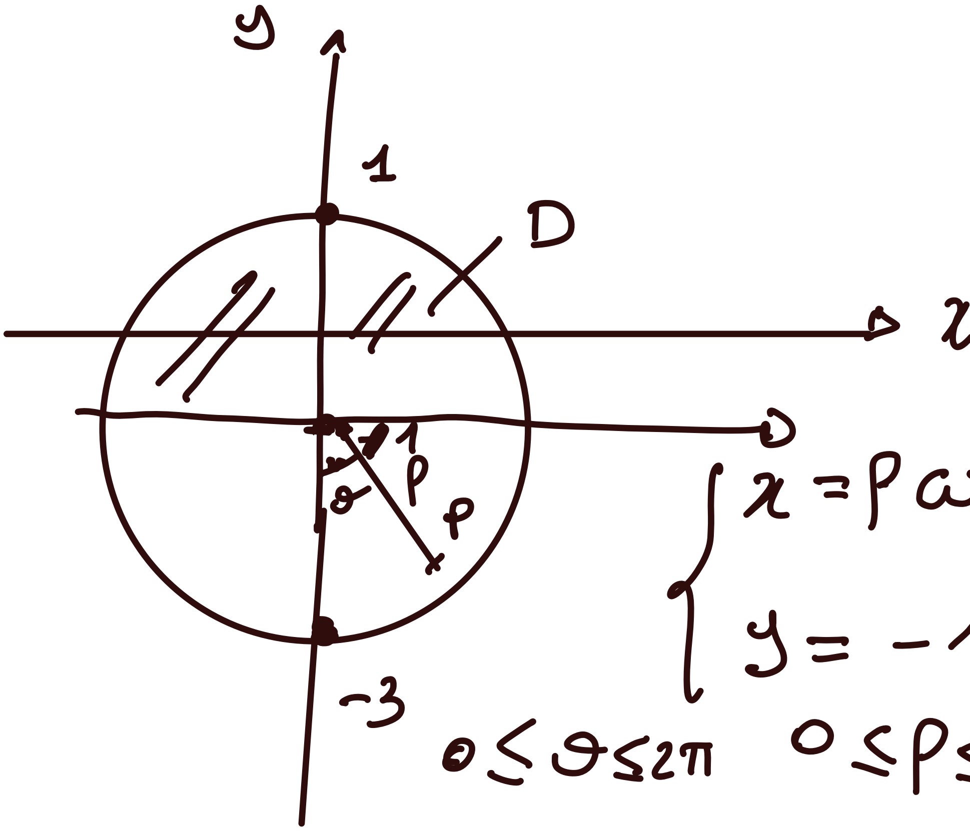
$$E = \{ (x,y,z) \in \mathbb{R}^3 : \underbrace{x^2+y^2}_{\alpha(x,y)} \leq z \leq \underbrace{3-2y}_{\beta(x,y)} \}$$

$$x^2+y^2 \leq 3-2y$$

$$\Longleftrightarrow \quad x^2+y^2+2y \leq 3$$

$$C = (0, -1)$$

$$R = 2$$



$$\begin{cases} x = \rho \cos\theta \\ y = -1 + \rho \sin\theta \end{cases}$$

$0 \leq \theta \leq 2\pi \quad 0 \leq \rho \leq 2$

$$VOL(E) = \iint_D \left[3 - 2y - x^2 - y^2 \right] dxdy$$

$$Vol(E) = \iint_D \left[\beta(x,y) - \alpha(x,y) \right]$$

$$\begin{cases} x = \rho \cos\theta \\ y = -1 + \rho \sin\theta \end{cases} \qquad T = \begin{Bmatrix} 0 \leq \rho \leq 2 \\ 0 \leq \theta \leq 2\pi \end{Bmatrix}$$

$$= \iint_T \rho\left[3 - 2(-1+\rho\sin\theta) - \rho^2\cos^2\theta - (-1+\rho\sin\theta)^2\right] d\rho\, d\theta$$

$$= \iint_T \rho\left[3 + 2 - \cancel{2\rho\sin\theta} - \rho^2\cos^2\theta - 1 - \rho^2\sin^2\theta + \cancel{2\rho\sin\theta}\right] d\rho\, d\theta$$

$$= \iint_T \rho\left[4 - \rho^2\right] d\rho\, d\theta =$$

$$= \int_0^2 d\rho \int_0^{2\pi} \rho(4-\rho^2)\, d\theta =$$

$$= 2\pi \int_0^2 \rho(4-\rho^2)\, d\rho$$

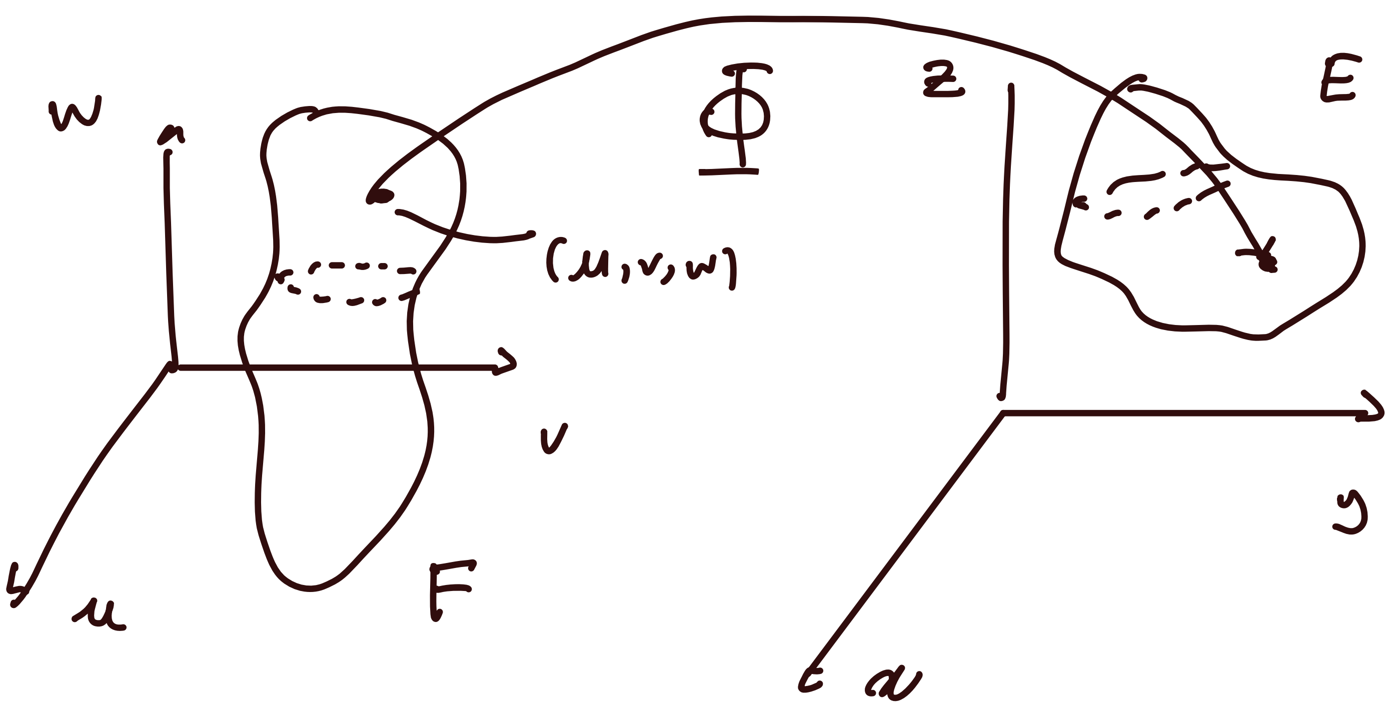
$$= -\pi \int_0^2 (-2\rho)(4-\rho^2)\, d\rho$$

$$= -\pi \left[\frac{(4-\rho^2)^2}{2} \right]_{\rho=0}^{\rho=2}$$

$$= -\frac{\pi}{2} \left[-16 \right] = 8\pi$$

Def $E \subseteq \mathbb{R}^3$ dominio regolare

$$\Phi = \Phi(u, v, w)$$



(x, y, z)

$$\Phi \equiv \begin{cases} x = x(u, v, w) \\ y = y(u, v, w) \\ z = z(u, v, w) \end{cases}$$

$$\det \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix}$$

DETERMINANTE JACOBIANO

$$\iiint_E f(x, y, z)\, dx\, dy\, dz =$$

$$\iiint_F f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \det \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du\, dv\, dw$$

TRASFORMAZIONE ALLE COORDINATE

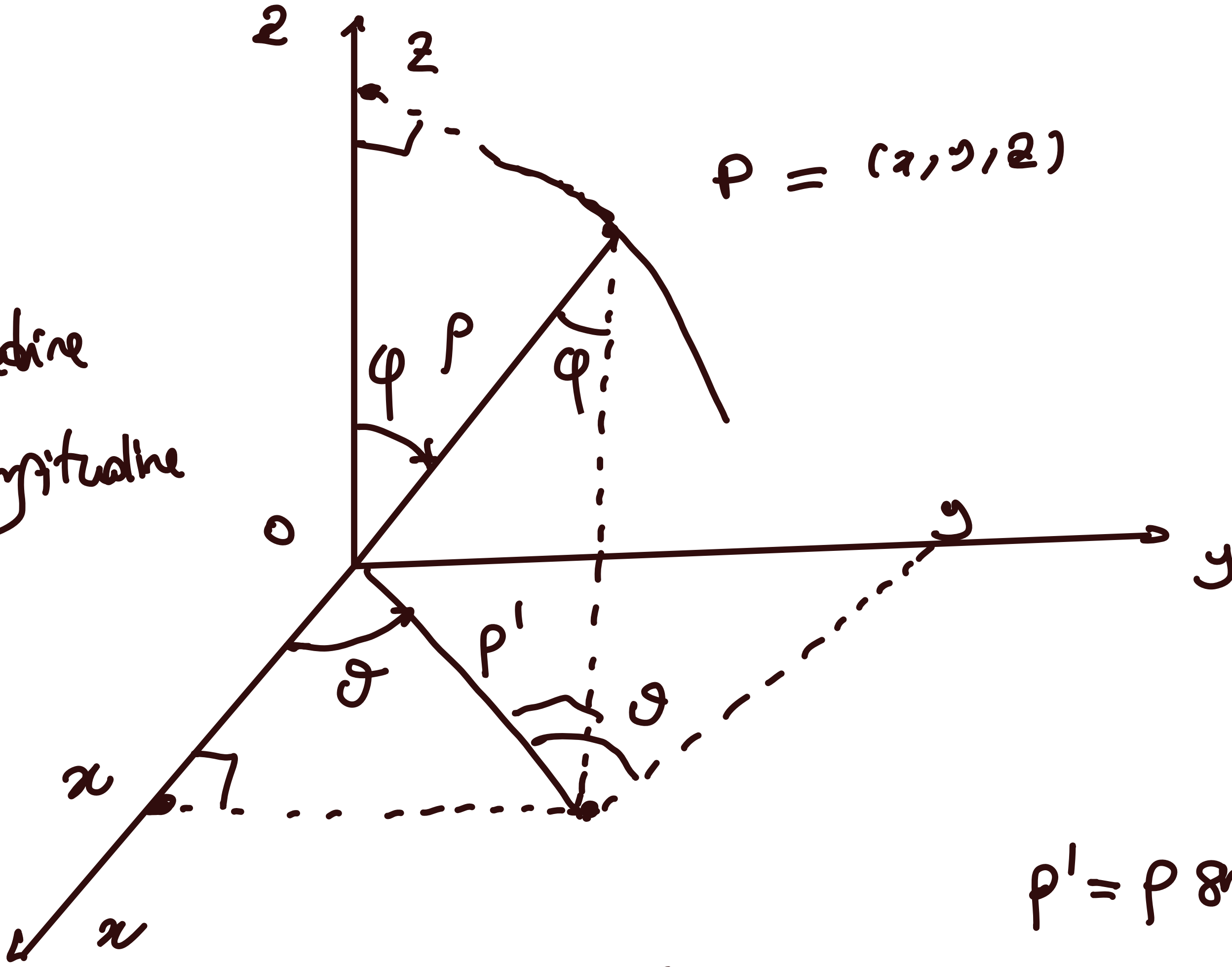
SFERICHE

$$\rho = \sqrt{x^2+y^2+z^2}$$

$\frac{\pi}{2} - \varphi$

latitudine

$\vartheta =$ longitudine



$$\rho' = \rho \sin\varphi$$

$$\begin{cases} x = \rho' \cos\vartheta \\ y = \rho' \sin\vartheta \\ z = \rho \cos\varphi \end{cases}$$

$$\begin{cases} x = \rho \sin\varphi \cos\vartheta \\ y = \rho \sin\varphi \sin\vartheta \\ z = \rho \cos\varphi \end{cases}$$

trasformazione delle coordinate sferiche

$x^2+y^2+z^2 = \rho^2$ sfera centrata in $(0,0,0) = 0$ e raggio $R=\rho$

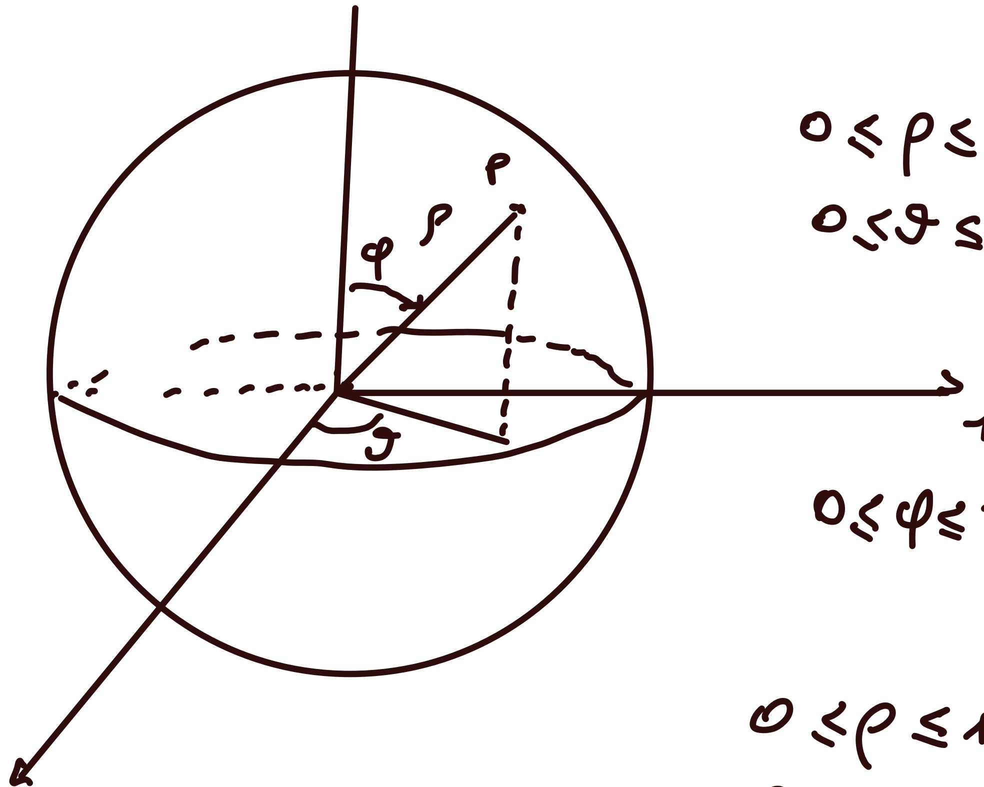
$$\det \frac{\partial(x,y,z)}{\partial(\underbrace{\rho}_{u},\underbrace{\vartheta}_{v},\underbrace{\varphi}_{w})} = \begin{vmatrix} x_\rho & x_\vartheta & x_\varphi \\ y_\rho & y_\vartheta & y_\varphi \\ z_\rho & z_\vartheta & z_\varphi \end{vmatrix} =$$

$$= \underbrace{\rho^2}\sin\varphi$$

ES.

$$\iiint_E \sqrt{\underbrace{x^2+y^2}+z^2}\,dx\,dy\,dz$$

E sfera centrata in O e raggio 1



$0 \leq \rho \leq 1$

$0 \leq \vartheta \leq 2\pi$

$0 \leq \varphi \leq \pi$

$0 \leq \rho \leq 1$

$0 \leq \vartheta \leq 2\pi$

$0 \leq \varphi \leq \pi$

$$= \iiint_F \rho \cdot \rho^2 \sin\varphi \; d\rho \, d\vartheta \, d\varphi$$

f.le di riduzione

$$= \iiint_F \rho^3 \sin\varphi \; d\rho \, d\vartheta \, d\varphi =$$

$$= \int_0^1 d\rho \int_0^{2\pi} d\vartheta \int_0^{\pi} \rho^3 \sin\varphi \, d\varphi$$

$$\int \sin\varphi \, d\varphi = -\cos\varphi + c$$

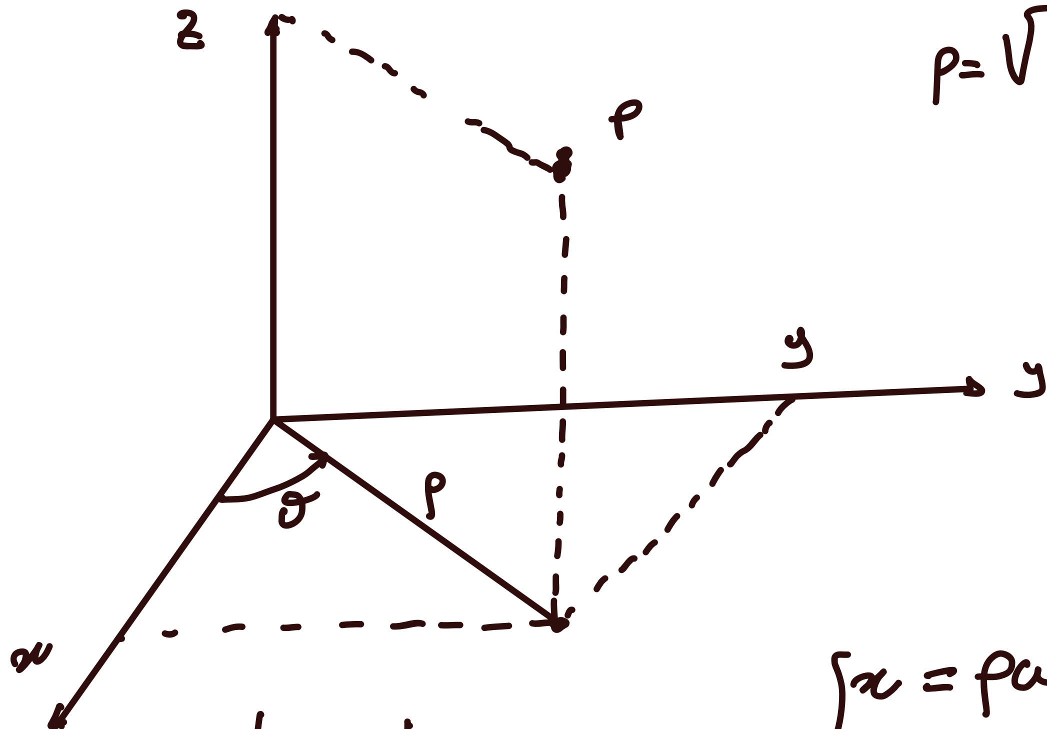
$$\int_0^{\pi} \sin\varphi \, d\varphi = -\left(\cos\varphi\right)_0^{\pi}$$

$= -(-1-1)$
$= 2$

$$= 2\int_0^1 \underbrace{\rho^3 \, d\rho}_{} \cdot \int_0^{2\pi} d\theta$$

$$= \cancel{4}\pi \left(\frac{\rho^4}{\cancel{4}}\right)_0^1 = \pi .$$

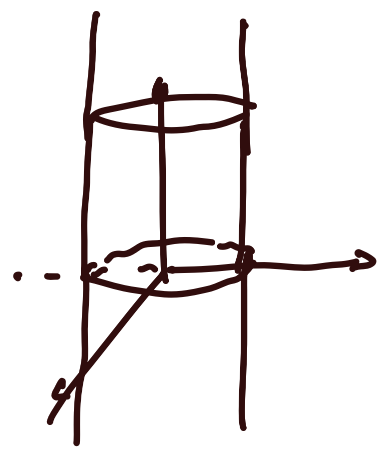
Trasformazione alle coordinate cilindriche



$\rho = \sqrt{x^2+y^2}$

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases}$$

$x^2+y^2 = \rho^2$



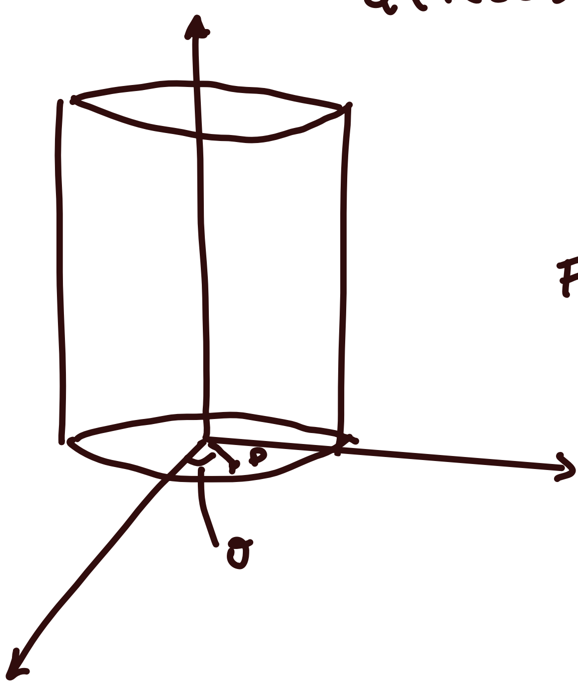
$$\frac{\partial(x,y,z)}{\partial(\rho,\vartheta,z)} = \begin{vmatrix} x_\rho & x_\vartheta & x_z \\ y_\rho & y_\vartheta & y_z \\ z_\rho & z_\vartheta & z_z \end{vmatrix} = \begin{vmatrix} \cos\vartheta \\ \\ \\ \end{vmatrix}$$

$$= \begin{vmatrix} \cos\vartheta & -\rho\sin\vartheta & 0 \\ \sin\vartheta & \rho\cos\vartheta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \rho$$

Es

$$\iiint_E xz\,dx\,dy\,dz$$

$E \equiv$ cilindro di raggio 1
altezza 1



$$F = \left\{ \begin{array}{l} 0 \le \vartheta \le \pi \\ 0 \le \rho \le 1 \\ 0 \le z \le 1 \end{array} \right|$$

$$\iiint_E xz\,dx\,dy\,dz = \iiint_F \rho\cos\vartheta \cdot z \cdot \rho \, d\rho\, d\vartheta\, dz$$

$$\iiint x^2 z \, dx\,dy\,dz$$

$$= \iiint_F \rho^2 z \cos\vartheta \, d\rho\, d\vartheta\, dz =$$

$$= \int_0^1 d\rho \int_0^{\pi} d\vartheta \int_0^1 \rho^2 z \cos\vartheta \, dz =$$

$$= \int_0^1 \rho^2 d\rho \int_0^{\pi} \cos\vartheta \, d\vartheta \int_0^1 z \, dz$$

$$= \frac{1}{6} \left(\sin\vartheta \right) = 0$$

1) Lunghezza di

$$\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$$

$t \in [0, \pi/2]$

$$x' = e^t \cos t - e^t \sin t$$

$$y' = e^t \sin t + e^t \cos t$$

$$(x')^2 + (y')^2 = e^{2t}\left[\cos^2 t + \sin^2 t \cancel{- 2 \sin t \cos t} + \sin^2 t + \cos^2 t \cancel{+ 2 \sin t \cos t}\right]$$

$$= 2e^{2t}$$

$$L(\gamma) = \int_0^{\pi/2} \sqrt{(x')^2 + (y')^2}\, dt =$$

$$= \sqrt{2} \int_0^{\pi/2} e^t\, dt = \sqrt{2}\left(e^{\pi/2} - 1\right).$$

$$\rho = a\vartheta \qquad \vartheta \in [0, \pi/4] \qquad \rho' = a$$

$$\rho = \rho(\vartheta) \qquad \vartheta \in [\vartheta_1, \vartheta_2]$$

$$L(\gamma) = \int_{\vartheta_1}^{\vartheta_2} \sqrt{\rho^2 + (\rho')^2}\, d\vartheta$$

$$= a\int_0^{\pi/4} \sqrt{1+\theta^2}\, d\theta$$

$$\int \sqrt{1+\theta^2}\, d\theta = \theta\sqrt{1+\theta^2} +$$

$$- \int \theta \cdot \frac{1}{\cancel{2}\sqrt{1+\theta^2}} \cdot \cancel{2}\theta\, d\theta$$

$$= \theta\sqrt{1+\theta^2} - \int \frac{1+\theta^2 - 1}{\sqrt{1+\theta^2}}\, d\theta$$

$$= \theta\sqrt{1+\theta^2} - \underbrace{\int \frac{(1+\theta^2)}{\sqrt{1+\theta^2}}\, d\theta}_{\substack{\parallel \\ \int \sqrt{1+\theta^2}\, d\theta}} + \int \frac{d\theta}{\sqrt{1+\theta^2}}$$

$$\int \sqrt{1+\theta^2}\, d\theta = \frac{\theta\sqrt{1+\theta^2}}{2} + \frac{1}{2}\int \frac{d\theta}{\sqrt{1+\theta^2}}$$

$$\int \frac{d\vartheta}{\sqrt{1+\vartheta^2}} = \log\left(\vartheta + \sqrt{1+\vartheta^2}\right) + c$$

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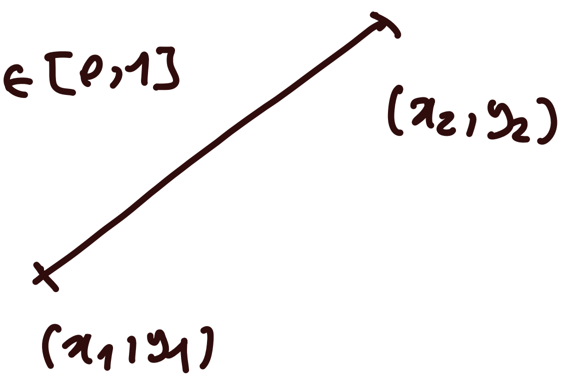
2) $$\int_\gamma \sqrt{x+2y}\, ds$$

$\gamma \equiv$ segmento di estremi $(0,0)$ e $(2,4)$

$$\begin{cases} x = t\,x_2 + (1-t)\,x_1 \\ y = t\,y_2 + (1-t)\,y_1 \end{cases} \qquad t \in [0,1]$$



$(x_1,y_1) = (0,0)$

$(x_2,y_2) = (2,4)$

$$= \begin{cases} x = 2t \\ y = 4t \end{cases} \qquad t \in [0,1]$$

$$\int_\gamma f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x')^2+(y')^2}\,dt$$

$$\begin{cases} x'=2 \\ y'=4 \end{cases} \qquad (x')^2+(y')^2 = 4+16 = 20$$

$$\sqrt{20} = 2\sqrt{5}$$

$$\int_\gamma \sqrt{\underset{=}{x}+2\underset{\uparrow}{y}}\,ds = \int_{\underset{a}{\underset{\shortparallel}{0}}}^{\overset{b}{\overset{\shortparallel}{1}}} \sqrt{2t+8t}\cdot 2\sqrt{5}\,dt$$

$$= 2\sqrt{5}\int_0^1 \sqrt{10}\cdot\sqrt{t}\,dt = 2\sqrt{5}\cdot\sqrt{10}\cdot\frac{2}{3}\left(t\sqrt{t}\right)\Big|_0^1$$

$$= \frac{4}{3}\sqrt{50}$$

$\underline{3}$

$$\int_\gamma z\,ds \qquad \gamma = \begin{cases} x = 3\cos t \\ y = 3\sin t \\ z = 4t \end{cases}$$

$$t\in[0,\pi]$$

$$x' = -3\sin t$$
$$y' = 3\cos t$$

$z' = 4$

$(x')^2 + (y')^2 + (z')^2 = 9 + 16 = 25$

$\int_\gamma z\, ds = 5 \int_0^\pi 4t\, dt$

4) $y'' - 2y' = e^x - 4$

OMOGENEA $y'' - 2y' = 0$

CARATT. $\lambda^2 - 2\lambda = 0 \quad \lambda_1 = 0, \ \lambda_2 = 2$

$y = c_1 + c_2 e^{2x}$

$e^{\bar{\lambda} x} p(x) \qquad e^{\bar{\lambda} x} (\cos \ldots + \sin \ldots)$

•) $y'' - 2y' = e^x \leftarrow y_1$

••) $y'' - 2y' = -4 \leftarrow y_2$

$y_1 + y_2 = \bar{y}$ int. partical. completu

$f(x) = e^x = 1 \cdot e^{1 \cdot x}$ $\bar{\lambda} = 1$ NO RADICE

$y_1 = a e^x$ $y_1' = y_1'' = a e^x$

$a e^x - 2a e^x = e^x$

$-a \cancel{e^x} = \cancel{e^x} \Leftrightarrow -a = 1 \Leftrightarrow \boxed{a = -1}$

$y_1 = -e^x$

••) $f(x) = -4 = -4 \cdot e^{0 \cdot x}$ $\bar{\lambda} = 0$ $\bar{E}$ RADICE!

$y_2 = ax$: $y_2' = a$, $y_2'' = 0$

$-2a = -4 \iff \boxed{a=2}$

$y_2 = 2x$

$y_0 = y_1 + y_2 = 2x - e^x$

Int. gen. COMPLETA : $\underbrace{y = c_1 + c_2 e^{2x} + 2x - e^x}$

$$\begin{cases} y'' - 2y' = e^x - 4 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

$y(0) = c_1 + c_2 - 1 = 1$

$c_1 + c_2 = 2$

$y' = 2c_2 e^{2x} + 2 - e^x$

$0 = y'(0) = 2c_2 + 1 \iff \boxed{c_2 = -\frac{1}{2}}$

$\left[\; c_1 = 2 - c_2 = 2 + \frac{1}{2} = \frac{5}{2} \; \right]$

SOL. P.MA CAUCHY:

$$y = \frac{5}{2} - \frac{1}{2}e^{2x} + 2x - e^{x}$$

$$y' = -y + \underbrace{e^{-x} \cdot (\cos x)}_{b(x)}$$

$$y' = a(x)y + b(x) \qquad A'(x) = a(x)$$

$$y(x) = e^{A(x)} \int b(x) e^{-A(x)} dx$$

$$a(x) = -1, \quad A(x) = -x$$

$$y(x) = e^{-x} \int \cos x \cdot \underbrace{e^{-x} \cdot e^{x}}_{=1} dx =$$

$$= e^{-x} \int \cos x \, dx = e^{-x} (\sin x + C)$$

$$y' = 2x\sqrt{1-y^2}$$

$$\frac{dy}{dx} = 2x\sqrt{1-y^2} \quad : \int \frac{dy}{\sqrt{1-y^2}} = \int 2x\,dx$$

$$1-y^2 \neq 0 \iff \underline{\underline{y \neq \pm 1}}$$

$$\arcsin y = x^2 + C$$

$y = \pm 1$ SONO INTEGRALI SINGOLARI:

$$y' = e^{x-y}\cos x = e^x \cdot e^{-y}\cos x$$

$$\frac{dy}{dx} = e^{-y} \cdot e^x \cos x$$

$$\int e^y\,dy = \int e^x \cos x\,dx$$

$$\left[e^y = \int e^x \cos x\,dx = \frac{e^x}{2}(\sin x + \cos x) + C \right]$$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx =$$

$$= e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$\int e^x \cos x = \frac{e^x}{2} (\sin x + \cos x) + C$$

$$\underline{\underline{y(0) = 1}}$$

$$e^{\overbrace{y(0)}^{=1}} = \frac{1}{2} + C$$

$$e = \frac{1}{2} + C \iff \underline{\underline{C = e - \tfrac{1}{2}}}$$

$$\left\| \; e^y = \frac{e^x}{2} (\sin x + \cos x) + e - \frac{1}{2} \; \right\|$$

$$y'' + y = tg\,x$$

$$y'' + y = 0 \qquad y = C_1 \cos x + C_2 \sin x$$

$$\lambda^2 + 1 = 0 \qquad \lambda = \pm i$$

$$y = C_1(x) \cos x + C_2(x) \sin x$$

$$\begin{cases} \cos x \; C_1'(x) + \sin x \; C_2'(x) = 0 \\ -\sin x \; C_1'(x) + \cos x \; C_2'(x) = tg\,x \end{cases}$$

$$C_1'(x) = \begin{vmatrix} 0 & \sin x \\ tg\,x & \cos x \end{vmatrix} = -\frac{\sin^2 x}{\cos x}$$

$$C_2'(x) = \begin{vmatrix} \cos x & 0 \\ -\sin x & tg\,x \end{vmatrix} = \sin x$$

$$C_2(x) = -\cos x$$

$$-\int \frac{\sin^2 x}{\cos x} = -\left[\int \frac{1-\cos^2 x}{\cos x}\,dx = \right.$$

$$\left. = \int \underbrace{\frac{1}{\cos x}}\,dx - \int \cos x\,dx \right]$$

$$\int \frac{1}{\cos x}\,dx =$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$t = \operatorname{tg}\frac{x}{2}$$

$$= \int \frac{\cancel{1+t^2}}{1-t^2}\cdot\frac{2}{\cancel{1+t^2}}\,dt$$

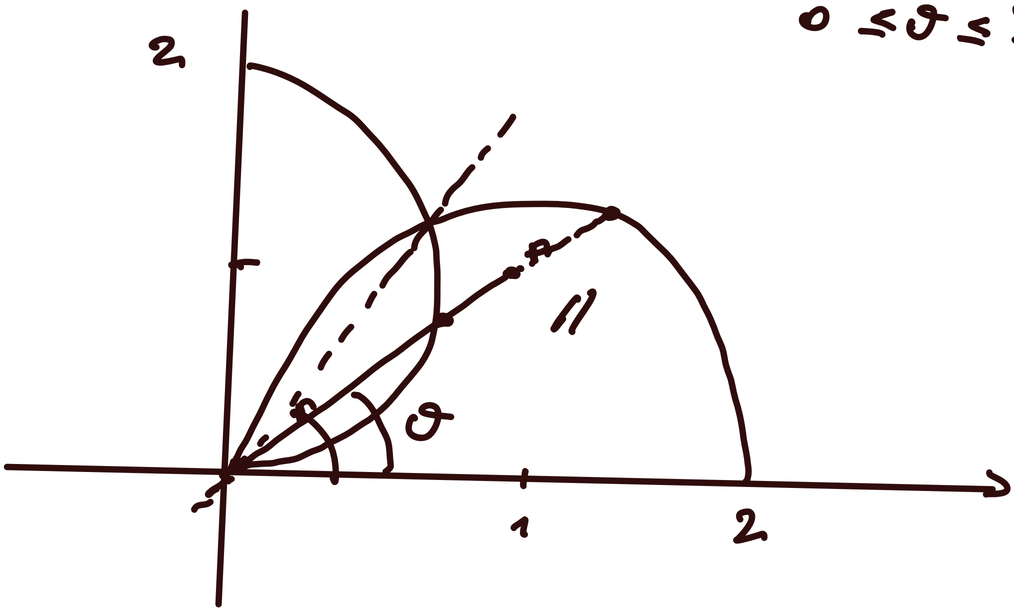
$$x = 2\operatorname{arctg} t$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$= -2\int \frac{dt}{t^2-1} = \dots$$

$$\iint \sqrt{x^2+y^2}\, dx\,dy$$

$0 \le \vartheta \le \frac{\pi}{4}$



$(0,1)$

$x^2+(y-1)^2=1$

$$\underbrace{\rho^2\cos^2\vartheta + \rho^2}\sin^2\vartheta - 2\rho\sin\vartheta + \cancel{1} = \cancel{1}$$

$$\rho^{\cancel{2}} = 2\cancel{\rho}\sin\vartheta \qquad \rho = 2\sin\vartheta$$

$$2\sin\vartheta \le \rho \le 2\cos\vartheta$$

$(x-1)^2 + y^2 = 1$

$T = \{(\rho,\vartheta) : 0 \le \vartheta \le \pi/4$
$2\sin\vartheta \le \rho \le 2\cos\vartheta\}$

$(\rho\cos\vartheta - 1)^2 + \rho^2\sin^2\vartheta = 1$

$$\rho^2\cos^2\vartheta + \cancel{1} - 2\rho\cos\vartheta + \rho^2\sin^2\vartheta = \cancel{1}$$

$$\rho^{\cancel{2}} = 2\cancel{\rho}\cos\vartheta \qquad \boxed{\rho = 2\cos\vartheta}$$

$$\iint_D \sqrt{x^2+y^2}\,dx\,dy = \iint_T \rho \cdot \rho \, d\rho \, d\vartheta =$$

$$= \iint_T \rho^2 \, d\rho \, d\vartheta$$

$$= \int_0^{\pi/4} d\vartheta \int_{2\sin\vartheta}^{2\cos\vartheta} \rho^2 \, d\rho = \frac{1}{3} \int_0^{\pi/4} \left(\rho^3\right)_{2\sin\vartheta}^{2\cos\vartheta} d\vartheta$$

$$= \frac{8}{3} \int_0^{\pi/4} \left[\cos^3\vartheta - \sin^3\vartheta\right] d\vartheta$$

$$\int \cos^3\vartheta \, d\vartheta = \int \cos\vartheta \, (1-\sin^2\vartheta) \, d\vartheta = \int \cos\vartheta - \int \sin^2\vartheta \cos\vartheta$$

$$= \sin\vartheta - \frac{\sin^3\vartheta}{3} + c$$