Lezioni del 13/10/23 e del 17/10/23

$f = f(x)$ $\qquad f: I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$

$f(x, y)$

Es. $f(x,y) = x + y \quad , (x,y) \in \mathbb{R}^2$

$f(0,0) = 0$

$x \quad y$

$f(1,2) = 1 + 2 = 3$

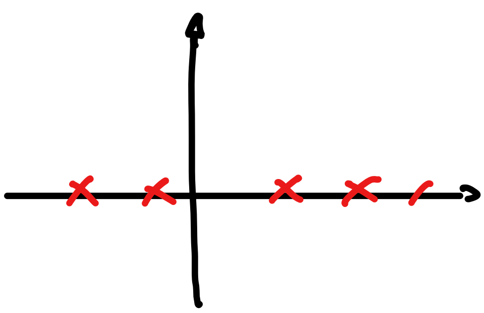
$f(x,y) = \dfrac{x}{y} \qquad y \neq 0$

$X = \{(x,y) \in \mathbb{R}^2 : \ y \neq 0\}$

$f(x,y) = x \log y \quad , \underline{\underline{y > 0}}$

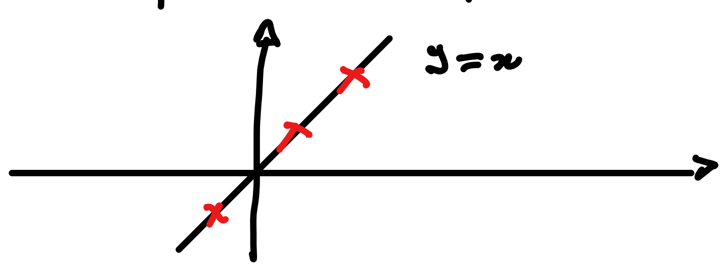
$X = \{(x,y) \in \mathbb{R}^2 : y > 0\}$

$f(2,1) = 2 \log 1 = 0$



$f(x,y) = \dfrac{1}{x - y}$

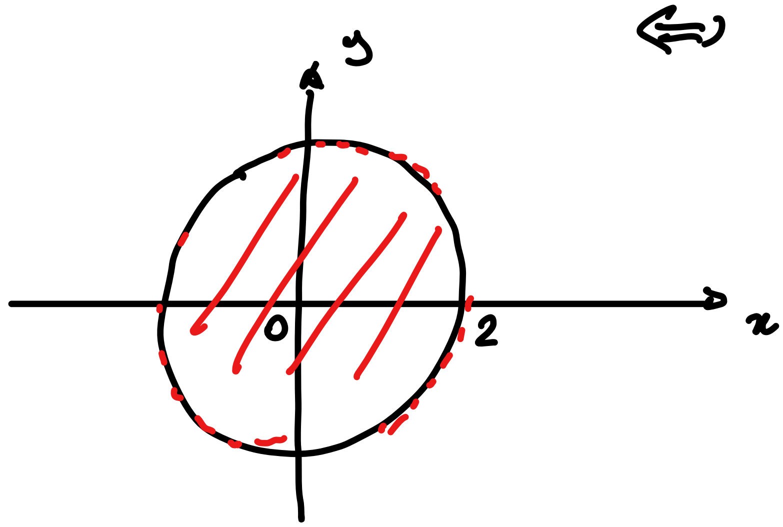
$x \neq y \iff y \neq x \qquad y = x$



$f(x,y) = \sqrt{4 - x^2 - y^2}$: $4 - x^2 - y^2 \geq 0$

$\Longleftrightarrow$ $x^2 + y^2 \leq 4$

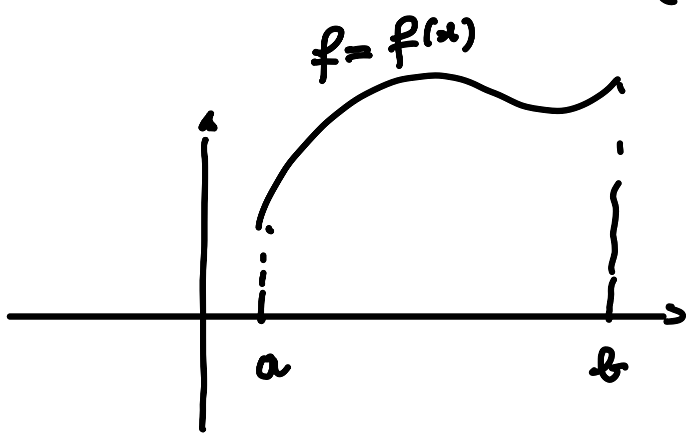
$x^2 + y^2 = 4$



$f(x,y) = \log(4 - x^2 - y^2)$: $x^2 + y^2 \underset{\neq}{<} 4$

$f = f(x,y)$, $f : X \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x,y) \in X \longrightarrow f(x,y) \in \mathbb{R}$



$G_f = \{ (x, f(x)) : x \in [a,b] \}$

$y = f(x)$

equazione del grafico di f

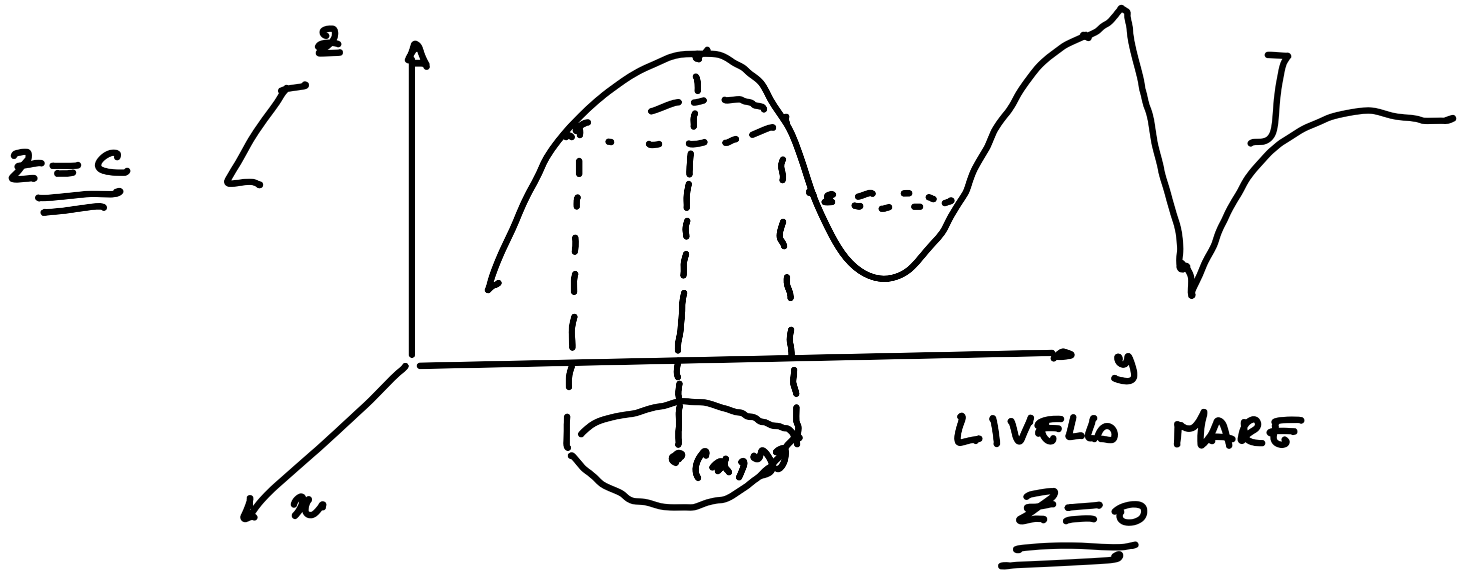
$f = f(x,y)$ $\quad G_f = \{ (x, y, f(x,y)) \in \mathbb{R}^3 : (x,y) \in X \}$

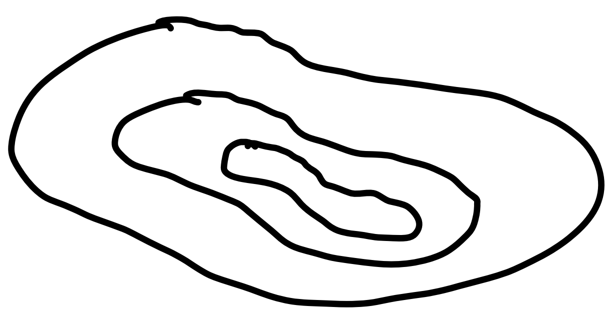
$\subseteq \mathbb{R}^3$

Equazione del grafico è $\quad z = f(x,y)$

superficie di $\mathbb{R}^3$
cartesiana

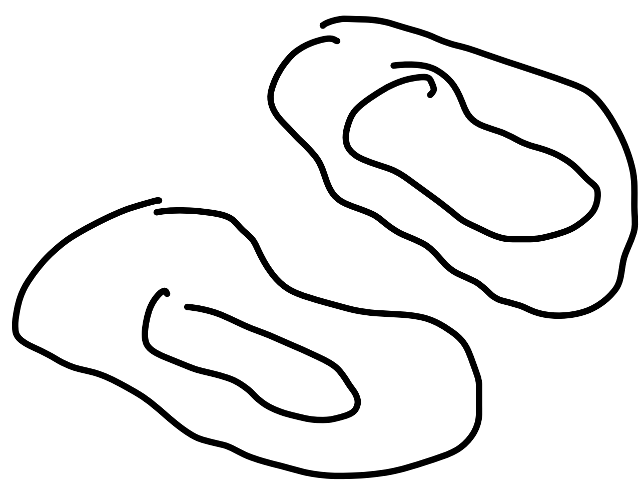
$f(x,y)$ = livello della superficie terrestre rispetto al livello del mare, nel punto $(x,y) \in X$





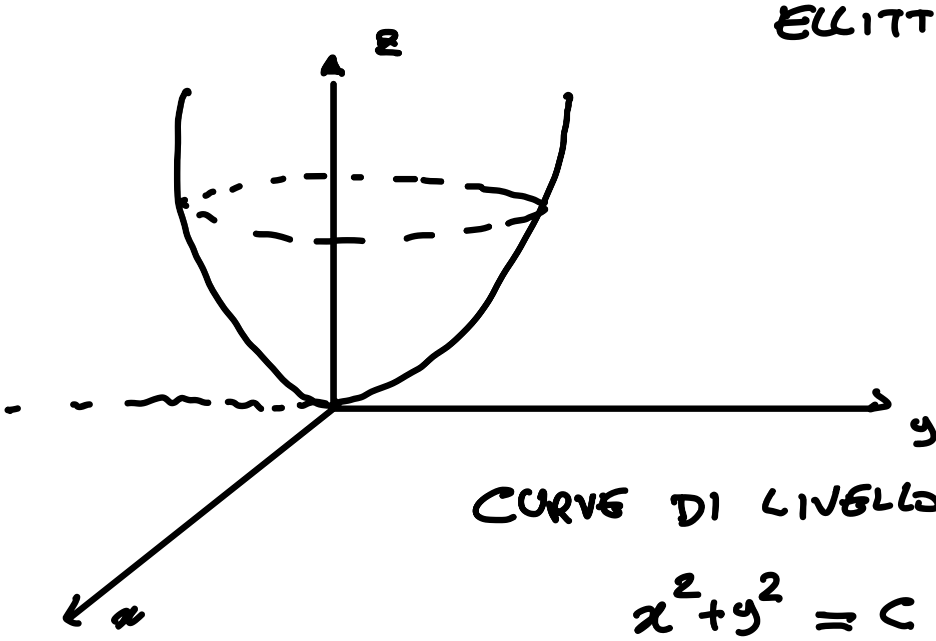
$f(x,y)$ = pressione atmosferica

ISOBARE



$f(x,y) = x^2+y^2$ $\qquad$ $z = x^2+y^2$ $\qquad$ PARABOLOIDE ELLITTICO

$f: \mathbb{R}^2 \longrightarrow \mathbb{R}$



CURVE DI LIVELLO

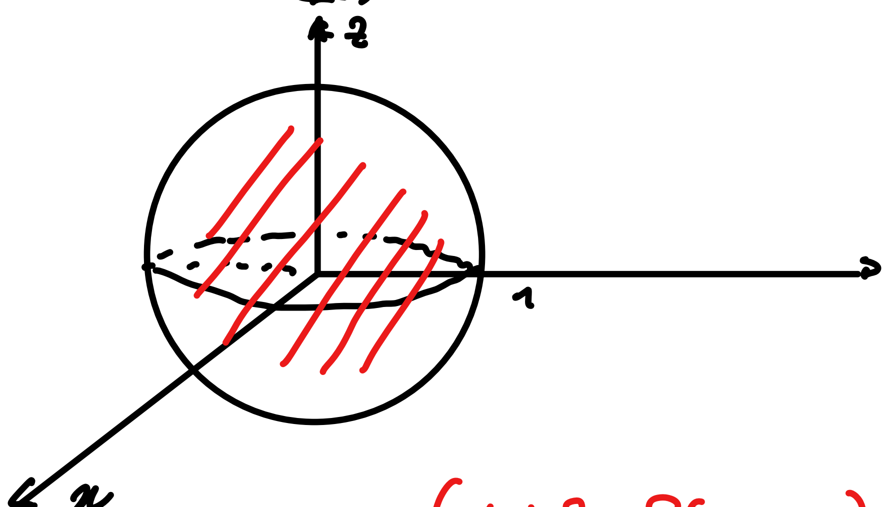
$x^2+y^2 = c$

$f(x,y,z)$ $\qquad$ $f: X \subseteq \mathbb{R}^3 \longrightarrow \mathbb{R}$

$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = R^2$

$$f(x,y,z) = \sqrt{\underbrace{1-x^2-y^2-z^2}_{\geq 0}}$$

$\iff x^2+y^2+z^2 \leqq 1$

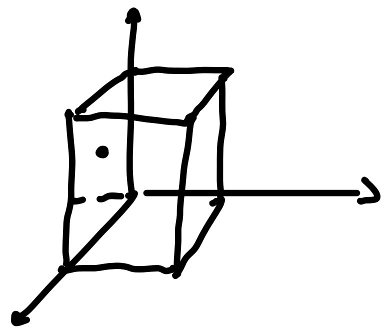


$(x,y,z,f(x,y,z)) \in \mathbb{R}^4$

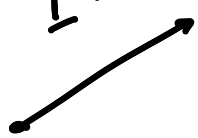
$t = f(x,y,z)$

$T(x,y,z,t)$

$t \in [t_0, t_1]$

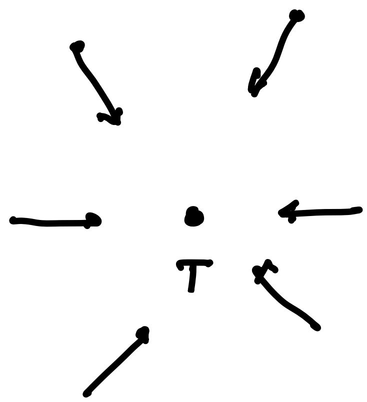


$\underline{F}(x,y,z)$



(x,y,z)

$$\underline{F}(x,y,z) = \big(F_1(x,y,z), F_2(x,y,z), F_3(x,y,z)\big)$$



$$\mathbb{R}^n = \{ (x_1, \dots, x_n) : x_i \in \mathbb{R} \ , \ \forall i = 1, \dots, n \}$$

$\underline{x} = (x_1, \dots, x_n)$

$\underline{y} = (y_1, \dots, y_n)$

$\lambda \in \mathbb{R}$

$\underline{x} + \underline{y} = (x_1 + y_1, \dots, x_n + y_n)$

$\lambda \underline{x} = (\lambda x_1, \lambda x_2, \dots, \lambda x_n)$

$\underline{0} = (0, \dots, 0)$

$$-\underline{x} = (-x_1, \dots, -x_n)$$

$$\underline{x} + (-\underline{x}) = \underline{0}$$

$$(\lambda + \mu)\underline{x} = \lambda \underline{x} + \mu \underline{x} \qquad \forall \lambda, \mu \in \mathbb{R}$$

$$\lambda(\underline{x} + \underline{y}) = \lambda \underline{x} + \lambda \underline{y}$$

$$1 \cdot \underline{x} = \underline{x} \qquad \forall \underline{x} \in \mathbb{R}^n \qquad (\mathbb{R}^n, +, \cdot)$$

spazio vettoriale

$x_1, \dots, x_k$ lin. indipendenti

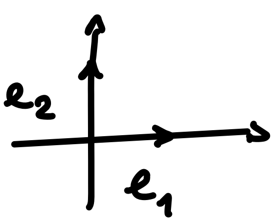
$$\lambda_1 \underline{x}_1 + \dots + \lambda_k \underline{x}_k = \underline{0} \iff \lambda_1 = \dots = \lambda_k = 0$$

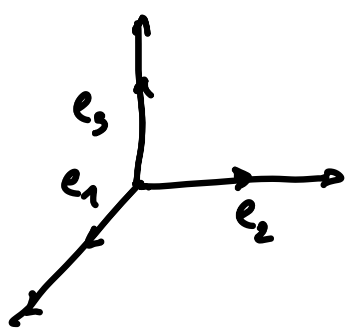
$$\underline{e}_1 = (1, 0, \dots, 0), \qquad \underline{e}_2 = (0, 1, 0, \dots, 0)$$

$$\dots \qquad \underline{e}_n = (0, 0, 0, \dots, 0, 1)$$

BASE CANONICA

$$\underline{x} = x_1 \underline{e}_1 + x_2 \underline{e}_2 + \dots + x_n \underline{e}_n$$





PRODOTTO SCALARE : $\underline{x} \cdot \underline{y} = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

$$\langle \underline{x}, \underline{y} \rangle$$

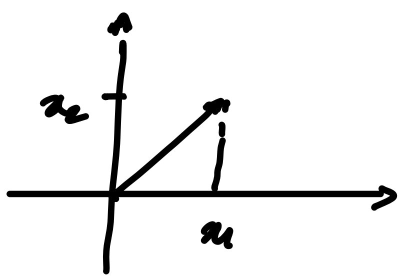
$$\underline{x} \cdot \underline{x} = x_1^2 + x_2^2 + \dots + x_n^2 \geq 0$$

$$\underline{x} \cdot \underline{x} = 0 \iff \underline{x} = \underline{0}$$

$\mathbb{R}^n$ <u>spazio euclideo</u>

$\underline{x} \in \mathbb{R}^n$ $\quad \|\underline{x}\|$ = NORMA EUCLIDEA DI $\underline{x}$

$$= \sqrt{\underline{x} \cdot \underline{x}} = \sqrt{x_1^2 + \dots + x_n^2} \geq 0$$

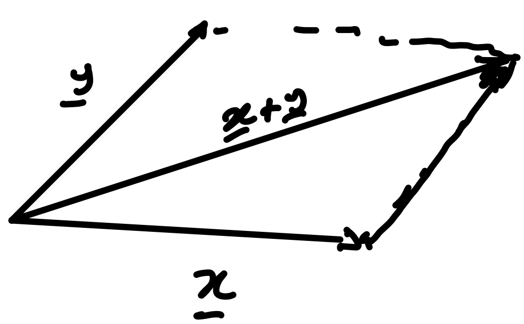


$$\|\underline{x}\| = 0 \iff \underline{x} = \underline{0}$$

$$\|\lambda \underline{x}\| = |\lambda| \, \|\underline{x}\| \quad \forall \underline{x} \in \mathbb{R}^n$$
$$\forall \lambda \in \mathbb{R}$$

$|x+y| \leq |x| + |y|$

$$\|\underline{x} + \underline{y}\| \leq \|\underline{x}\| + \|\underline{y}\|$$



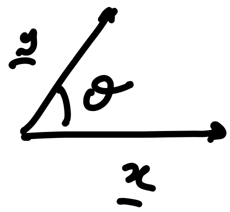
DISUGUAGLIANZA DI CAUCHY-SCHWARTZ

$$|\underline{x} \cdot \underline{y}| \leq \|\underline{x}\| \cdot \|\underline{y}\| \quad \forall \underline{x}, \underline{y} \in \mathbb{R}^n$$

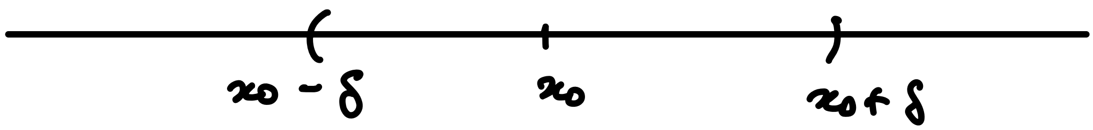
$\underline{x} \perp \underline{y} \iff \underline{x} \cdot \underline{y} = 0$ $\qquad$ $\{\underline{e}_1, \dots, \underline{e}_n\} =: B$

BASE ORTONORMALE

$\underline{x} \cdot \underline{y} = \|\underline{x}\| \cdot \|\underline{y}\| \cos\theta$



$\mathbb{R}$ $\qquad$ $]x_0-\delta, x_0+\delta[\; = I_\delta(x_0)$



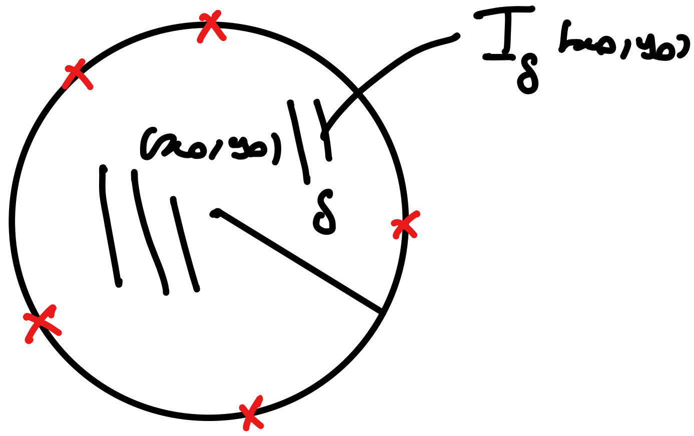
$\mathbb{R}^n$ $\quad$ $(n=2,\ n=3)$

$x_0 \in \mathbb{R}^n$, $\delta > 0$: si dice intorno sferico centrato in x_0 e di raggio δ

$$I_\delta(x_0) = \{x \in \mathbb{R}^n : \|x - x_0\| < \delta\}$$

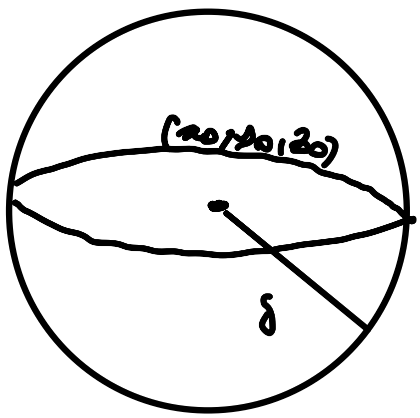
$n=2$ $\qquad$ (x_0, y_0)

$$I_\delta(x_0,y_0) = \{(x,y) \in \mathbb{R}^2 : \underbrace{\sqrt{(x-x_0)^2+(y-y_0)^2} < \delta}_{(x-x_0)^2+(y-y_0)^2 < \delta^2}\}$$



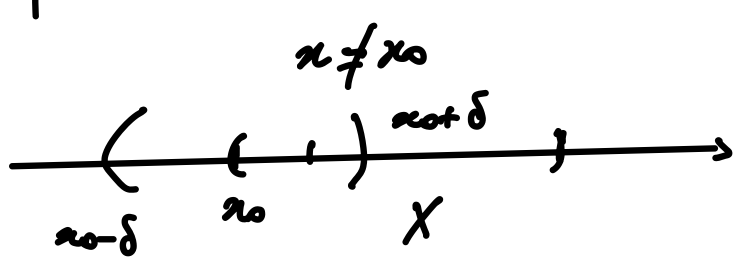
$n=3$

$$I_\delta(x_0,y_0,z_0) = \{ (x,y,z) \in \mathbb{R}^3 : (x-x_0)^2+(y-y_0)^2+(z-z_0)^2 < \delta^2 \}$$



Punto di accumulazione per $X \subseteq \mathbb{R}$

$x_0 \in \mathbb{R}$



$$\forall \delta > 0 \ \exists\, x \in X, \ x \neq x_0, \ |x-x_0| < \delta$$

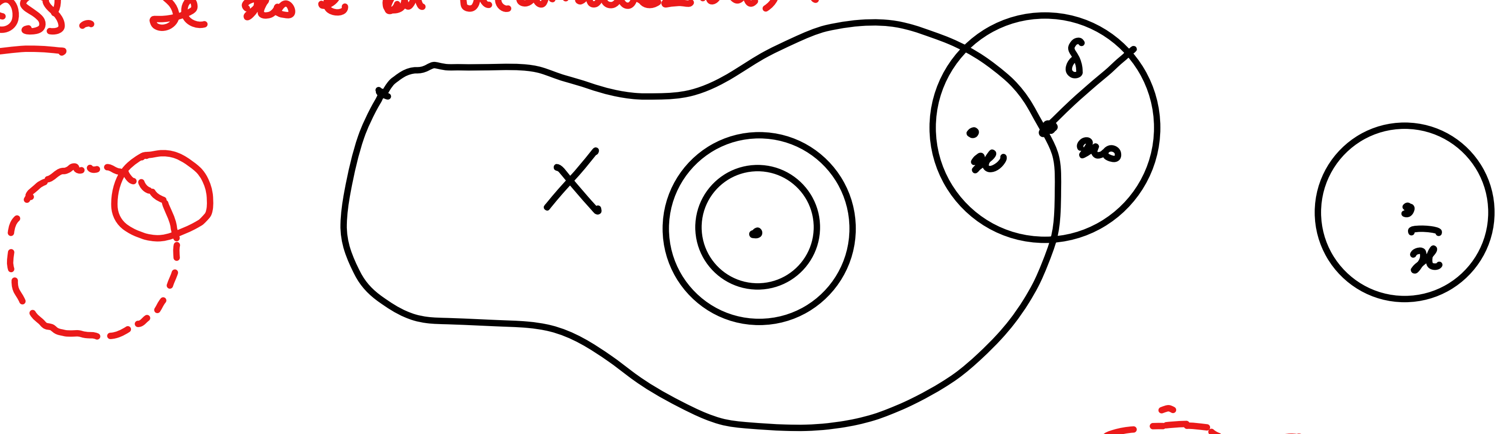
Def $X \subseteq \mathbb{R}^m$, $x_0 \in \mathbb{R}^m$: x_0 è di accumulazione per X se in ogni intorno di x_0 cade almeno un punto $x \in X$

$x \neq x_0$

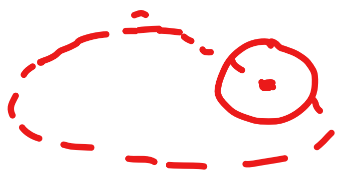
$\iff \forall I_\delta(x_0) \quad \exists x \in X, \ x \neq x_0, \quad x \in I_\delta(x_0)$

$\forall \delta > 0 \quad \exists x \in X, \ x \neq x_0 : \|x - x_0\| < \delta$

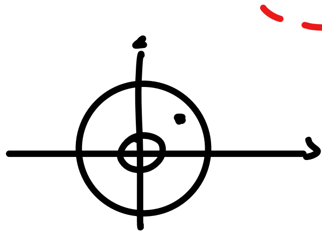
OSS. Se x_0 è di accumulazione, non è detto che $x_0 \in X$!!

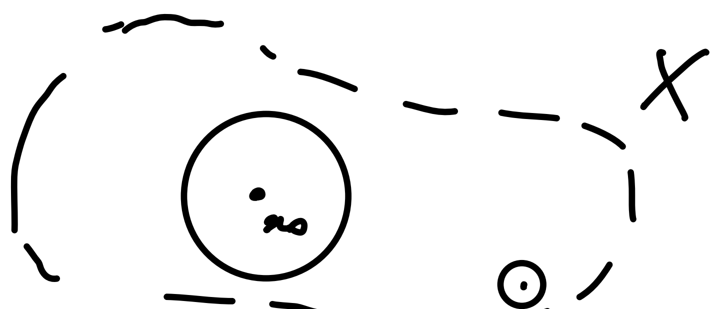


$X_1 = X \cup \{\bar{x}\}$



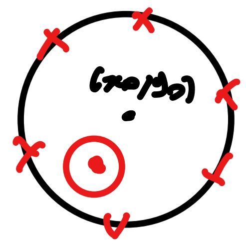
Def $X = \mathbb{R}^2 \setminus \{(0,0)\}$

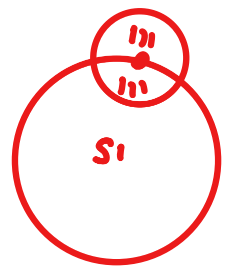




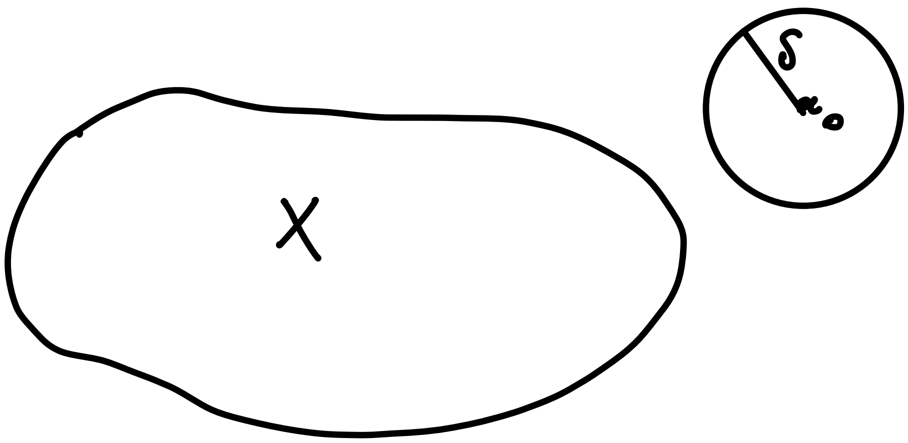
Si dice che x_0 è interno ad

$\exists \delta > 0 : I_\delta(x_0) \subseteq X$ (X se

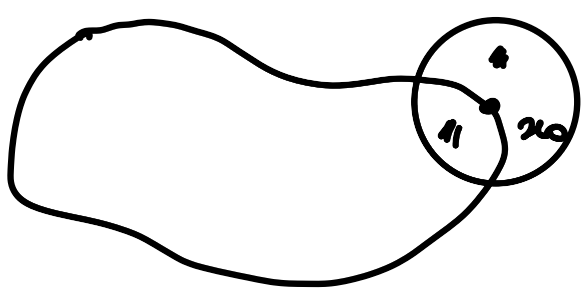




$x_0 \in \mathbb{R}^n$ si dice esterno ad X se è interno al suo complementare $X^c = \mathbb{R}^n \setminus X$

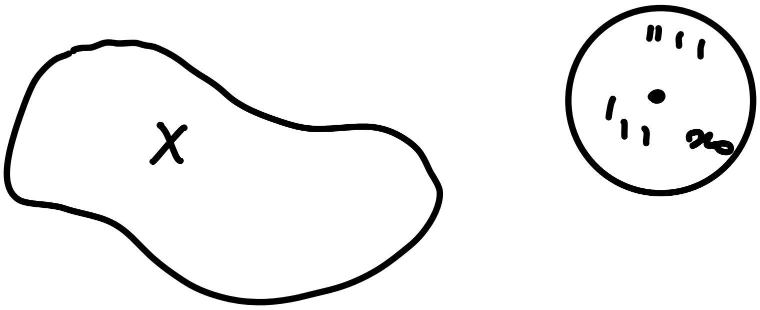


$x_0 \in \mathbb{R}^n$ è di frontiera per X se non è né interno né esterno ad X : in ogni intorno di x_0 cadono sia punti di X che punti di X^c



$x_0 \in \mathbb{R}^n$ di frontiera per X: o x_0 è di accumulazione oppure non lo è

$$X_1 = X \cup \{x_0\}$$

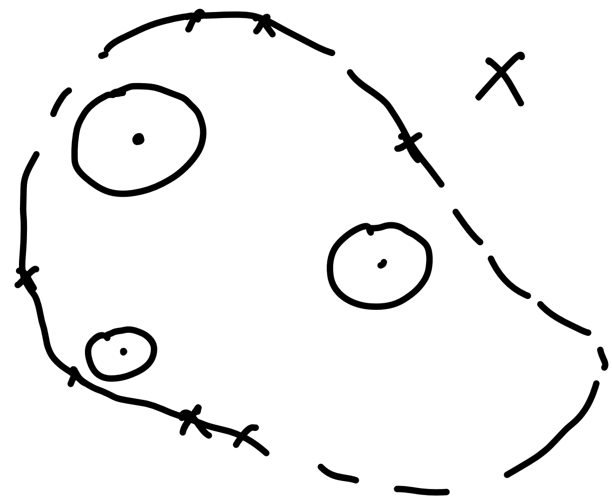


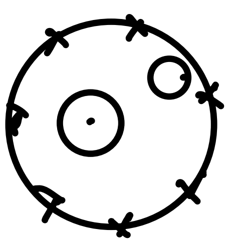
x_0 è di frontiera!
No di accumulazione per X_1

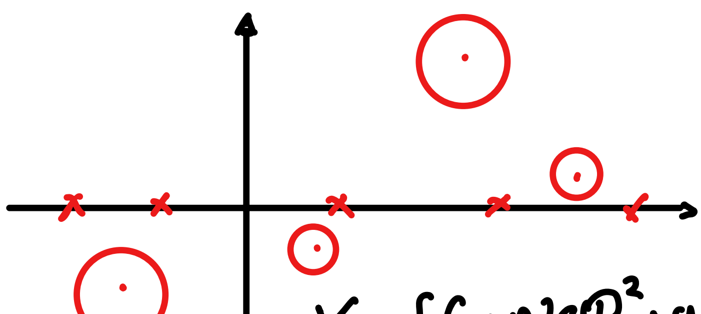
Se x_0 è di frontiera ma <u>non</u> è di accumulazione, si dice che x_0 è <u>isolato</u>.

<u>Def</u>.

$X \subseteq \mathbb{R}^n$ si dice <u>aperto</u> se ogni punto di X è interno ad X

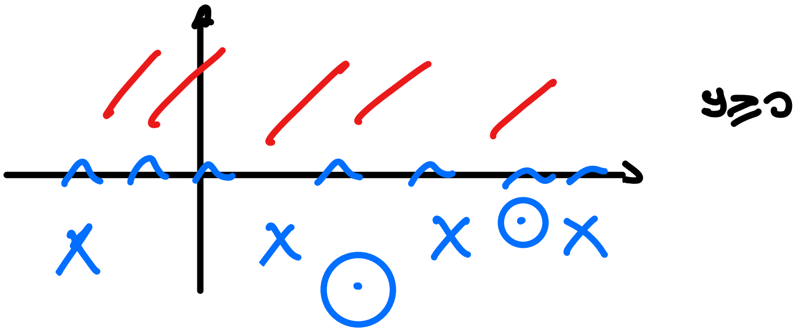


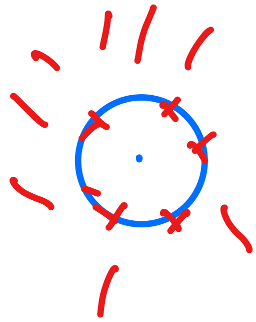


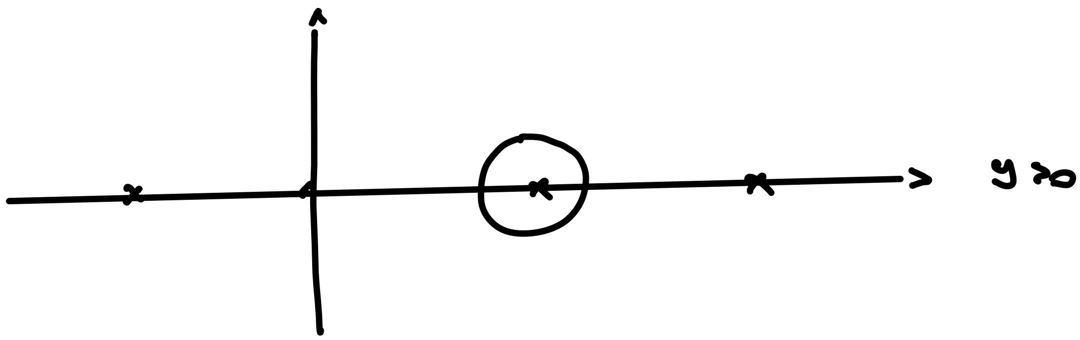


$X = \{(x,y) \in \mathbb{R}^2 : y \neq 0\}$

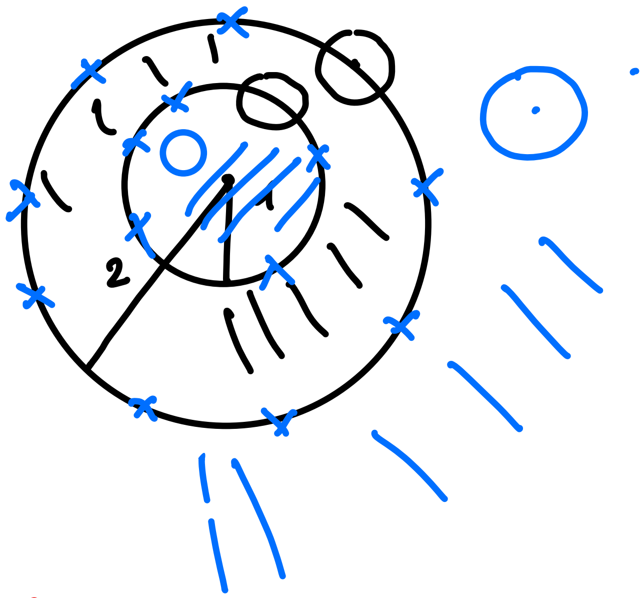
Def $X \subseteq \mathbb{R}^n$ chiuso se X^c è aperto



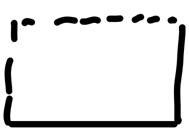




$$E = \{ (x,y) \in \mathbb{R}^2 : \ 1 \leq x^2 + y^2 \leq 9 \}$$



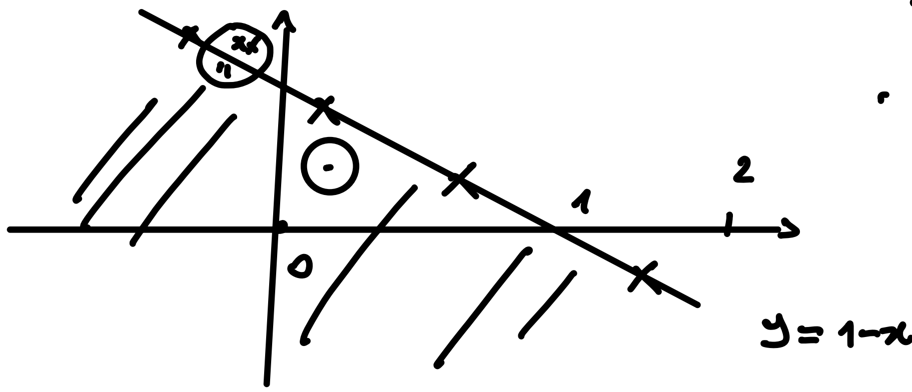
$$E_1 = \{ (x,y) \in \mathbb{R}^2 : \ 1 < x^2 + y^2 \leq 9 \}$$



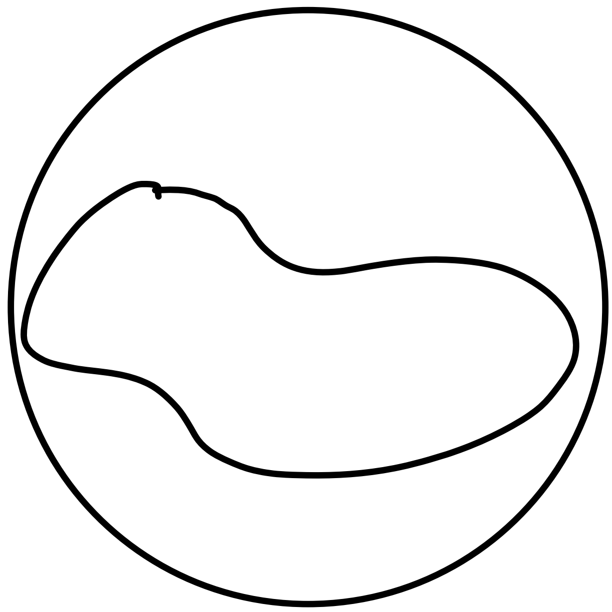
$E = \{ (x,y) \in \mathbb{R}^2 : \quad x + y < 1 \}$ aperto

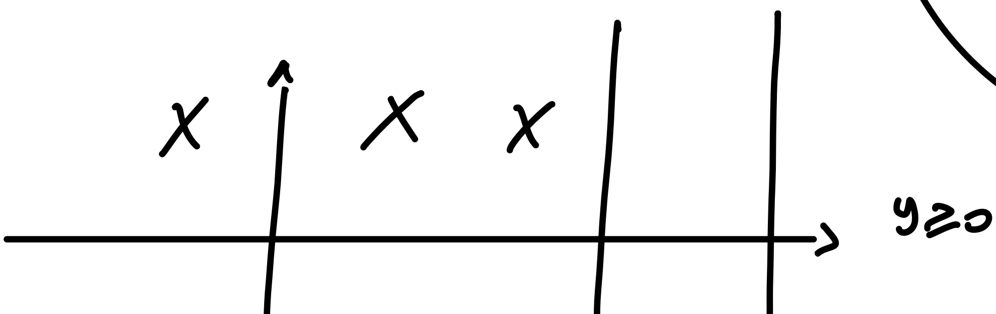
$x + y < 1 \iff y < 1 - x$

$\cancel{0 < 1-2 = -1}$

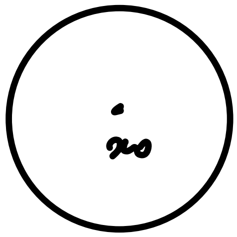


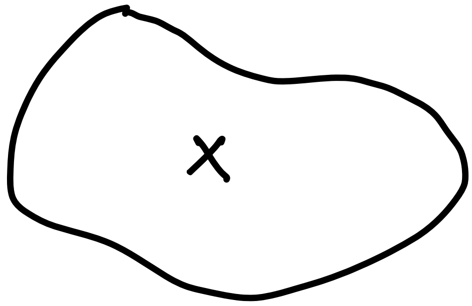
<u>Def.</u> $X \subseteq \mathbb{R}^n$ si dice limitato se $\exists\, I_\delta(x_0)$ tale che $X \subseteq I_\delta(x_0)$

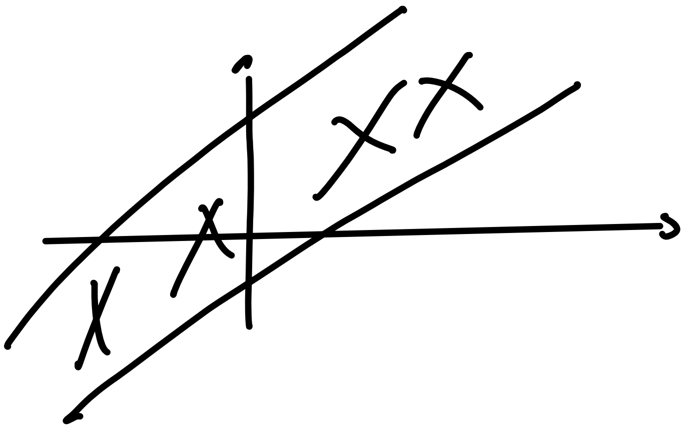




<u>Def.</u> $X \subseteq \mathbb{R}^n$ compatto se è chiuso e limitato





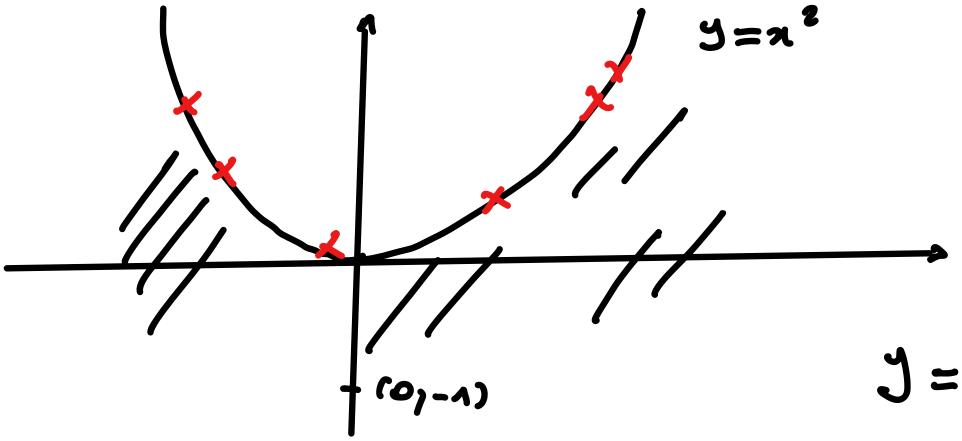


$$f(x,y) = \log(x^4 - y^2)$$

$$x^4 - y^2 > 0 \iff (x^2 - y)(x^2 + y) > 0$$

$$\iff \begin{cases} x^2 - y > 0 \\ x^2 + y > 0 \end{cases} \cup \begin{cases} x^2 - y < 0 \\ x^2 + y < 0 \end{cases}$$

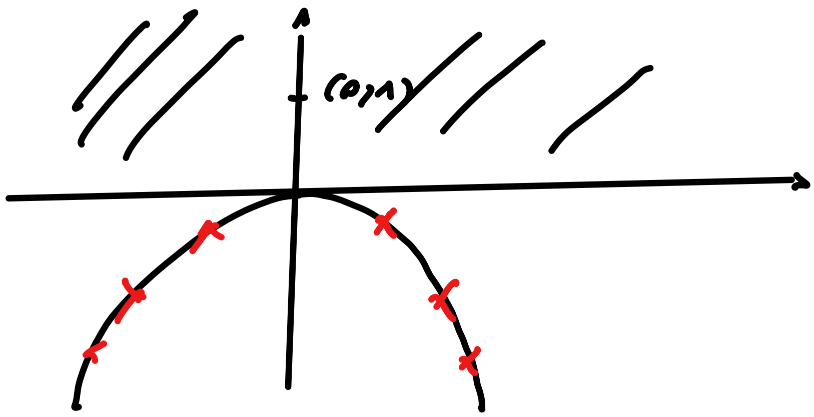
$$\iff \begin{cases} y < x^2 \\ y > -x^2 \end{cases} \cup \underbrace{\begin{cases} y > x^2 \\ y < -x^2 \end{cases}}_{\emptyset}$$

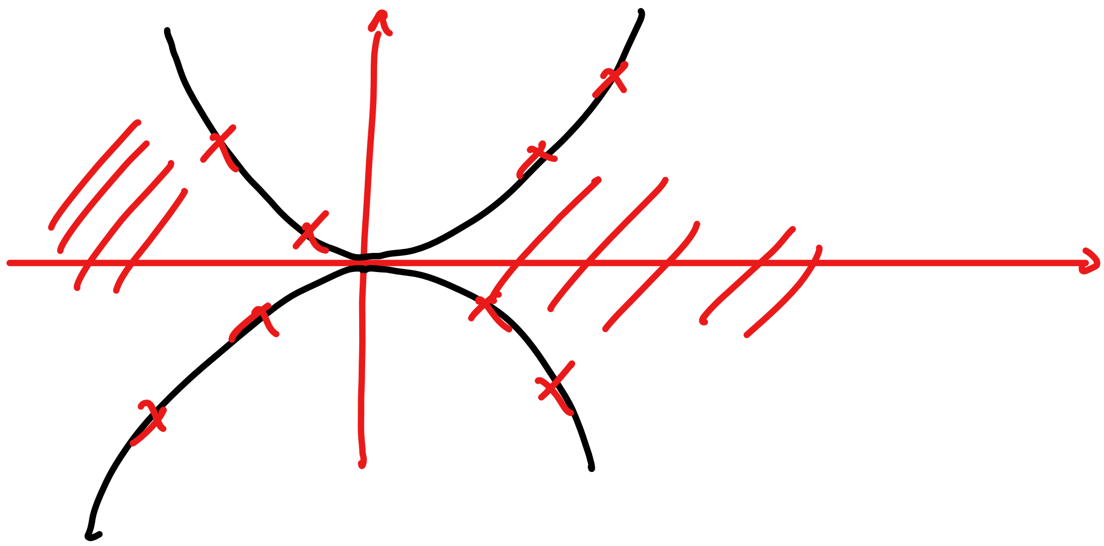


$y = -1 < 0 = x^2$

VERA

$y > -x^2$





ES.

$$f(x,y) = \log(y^4 - x^2).$$