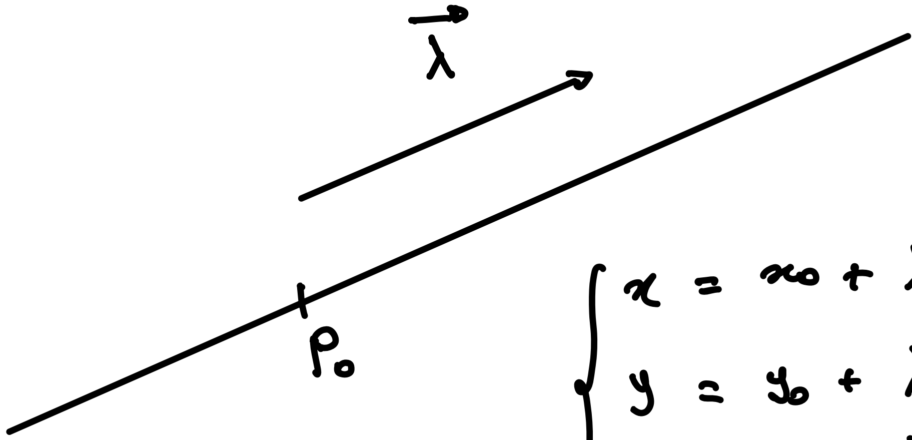
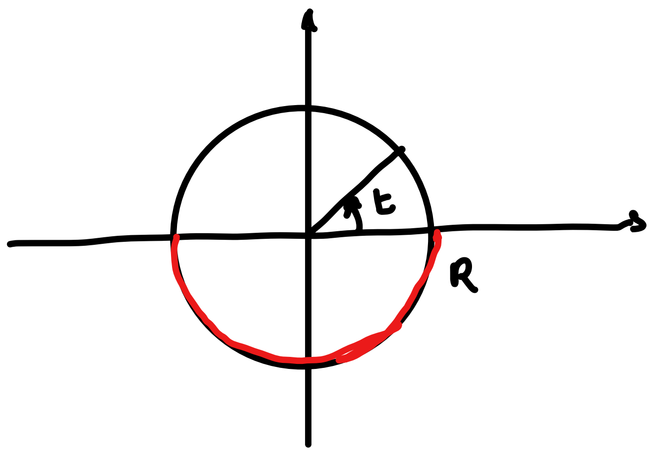
# Curve equivalenti




$$\begin{cases} x = x_0 + \lambda_1 t \\ y = y_0 + \lambda_2 t \\ z = z_0 + \lambda_3 t \end{cases}$$




$$\begin{cases} x = R\cos t \\ y = R\sin t \end{cases}$$

$t \in [0, 2\pi]$

$$\begin{cases} x = R\cos s \\ y = R\sin s \end{cases}$$

$s \in [0, 4\pi]$

$$\begin{cases} x = R\cos t \\ y = R\sin t \end{cases}$$

$t \in [-\pi, 0]$

$$\begin{cases} x = R\cos 2s \\ y = R\sin 2s \end{cases}$$

$s \in [\frac{\pi}{2}, \pi]$

**Def.** 
$$\varphi : I \longrightarrow \mathbb{R}^2 \quad (\mathbb{R}^3)$$
$$t \longrightarrow \varphi(t)$$

$$\psi : J \longrightarrow \mathbb{R}^2 \quad (\mathbb{R}^3)$$
$$s \longrightarrow \psi(s)$$

curve regolari:

SI DICE CHE $\varphi$ e $\psi$ sono EQUIVALENTI

SE ESISTE

$$g : I \longrightarrow J$$ di classe $C^1$ invertibile
$$s = g(t)$$

$$g'(t) \neq 0 \quad \forall t \in I$$

ed inoltre

$$\varphi(t) = \psi(g(t)) .$$

**OSS.** Abbiamo che

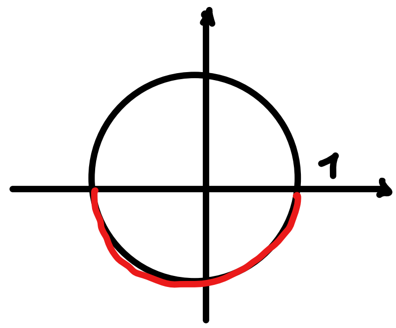
$$g'(t) \gneq 0$$ oppure

$$g'(t) \lneq 0 .$$

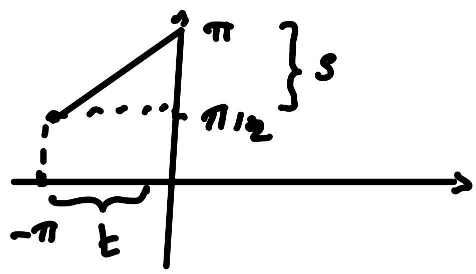
$g(t)$ "CAMBIAMENTO AMMISSIBILE DI PARAMETRO"

ES.

$$\varphi(t) = \begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad t \in [-\pi, 0]$$




$$\psi(s) = \begin{cases} x = \cos(2s) \\ y = \sin(2s) \end{cases} \qquad s \in \left[\frac{\pi}{2}, \pi\right]$$




$$s = g(t) = \underbrace{\frac{t}{2} + \pi}, \quad t \in [\underbrace{-\pi, 0}]$$

OSS. DUE CURVE EQUIVALENTI HANNO LO STESSO SOSTEGNO!!

Verifichiamo che $\qquad \varphi(t) = \psi(g(t))$

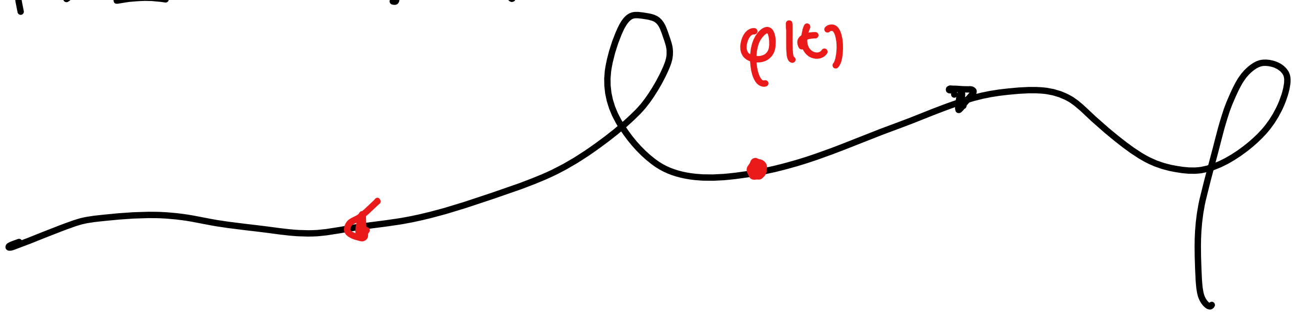
$$\psi(g(t)) = (\cos 2g(t), \sin 2g(t)) =$$

$$= (\cos(t + 2\pi), \sin(t + 2\pi))$$

$$= (\cos t, \sin t) = \varphi(t)$$

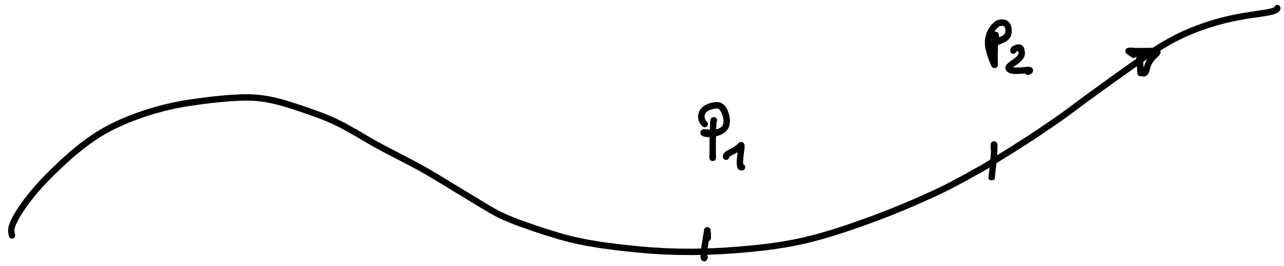
# ORIENTAMENTO DI UNA CURVA

$\varphi : I \longrightarrow \mathbb{R}^2 (\mathbb{R}^3)$




"Si dice che un punto $P_1 = \varphi(t_1)$ precede $P_2 = \varphi(t_2)$ nel verso di percorrenza ($\gamma$ orientata) indotto dal parametro $t$ se

$$\underline{\underline{t_1 < t_2}}$$ ".




ES. (1)

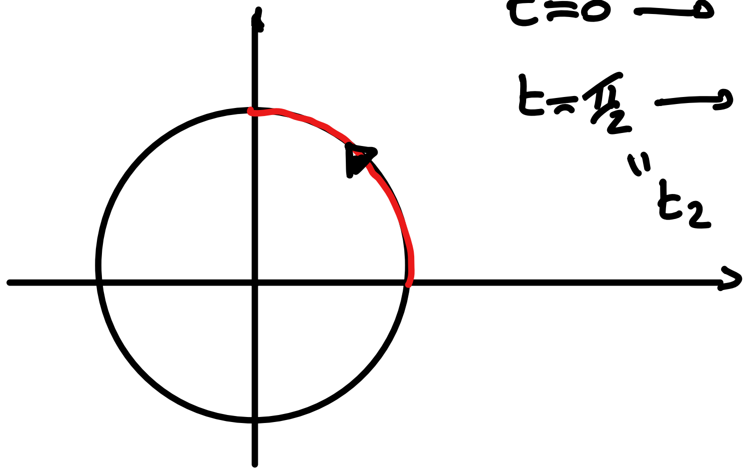
$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad t \in [0, \tfrac{\pi}{2}]$$

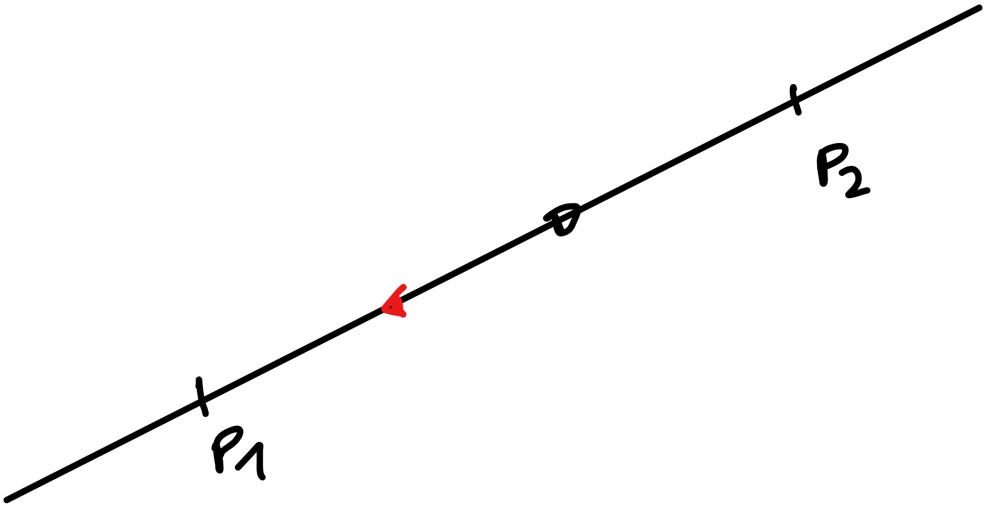
$t_1$

$t = 0'' \longrightarrow (1,0)$ preade

$t = \frac{\pi}{2} \longrightarrow (0,1)$

'' $t_2$

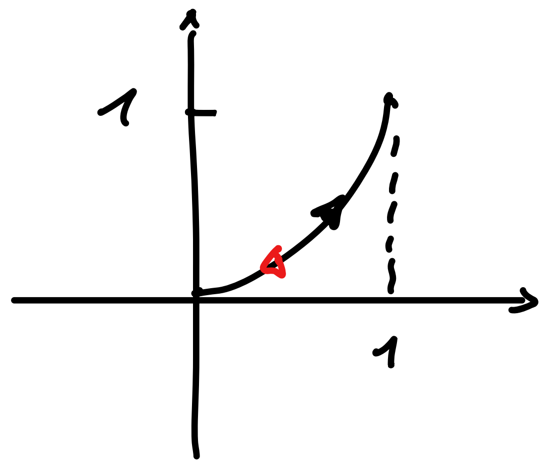





$$\begin{cases} x = t x_2 + (1-t) x_1 \\ y = t y_2 + (1-t) y_1 \end{cases} \qquad t \in [0,1]$$

$$\begin{cases} x = t x_1 + (1-t) x_2 \\ y = t y_1 + (1-t) y_2 \end{cases} \qquad t \in [0,1]$$

$y = x^2$ $\qquad x \in [0,1]$




$$\begin{cases} x = t \\ y = t^2 \end{cases} \qquad t \in [0,1]$$

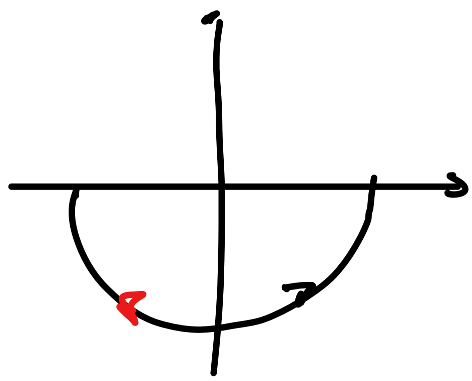
$t = 0 \longrightarrow (0,0)$ precede

$t = 1 \longrightarrow (1,1)$

$$\begin{cases} x = -s \\ y = s^2 \end{cases}$$

$s \in [-1, 0]$

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad | \quad t \in [-\pi, 0]$$



$$\begin{cases} x = \cos s \\ y = -\sin s \end{cases} \qquad s \in [0, \pi]$$

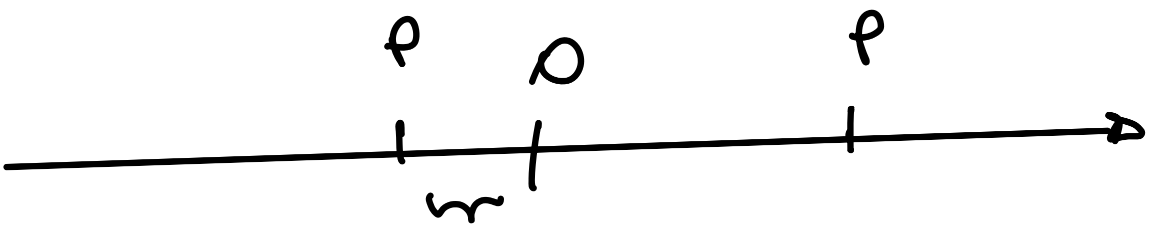
$\odot$

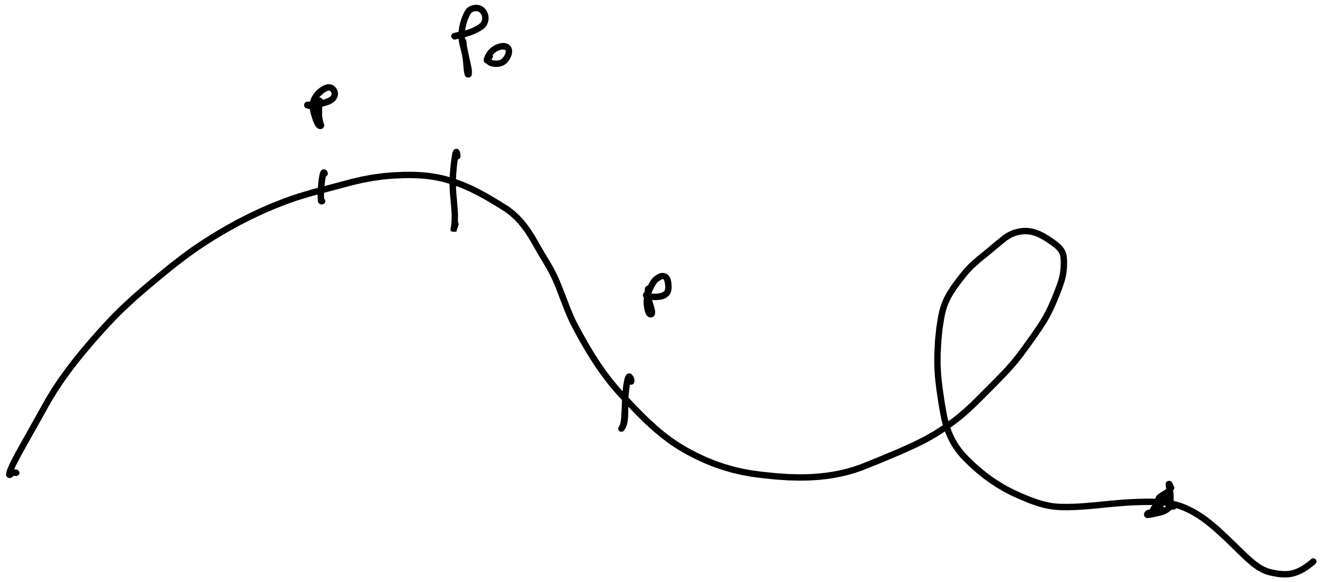
$s = g(t) = -t \qquad g'(t) = -1$

<u>OSS</u> Se $\varphi$ e $\psi$ sono equivalenti, esse inducono lo <u>stesso</u> orientamento se

$$g'(t) \underset{\neq}{>} 0$$

ASCISSA CURVILINEA



$s(P)$ = "ASCISSA CURVILINEA DI P"

$$= \begin{cases} |\overset{\frown}{P_0P}| & \text{se } P \text{ segue } P_0 \\ -|\overset{\frown}{P_0P}| & \text{se } P \text{ precede } P_0 \end{cases}$$

$\varphi(t)$ rappresentazione parametrica "REGOLARE"

ASCISSA CURVILINEA : $t_0 \in [a,b]$

$$s(t) = \int_{t_0}^{t} \|\varphi'(\tau)\| d\tau \quad , t \in [a,b]$$

$P_0 = \varphi(t_0)$ ORIGINE DELLE ASCISSE

$P = \varphi(t)$ : se $P$ segue $P_0$, si ha

$$t > t_0$$

$$\Rightarrow \quad s(t) = \int_{t_0}^{t} \|\varphi'(\tau)\| d\tau = \text{lunghezza dell'arco } \overset{\frown}{P_0P}$$

: se $P$ precede $P_0$, si ha

$t < t_0$

$$s(t) = -\int_t^{t_0} \underbrace{\|\varphi'(\tau)\| d\tau}_{L(\widehat{P_0 P})}$$

$= -$ (lunghezza di $\widehat{P_0 P}$)

<u>OSS.</u> $S'(t) = \|\varphi'(t)\| \not> 0$ (TEOR. FOND. CALCOLO)

$s = s(t)$ strett. crescente

$s : t \in [a, b] \longrightarrow s(t) \in \underbrace{[s(a), s(b)]}$

str. crescente

$t = t(s)$ inversa di $s(t)$

$$t = t(s) : s \in [s(a), s(b)] \xrightarrow{t(s)} t \in [a, b]$$

$$\psi(s) = \varphi(\underbrace{t(s)}_{\text{camb. ammissibile}}) \quad : \quad \psi \text{ e } \varphi \ \underline{\underline{\text{equivalenti}}}$$

$$s'(t) = \|\varphi'(t)\|$$

$$\psi'(s) = \varphi'(t(s)) \cdot \frac{dt}{ds}$$

$$= \varphi'(t(s)) \cdot \frac{1}{\underbrace{s'(t(s))}}$$

$$= \varphi'(t(s)) \cdot \frac{1}{\|\varphi'(t(s))\|}$$

$$\Longrightarrow \quad \|\psi'(s)\| = 1.$$

**<u>Es.</u>**

$$\begin{cases} x = \cos t \\ y = \sin t \\ z = t \end{cases} \qquad t \in [0, 2\pi]$$

$$t_0 = 0$$

$$P_0 = (1, 0, 0)$$

$$\begin{cases} x' = -\sin t \\ y' = \cos t \\ z' = 1 \end{cases}$$

$$s = s(t) = \int_0^t \overbrace{\|\varphi'(\tau)\|}^{\sqrt{2}} d\tau = \sqrt{2}\, t \qquad t \in \underbrace{[0, 2\pi]}$$

$t = s/\sqrt{2}$

$$\begin{cases} x = \cos \frac{s}{\sqrt{2}} \\ y = \sin s/\sqrt{2} \\ z = s/\sqrt{2} \end{cases}$$

$s \in [0, 2\sqrt{2}\pi]$

?

$$\begin{cases} x'(s) = -\frac{1}{\sqrt{2}} \sin\left(s/\sqrt{2}\right) \\ y'(s) = \frac{1}{\sqrt{2}} \cos\left(s/\sqrt{2}\right) \\ z'(s) = \frac{1}{\sqrt{2}} \end{cases}$$

$$(x')^2 + (y')^2 + (z')^2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$\| \varphi'(s) \| = 1$$