Derivata di Funzioni Composte

- $D[f \circ g] = D[f(g(x))] = f'(g(x)) \cdot g'(x)$

- $f(g(x)) = [g(x)]^m \qquad m \in \mathbb{N}$

$$D[[g(x)]^m] = m[g(x)]^{m-1} \cdot g'(x)$$

$f(x) = x^m$

$f'(x) = m x^{m-1}$

- $f(g(x)) = [g(x)]^d \qquad d \in \mathbb{R} \text{ e } d \neq 0$

$$D[[g(x)]^d] = d \cdot [g(x)]^{d-1} \cdot g'(x)$$

Esempio

$$f(x) = (11x^3 - 5x^2 + \log x)^3$$

$$f'(x) = 3(11x^3 - 5x^2 + \log x)^2 \cdot (11 \cdot 3x^2 - 5 \cdot 2x + \frac{1}{x})$$

- $f(g(x)) = \sqrt[m]{g(x)}$

$$D[f(g(x))] = D[\sqrt[m]{g(x)}] = \frac{1}{m\sqrt[m]{[g(x)]^{m-1}}} \cdot g'(x)$$

$$\sqrt[m]{g(x)} = [g(x)]^{\frac{1}{m}}$$

Esempio

$$f(x) = \sqrt{3x^5 - e^x}$$

$$f'(x) = \frac{1}{2\sqrt{(3x^5 - e^x)}} \cdot (12x^3 - e^x) = \frac{12x^3 - e^x}{2\sqrt{3x^5 - e^x}}$$

$$f(x) = (3x^5 - e^x)^{1/2}$$

$$f'(x) = \frac{1}{2} \cdot (3x^5 - e^x)^{\frac{1}{2}-1} \cdot (12x^3 - e^x) = \frac{1}{2}(3x^5 - e^x)^{\frac{-1}{2}}(12x^3 - e^x) =$$

$$= \frac{1}{2} \cdot \frac{1}{2\sqrt{3x^5 - e^x}} \cdot (12x^3 - e^x)$$

$$\frac{1}{(3x^5 - e^x)^{1/2}} = \frac{1}{\sqrt{3x^5 - e^x}}$$

- $f(g(x)) = a^{g(x)} \qquad a > 0$

$$D[a^{g(x)}] = a^{g(x)} \cdot \log a \cdot g'(x)$$

$$D[a^x] = a^x \cdot \log a$$

CASO $a = e$

$$D[e^{g(x)}] = e^{g(x)} \cdot \log e \cdot g'(x) = e^{g(x)} \cdot g'(x)$$

$\log e = 1$

Esempio

$$f(x) = 2^{3x^5 - e^x}.$$

$$f'(x) = 2^{3x^4 - e^x} \cdot \log 2 \cdot (12x^3 - e^x)$$

$$f(x) = e^{3x^4 - e^x}$$

$$f'(x) = e^{3x^4 - e^x} \cdot (12x^3 - e^x)$$

- $$f(g(x)) = \log_a g(x) \qquad a > 0 \text{ e } a \neq 1$$

$$D[f(g(x))] = \frac{1}{g(x)} \cdot \log_a e \cdot g'(x)$$

$$= \frac{g'(x)}{g(x)} \cdot \log_a e$$

$$D[\log_a x] = \frac{1}{x} \cdot \log_a e$$

CASO $a = e$

$$D[f(g(x))] = D[e^{g(x)}] = \frac{1}{g(x)} \cdot \underbrace{\log_e e}_{1} \cdot g'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$= \frac{g'(x)}{g(x)}$$

ESEMPI

$$f(x) = \log_{\frac{1}{2}} (x^2 + 5x + 7)$$

$$f'(x) = \frac{1}{x^2 + 5x + 7} \cdot \log_{\frac{1}{2}} e \cdot (2x + 5) = \frac{2x + 5}{x^2 + 5x + 7} \cdot \log_{\frac{1}{2}} e$$

$$f(x) = \log(5x^2 - 3x + 5^x)$$

$f(x) = \log(g(x))$

$$f'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$f'(x) = \frac{1}{5x^2 - 3x + 5^x} \cdot (10x - 3 + 5^x \cdot \log 5)$$

$$= \frac{10x - 3 + \log 5 \cdot 5^x}{5x^2 - 3x + 5^x}$$

funzione + esterna = logaritmo

$$f(x) = \log\left(\frac{5x^3 - 6x^2}{3x - 1}\right) = \frac{1}{\frac{5x^3 - 6x^2}{3x - 1}} \cdot \left(\frac{5x^3 - 6x^2}{3x - 1}\right)' =$$

$$= \frac{3x - 1}{5x^3 - 6x^2} \cdot \left(\frac{5x^3 - 6x^2}{3x - 1}\right)'$$

$$\frac{1}{\frac{5x^3 - 6x^2}{3x - 1}} = 1 \cdot \frac{3x - 1}{5x^3 - 6x^2}$$

$$D\left[\frac{f}{g}\right] = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \frac{3x - 1}{5x^3 - 6x^2}$$

$$= \frac{3x - 1}{5x^3 - 6x^2} \cdot \frac{(5x^3 - 6x^2)'(3x - 1) - (5x^3 - 6x^2)(3x - 1)'}{(3x - 1)^2} =$$

$$= \frac{3x - 1}{5x^3 - 6x^2} \cdot \frac{(5 \cdot 3 \cdot x^2 - 6 \cdot 2x)(3x - 1) - (5x^3 - 6x^2)(3)}{(3x - 1)^2}$$

Funz. + esterna =

rapporto

$$D\left[\frac{f}{g}\right] = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$f(x) = \frac{5x^3 - \log(11x-2)}{x^2-3x+1}$$

$$f'(x) = \frac{(5x^3 - \log(11x-2))' \cdot (x^2-3x+1) - (5x^3 - \log(11x-2)) \cdot (x^2-3x+1)'}{(x^2-3x+1)^2}$$

$$= \frac{\left(15x^2 - \frac{11}{11x-2}\right)(x^2-3x+1) - (5x^3 - \log(11x-2)) \cdot (2x-3)}{(x^2-3x+1)^2}$$

$$D[f \cdot g] = f' \cdot g + f \cdot g'$$

1) $f(x) = \log(x^2+5x+7) + e^{3x+1} \cdot \sqrt{x^2+4}$

$$f'(x) = \frac{1}{x^2+5x+7} \cdot (2x+5) + (e^{3x+1})' \sqrt{x^2+4} + e^{3x+1} (\sqrt{x^2+4})'$$

$$(x^2+4)^{\frac{1}{2}}$$

$$= \frac{2x+5}{x^2+5x+7} + \left(e^{3x+1}\cdot 3\right)\sqrt{x^2+4} + e^{3x+1}\cdot\frac{1}{2}\cdot(x^2+4)^{-\frac{1}{2}}\cdot(2x)$$

$$= \frac{2x+5}{x^2+5x+7} + 3e^{3x+1}\cdot\sqrt{x^2+4} + e^{3x+1}\cdot\frac{x}{(x^2+4)^{1/2}}$$

$$= \frac{2x+5}{x^2+5x+7} + 3e^{3x+1}\cdot\sqrt{x^2+4} + e^{3x+1}\cdot\frac{x}{\sqrt{x^2+4}}$$

30) $f(x) = \dfrac{e^{5x+\sqrt{x}}}{3x+1} + e^{6x^2-3x}\cdot\log(5x^2+6)$

$$f'(x) = \frac{\left(e^{5x+\sqrt{x}}\right)'(3x+1) - \left(e^{5x+\sqrt{x}}\right)(3x+1)'}{(3x+1)^2} +$$

$$+ \left(e^{6x^2-3x}\right)'\cdot\log(5x^2+6) + e^{6x^2-3x}\cdot\left(\log(5x^2+6)\right)' =$$

$$= f'(x) = \frac{\left(e^{5x+\sqrt{x}}\cdot\left(5+\frac{1}{2\sqrt{x}}\right)\right)(3x+1) - 3e^{5x+\sqrt{x}}}{(3x+1)^2} +$$

$$+ \left(e^{6x^2-3x}\cdot(12x-3)\right)\cdot\log(5x^2+6) + e^{6x^2-3x}\cdot\frac{10x}{5x^2+6}$$

$\sqrt[n]{x}$ $m=2$

$$D[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$D[x^{1/2}] = .$

$$D[x^{1/2}] = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}}$$

$$= \frac{1}{2\sqrt{x}}$$