Esercitazione

16) $$\lim_{x\to 0}\left(\underbrace{\frac{\log(x+1)}{x}}_{\frac{0}{0}}+\underbrace{\frac{e^x-1}{x}}_{\frac{0}{0}}\right)=$$

$$\lim_{x\to 0}\frac{\log(x+1)}{x}=1$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

$$=(1+1)=2$$

17) $$\lim_{x\to+\infty}\frac{3x^5+5x^8}{e^{3x+1}}=\lim_{x\to+\infty}\frac{5x^8}{e^{3x}}=\frac{+\infty}{+\infty}=0^+=0$$

Limite notevole

$$\lim_{x\to+\infty}\frac{x^\alpha}{a^x}=0 \qquad \alpha>0,\ a>0$$

21) $$\lim_{x\to 0}\left(\underbrace{\frac{\log(x+1)}{x}}_{1}\right)^2=(1)^2=1$$

$$\lim_{x\to 0}\left(\underbrace{\frac{\log_2(x+1)}{x}}_{\log_2 e}\right)^2=(\log_2 e)^2=\log_2 e\cdot\log_2 e=\log_2^2 e$$

$$(\log_2 e)^2 \neq \log_2 e^2$$

$a=2$

$$\lim_{x\to 0}\frac{\log_a(x+1)}{x}=\log_a e$$

25) $$\lim_{x\to 0}\left(\frac{\log(4x^5+1)}{4x^5} + \frac{e^{3x}-1}{3x} + \log(5x^2+4)\right) =$$

$$\frac{\log(4x^5+1)}{4x^5} \to \frac{0}{0} \qquad \frac{e^{3x}-1}{3x} \to \frac{0}{0}$$

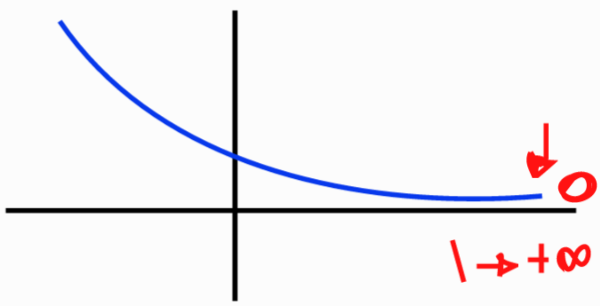
$$\lim_{x\to 0}\frac{\log(x+1)}{x} = 1 \qquad \lim_{x\to 0}\frac{e^{x}-1}{x} = 1$$

$$\lim_{x\to 0}\left(\underbrace{\frac{\log(4x^5+1)}{4x^5}}_{\downarrow\, 1} + \underbrace{\frac{e^{3x}-1}{3x}}_{\downarrow\, 1} + \underbrace{\log(5x^2+4)}_{\log 4}\right) =$$

$$= 1 + 1 + \log 4 = 2 + \log 4$$

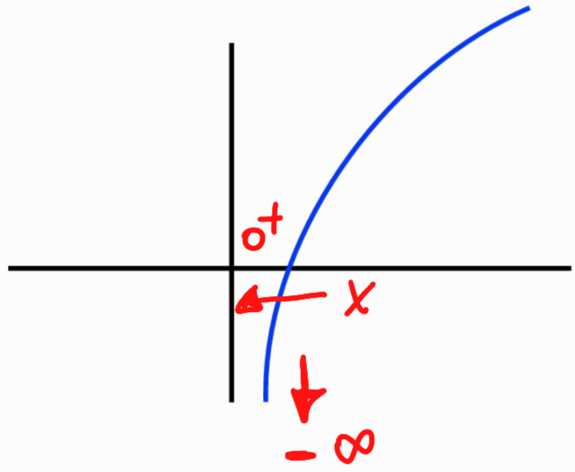
$$\lim_{x\to+\infty}\left(\frac{1}{2}\right)^{\underbrace{\sqrt{x}}_{\downarrow\, x^{1/2}} + 7x^3 + 4x^2} =$$

$$= \lim_{x\to+\infty}\left(\frac{1}{2}\right)^{7x^3} = \left(\frac{1}{2}\right)^{7(+\infty)^3} = \left(\frac{1}{2}\right)^{+\infty} = 0$$



• $\lim\limits_{x\to 2^+} \log_3\left(x^2-4\right) = \log_3\left(0^+\right) = -\infty$

$x \to 2^+ \downarrow 2.1 \qquad x^2-4 \downarrow 4^+ \quad \underbrace{\quad}_{0.41}$



Prova A

Es. 1

$$f(x) = \log\left(\frac{x^2-6x+8}{3-x}\right)$$

$2^- \to 2 \qquad 3 \qquad 4$

$$E[f(x)] = \left\{x \in \mathbb{R} : \frac{x^2-6x+8}{3-x} > 0\right\} =]-\infty, 2[\ \cup\ ]3,4[$$

$2 \quad 4$

$$\frac{x^2-6x+8}{3-x} > 0 \iff \begin{cases} x^2-6x+8>0 \\ 3-x>0 \end{cases} \cup \begin{cases} x^2-6x+8<0 \\ 3-x<0 \end{cases}$$

3

• $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \log\left(\frac{x^2-6x+8}{3-x}\right) = \lim\limits_{x\to-\infty} \log\left(\frac{\cancel{x^2}}{-\cancel{x}}\right) =$

$= \lim\limits_{x\to-\infty} \log(-x) = \log(-(-\infty)) =$

$$= \log(+\infty) = +\infty$$

0

- $$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \log\left(\frac{x^2-6x+8}{3-x}\right) = \log\left(\frac{(2^-)^2-6(2^-)+8}{3-2^-}\right)$$

1.9   1

$$3.61 - 11.4$$

$$= \log\left(\frac{-7.79+8}{1}\right) = \log\left(\frac{0^+}{1}\right) = \log(0^+)$$

$$= -\infty$$

-10

$9 - 18 + 8$

$9 - 6\cdot 3 + 8$

- $$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \log\left(\frac{x^2-6x+8}{3-x}\right) = \log\left(\frac{-1}{3-3.1}\right) =$$

3.1

$$= \log\left(\frac{-1}{0^-}\right) = \log(+\infty) = +\infty$$

0

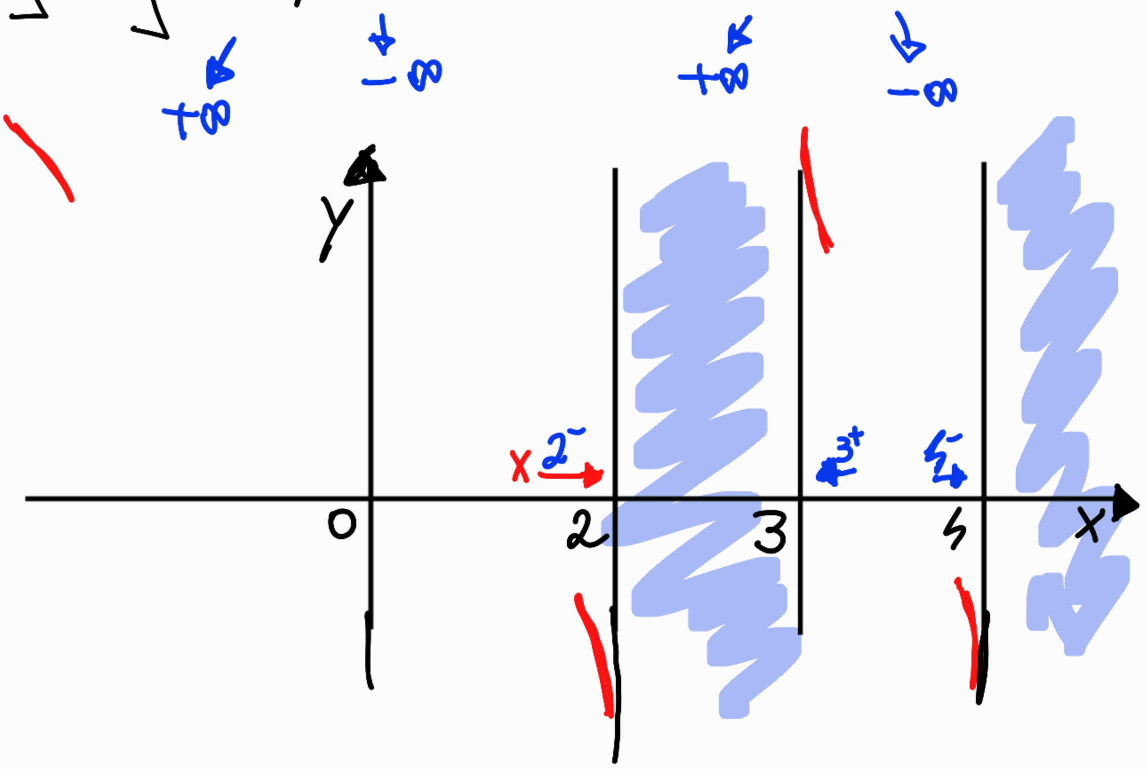
$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} \log\left(\frac{x^2-6x+8}{3-x}\right) = \log\left(\frac{(5^-)^2-6(5^-)+8}{-1}\right) =$$

3.9 -1

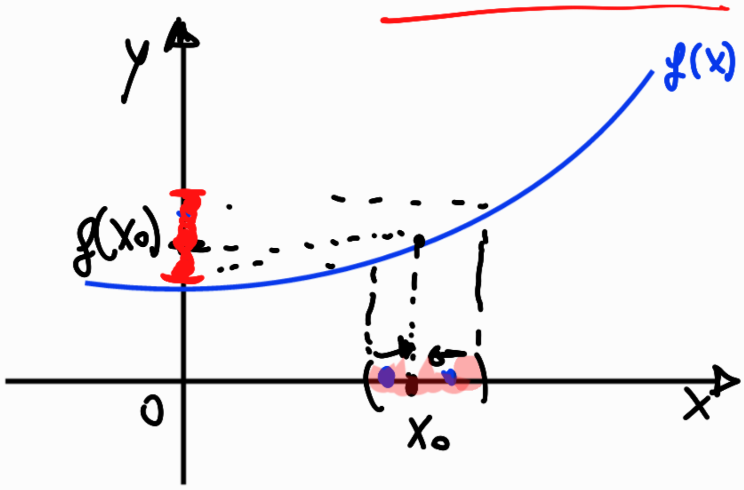
$$= \log\left(\frac{-0.19}{-1}\right) = \log\left(\frac{0^-}{-1}\right) = \log(0^+)$$

$= -\infty$

$E[f(x)] =]-\infty, 2[\cup]3, 5[$



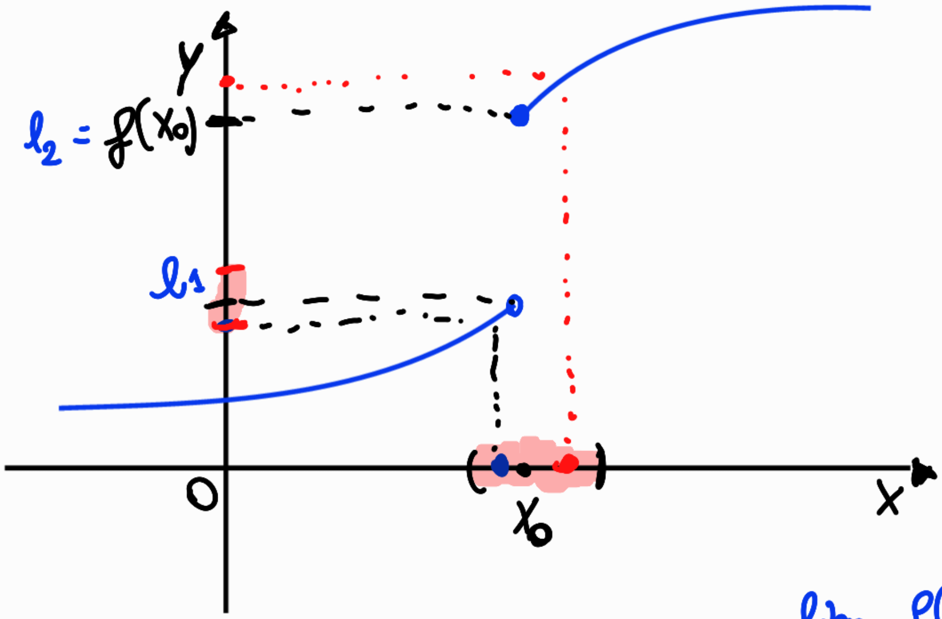
Continuità



$$\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^+} f(x) = f(x_0)$$

$X =]-\infty, +\infty[$

$x_0 \in X$



$X =]-\infty, +\infty[$

$x_0 \in X$

$$\lim_{x\to x_0^-} f(x) = l_1 \neq \lim_{x\to x_0^+} f(x) = l_2 = f(x_0)$$

x_0 è un punto di discontinuità

Definizione di funzione continua in un punto

Sia $f: X \subseteq \mathbb{R} \to \mathbb{R}$ e $x_0 \in X$.

(non ha senso valutare la continuità in punti $\notin X$)

Si dice che f è continua in x_0 se:

$$\boxed{\forall J \in \mathcal{I}(f(x_0)),\ \exists\, I \in \mathcal{I}(x_0) : \forall x\,(X \cap I) \Rightarrow f(x) \in J}$$

intorno dell'immagine $f(x_0)$

famiglia degli intorni di $f(x_0)$

intorno di x_0

famiglia degli intorni di x_0