Esercitazione sui limiti

1) $$\lim_{x\to+\infty}\left(\frac{3x^2+5x^5-6x}{5x^3-7}+\log(x^3+1)\right)=$$

$$=\lim_{x\to+\infty}\left(\frac{5x^{\cancel{5}}\,x^2}{\cancel{5x^3}_{\,1}}+\log(x^3+1)\right)=$$

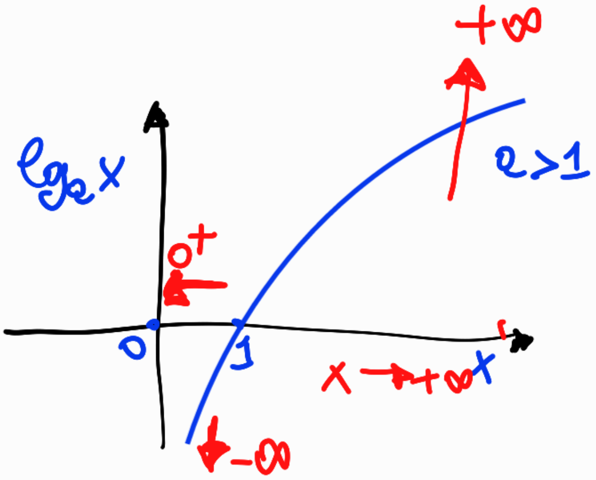
$$=\lim_{x\to+\infty}\left(\frac{5}{5}\underbrace{x^2}_{+\infty}+\log(\underbrace{x^3+1}_{(+\infty)})\right)=$$

$$\boxed{\frac{x^5}{x^3}=x^{5-3}=x^2}$$

$$=\frac{5}{5}\underbrace{(+\infty)^2}_{+\infty}+\log\left(\underbrace{(+\infty)^3}_{+\infty}+1\right)=\underbrace{\frac{5}{5}(+\infty)}_{+\infty}+\log(\underbrace{+\infty+\cancel{1}}_{+\infty})=$$

$$=+\infty+\underbrace{\log(+\infty)}_{+\infty}=$$

$$=+\infty+\infty=+\infty$$



2) $$\lim_{x\to+\infty}\left(\frac{5x^3-5x^2-4x}{8x^5-4x^3+2}+\log(3x^5+4x^3)+e^{7x+1}\right)=$$

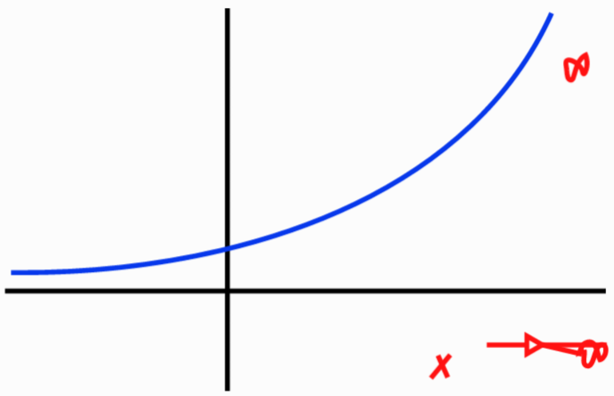
$$=\lim_{x\to+\infty}\left(\frac{5\cancel{x^3}^{\,1}}{8x^{\cancel{5}\,2}}+\log(3x^5)+e^{7x}\right)=$$

$$\boxed{\frac{x^3}{x^5}=x^{-2}=\frac{1}{x^2}}$$

$$=\lim_{x\to+\infty}\left(\frac{5}{8}\cdot\frac{1}{x^2}+\log(3x^5)+e^{7x}\right)=$$

$$=\frac{5}{8}\cdot\underbrace{\underbrace{\frac{1}{\underbrace{(+\infty)^2}_{\to+\infty}}}_{\to 0^+}}_{0}+\log\underbrace{\left(3\underbrace{(+\infty)^5}_{+\infty}\right)}_{+\infty}+e^{\overbrace{7(+\infty)}^{+\infty}}=0+\log(+\infty)+e^{+\infty}=$$

$$=\underbrace{\log(+\infty)}_{+\infty}+\underbrace{e^{+\infty}}_{+\infty}=+\infty+\infty=+\infty$$



5) $$\lim_{x\to+\infty}\left(\frac{\log(3x^2+3)}{3x^2+3}+e^{4x^3-5x+1}\right)=$$

Non serve

$$\lim_{x\to0}\frac{\log(x+1)}{x}=1$$

$$=\lim_{x\to+\infty}\left(\frac{\log(3x^2)}{3x^2}+e^{4x^3}\right)=$$

$$= \lim_{x\to+\infty} \left(\frac{\log(\overset{y}{3x^2})}{(3x^2)^1} + e^{5x^3} \right) =$$

$\to 0$

Posto $\boxed{3x^2 = y}$

$$\lim_{y\to+\infty} \frac{\log(y)}{y}$$

$$\boxed{\lim_{x\to+\infty} \frac{\log(3x^2)}{3x^2} = \boxed{\lim_{y\to+\infty} \frac{\log(y)}{y^1}} = 0}$$

$x \to +\infty$

$3x^2 = y$

$y = 3\underset{+\infty}{x}^2 \to +\infty$

$x \to +\infty$

$y = 3x^2 = 3(+\infty) = +\infty$

$$= \lim_{x\to+\infty} \left(\frac{\log(3x^2)}{3x^2} + e^{5x^3} \right) = 0 + e^{\overbrace{5\overbrace{(+\infty)^3}^{+\infty}}^{+\infty}} = e^{+\infty} = +\infty$$

$(3x^2)^1$

$$\boxed{\lim_{x\to\infty} \frac{\log(x)}{\cancel{x^\alpha}} = 0}$$

$\alpha > 0$

$\alpha \neq 2$

$3x^2$ $\quad \alpha = 1$

$$\lim_{y\to+\infty} \frac{\log(y)}{y^\alpha} = 0 \qquad \alpha > 0$$

$3x^2 = y$

$$\lim_{x\to+\infty} \frac{\log(x)}{x^\alpha} = 0 \; ; \; \lim_{x\to+\infty} \frac{\log(3x^2)}{(3x^2)^1} = 0$$

⑥

• $\lim_{x\to 0^+} (5x^3+4x^2+2x) \cdot \log(x^3+2x^2) = 0^+ \cdot \log(0^+) =$

$+\infty$ $\quad$ $-\infty$

$= 0^+(-\infty)$ f.i.

$f(x) = \log x$

$E[f(x)] =]0, +\infty[$

$= \lim_{x\to 0^+} (2x) \cdot \log(2x^2) =$

$= \lim_{x\to 0^+} \underbrace{(2x)}_{0} \underbrace{\log(2x^2)}_{-\infty} = 0$

$$\boxed{\lim_{x\to 0^+} x^{\alpha} \log_a x = 0 \quad \alpha > 0}$$

$E[f(x)] = \mathbb{R} - \{-4, 0\}$

• 8) $\lim_{x\to -4^+} \frac{3x^2+5x+2}{x^2+4x} =$

$\boxed{(-3.9)}$

$= \frac{3(-4)^2+5(-4)+2}{(-4^+)^2+4(-4^+)} = \frac{48-20+2}{+} =$

$-4 \quad -4^+ \quad 0 \quad \mathbb{R}$

-3.99

$+$

$= \frac{30}{0^-} = -\infty$

$0 \quad 3^- \quad 3 \quad 3^+$

$2.9 \quad 3.1$

$$\lim_{x\to -4^+} \frac{3x^2+5x+2}{x^2+4x} = \frac{30}{(-3.9)^2+4(-3.9)} = \frac{30}{+15.21-15.6}$$

$$= \frac{30}{-39} \simeq \frac{30}{0^-}$$

$$E[f(x)] = \mathbb{R} - \{-4, 0\}$$

9) $$\lim_{x\to -4^-} \frac{3x^2+5x+2}{x^2+4x} =$$

$$= \frac{3(\overset{16}{-4})^2+5(-4)+2}{(-4^-)^2+4(-4^-)} = \frac{48-20+2}{+16.81-16.4} =$$

-4.1

$$= \frac{30}{0.41} = \frac{30}{0^+} = +\infty$$

-4^- $\quad -4 \quad 0$

-4.1