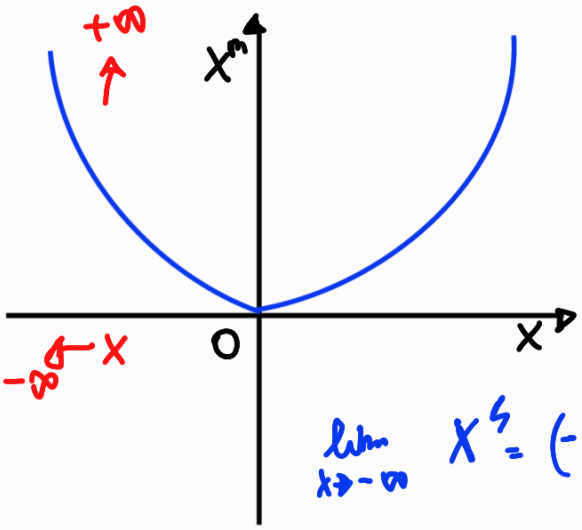
Limiti delle funzioni elementari

- **Funzione potenza** $f(x) = x^m$, $m \in \mathbb{N}$ $\quad X =]-\infty, +\infty[$ (estremo $\dagger$ incluso, estremo 2 incluso)

m pari

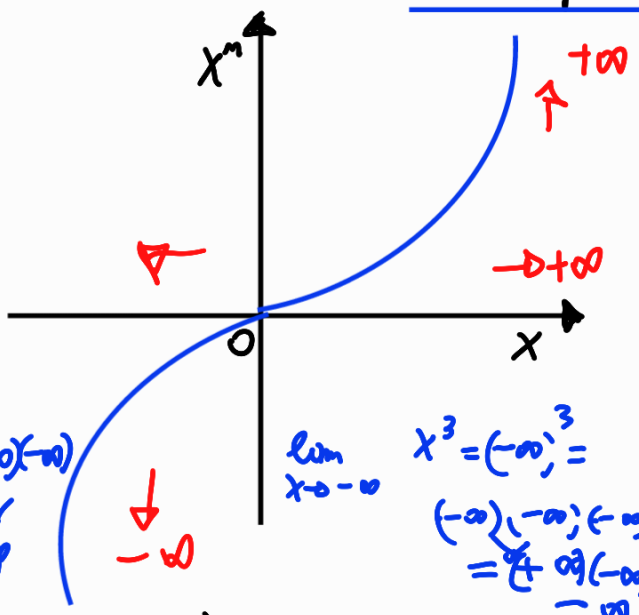


$\lim_{x \to -\infty} x^4 = (-\infty)^4 = (-\infty)(-\infty)(-\infty)(-\infty) = (+\infty)(+\infty) = +\infty$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^m = +\infty$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^m = +\infty$

m dispari



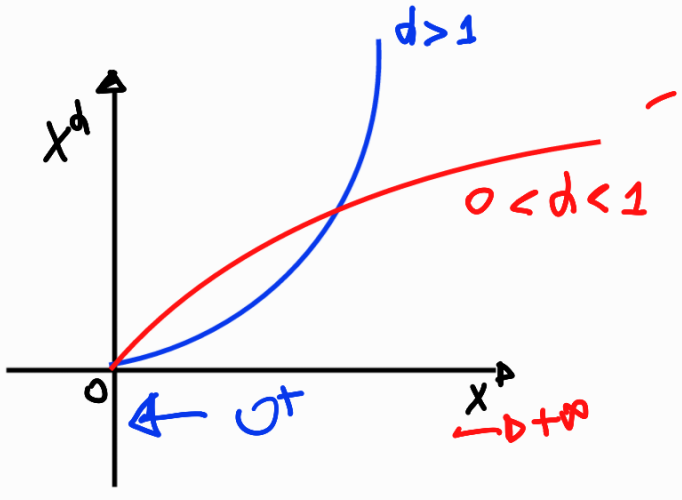
$\lim_{x \to -\infty} x^3 = (-\infty)^3 = (-\infty)(-\infty)(-\infty) = (+\infty)(-\infty) = -\infty$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^m = -\infty$

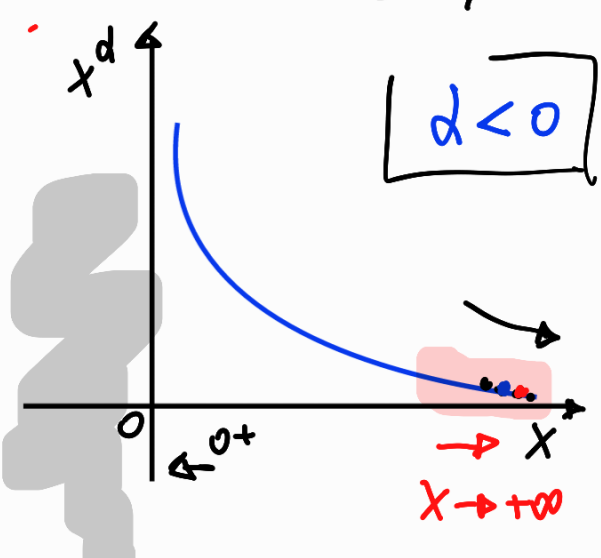
$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^m = +\infty$

- **Funzione potenza ad esponente reale** $f(x) = x^\alpha$, $\alpha \in \mathbb{R}$, $\alpha \neq 0$

$X = [0, +\infty[$ $\quad \alpha > 0$



$X =]0, +\infty[$



$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^\alpha = f(0) = 0^\alpha = 0$

$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} x^\alpha = +\infty$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^\alpha = +\infty$

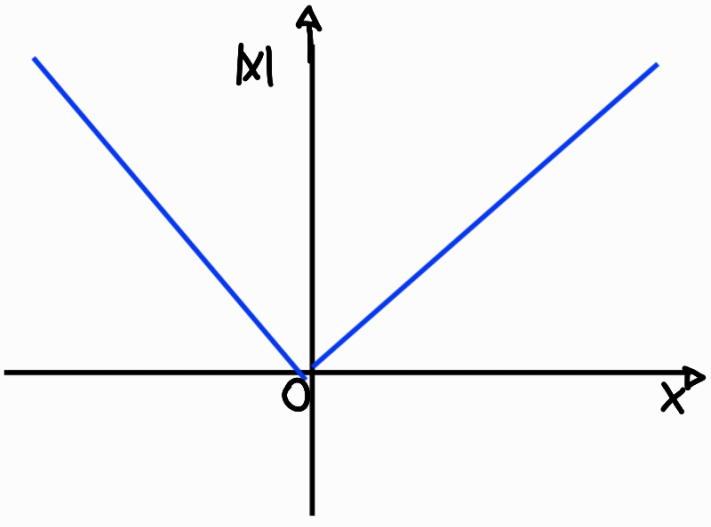
$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} x^\alpha = 0$

$\lim_{x\to 0^+} x^{2.3} = 0^{2.3} = 0$

$\lim_{x\to +\infty} x^{2.3} = (+\infty)^{2.3} = +\infty$

$\lim_{x\to 0^+} x^{-2} = \lim_{x\to 0^+} \frac{1}{x^2} = \frac{1}{(0^+)^2} = \frac{1}{+\infty} = 0$

- **Funzione valore assoluto** $f(x) = |x|$ $\quad X =]-\infty, +\infty[$

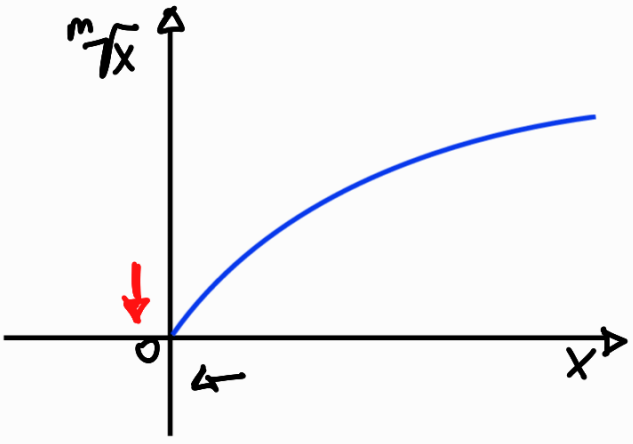


$\lim_{x\to +\infty} \frac{1}{x^2} = \frac{1}{(+\infty)^2} = \frac{1}{+\infty} = 0$

$\lim_{x\to -\infty} |x| = +\infty$

$\lim_{x\to +\infty} |x| = +\infty$

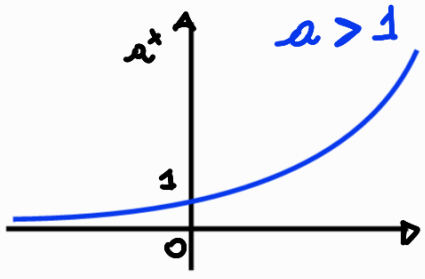
Funzione radice: $f(x) = \sqrt[n]{x}$, $n \in \mathbb{N}$ $\quad X = [0, +\infty[$



$\lim_{x\to 0^+} \sqrt[n]{x} = f(0) = \sqrt[n]{0} = 0$

$\lim_{x\to +\infty} \sqrt[n]{x} = +\infty$

- **Funzione esponenziale** $f(x) = a^x$, $a > 0$ $\quad X =]-\infty, +\infty[$

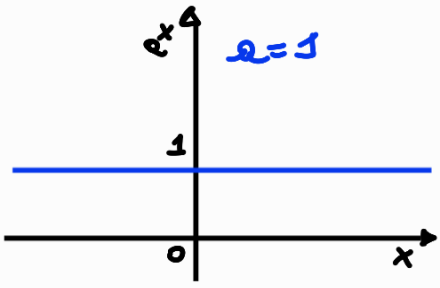


$\lim_{x \to -\infty} a^x = 0$

$\lim_{x \to +\infty} a^x = +\infty$

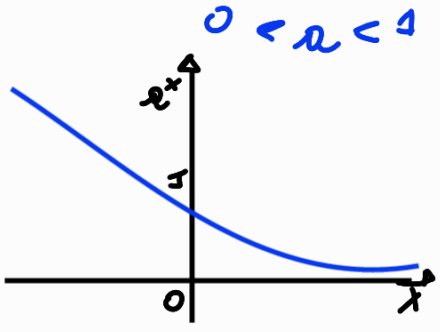
$3^{-\infty} = 0$

$e^{-\infty} = 0$



$\lim_{x \to -\infty} a^x = 1$

$\lim_{x \to +\infty} a^x = 1$

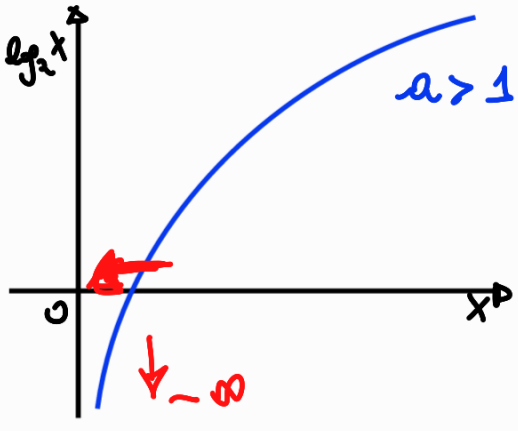


$\lim_{x \to -\infty} a^x = +\infty$

$\lim_{x \to +\infty} a^x = 0$

$\left(\frac{1}{2}\right)^{-\infty} = +\infty$

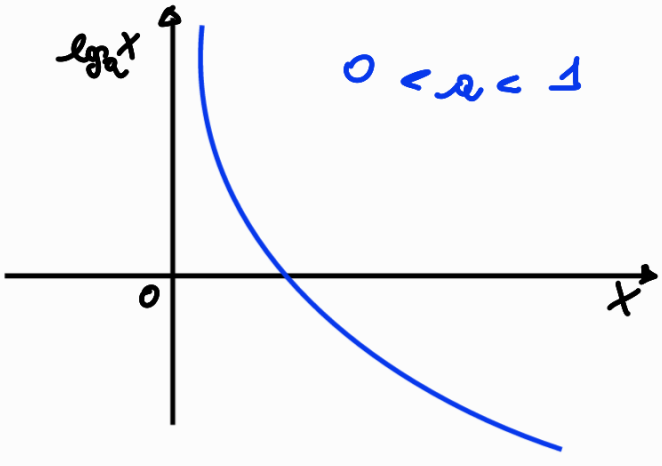
Funzione Logaritmica $f(x) = \log_a x$, $a > 0$ e $a \neq 1$ $\quad X =]0, +\infty[$



$\lim_{x \to 0^+} \log_a x = -\infty$

$\boxed{\log_a(0^+) = -\infty}$

$\lim_{x \to +\infty} \log_a x = +\infty$



$\lim_{x \to 0^+} \log_a x = +\infty$

$\lim_{x \to +\infty} \log_a x = -\infty$

NOTA

Vedi le trigonometriche !

Operazioni in $\mathbb{R}$ ampliato

Somma

- $+\infty + (+\infty) = +\infty + \infty = +\infty$
- $-\infty + (-\infty) = -\infty - \infty = -\infty$
- $a + (+\infty) = a + \infty = +\infty$
  $a \in \mathbb{R}$
- $a + (-\infty) = a - \infty = -\infty$

Es.

$-5 + \infty = +\infty$

$\sqrt{2} + \infty = +\infty$

$+2 - \infty = -\infty$

$-5 - \infty = -\infty$

Differenza

- $+\infty - (+\infty) = +\infty - \infty$ non definita *
- $+\infty - (-\infty) = +\infty + \infty = +\infty$
- $(-\infty) - (-\infty) = -\infty + \infty$ non definita *
- $(-\infty) - (+\infty) = -\infty - \infty = -\infty$
- $a - (+\infty) = a - \infty = -\infty$
- $a - (-\infty) = a + \infty = +\infty$

$a \in \mathbb{R}$

$5 - (+\infty) = 5 - \infty = -\infty$

$-3 - (+\infty) = -3 - \infty = -\infty$

$5 - (-\infty) = 5 + \infty = +\infty$

$-3 - (-\infty) = -3 + \infty = +\infty$

Prodotto

$(+\infty)(+\infty) = +\infty$

$(-\infty)(-\infty) = +\infty$

$(+\infty)(-\infty) = -\infty$

$(-\infty)(+\infty) = -\infty$

Se $a > 0$

$a(+\infty) = +\infty$

$a(-\infty) = -\infty$

Se $a < 0$

$a(+\infty) = -\infty$

$a(-\infty) = +\infty$

Esempio

$\frac{1}{2} \cdot (+\infty) = +\infty$

$-\sqrt{2}(+\infty) = -\infty$

$7(-\infty) = -\infty$

$-\frac{5}{3}(-\infty) = +\infty$

Se $a = 0$

$a(\pm\infty)$ non è definita *