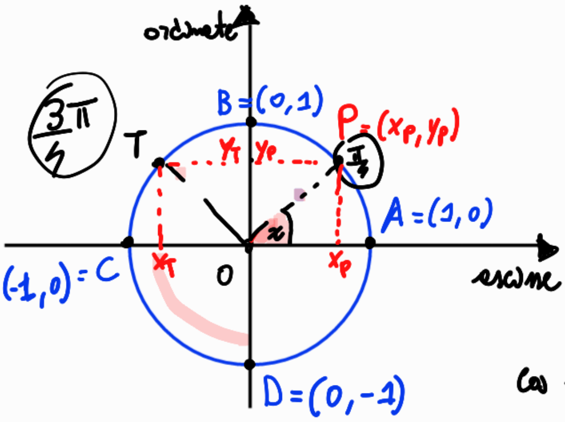
Trigonometria - Parte II



X = un angolo (misurato in senso antiorario)

$\boxed{\cos X = X_p}$

$\boxed{\operatorname{sen} X = Y_p}$

$\boxed{tg X = \frac{\operatorname{sen} X}{\cos X}}$

$\boxed{cotg X = \frac{\cos X}{\operatorname{sen} X}} = \frac{1}{tg X} = (tg x)^{-1}$

$\cos X = 0$

$X = \frac{\pi}{2} + K\pi$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad$ $\operatorname{sen} \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos \frac{3}{4}\pi = -\frac{\sqrt{2}}{2}$ $\quad$ $\operatorname{sen} \frac{3}{4}\pi = \frac{\sqrt{2}}{2}$

x (angolo in gradi)	x (angolo in radianti)	cos x	sen x	tg x	cotg x
0°	0	1	0	0	∄
90°	$\pi/2$	0	1	∄	0
180°	π	-1	0	0	∃
270°	$\frac{3}{2}\pi$	0	-1	∄	0
360°	2π	1	0	0	∄
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$

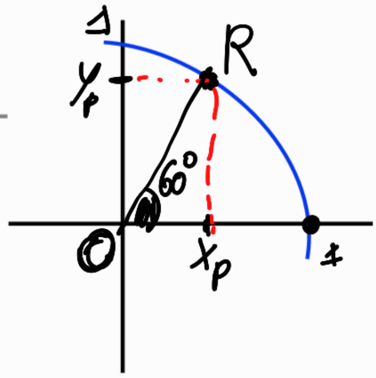
$\frac{0}{-1} = 0$

$\frac{1}{0}$

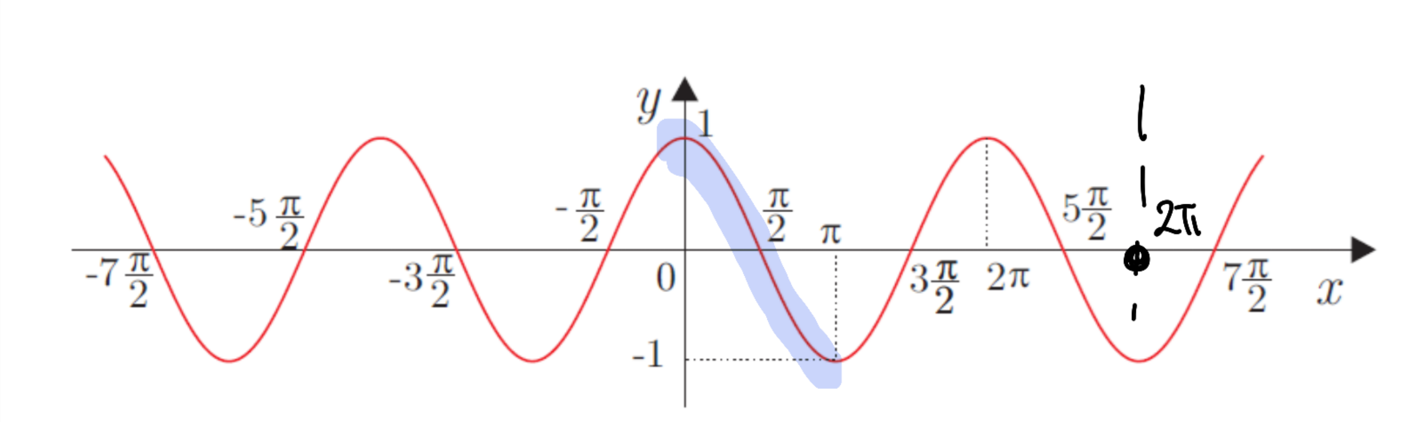
$\frac{1}{0}$

$tg x = \frac{\operatorname{sen} x}{\cos x}$

$\frac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}} = \frac{1}{\sqrt{3}}$

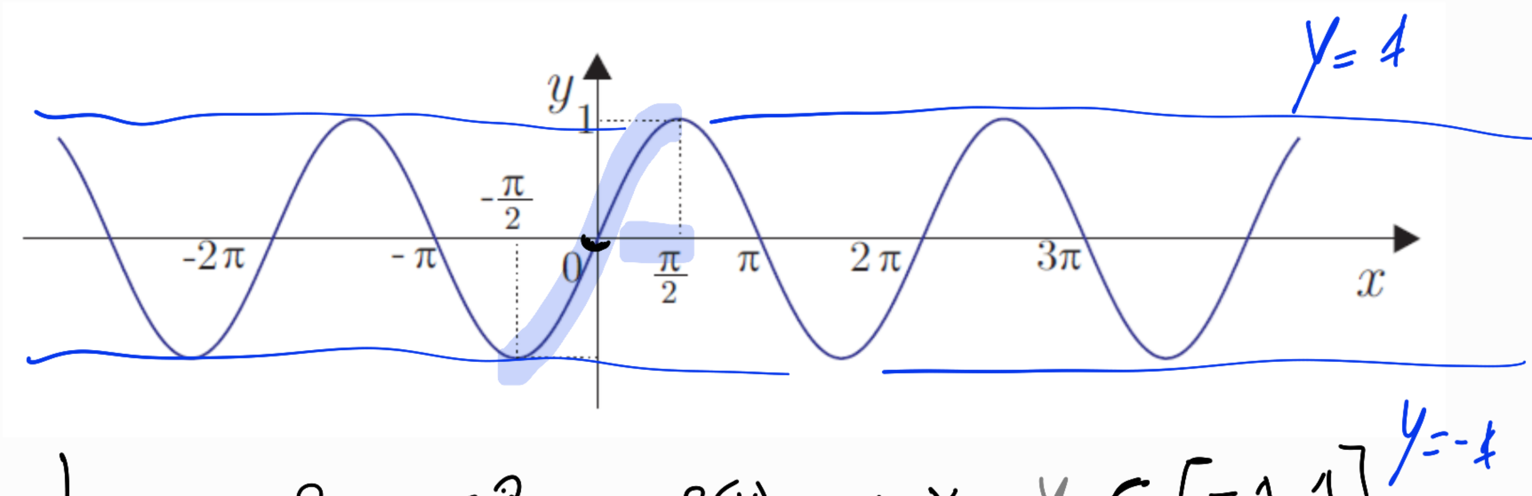


Funzione Coseno



Legge: $f: \underbrace{x}_{\text{angolo espresso in radianti}} \in \mathbb{R} \rightarrow f(x) = \cos x = \underbrace{x_P}_{P \in \text{circonferenza trigonometrica}} \in [-1, 1]$

Funzione Seno

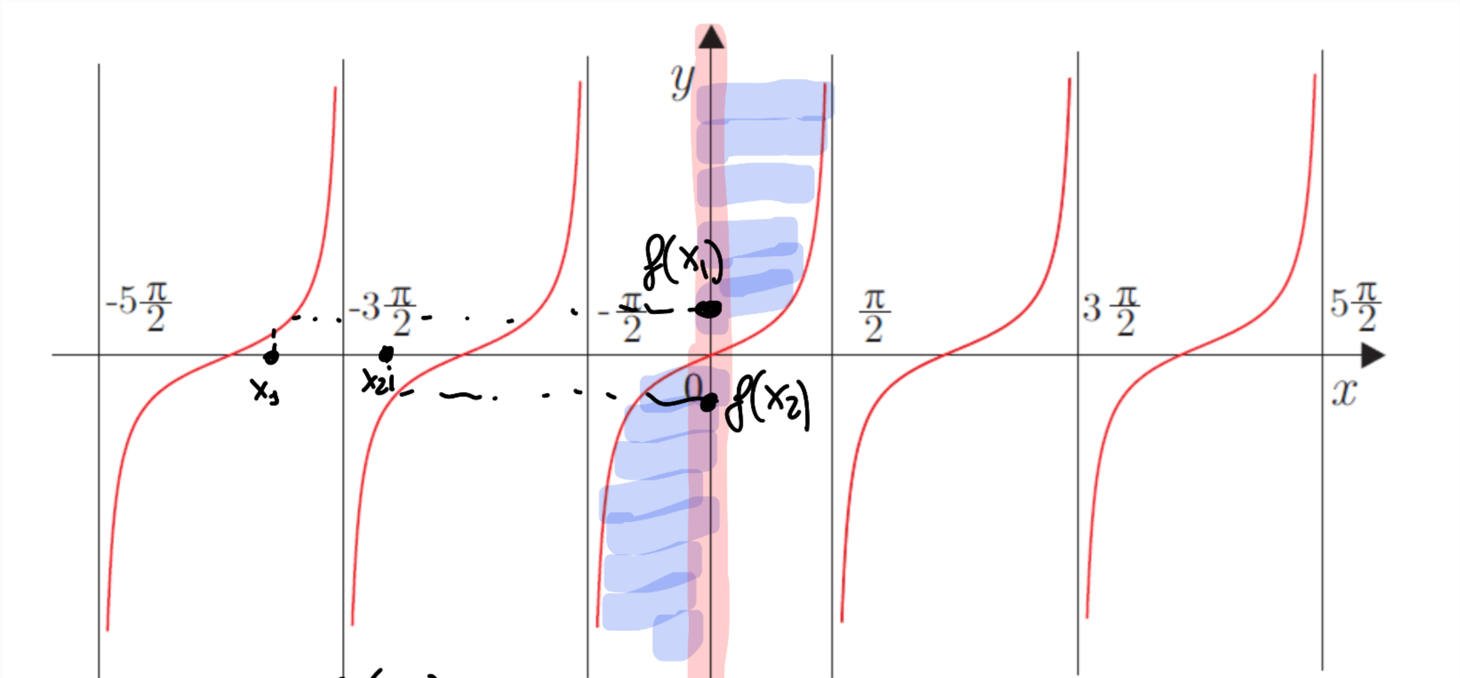


Legge: $f: x \in \mathbb{R} \to f(x) = \operatorname{sen} x = y_P \in [-1, 1]$

x ↓ angolo in radianti

y_P ↓ $P \in$ circonferenza goniometrica

Funzione tangente (Periodica di periodo π)

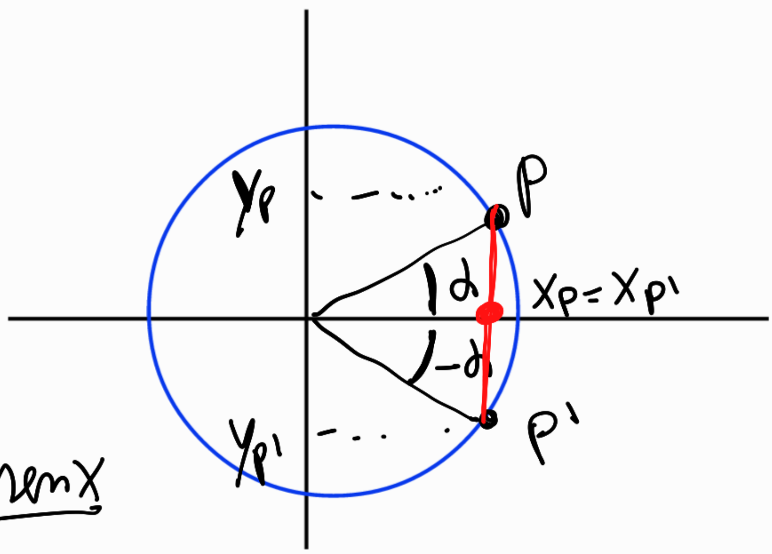


$x_1 < x_2 \qquad f(x_1) > f(x_2)$

Legge:

$$f: x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \right\} \to f(x) = tg\,x = \frac{sen\,x}{cos\,x} \in \mathbb{R}$$

$$Y =]-\infty, +\infty[$$



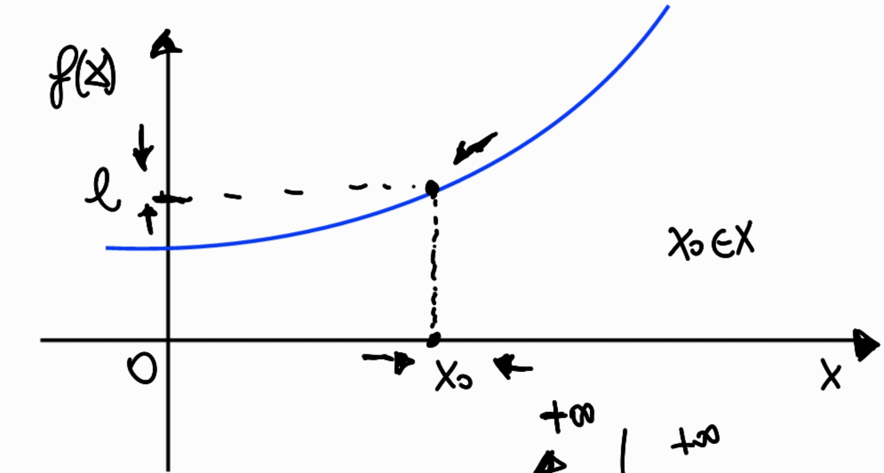
$$tg\,x = \frac{sen\,x}{cos\,x}$$

Limiti → (Riguardano l' <u>immagine</u> di una funzione)

$\Downarrow$

(hanno a che fare con il codominio)

Esempi grafici



limite sinistro

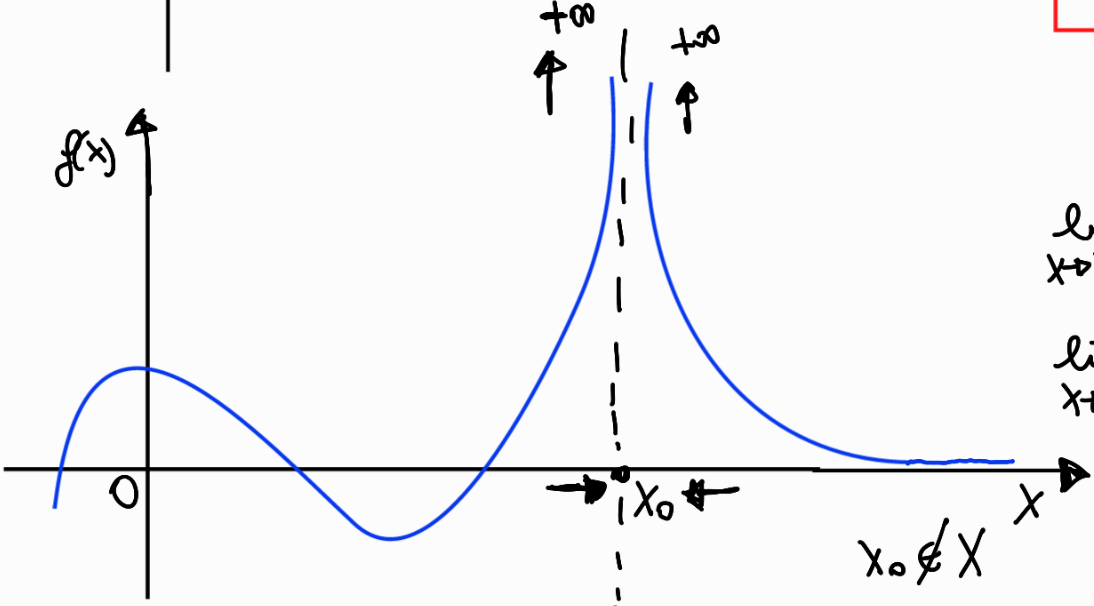
$$\lim_{x \to x_0^-} f(x) = l$$

uguali

limite destro

$$\lim_{x \to x_0^+} f(x) = l$$

$$\boxed{\exists \lim_{x \to x_0} f(x) = l}$$

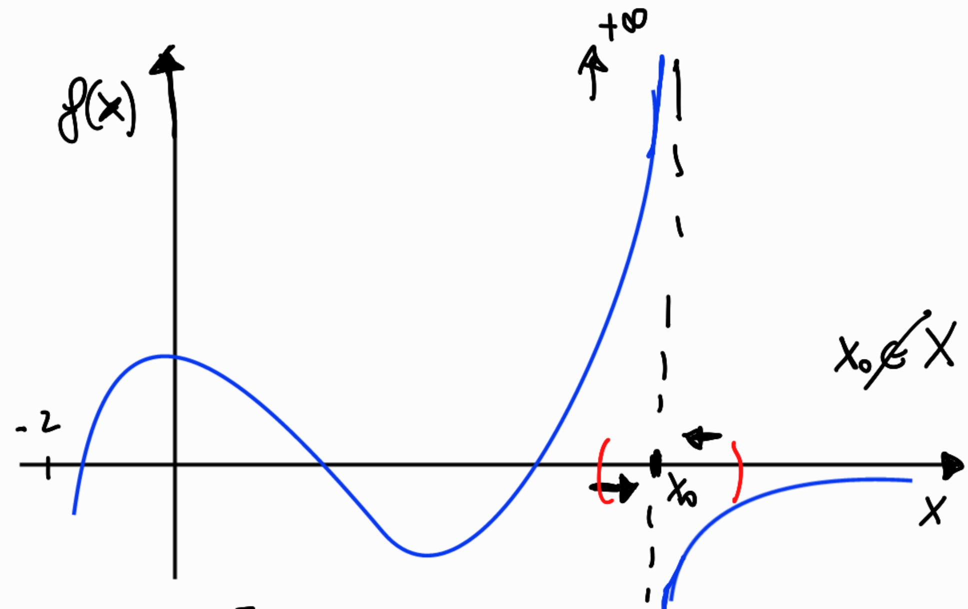


$$\lim_{x \to x_0^-} f(x) = +\infty$$

uguali

$$\lim_{x \to x_0^+} f(x) = +\infty$$

$$\boxed{\exists \lim_{x \to x_0} f(x) = +\infty}$$



$$\lim_{x \to x_0^-} f(x) = +\infty$$

diversi

$$\lim_{x \to x_0^+} f(x) = -\infty$$

$$\boxed{\nexists \lim_{x \to x_0} f(x)}$$

$E[f(x)] =]-2, x_0[\cup]x_0, +\infty[$ $\quad -\infty$

Definizione (generale) di limite

Sia $f: X \subseteq \mathbb{R} \to \mathbb{R}$ e sia $x_0 \in \overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$.

Pertanto:

- se $x_0 \in \mathbb{R}$, x_0 è di accumulazione per X (dominio di f).
- se $x_0 = +\infty$, il dominio X di f è illimitato superiormente:
  $\sup X = +\infty$
- se $x_0 = -\infty$, il dominio X di f è illimitato inferiormente:
  $\inf X = -\infty$.

Si dice che la funzione f è regolare in $x_0 \in \overline{\mathbb{R}}$ e ammette limite $\ell \in \overline{\mathbb{R}}$ in x_0 e si scrive:

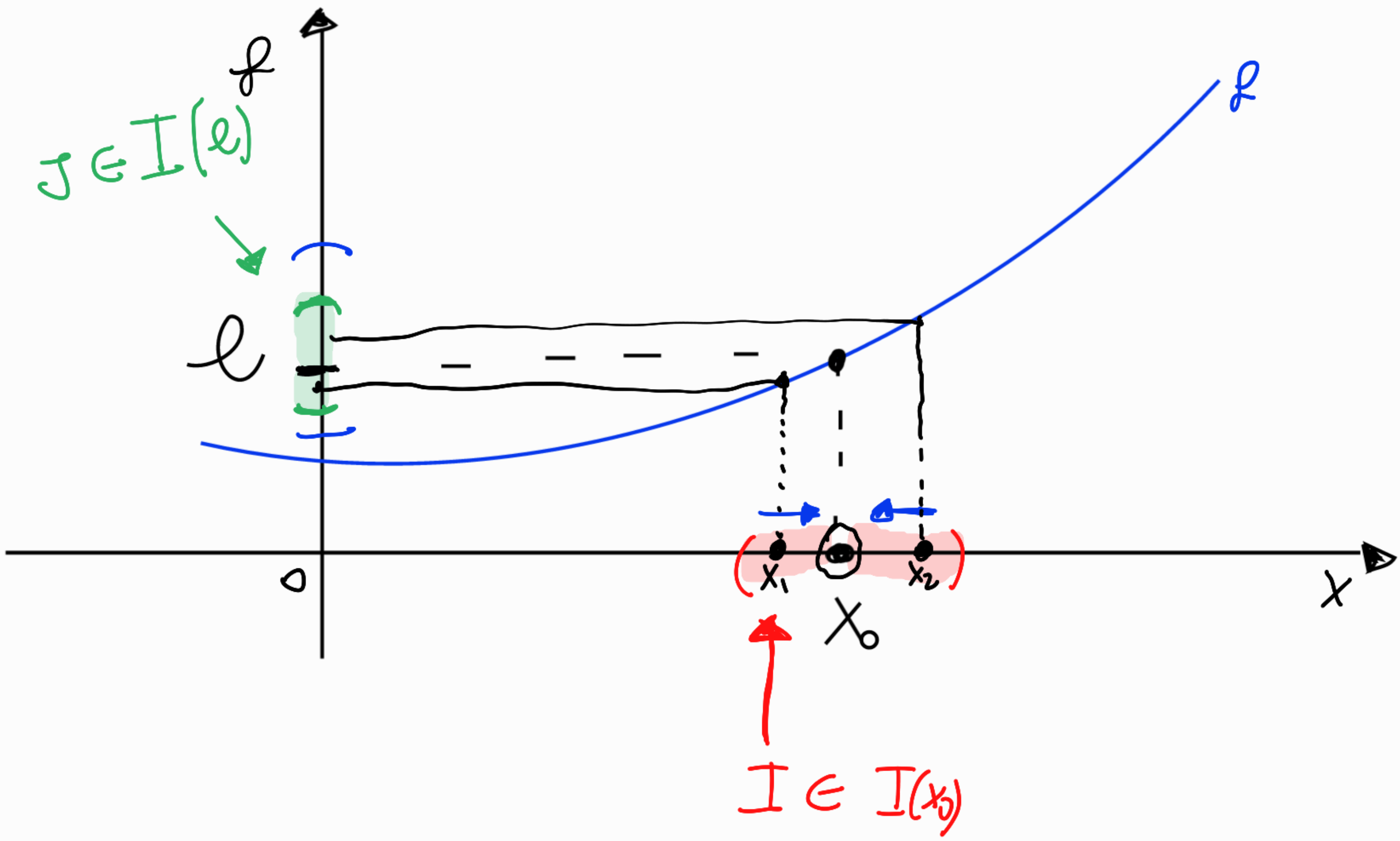
$$\lim_{x \to x_0} f(x) = \ell \in \overline{\mathbb{R}} = \begin{cases} \ell \in \mathbb{R} \\ +\infty \\ -\infty \end{cases}$$

se:

$$\underbrace{\forall J \in \mathcal{I}(\ell)}_{\text{(intorno di } \ell)}, \ \exists \underbrace{I \in \mathcal{I}(x_0)}_{\text{(intorno di } x_0)} : f(x) \in J \ \ \forall x \in X \cap (I - \{x_0\})$$

(famiglia degli intorni di ℓ)

(famiglia degli intorni di x_0)



$$\forall x \in X \cap (I - \{x_0\})$$

$$x_0 \in \overline{\mathbb{R}} = \begin{cases} x_0 \in \mathbb{R} \\ +\infty \\ -\infty \end{cases}$$

$$\ell \in \overline{\mathbb{R}} = \begin{cases} \ell \in \mathbb{R} \\ +\infty \\ -\infty \end{cases} \qquad 3^2 = 9$$

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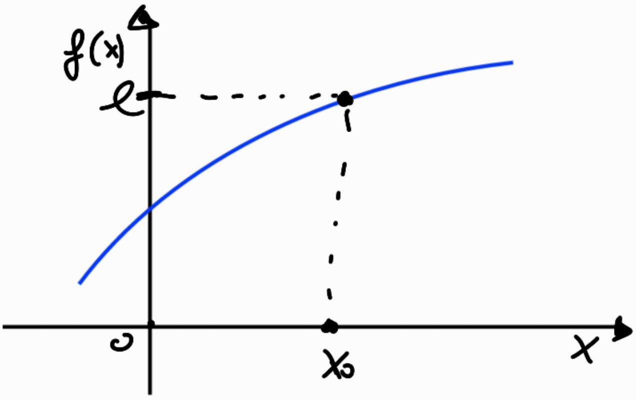
9 CASI → in base alla combinazione dei valori di x_0 e ℓ.

CASO 1

$x_0 \in \mathbb{R}$ (valore finito)

$\ell \in \mathbb{R}$ (valore finito)



$$\lim_{x \to x_0} f(x) = \ell \in \mathbb{R}$$

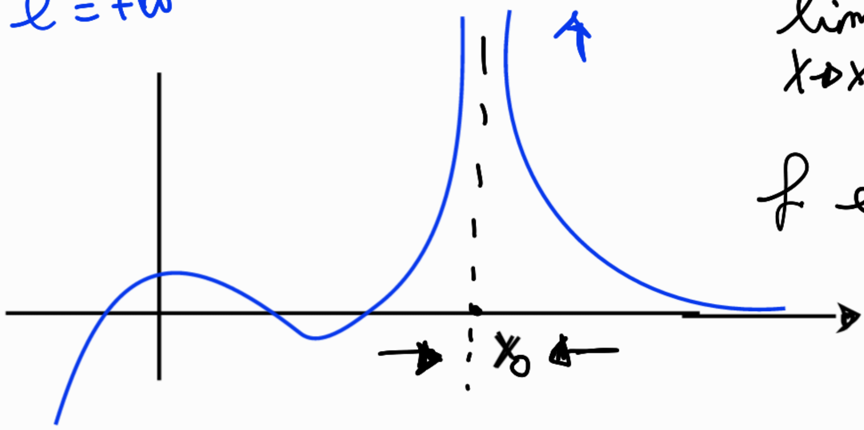
f è convergente in $x_0 \in \mathbb{R}$.

CASO 2

$x_0 \in \mathbb{R}$ (valore finito)

$\ell = +\infty$

$+\infty$



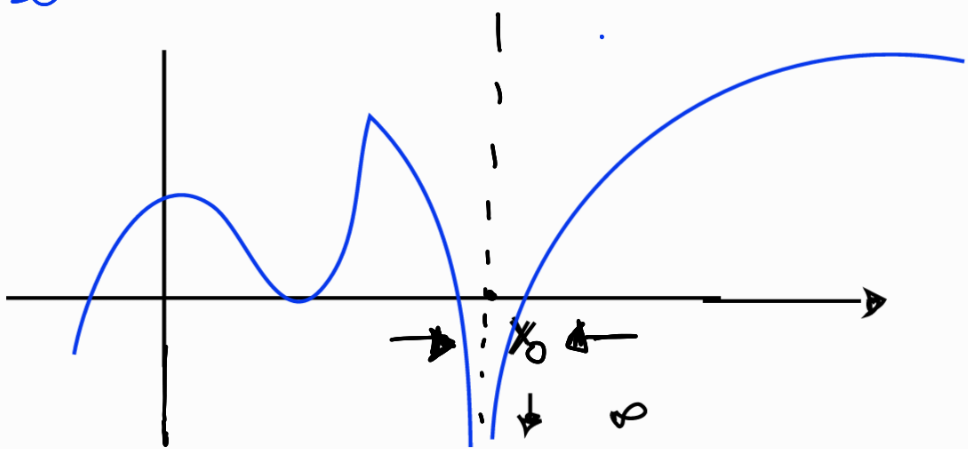
$$\lim_{x \to x_0} f(x) = +\infty$$

f è divergente positivamente in x_0

CASO 3

$x_0 \in \mathbb{R}$ (valore finito)

$\ell = -\infty$



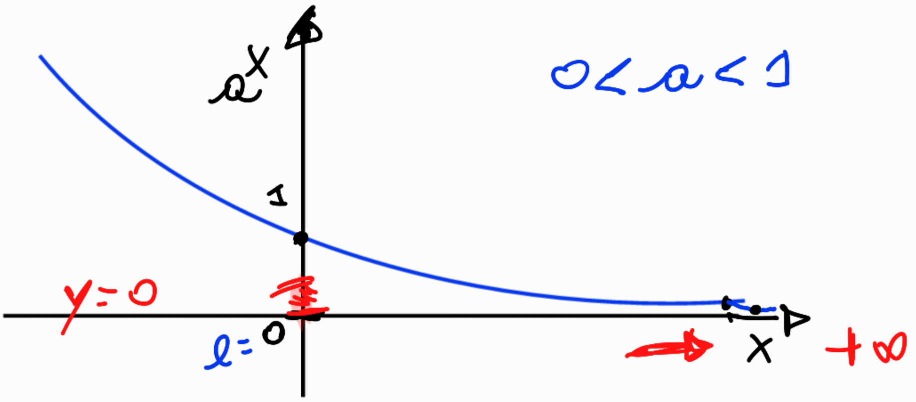
$$\lim_{x \to x_0} f(x) = -\infty$$

f divergente negativamente in x_0

CASO 4

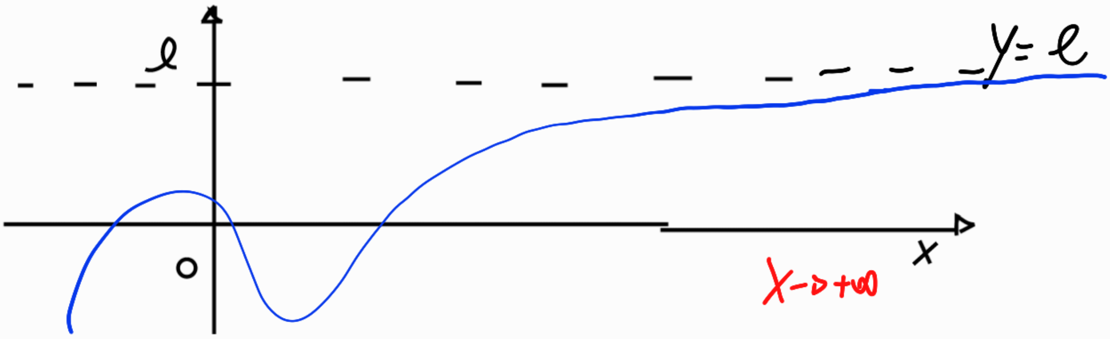
$x_0 = +\infty$

$\ell \in \mathbb{R}$ (valore finito)



$$\lim_{x \to +\infty} f(x) = \ell \in \mathbb{R}$$

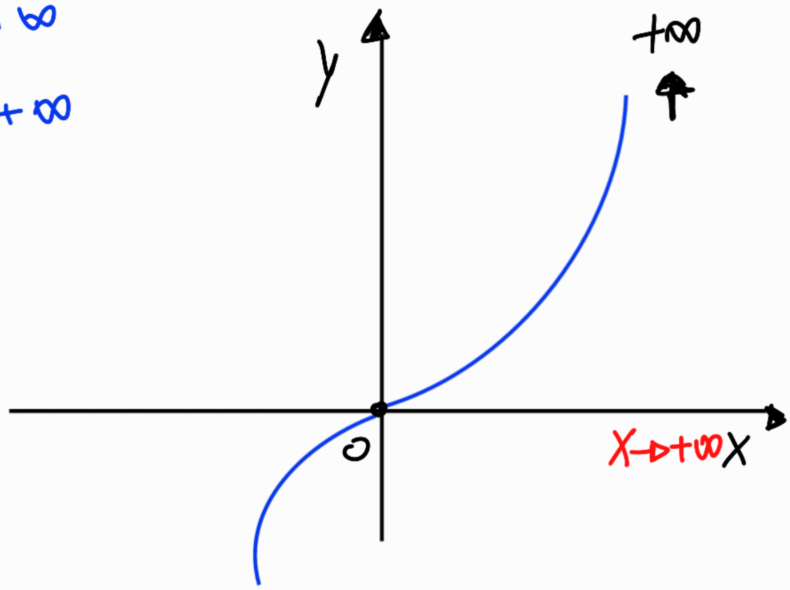
f è convergente in $+\infty$



CASO 5

$x_0 = +\infty$

$\ell = +\infty$



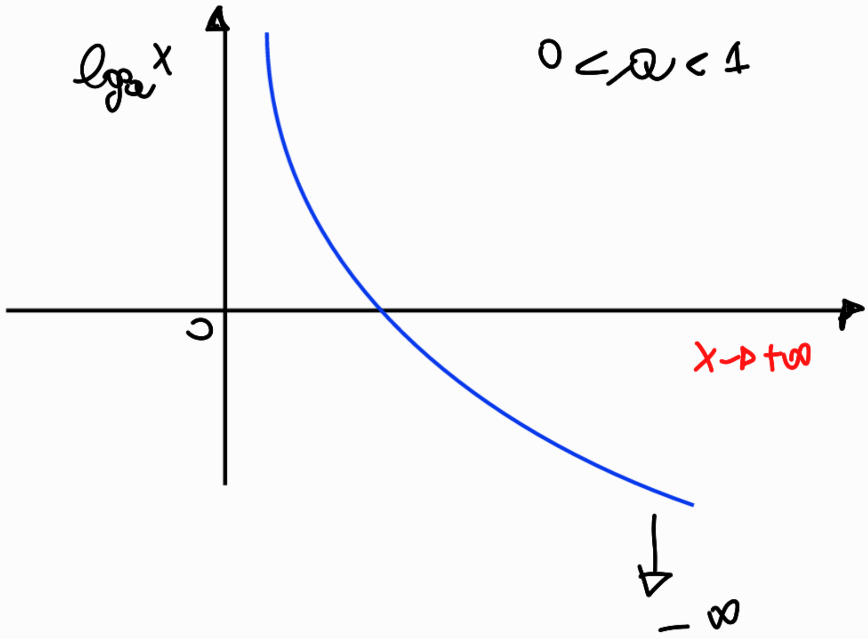
$$\lim_{x \to +\infty} f(x) = +\infty$$

f. divergente positivamente in $+\infty$

CASO 6

$x_0 = +\infty$

$\ell = -\infty$



$0 < a < 1$

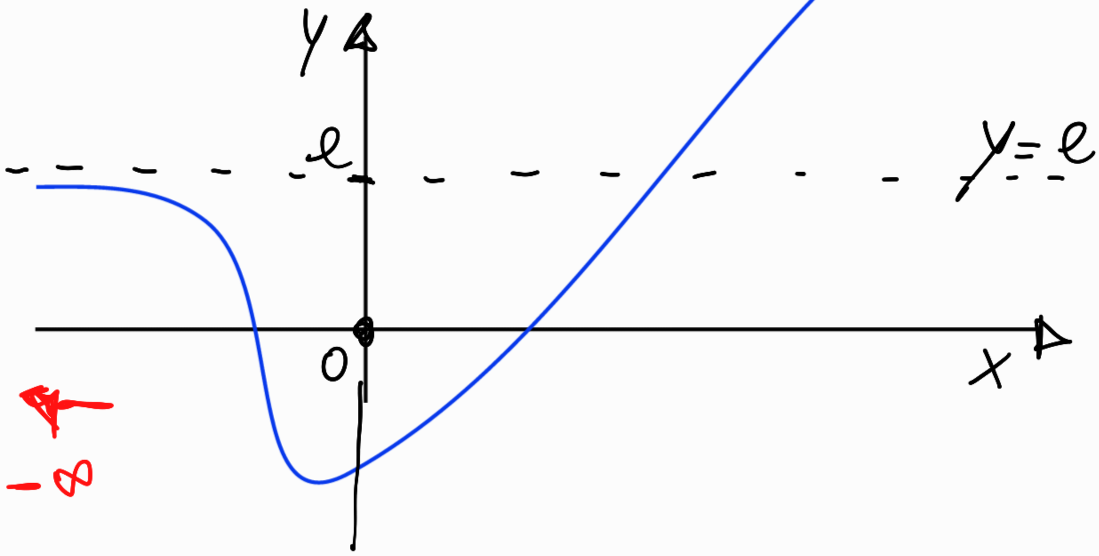
$$\lim_{x \to +\infty} f(x) = -\infty$$

f. diverg. negativamente in $+\infty$

CASO 7

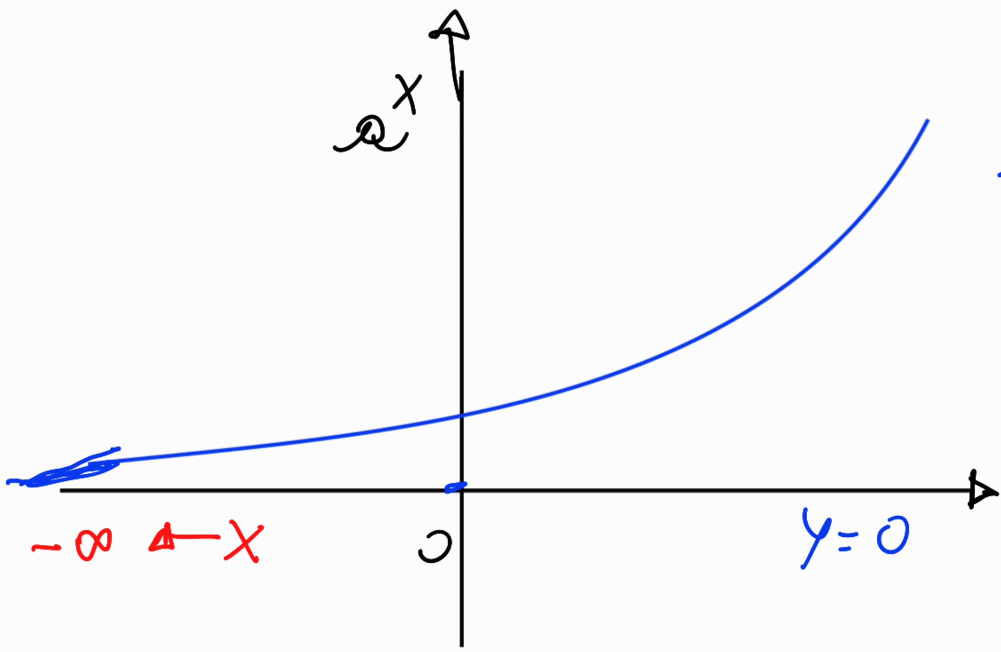
$x_0 = -\infty$

$l \in \mathbb{R}$ (valore finito)



$$\lim_{x \to -\infty} f(x) = l \in \mathbb{R}$$

f. è convergente in $-\infty$



$a > 1$

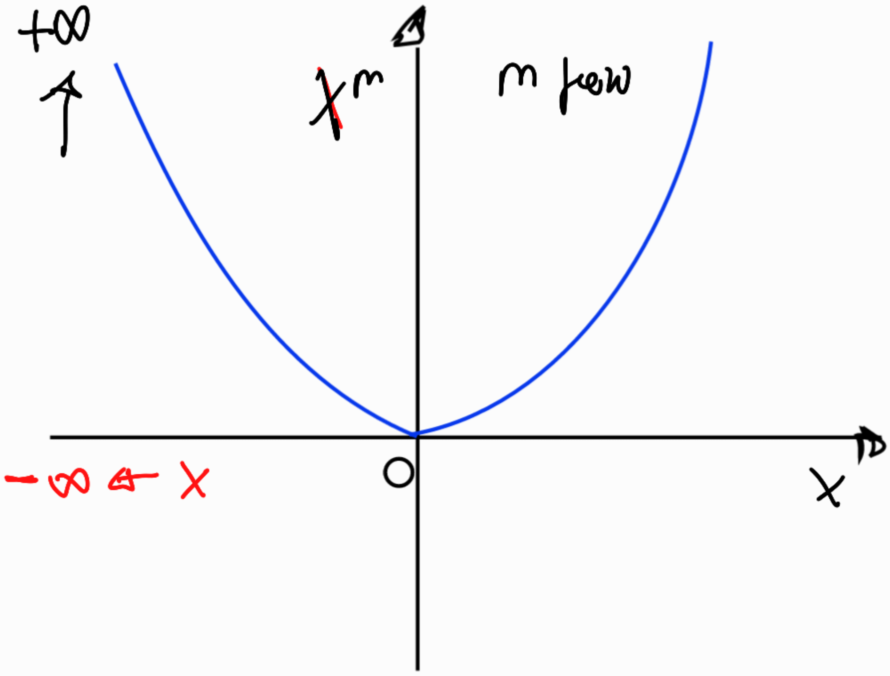
$$\lim_{x \to -\infty} f(x) = l = 0$$

$l = 0$

CASO 8

$x_0 = -\infty$

$\ell = +\infty$



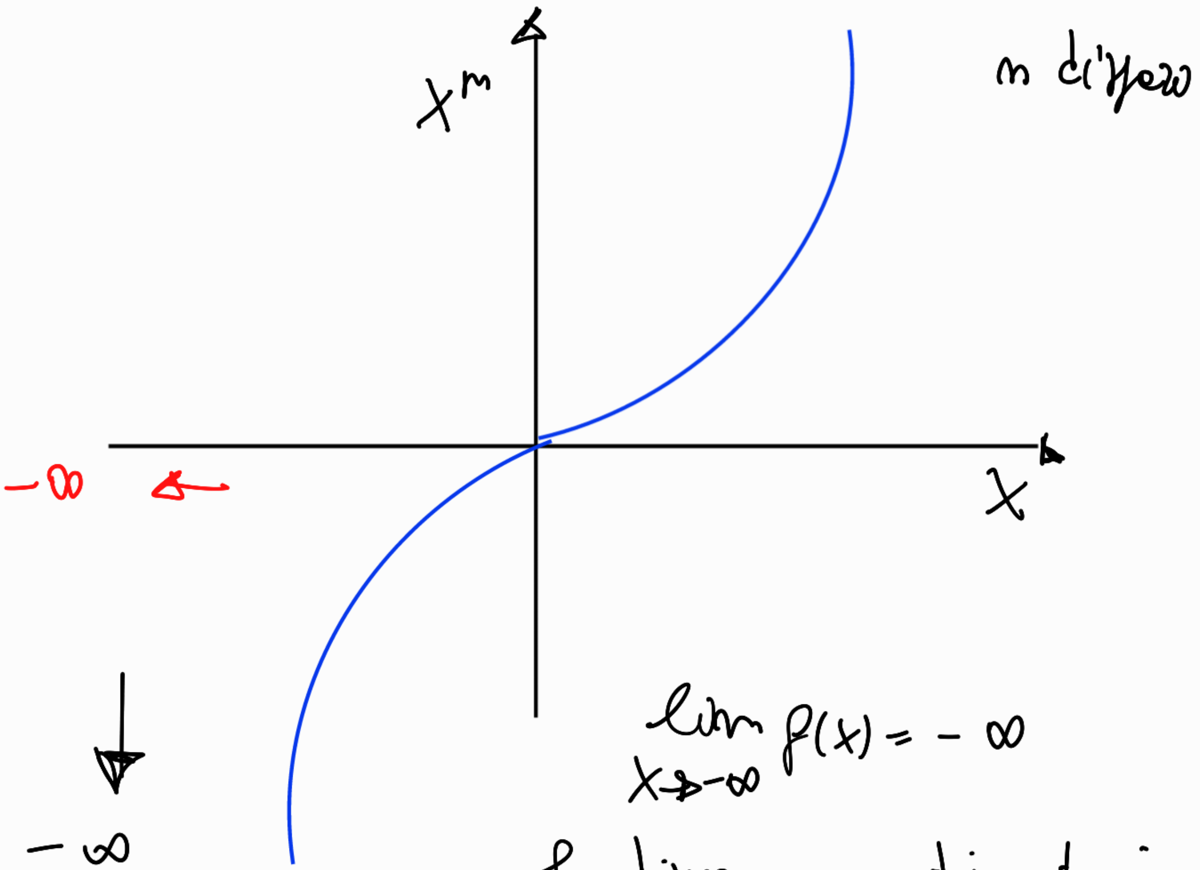
$$\lim_{x \to -\infty} f(x) = +\infty$$

f. divergente positivamente in $-\infty$

CASO 9

$x_0 = -\infty$

$\ell = -\infty$



$$\lim_{x \to -\infty} f(x) = -\infty$$

f. diverg. negativamente in $-\infty$