- Disequazioni con valore assoluto
- Disequazioni esponenziali e logaritmiche
- Campi di esistenza

Disequazioni con valore assoluto

$P(x)$ è un polinomio

$$|P(x)| \gtreqless K \qquad K \in \mathbb{R}$$

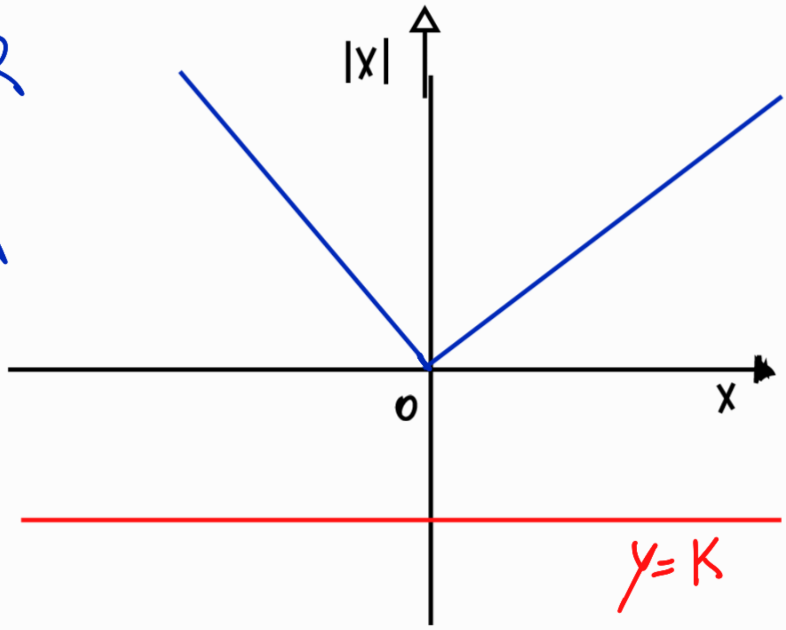
$\boxed{\text{CASO 1: } K < 0}$

$|P(x)| > K \iff \forall x \in \mathbb{R}$

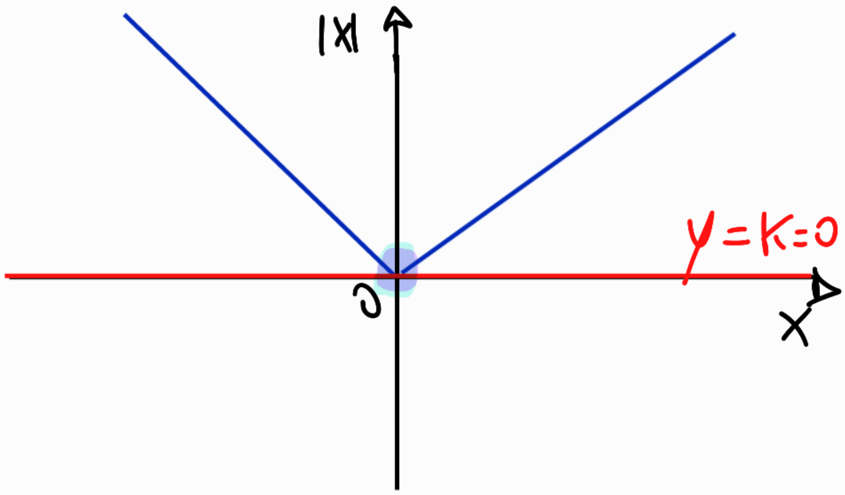
$|P(x)| \geq K \iff \forall x \in \mathbb{R}$

$|P(x)| < K \iff \emptyset$

$|P(x)| \leq K \iff \emptyset$



$\boxed{\text{CASO 2: } K = 0}$



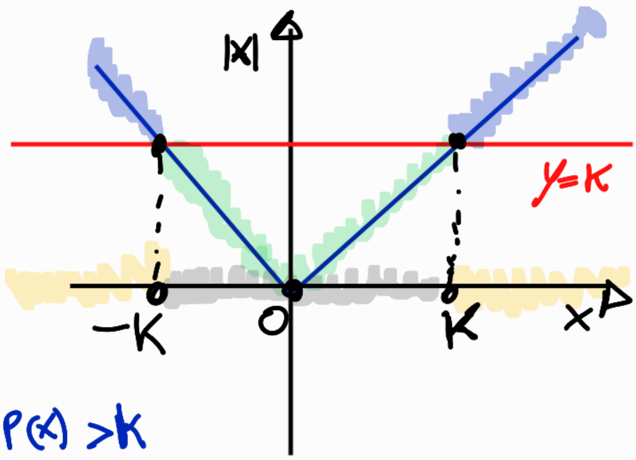
$$|P(x)| > 0 \iff \forall x \in \mathbb{R} - \{x : P(x) = 0\}$$

$$|P(x)| \geq 0 \iff \forall x \in \mathbb{R}$$

$$|P(x)| < 0 \iff \emptyset$$

$$|P(x)| \leq 0 \iff x = x^* : P(x^*) = 0$$

$$\boxed{\text{CASO} : K > 0}$$



$$|P(x)| > K \iff P(x) < -K \ \vee\ P(x) > K$$

$$|P(x)| \geq K \iff P(x) \leq -K \ \vee\ P(x) \geq K$$

$$|P(x)| < K \iff -K < \boxed{P(x) < K} \iff \begin{cases} P(x) < K \\ P(x) > -K \end{cases}$$

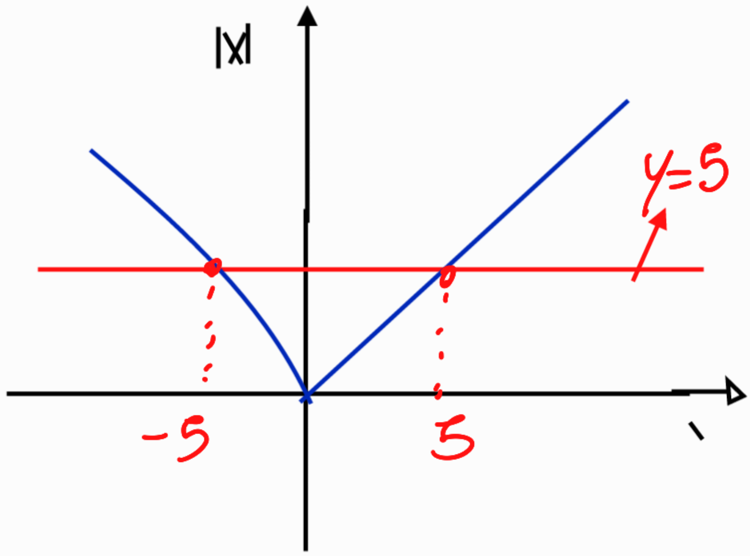
$$|P(x)| \leq K \iff -K \leq P(x) \leq K \iff \begin{cases} P(x) \leq K \\ P(x) \geq -K \end{cases}$$

Exemplo

- $|x-1| < 0 \Leftrightarrow \emptyset$

$|x-2| > 0 \Leftrightarrow \mathbb{R} - \{2\} \Leftrightarrow]-\infty, 2[\cup]2, +\infty[$

$|x-1| < -5 \Leftrightarrow \emptyset$



$|x-1| > 5$

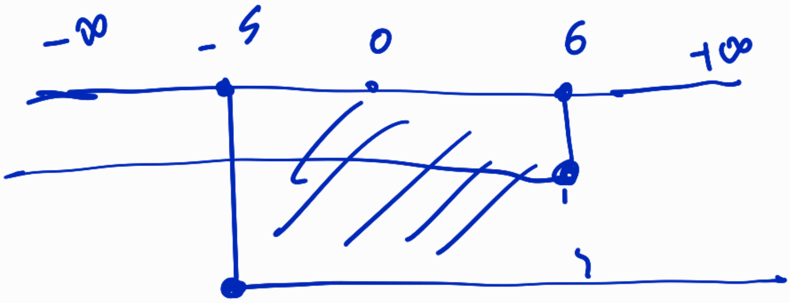
$\Updownarrow$

$x-1 < -5 \quad \vee \quad x-1 > 5$

$x < -4 \quad \vee \quad x > 6$

$|x-1| \leq 5 \Leftrightarrow -5 \leq x-1 \leq 5 \Leftrightarrow$

$\begin{cases} x-1 \leq 5 \\ x-1 \geq -5 \end{cases} \Leftrightarrow \begin{cases} x \leq 6 \\ x \geq -4 \end{cases}$



$-4 \leq x \leq 6$

$$-5 \le x - \cancel{1} \le 5 \iff -5 \le x \le 6$$

$$-5 \le x - \cancel{1} \le 5$$

$$-5 \le x \le 6$$

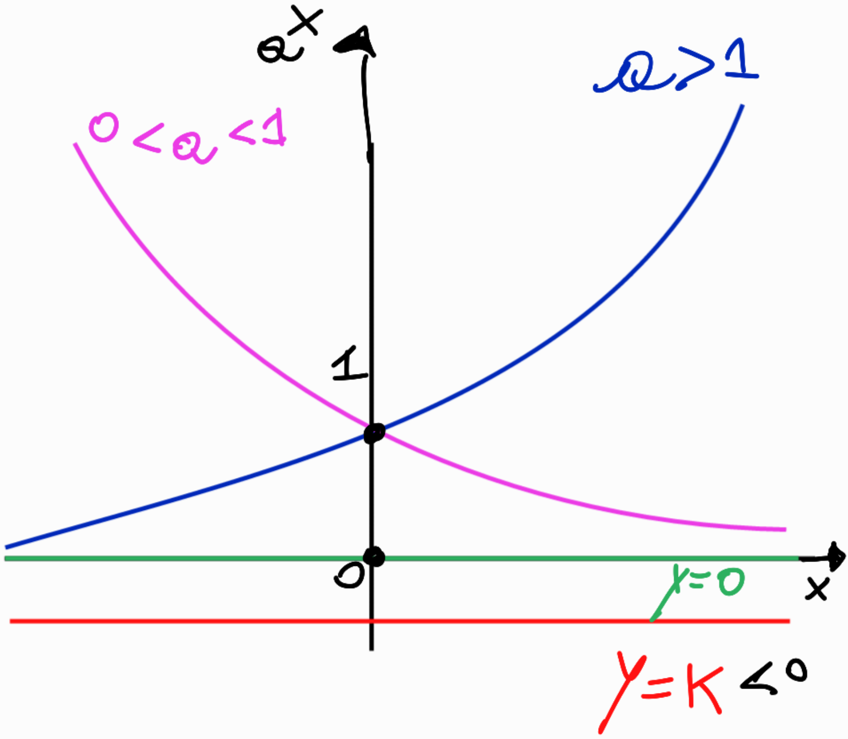
$$|3x^2 - 4x + 5| < 3 \iff \begin{cases} 3x^2 - 4x + 5 < 3 \\ \quad // \quad > -3 \end{cases}$$

Disequazioni Esponenziali

$$\boxed{a^x \gtrless k}$$

$k \in \mathbb{R}$

$a > 0$



- $\boxed{\text{CASO } k < 0}$

oppure

$\boxed{\text{CASO } k = 0}$

- $a^x > k \iff \forall x \in \mathbb{R}$

- $a^x \geq K \iff \forall x \in \mathbb{R}$
- $a^x < K \iff \emptyset$
- $a^x \leq K \iff \emptyset$

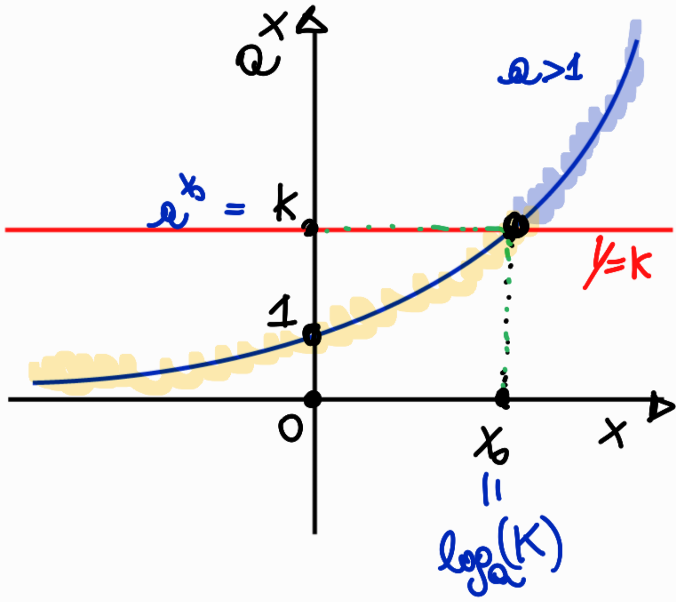
Esempio

$$f(x) = \sqrt{\left(\frac{1}{2}\right)^x + 2} \qquad E[f(x)] =]-\infty, +\infty[$$

$$\left(\frac{1}{2}\right)^x + 2 \geq 0 \iff \left(\frac{1}{2}\right)^x \geq -2 \iff \forall x \in \mathbb{R}$$

$$\left(\frac{1}{2}\right)^x \geq 0 \quad \forall x \in \mathbb{R}$$

$\boxed{\text{CASO: } K > 0}$ e $\boxed{a > 1}$



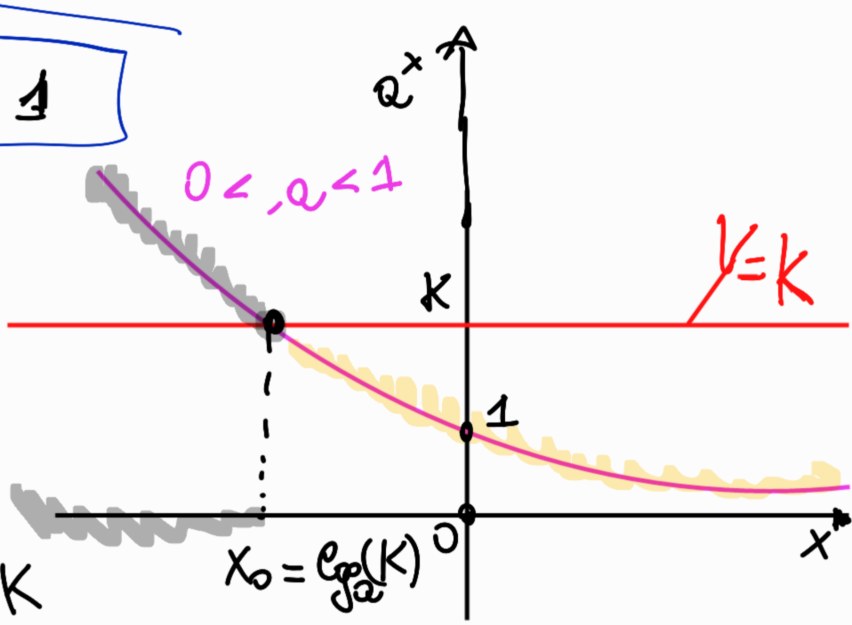
- $a^x > K \iff x > x_0 \iff x > \log_a K$
- $a^x \geq K \iff x \geq \log_a K$
- $a^x < K \iff x < \log_a K$
- $a^x \leq K \iff x \leq \log_a K$

$f: x \to a^x \qquad a^{x_0} \to x_0$

$a^{x_0} = K \longrightarrow x_0$

$x_0 = \log_a(K)$

$\boxed{\text{CASO: } K > 0}$ e $\boxed{0 < a < 1}$



- $a^x > K \iff x < \log_a K$
- $a^x \geq K \iff x \leq \log_a K$
- $a^x < K \iff x > \log_a K$
- $a^x \leq K \iff x \geq \log_a K$

Esempio

- $3^x \geq -5 \iff \forall x \in \mathbb{R}$
- $3^x > 0 \iff \forall x \in \mathbb{R}$
- $\left(\frac{1}{2}\right)^x < 0 \iff \emptyset$

- $5^x > 25 \iff x > \log_5(25) = \log_5(5^2) =$

$= 2\underbrace{\log_5 5}_{=1} =$

$= 2 \cdot 1 = 2$

$x > 2$

$5^2 = 25 \Rightarrow 2 = \log_5 25$

- $8^{x+1} \geq 2^{x^2} \iff (2^3)^{x+1} \geq 2^{x^2} \iff$

$(2^3)^{x+1} = 2^{3\cdot(x+1)}$

$2^{3(x+1)} \geq 2^{x^2} \iff 2^{3x+3} \geq 2^{x^2} \iff$

$3x+3 \geq x^2 \iff -x^2 + 3x + 3 \geq 0$

$\iff x^2 - 3x - 3 \leq 0$

$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(-3) = 9 + 12 = 21 > 0$

$$x_{1,2} = \frac{3 \pm \sqrt{21}}{2}$$

$$x_1 = \frac{3 - \sqrt{21}}{2}$$

$$x_2 = \frac{3 + \sqrt{21}}{2}$$

$$\frac{3 - \sqrt{21}}{2} \leq x \leq \frac{3 + \sqrt{21}}{2}$$

$$\left[\frac{3 - \sqrt{21}}{2} , \frac{3 + \sqrt{21}}{2} \right]$$

Proprietà dei logaritmi

- Esempio $\log_2 4 = 2$ perché $2^2 = 4$

  $\log_2 8 = 3$ perché $2^3 = 8$

- $\log_5 1 = 0$ perché $5^0 = 1$

- $\boxed{\log_a 1 = 0}$ perché $a^0 = 1$

CASO PARTICOLARE

$\boxed{\log 1 = 0}$ $\qquad e^0 = 1$

- $\log_5 5 = 1$ perché $5^1 = 5$
- $\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right) = 1$ $\qquad$ " $\left(\frac{1}{2}\right)^1 = \frac{1}{2}$

- $\boxed{\log_a a = 1}$

CASO PARTICOLARE

$\boxed{\log e = 1}$

- $\boxed{\log_a\left(\frac{1}{a}\right) = -1}$

$$a^{-1} = \frac{1}{a}$$

- $\boxed{\log_{\left(\frac{1}{a}\right)}(a) = -1}$

CASO PARTICOLARE

$\boxed{\log\left(\frac{1}{e}\right) = -1}$

SOMMA

- $\boxed{\log_a b + \log_a c = \log_a(b \cdot c)}$

DIFFERENZA

- $\boxed{\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)}$

- $\log_a(b^c) = c \cdot \log_a b$

CAMBIO BASE

- $$\boxed{\log_a b = \frac{\log_c(b)}{\log_c(a)}}$$

CASO PARTICOLARE base nuova $= e$

$$\log_a b = \frac{\log_e b}{\log_e a} = \boxed{\frac{\ln(b)}{\ln(a)}}$$

Esempio

$$\log_2 4 = \frac{\ln 4}{\ln 2} = 2$$

$$\log_5 25 = \frac{\ln 25}{\ln 5} = 2$$

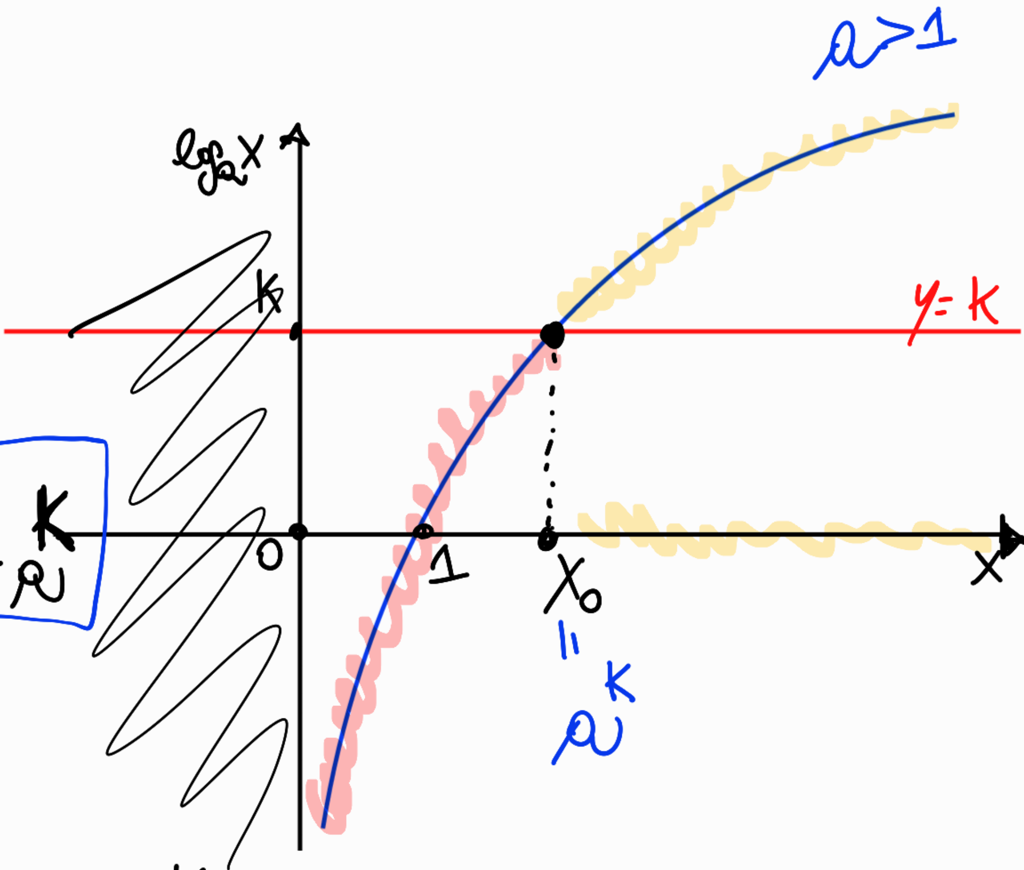
$$\log_5\left(\frac{1}{5}\right) = \frac{\ln\left(\frac{1}{5}\right)}{\ln(5)} = -1$$

Disequazioni logaritmiche

$$\boxed{\log_a x \ \substack{\geq \\ \leq}\ K} \qquad K \in \mathbb{R}$$

(qualunque sia K, il criterio è lo stesso)

• CASO $a > 1$



$x_0 \to \log_a x_0$

$\log_a x_0 = K \to \boxed{x_0 = a^K}$

1) $\log_a x > K \iff x > a^K$

2) $\log_a x \geq K \iff x \geq a^K$ $\Rightarrow \begin{cases} x < a^K \\ x > 0 \end{cases}$

3) $\log_a x < K \iff 0 < x < a^K$

4) $\log_a x \leq K \iff 0 < x \leq a^K \Rightarrow \begin{cases} x \leq a^K \\ x > 0 \end{cases}$

Esempio

- $\log_2 X < 5 \iff \begin{cases} X < 2^5 \\ X > 0 \end{cases} \iff \begin{cases} X < 32 \\ X > 0 \end{cases} \iff$

$\iff 0 < X < 32 \iff]0, 32[$

- $\log\left(1 + \frac{1}{X}\right) \leq 1 \iff \begin{cases} 1 + \frac{1}{X} \leq e \\ 1 + \frac{1}{X} > 0 \end{cases}$

$$\begin{cases} \frac{X+1}{X} - e \leq 0 \\ \frac{X+1}{X} > 0 \end{cases}$$

$$\begin{cases} \frac{X + 1 - e \cdot X}{X} \leq 0 \\ \frac{X+1}{X} > 0 \end{cases} ; \begin{cases} \frac{(1-\overset{2.7}{e})X + 1}{X} \leq 0 \\ \frac{X+1}{X} > 0 \end{cases}$$

$$\begin{cases} \dfrac{(e-1)x-1}{x} \geq 0 \\ \dfrac{x+1}{x} > 0 \end{cases}$$

$$S =]-\infty, -1[\; \cup \;]\frac{1}{e-1}, +\infty[$$

Campi di esistenza

- $f(x) = \dfrac{x^3+5x^2}{3x^2-2}$

$$E[f(x)] = \{ x \in \mathbb{R} : \; 3x^2-2 \neq 0 \}$$

- $f(x) = \sqrt[7]{\dfrac{x^2-2x+2}{x^2-4}}$

$$E[f(x)] = \{ x \in \mathbb{R} : \; \frac{x^2-2x+2}{x^2-4} \geq 0 \}$$

- $f(x) = \log\left(\dfrac{3x^2+2x}{x+1}\right)$

$$E[f(x)] = \left\{ x \in \mathbb{R} : \frac{3x^2+2x}{x+1} > 0 \right\}$$

- $$f(x) = \sqrt[3]{\frac{2x}{x^2-4}}$$

$$E[f(x)] = \left\{ x \in \mathbb{R} : x^2-4 \neq 0 \right\}$$

- $$f(x) = \sqrt{|5x+3| - 5}$$

$$E[f(x)] = \left\{ x \in \mathbb{R} : |5x+3| - 5 \geq 0 \right\}$$

$$|5x+3| \geq 5 \iff 5x+3 \leq -5 \ \vee \ 5x+3 > 5$$

- $$f(x) = e^{3x+5} - \log(x^2)$$

$$E[f(x)] = \left\{ x \in \mathbb{R} : x \neq 0 \right\}$$