- Sistemi di disequazioni
- Disequazioni fratte
- Disequazioni con valore assoluto

Sistemi di disequazioni (1 sola risolvibile)

$$\begin{cases} x^2 - 4x + 3 \geq 0 & \text{(II grado)} \\ 4 - 8x > 0 & \text{(I grado)} \end{cases}$$

1ª diseq) $x^2 - 4x + 3 \geq 0$

eq. ass. $x^2 - 4x + 3 = 0$

$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(3) = 16 - 12 = 4 > 0$

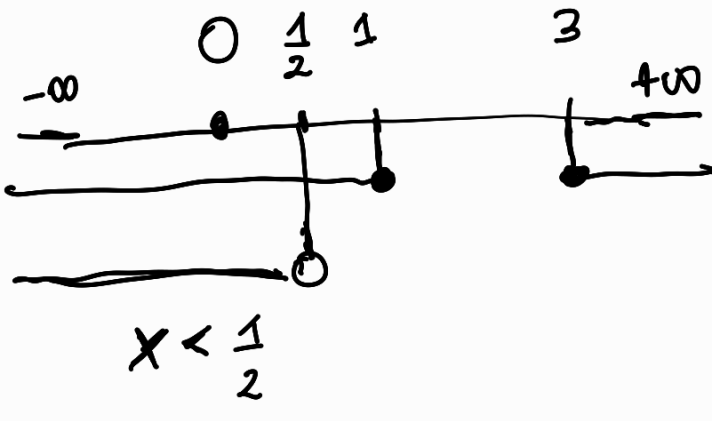
$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{4 \pm 2}{2}$ $\quad x_1 = 1, \quad x_2 = 3$

$x^2 - 4x + 3 \geq 0 \iff x \leq 1 \quad \vee \quad x \geq 3$

2ª diseq.) $4 - 8x > 0 \iff 8x - 4 < 0 \iff x < \frac{1}{2}$

$$\begin{cases} x^2 - 4x + 3 \geq 0 \\ 4 - 8x > 0 \end{cases} \iff$$

$$\iff \begin{cases} x \leq 1 \quad \vee x \geq 3 \\ x < \frac{1}{2} \end{cases}$$



$$S =]-\infty, \frac{1}{2}[$$

Equazioni fratte

$$\frac{P(x)}{Q(x)} = 0 \iff \begin{cases} P(x) = 0 \\ Q(x) \neq 0 \end{cases}$$

$$\frac{x}{x-2} = 0 \iff \begin{cases} x = 0 \\ x - 2 \neq 0 \end{cases} \iff \begin{cases} x = 0 \\ x \neq 2 \end{cases}$$

$$\frac{3}{0} = K \qquad \frac{0}{3} = 0$$

Disequazioni fratte

$$\frac{P(x)}{Q(x)} \gtrless 0$$

$\frac{+3}{+6} > 0$ $\quad \frac{-2}{-5} > 0$

$\frac{-5}{+2} < 0$ $\quad \frac{+10}{-12} < 0$

Criterio

1) $\frac{P(x)}{Q(x)} > 0 \iff \begin{cases} P(x) > 0 \\ Q(x) > 0 \end{cases} \cup \begin{cases} P(x) < 0 \\ Q(x) < 0 \end{cases}$

2) $\frac{P(x)}{Q(x)} \geq 0 \iff \begin{cases} P(x) \geq 0 \\ Q(x) > 0 \end{cases} \cup \begin{cases} P(x) \leq 0 \\ Q(x) < 0 \end{cases}$

3) $\frac{P(x)}{Q(x)} < 0 \iff \begin{cases} P(x) > 0 \\ Q(x) < 0 \end{cases} \cup \begin{cases} P(x) < 0 \\ Q(x) > 0 \end{cases}$

4) $\frac{P(x)}{Q(x)} \leq 0 \iff \begin{cases} P(x) \geq 0 \\ Q(x) < 0 \end{cases} \cup \begin{cases} P(x) \leq 0 \\ Q(x) > 0 \end{cases}$

Esempio

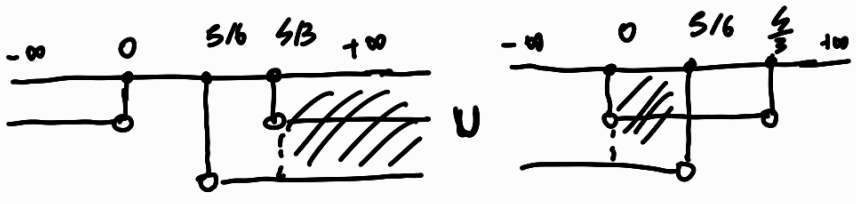
$$\frac{3x^2-4x}{6x-5} > 0 \iff \underbrace{\begin{cases} 3x^2-4x > 0 \\ 6x-5 > 0 \end{cases}}_{S_1} \cup \underbrace{\begin{cases} 3x^2-4x < 0 \\ 6x-5 < 0 \end{cases}}_{S_2} \iff$$

$3x-4=0$

$x=0$ $\quad$ $x=\frac{4}{3}$

$$\iff \begin{cases} x(3x-4) > 0 \\ x > \frac{5}{6} \end{cases} \cup \begin{cases} x(3x-4) < 0 \\ x < \frac{5}{6} \end{cases}$$

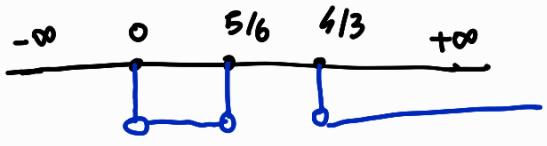
$$\iff \begin{cases} x < 0 \ \vee \ x > \frac{4}{3} \\ x > \frac{5}{6} \end{cases} \cup \begin{cases} 0 < x < \frac{4}{3} \\ x < \frac{5}{6} \end{cases}$$



$x > \frac{4}{3} \quad \vee \quad 0 < x < \frac{5}{6}$

Unione

- $x > \frac{4}{3}$
- $0 < x < \frac{5}{6}$



$0 < x < \frac{5}{6} \quad \vee \quad x > \frac{4}{3}$

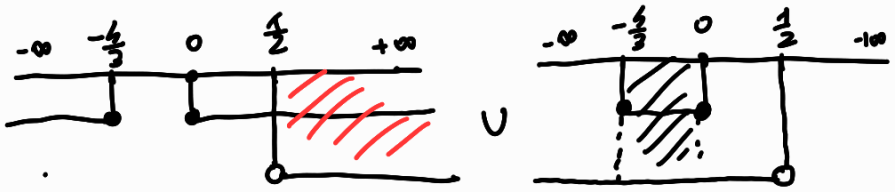
$$S =]0, \frac{5}{6}[\cup]\frac{4}{3}, +\infty[$$

- $\frac{3x^2+4x}{2x-1} \geq 0 \iff \begin{cases} 3x^2+4x \geq 0 \\ 2x-1 > 0 \end{cases} \cup \begin{cases} 3x^2+4x \leq 0 \\ 2x-1 < 0 \end{cases} \iff$

$x \geq 0$; $3x+4 \geq 0$, $x \geq -\frac{4}{3}$

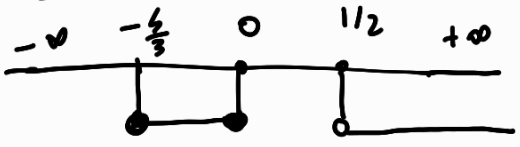
$$\iff \begin{cases} x(3x+4) \geq 0 \\ x > \frac{1}{2} \end{cases} \cup \begin{cases} x(3x+4) \leq 0 \\ x < \frac{1}{2} \end{cases}$$

$$\begin{cases} x \leq -\frac{4}{3} \vee x \geq 0 \\ -5 < x \ \frac{1}{2} \end{cases} \cup \begin{cases} -\frac{4}{3} \leq x \leq 0 \\ x < \frac{1}{2} \end{cases}$$



$x > \frac{1}{2}$ $\qquad -\frac{4}{3} \leq x \leq 0$

Unione



$$-\frac{4}{3} \leq x \leq 0 \ \vee \ x > \frac{1}{2}$$

$$S = [-\frac{4}{3}, 0] \cup]\frac{1}{2}, +\infty[$$

- $\frac{-x^2+x+6}{3x+2} > 0 \iff$ Cambio segno

$$\iff \frac{x^2-x-6}{3x+2} < 0$$

$$\frac{-x^2+2x}{-2x^3+5x-2} \geq 0$$

$$\frac{A}{B} \cdot \frac{(-1)}{(-1)} = \frac{A}{B} \cdot 1 = \frac{A}{B}$$

$$\frac{x^2-2x}{2x^3-5x+2} \geq 0$$

S1 S2

$$\frac{x^2-x-6}{3x+2} < 0 \iff \begin{cases} x^2-x-6 > 0 \\ 3x+2 < 0 \end{cases} \cup \begin{cases} x^2-x-6 < 0 \\ 3x+2 > 0 \end{cases} \iff$$

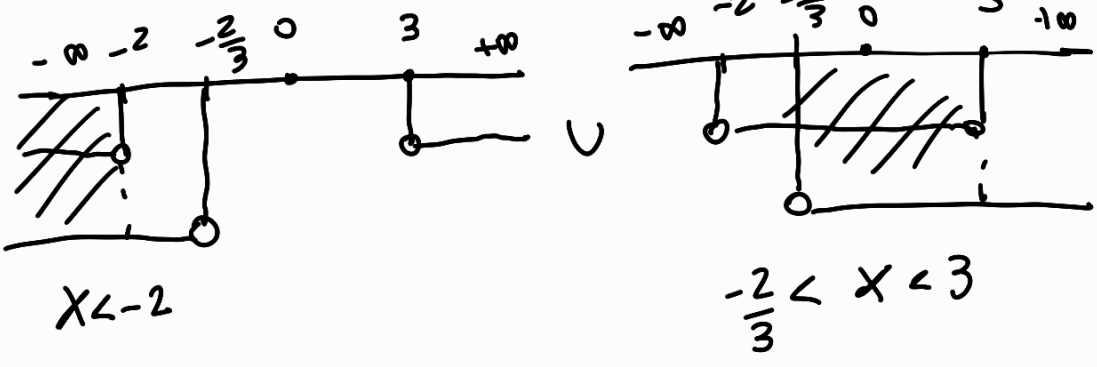
$$\Leftrightarrow \begin{cases} x^2 - x - 6 > 0 \\ x < -\frac{2}{3} \end{cases} \cup \begin{cases} x^2 - x - 6 < 0 \\ x > \frac{-2}{3} \end{cases}$$

1ª eq. ensaiada: $x^2 - x - 6 = 0$

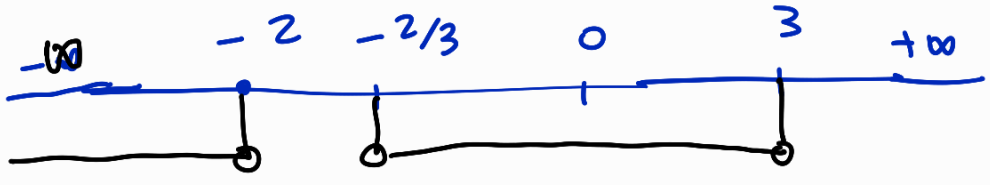
$\Delta = b^2 - 4ac = (-1)^2 - 4(1)(-6) = 1 + 24 = 25 > 0$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 5}{2}$ → $x_1 = -\frac{4}{2} = -2$; $x_2 = \frac{6}{2} = 3$

$$S1 \cup S2 \Leftrightarrow \begin{cases} x < -2 \ \vee \ x > 3 \\ x < -\frac{2}{3} \end{cases} \cup \begin{cases} -2 < x < 3 \\ x > -\frac{2}{3} \end{cases}$$



$x < -2$ $\qquad -\frac{2}{3} < x < 3$



$x < -2$

$-\frac{2}{3} < x < 3$

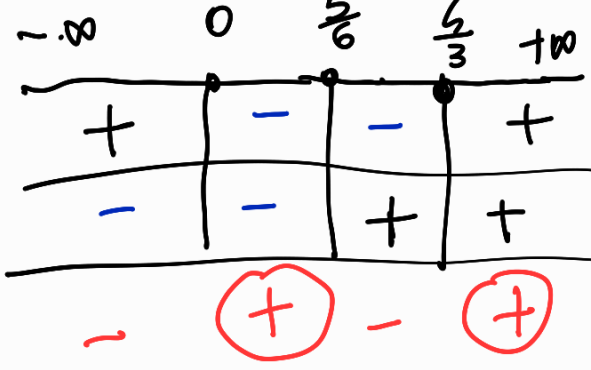
$x < -2 \ \vee \ -\frac{2}{3} < x < 0$

$$S =]-\infty, -2[\ \cup \ ]-\frac{2}{3}, 3[$$

Studio del segno

- $\frac{3x^2-4x}{6x-5} > 0 \iff 0 < x < \frac{5}{6} \vee x > \frac{4}{3}$

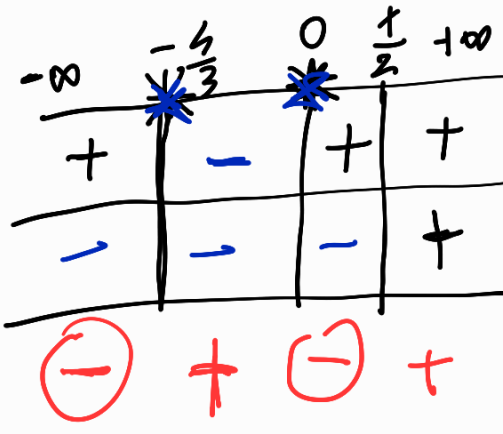
$S =]0, \frac{5}{6}[\cup]\frac{4}{3}, +\infty[$



(+) $3x^2-4x>0$; $x<0 \vee x>\frac{4}{3}$

$6x-5>0$; $x>\frac{5}{6}$

- $\frac{3x^2+4x}{2x-1} \leq 0 \iff x \leq -\frac{4}{3} \vee 0 \leq x < \frac{1}{2}$



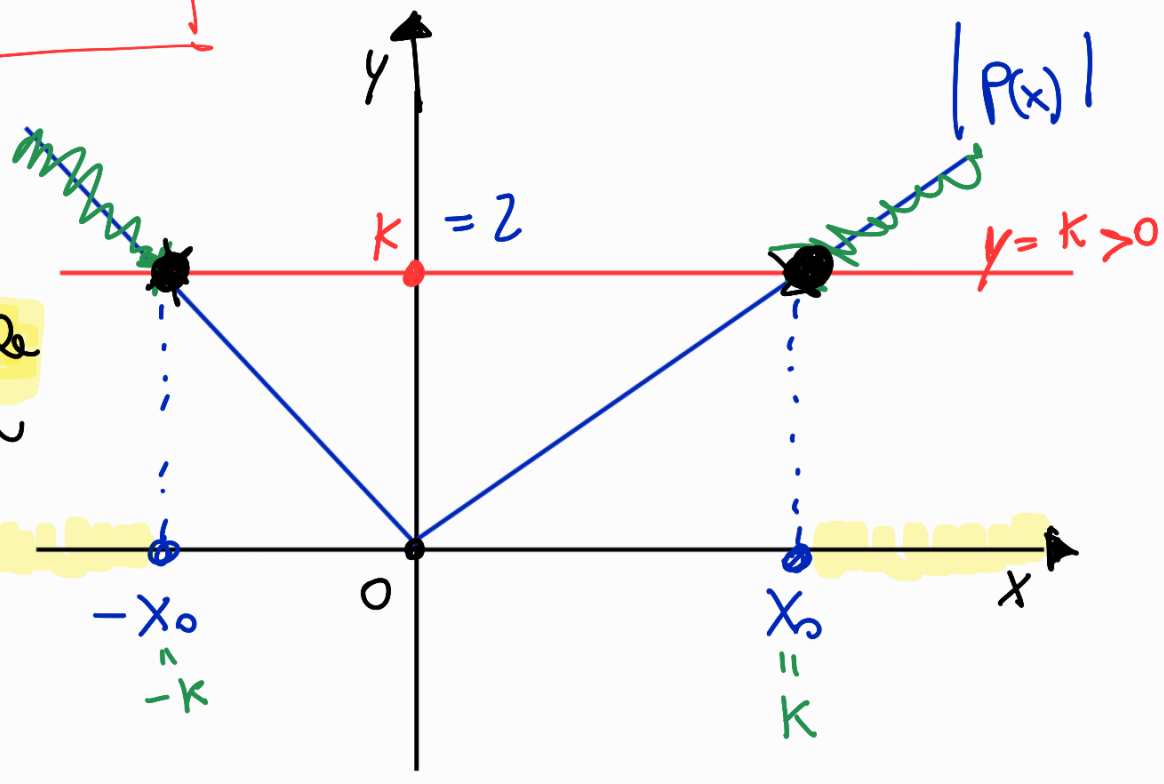
(−) $3x^2+4x \geq 0$; $x \leq -\frac{4}{3} \vee x \geq 0$

$2x-1>0$; $x>\frac{1}{2}$

Disequazioni con valore assoluto

- Confronto tra una funzione di tipo valore assoluto e una numerica

$$|P(x)| \gtrless K \qquad K \in \mathbb{R}$$



da/le zone gialle rappresentano la "soluzione"!

- $|P(x)| > K \iff x < -x_0 \quad \vee \quad x > x_0$

$\iff x < -K \quad \vee \quad x > K.$

Es. $|x+3| > 2 \iff x+3 < -2 \quad \vee \quad x+3 > 2$

$\Leftrightarrow \quad x < -3-2 \quad \vee \quad x > -3+2 \quad \Leftrightarrow$

$\Leftrightarrow \quad x < -5 \quad \vee \quad x > -1$

$S =]-\infty, -5[\cup]-1, +\infty[$

Graficamente