Sistemi di disequazioni (di una sola variabile)

$$\begin{cases} P_1(x) \gtreqless 0 \\ P_2(x) \gtreqless 0 \\ \vdots \\ P_m(x) \gtreqless 0 \end{cases}$$

è un insieme di due o più disequazioni nella medesima incognita (x)

$P_i(x)$, $i = 1, \dots, m$, è un polinomio o un rapporto tra polinomi.

Risolvere un sistema di disequazioni (di una sola variabile) equivale a trovare l'insieme delle $x \in \mathbb{R}$ tali che tutte le disequazioni siano soddisfatte "congiuntamente".

Operazione di riferimento: intersezione insiemistica

Esempio

$$\begin{cases} 3x^2 + 7x + 4 \geq 0 \\ x^2 - 2x - 3 < 0 \end{cases} \iff \begin{cases} x \leq -4/3 \ \vee \ x \geq -1 \\ -1 < x < 3 \end{cases}$$

1^a eq; eq. ass. $3x^2 + 7x + 4 = 0$

$$\Delta = b^2 - 4ac = 49 - 4(3)(4) = 1 > 0$$

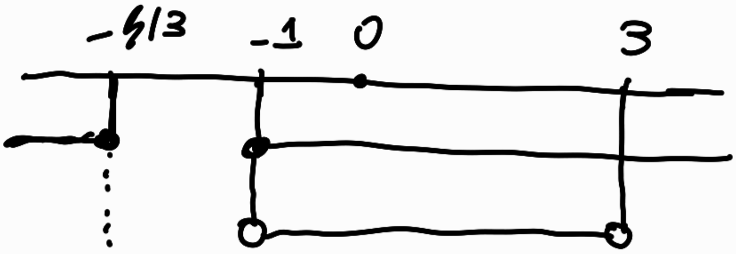
$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-7 \pm 1}{6} \rightarrow x_1 = -\frac{4}{3}, \quad x_2 = -1$$

2ª eq.; eq. ass. $x^2-2x-3=0$

$$\Delta = b^2-4ac = 4-4(-3) = 16 > 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{2 \pm 4}{2} \quad \begin{cases} x_1 = -1 \\ x_2 = 3 \end{cases}$$

$$\begin{cases} x \leq -4/3 \ \vee \ x \geq -1 \\ -1 < x < 3 \end{cases}$$



$-1 < x < 3$

$$S = \{x \in \mathbb{R} : -1 < x < 3\} =]-1, 3[$$

Esempio

$$\begin{cases} 2 + \dfrac{1}{x-1} \geq \dfrac{1}{x+1} \\ -4x^2 + 7 < 0 \end{cases} \iff \begin{cases} \dfrac{2}{1} + \dfrac{1}{x-1} - \dfrac{1}{x+1} \geq 0 \\ 4x^2 - 7 > 0 \end{cases}$$

1ª dis. $$\frac{2(x-1)(x+1) + 1\cdot(x+1) - 1\cdot(x-1)}{(x-1)(x+1)} \geq 0$$

$$\frac{2(x^2-1) + \cancel{x}+1 - \cancel{x}+1}{(x-1)(x+1)} \geq 0$$

$$\frac{2x^2 - \cancel{2} + \cancel{2}}{(x-1)(x+1)} \geq 0 \iff \frac{2x^2}{(x-1)(x+1)} \geq 0$$

$$\Leftrightarrow \frac{x^2}{(x-1)(x+1)} \geq 0 \Leftrightarrow \frac{x \cdot x}{(x-1)(x+1)} \geq 0$$

$$+ \left\| \begin{array}{l} x \geq 0 \\ x \geq 0 \\ x-1>0;\ x>1 \\ x+1>0;\ x>-1 \end{array} \right.$$

		-1		0		1	
	−		−	•	+		+
	−		−		+		+
	−		−		−		+
	−		+		+		+
	+		−	×	−		+

$x=0$

$$\frac{x^2}{(x-1)(x+1)} \geq 0 \Leftrightarrow x<-1 \vee x>1 \vee x=0$$

2^a dis. $4x^2-7>0 \Leftrightarrow x<-\frac{\sqrt{7}}{2} \vee x>\frac{\sqrt{7}}{2}$

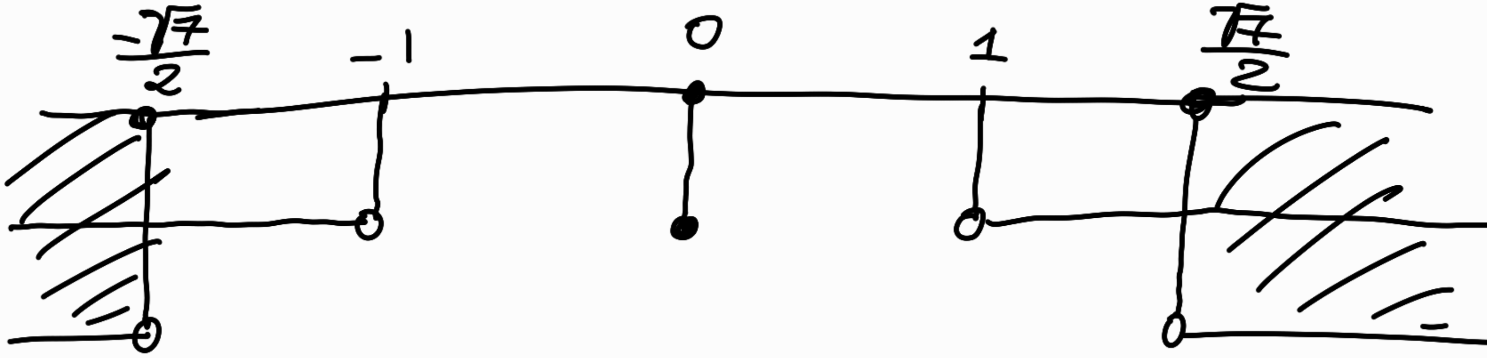
eq. ass. $4x^2-7=0$

$$\frac{\cancel{4}x^2}{\cancel{4}}=\frac{7}{4} \Leftrightarrow x^2=\frac{7}{4} \Leftrightarrow x_{1,2}=\pm\sqrt{\frac{7}{4}}$$

$$x_{1,2}=\pm\frac{\sqrt{7}}{2}$$

$$\begin{cases} x<-1 \vee x>1 \vee x=0 \\ x<-\frac{\sqrt{7}}{2} \vee x>\frac{\sqrt{7}}{2} \end{cases}$$

$$\sqrt{\frac{7}{4}}$$



$$S = \left\{ x \in \mathbb{R} : \ x < -\frac{\sqrt{7}}{2} \ \lor \ x > \frac{\sqrt{7}}{2} \right\} =$$

$$= \left] -\infty, -\frac{\sqrt{7}}{2} \right[\ \cup \ \left] \frac{\sqrt{7}}{2}, +\infty \right[$$

$$\begin{cases} \dfrac{10}{x^2+1} > 6 - x^2 \\ \\ -5x^2 + \sqrt{2}\,x - 8 \leq 0 \end{cases}$$

Soluzione

$$S = \{ x \in \mathbb{R} : \ x < -2 \ \lor \ -1 < x < 1 \ \lor \ x > 2 \}$$

Nota

x^4 ; $x^2 = t$; $x^4 = t^2$

Esempio

$t > 1 \iff x^2 > 1$

$$ax^4 + bx^2 + c = 0 \qquad \boxed{x^2 = t}$$

$$at^2 + bt + c = 0$$

$t_{1,2}$

$t_1 = 4$

$t_2 = 9$

$$t_1 = 4 = x^2$$

Esercitazione

1) $A = \{ x \in \mathbb{R} : x^2 - 5x + 4 \leq 0 \} = [1, 4]$

$B =]2, 5[$

determinare: $A \cup B$; $A \cap B$; $A \setminus B$; $B \setminus A$.

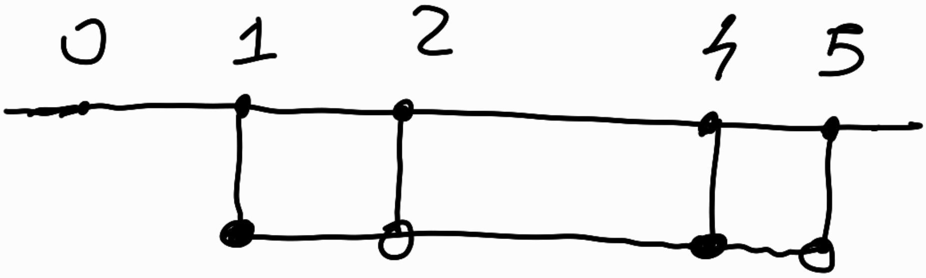
$x^2 - 5x + 4 = 0$

$\Delta = b^2 - 4ac = 25 - 16 = 9$

$x_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{5 \pm 3}{2}$ $\nearrow x_1 = 1$, $\searrow x_2 = 4$

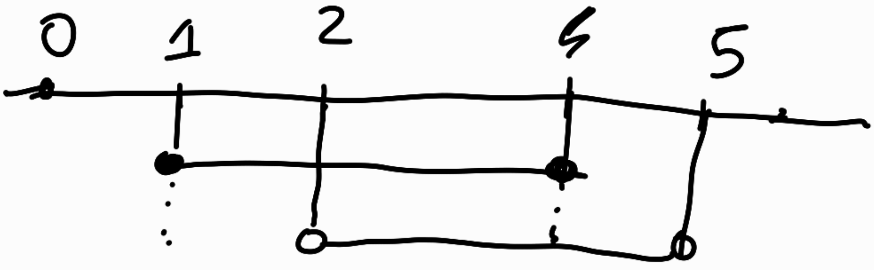
$A = [1, 4]$

$B =]2, 5[$



$A \cup B = [1, 5[$

$A \cap B =]2, 4]$



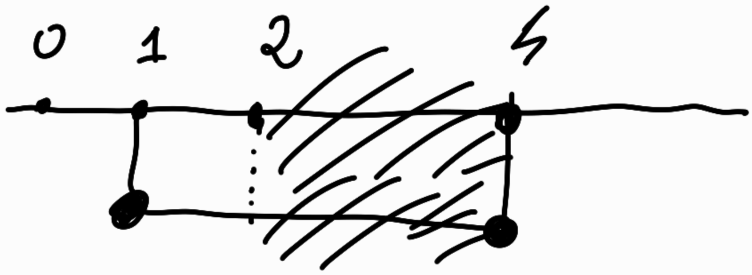
$A = [1, 4]$

$A \cap B =]2, 4]$

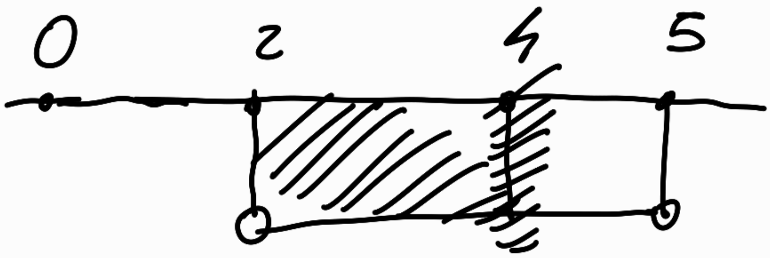
$B =]2, 5[$

$A \setminus B = A - (A \cap B) =$

$= [1, 2]$



$B \setminus A = B - (A \cap B)$



$B =]2, 5[$

$A \cap B =]2, 4]$

$B \setminus A =]4, 5[$

3) $P_1 = (1, -2) = (x_1, y_1)$

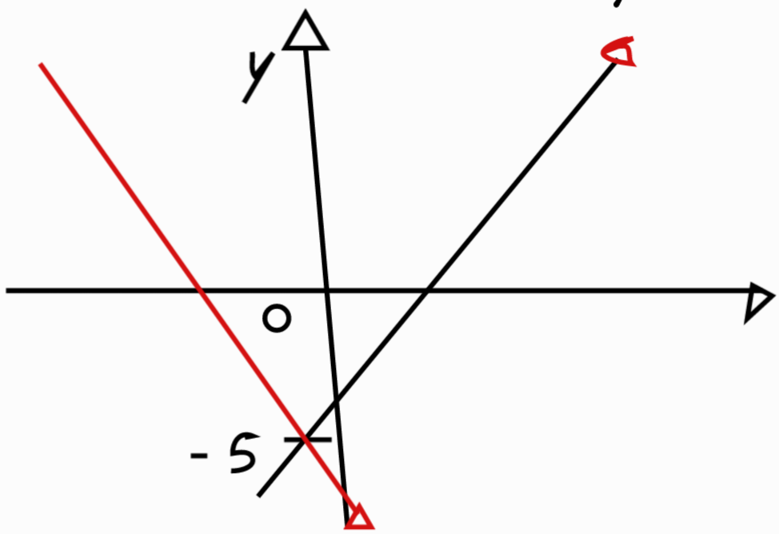
$P_2 = (4, 7) = (x_2, y_2)$

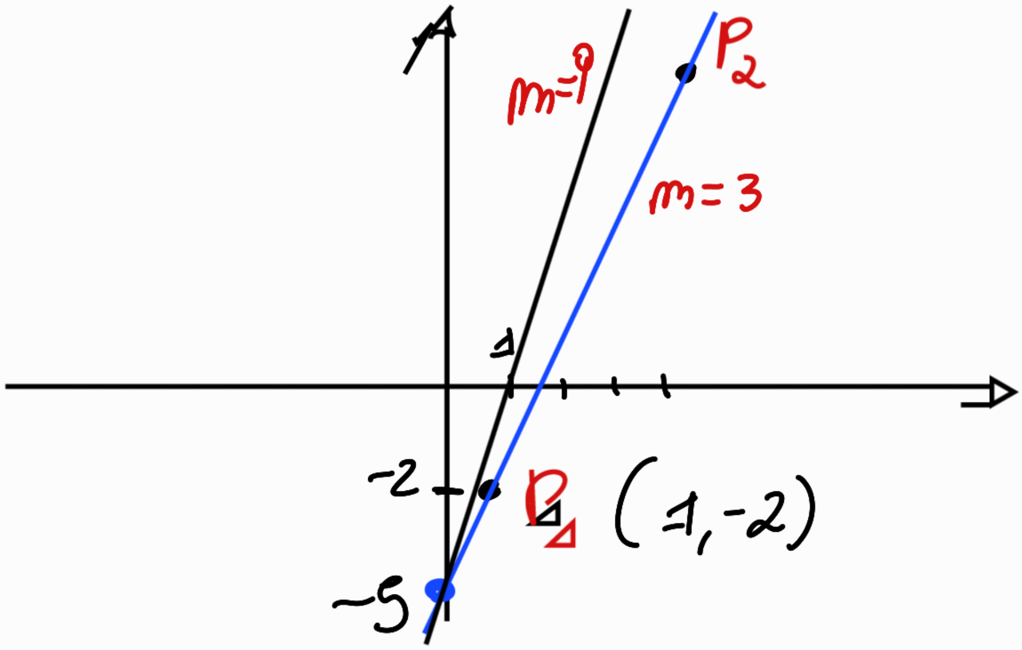
$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \iff \frac{y - (-2)}{7 - (-2)} = \frac{x - 1}{4 - 1}$$

$$\frac{y + 2}{9} = \frac{x - 1}{3} \iff y + 2 = \overset{3}{\cancel{9}} \cdot \frac{x - 1}{\underset{1}{\cancel{3}}}$$

$$y + 2 = 3(x - 1) \iff y = 3x - 3 - 2$$

$$y = 3x - 5$$





$y = 3x - 5$

$y = -5$

3 $P = (-1, 4)$ $\quad r: \ 4y - 8x + 3 = 0$

$s: \quad y - y_P = m_s(x - x_P)$

$s \parallel r: \quad m_s = m_r$ $\qquad s \perp r: \ m_s = \dfrac{-1}{m_r}$

$r: \ \boxed{4y - 8x + 3 = 0}$

$4y = 8x - 3$

$y = \dfrac{8x - 3}{4} \ ; \ y = \dfrac{\overset{2}{\cancel{8x}}}{\cancel{4}} - \dfrac{3}{4} = 2x - \dfrac{3}{4}$

$\boxed{y = 2x - \dfrac{3}{4}}$ $\qquad m_r = 2$

$ax + by + c = 0$ (eq. retta forma implicita)

$m = -\dfrac{a}{b}$

$4y - 8x + 3 = 0$

$-8x + 4y + 3 = 0$

$8x - 4y - 3 = 0$ $\qquad m = -\dfrac{a}{b} = -\dfrac{8}{-4} = +2$

$P = (-1, 4)$

$m_r = m_s = 2$

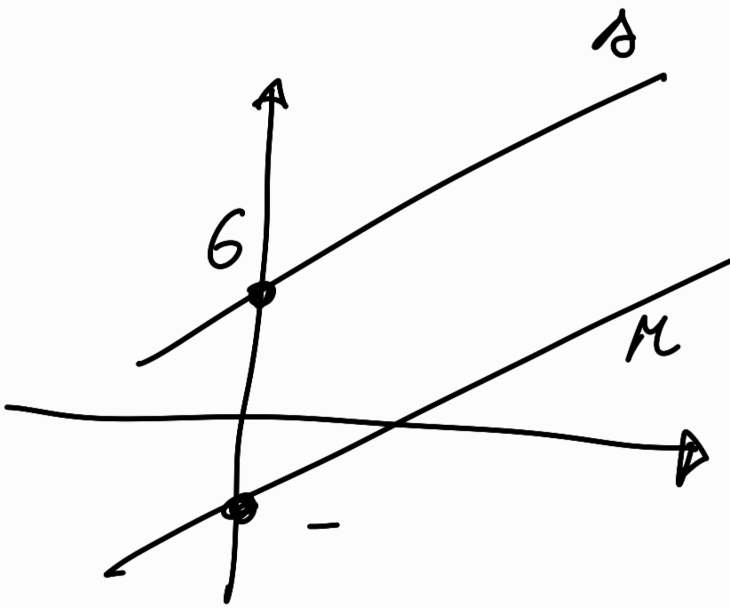
$y - y_P = m_s(x - x_P)$

$y - 4 = 2(x - (-1))$

$y = +4 + 2(x+1)$

$y = 4 + 2x + 2$

$y = 2x + 6$



12 $\quad Kx^2 - 2x + 3K \geq 0$

Trovare K affinché $X = \emptyset$

$$\boxed{\Delta < 0}$$

$$\Delta = b^2 - 4ac = 4 - 4(K)(3K) =$$

$$= 4 - 12K^2$$

$$4 - 12K^2 < 0$$

$$\boxed{12K^2 - 4 > 0} \longrightarrow \Delta < 0$$

$$4(3K^2 - 1) > 0 \iff 3K^2 - 1 > 0$$

eq. ass. $3K^2 - 1 = 0$

$$K^2 = \frac{1}{3}$$

$$K_{1,2} = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \pm \frac{\sqrt{3}}{(\cancel{\sqrt{3}})^{\cancel{2}}} = \pm \frac{\sqrt{3}}{3}$$

$$K_{1,2} = \pm \frac{1}{\sqrt{3}}$$

$$S_K = \left\{ K \in \mathbb{R} : K < -\frac{1}{\sqrt{3}} \cup K > \frac{1}{\sqrt{3}} \right\}$$