

Diseguazioni di II grado

$$ax^2 + bx + c \begin{matrix} \geq 0 \\ \leq 0 \end{matrix}$$

$$\begin{array}{|l} a, b, c \in \mathbb{R} \\ a > 0 \\ \hline a = 0 \end{array}$$

Siamo:

- $ax^2 + bx + c = 0$ l'equazione associata (*)
- $\Delta = b^2 - 4ac$ il discriminante (o delta)
- \bar{X} l'insieme delle soluzioni dell'equazione associata (*)
- S l'insieme delle soluzioni della disuguaglianza

CASO 1 $ax^2 + bx + c > 0$

1.1) $\Delta < 0$; $\bar{X} = \emptyset$; $S = \mathbb{R} =]-\infty, +\infty[$

1.2) $\Delta = 0$; $\bar{X} = \{x^*\}$; $S = \mathbb{R} - \{x^*\}$

$$S =]-\infty, x^*[\cup]x^*, +\infty[$$

$$1.3) \Delta > 0; X = \{x_1, x_2\}, x_1 < x_2;$$

$$S = \{x \in \mathbb{R} : x < x_1 \vee x > x_2\} = \\ =]-\infty, x_1[\cup]x_2, +\infty[$$

CASO 2

$$\underline{ax^2 + bx + c \geq 0}$$

$$2.1) \Delta < 0; X = \emptyset; S = \mathbb{R} =]-\infty, +\infty[$$

$$2.2) \Delta = 0; X = \{x^*\}; S = \mathbb{R} =]-\infty, +\infty[$$

$$2.3) \Delta > 0; X = \{x_1, x_2\}, x_1 < x_2;$$

$$S = \{x \in \mathbb{R} : x \leq x_1 \vee x \geq x_2\} = \\ =]-\infty, x_1] \cup [x_2, +\infty[$$

CASO 3

$$\underline{ax^2 + bx + c < 0}$$

$$3.1) \Delta < 0; X = \emptyset; S = \emptyset$$

$$3.2) \Delta = 0; \quad \underline{X} = \{x^*\}; \quad \underline{S} = \emptyset$$

$$3.3) \Delta > 0; \quad \underline{X} = \{x_1, x_2\}, \quad x_1 < x_2;$$

$$S = \{x \in \mathbb{R} : x_1 < x < x_2\} =]x_1, x_2[$$

$$\text{CASO 4} \quad ax^2 + bx + c \leq 0 \quad 0 \leq 0$$

$$4.1) \Delta < 0; \quad \underline{X} = \emptyset; \quad S = \emptyset$$

$$4.2) \Delta = 0; \quad \underline{X} = \{x^*\}; \quad S = \{x^*\} \quad \underline{\text{singolo}}$$

$$4.3) \Delta > 0; \quad \underline{X} = \{x_1, x_2\}, \quad x_1 < x_2 \uparrow$$

$$S = \{x \in \mathbb{R} : x_1 \leq x \leq x_2\} = [x_1, x_2] \\ (\text{completo})$$

Esempio

$$3x^2 + 11x - 4 \leq 0$$

$$\text{eq. associata} \quad 3x^2 + 11x - 4 = 0$$

$$\Delta = b^2 - 4ac = 121 - 4(3)(-4) = 121 + 48 = 169 > 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-11 \pm \sqrt{169}}{6} = \frac{-11 \pm 13}{6}$$

$$x_1 = -4; \quad x_2 = 1/3$$

$$S = \{x \in \mathbb{R} : -4 \leq x \leq 1/3\} = \left[-4; \frac{1}{3}\right]$$

Exemplar

$$x^2 + 5x + 7 \leq 0$$

eq. eqs. $x^2 + 5x + 7 = 0$

$$\Delta = b^2 - 4ac = 25 - 4(7) = 25 - 28 = \underline{-3} < 0$$

$$S = \emptyset$$

Exemplar

$$-2x^2 + 3x - 3 \geq 0 \Leftrightarrow \underline{2x^2 - 3x + 3 \leq 0}$$

eq. eqs. $2x^2 - 3x + 3 = 0$

$$\Delta = 9 - 4(2)(3) = 9 - 24 = \underline{\underline{-15}} < 0$$

$$\tilde{X} = \emptyset; \quad S = \emptyset$$

Esempio

$$-x^2 + 7x - 13 < 0 \Leftrightarrow x^2 - 7x + 13 > 0$$

eq. associata: $x^2 - 7x + 13 = 0$

$$\Delta = 49 - 4(13) = 49 - 52 = -3 < 0 \Rightarrow X = \emptyset$$

$$S = \mathbb{R} =] - \infty, +\infty [$$

~ 0 ~ 0 ~ 0 ~ 0

$$\boxed{ax^2 + bx + c = 0}$$

CASO $c = 0$ (Manca il termine noto)

$$ax^2 + bx = 0 \quad (\text{EQUAZIONI SPURIE})$$

$$(ax + b)x = 0$$

$$\begin{cases} \rightarrow x_1 = 0 \\ \rightarrow x_2 = -\frac{b}{a} \end{cases}$$

CASO $b = 0$ (Manca il termine lineare)

$$ax^2 + c = 0 \quad (\text{EQUAZIONE PURA})$$

$$x^2 = -\frac{c}{a}$$

$$x_{1,2} = \pm \sqrt{-\frac{c}{a}}$$

Esempio

$$\bullet \quad 3x^2 + 1 = 0$$

$$\frac{3}{3}x^2 = -\frac{1}{3} \Leftrightarrow x^2 = -\frac{1}{3} \Leftrightarrow x_{1,2} = \pm \sqrt{-\frac{1}{3}}$$

$$S = \emptyset$$

$$\Delta < 0$$

$$\Delta = b^2 - 4ac =$$

$$= -4ac = -12 < 0$$

$$\bullet \quad 3x^2 + 1 \geq 0$$

$$S = \mathbb{R} =]-\infty; +\infty[$$

CASO 2

$$\frac{P(x)}{Q(x)} \geq 0$$

$$Q(x) \neq 0$$

$$\frac{P(x)}{Q(x)} \geq 0 \Leftrightarrow \left\{ \begin{array}{l} P(x) \geq 0 \\ Q(x) > 0 \end{array} \right. \cup \left\{ \begin{array}{l} P(x) \leq 0 \\ Q(x) < 0 \end{array} \right.$$

CASO 3

$$\frac{P(x)}{Q(x)} < 0$$

$$Q(x) \neq 0$$

$$\frac{P(x)}{Q(x)} < 0 \Leftrightarrow \left\{ \begin{array}{l} P(x) > 0 \\ Q(x) < 0 \end{array} \right. \cup \left\{ \begin{array}{l} P(x) < 0 \\ Q(x) > 0 \end{array} \right.$$

CASO 4

$$\frac{P(x)}{Q(x)} \leq 0$$

$$Q(x) \neq 0$$

$$\frac{P(x)}{Q(x)} \leq 0 \Leftrightarrow \left\{ \begin{array}{l} P(x) \geq 0 \\ Q(x) < 0 \end{array} \right. \cup \left\{ \begin{array}{l} P(x) \leq 0 \\ Q(x) > 0 \end{array} \right.$$

Ejemplo

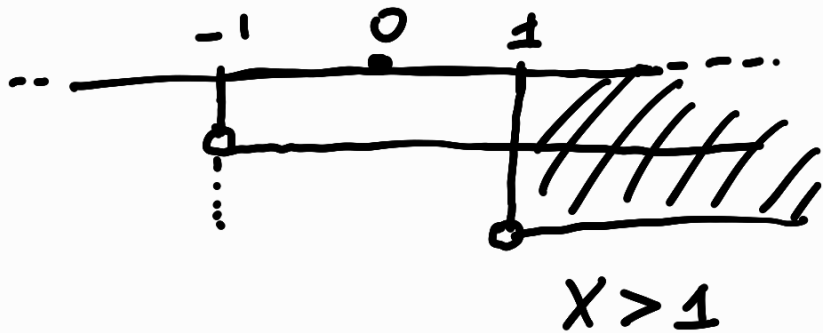
$$\frac{x+1}{x-1} > 0 \Leftrightarrow \left\{ \begin{array}{l} x+1 > 0 \\ x-1 > 0 \end{array} \right. \cup \left\{ \begin{array}{l} x+1 < 0 \\ x-1 < 0 \end{array} \right.$$

$$\Leftrightarrow \left\{ \begin{array}{l} x > -1 \\ x > 1 \end{array} \right\} \cup \left\{ \begin{array}{l} x < -1 \\ x < 1 \end{array} \right\}$$

$$x > 1 \vee x < -1 \Leftrightarrow$$

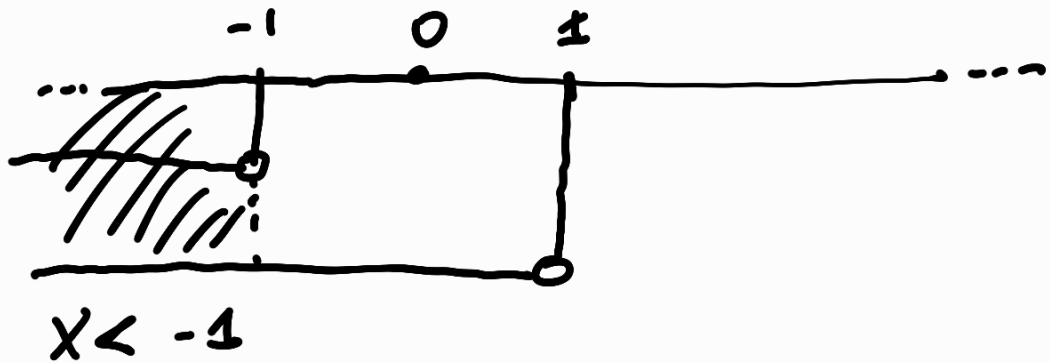
$$\Leftrightarrow x < -1 \vee x > 1$$

$$S_1: \left\{ \begin{array}{l} x > -1 \\ x > 1 \end{array} \right\}$$



$$S_1 = \{x > 1\}$$

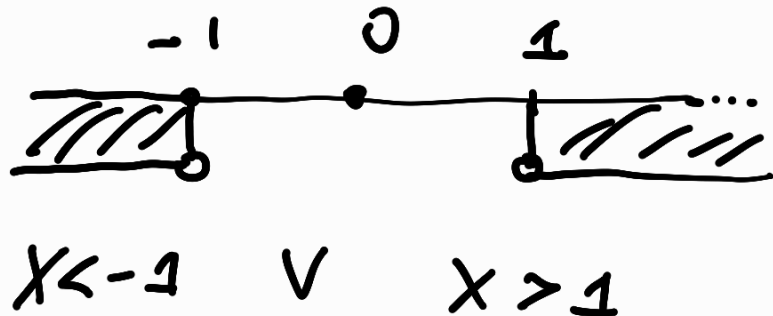
$$S_2: \left\{ \begin{array}{l} x < -1 \\ x < 1 \end{array} \right\}$$



$$S_2 = \{x < -1\}$$

Unione

$$\{x > 1\} \cup \{x < -1\}$$



$$S = \{x \in \mathbb{R}: x < -1 \vee x > 1\} =]-\infty, -1[\cup]1, +\infty[$$

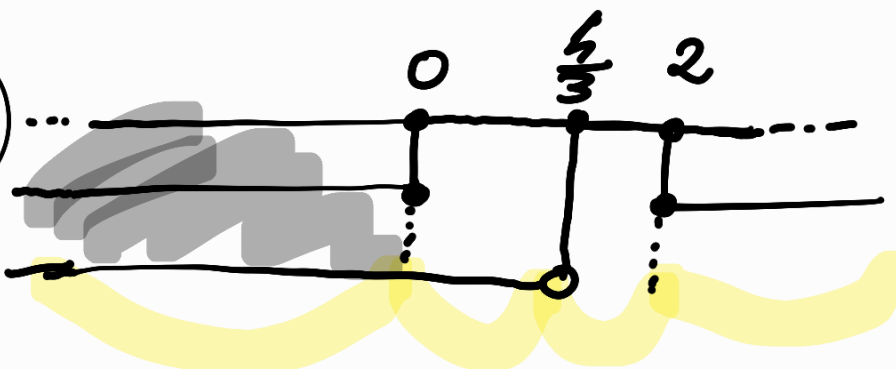
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$$\frac{x^2 - 2x}{3x - 4} \leq 0 \Leftrightarrow \begin{cases} x^2 - 2x \geq 0 \\ 3x - 4 < 0 \end{cases} \cup \begin{cases} x^2 - 2x \leq 0 \\ 3x - 4 > 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 2 \end{cases} \begin{cases} x(x-2) \geq 0 \\ x < 4/3 \end{cases} \cup \begin{cases} x(x-2) \leq 0 \\ x > 4/3 \end{cases} \Leftrightarrow$$

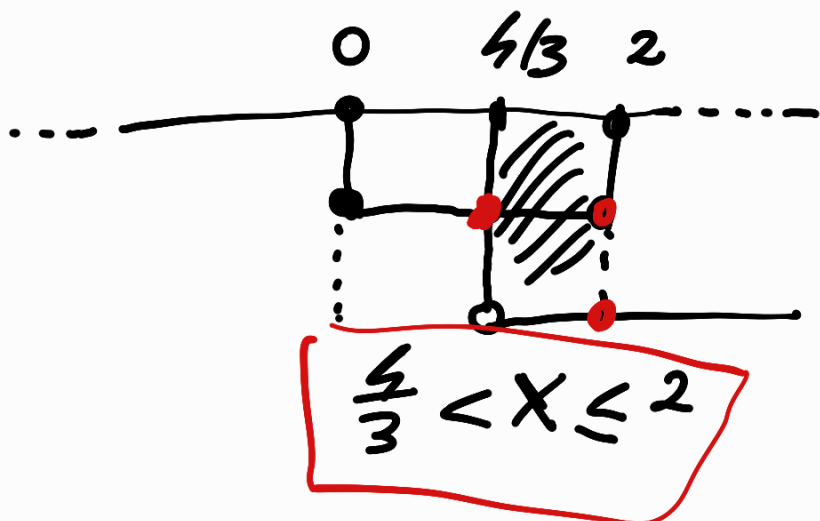
$$\Leftrightarrow \begin{cases} x \leq 0 \vee x \geq 2 \\ x < 4/3 \end{cases} \cup \begin{cases} 0 \leq x \leq 2 \\ x > 4/3 \end{cases}$$

$$S_1: \begin{cases} x \leq 0 \vee x \geq 2 \\ x < 4/3 \end{cases}$$



$$S_1 = \{ x \in \mathbb{R} : x \leq 0 \} \quad x \leq 0$$

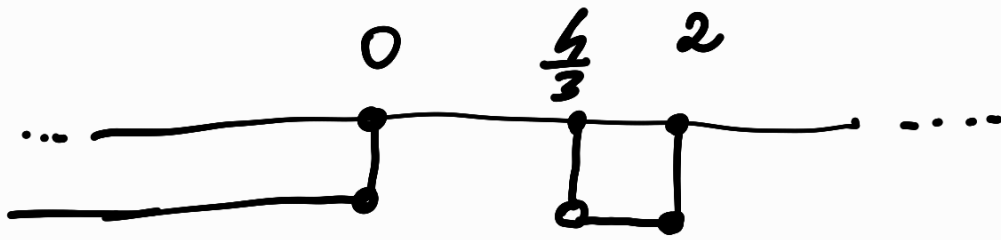
$$S_2: \begin{cases} 0 \leq x \leq 2 \\ x > 4/3 \end{cases}$$



$$S_2 = \{ x : \frac{4}{3} < x \leq 2 \}$$

Unione

$$\{x \leq 0\} \cup \left\{ \frac{4}{3} < x \leq 2 \right\}$$



$$\begin{aligned} S &= \{x \in \mathbb{R} : x \leq 0 \vee \frac{4}{3} < x \leq 2\} = \\ &=]-\infty, 0] \cup]\frac{4}{3}, 2] \end{aligned}$$

Esempio

$$\frac{x^2 + 5x - 6}{-x + 3} \leq 0$$

$$\text{I) } \frac{x^2 + 5x - 6}{-x + 3} \leq 0 \Leftrightarrow \begin{cases} x^2 + 5x - 6 \geq 0 \\ -x + 3 < 0 \end{cases} \cup \begin{cases} x^2 + 5x - 6 \leq 0 \\ -x + 3 > 0 \end{cases}$$

$$\text{II) } (-1) \cdot \frac{x^2 + 5x - 6}{-x + 3} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{x^2 + 5x - 6}{x - 3} \geq 0$$

I метод

$$\frac{x^2 + 5x - 6}{-x + 3} \leq 0$$

$$\left\{ \begin{array}{l} x^2 + 5x - 6 \geq 0 \\ -x + 3 < 0 \end{array} \right.$$

$$\cup \left\{ \begin{array}{l} x^2 + 5x - 6 \leq 0 \\ -x + 3 > 0 \end{array} \right.$$

eq. нулей $x^2 + 5x - 6 = 0$

$$\downarrow x - 3 < 0$$

$$\Delta = 25 + 24 = 49 > 0$$

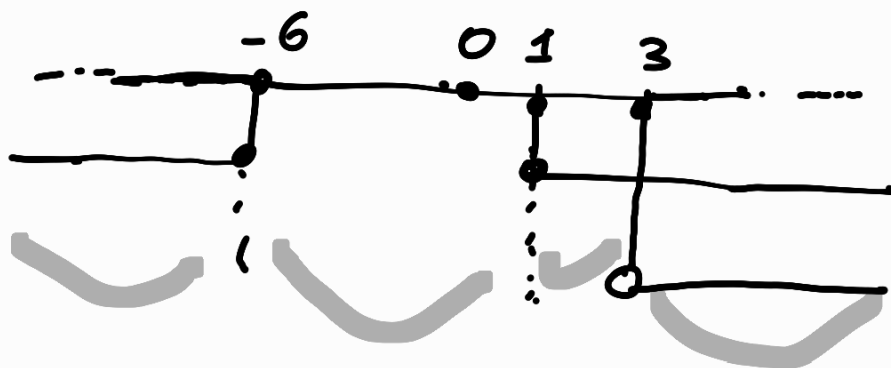
$$x < 3$$

$$x_{1,2} = \frac{-5 \pm 7}{2} \begin{cases} x_1 = -6 \\ x_2 = 1 \end{cases}$$

$$\left\{ \begin{array}{l} x \leq -6 \cup x \geq 1 \\ x > 3 \end{array} \right.$$

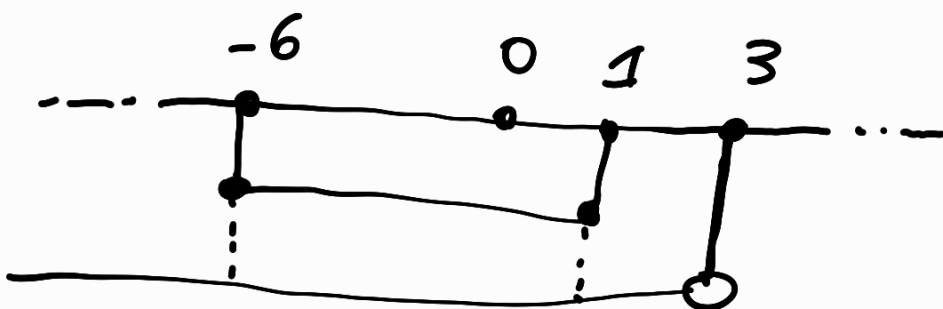
$$\cup \left\{ \begin{array}{l} -6 \leq x \leq 1 \\ x < 3 \end{array} \right.$$

$$S_1: \left\{ \begin{array}{l} x \leq -6 \cup x \geq 1 \\ x > 3 \end{array} \right.$$

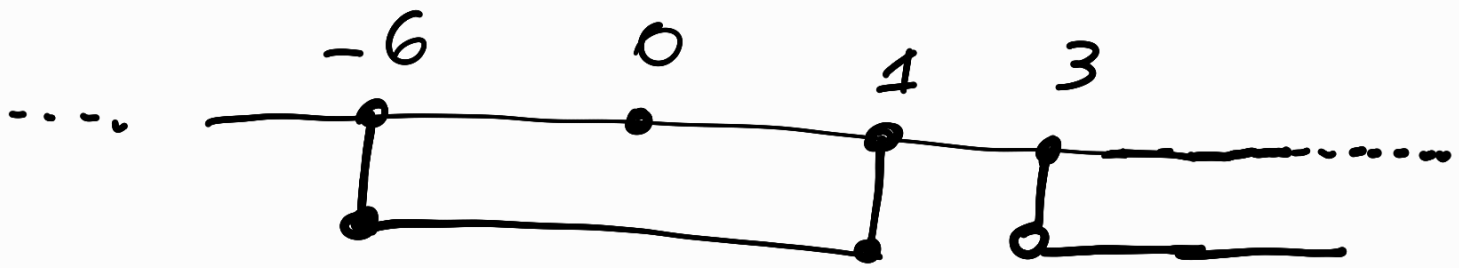


$$S_1 = \{ x > 3 \}$$

$$S_2: \left\{ \begin{array}{l} -6 \leq x \leq 1 \\ x < 3 \end{array} \right.$$



$$S_2 = \{-6 \leq x \leq 1\}$$

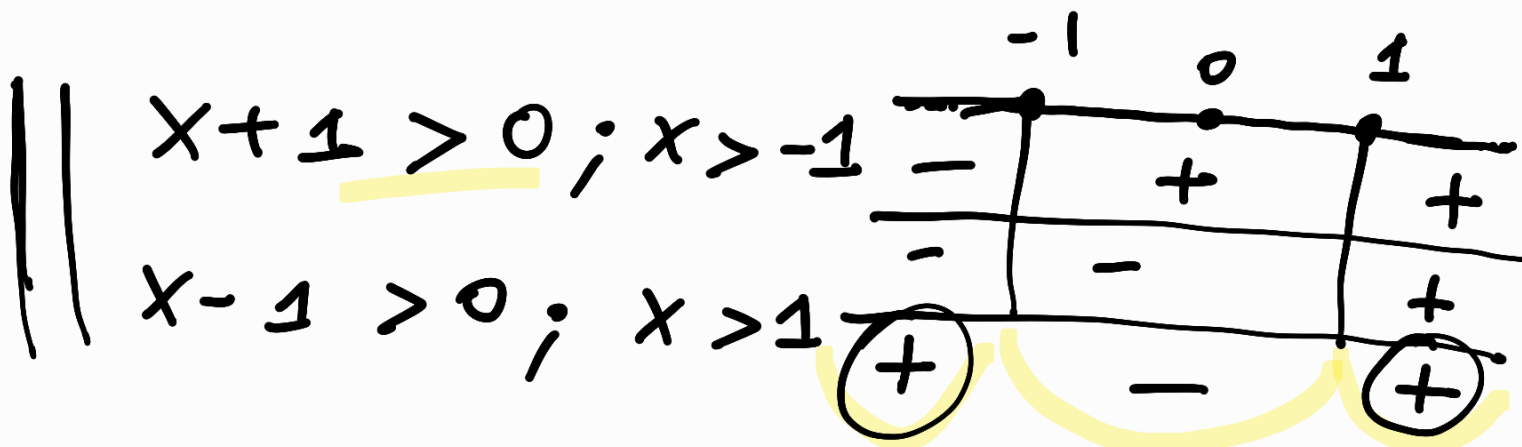


$$S = \{x \in \mathbb{R} : -6 \leq x \leq 1 \vee x > 3\}$$

$$= [-6, 1] \cup]3, +\infty[$$

Studio del segno

$$\frac{x+1}{x-1} > 0$$



$$S = \{x \in \mathbb{R} : x < -1 \vee x > 1\}$$

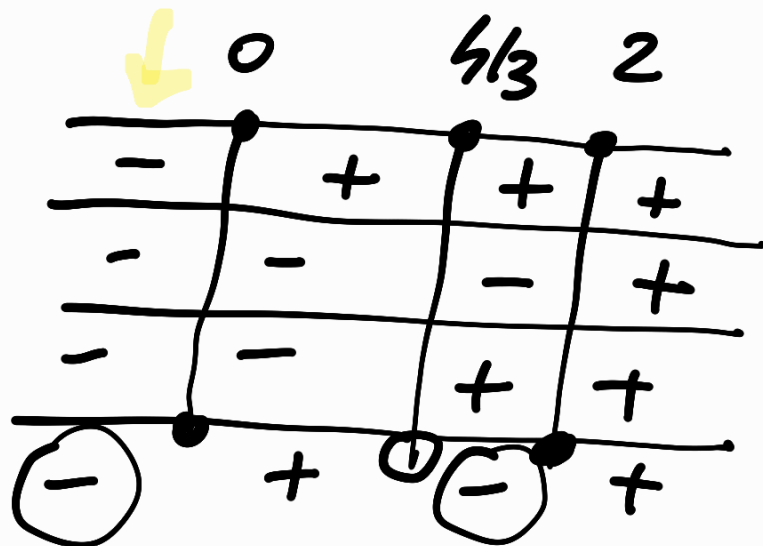
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$$\frac{x^2 - 2x}{3x - 4} \leq 0$$

$$\begin{cases} x^2 - 2x \geq 0; \\ 3x - 4 > 0 \end{cases}$$

$$\frac{x^2 - 2x}{3x - 4} \leq 0 \Leftrightarrow \frac{x(x-2)}{3x-4} \leq 0$$

$$\begin{cases} x \geq 0 \\ x - 2 \geq 0; x \geq 2 \\ 3x - 4 > 0; x > \frac{4}{3} \end{cases}$$



$$S = \{x \in \mathbb{R} : x < 0 \vee \frac{4}{3} < x \leq 2\} = [-\infty, 0] \cup \left] \frac{4}{3}, 2 \right]$$