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COURSE SYLLABUS
Scientific Computing (part 2) – IA:ML&BD

Teaching Material

Web Learning service: https://elearning.uniparthenope.it/.
Course name: SCIENTIFIC COMPUTING - part 2 (EN - since A.Y. 2022-2023)
URL: https://elearning.uniparthenope.it/course/view.php?id=2025 .

DETAILED SYLLABUS

SC2_00 – *Introduction to Part 2 of the Course*

[SC2_00] Informations about the course page on the e-Learning platform and related contents, about Teams classes and codes. In addition the course aim is explained, also supported by several MATLAB examples.

SC2_01 – *Useful topics and tools*

[SC2_01a] Recap: the **MATLAB Editor** (m-files). Sections and "Run Section". Different ways to execute a script. What is a "Live script". Examples. Advanced use of the **MATLAB Live Editor** (mlx-files): insert a Task (example: Create a Plot), insert a math formula (by templates or by LaTeX); insert a Control (examples: numeric slider, edit field, ...).

[SC2_01b] **Brief notes on complex numbers**. Cartesian and polar coordinate systems in the plane; passing from a coordinate system to another. Complex numbers. Real part, imaginary part, modulus and argument of a complex number. Representation of complex numbers: algebraic form, trigonometric form and exponential form. The Principal Argument of a complex number. The complex conjugate of a complex number. MATLAB function: **real()**, **imag()**, **abs()**, **angle()**, **conj()**, and examples. Operations on complex numbers (algebraic addition, product, reciprocal). Integer power of a complex number. The nth roots of a complex number: mathematical formula, the MATLAB **roots()** function. The nth roots of unity and their applications. Primitive n^{th} roots of unity: definition, examples, properties and applications. Complex-valued functions of a real variable. Examples and plots with MATLAB. Complex-valued functions of a complex variable. Examples and 3D plots.

[SC2_01c] **Symbolic Computations in MATLAB** (required the *Symbolic Math Toolbox*). Symbolic objects (constants, variables, functions and formulas). Comparison between numeric and symbolic objects. The function **simplify()** for both m-files and mlx-files, and insert task "Simplify Symbolic Expressions" in a Live Script. Facilities provided by *Symbolic Math Toolbox*: solve equations and systems by **solve()**, limits by **limit()**, summation of series by **symsum()**, Taylor expansion by **taylor()**, ordinary and partial derivatives by **diff()**, gradient by **gradient()**, Jacobian matrix by **jacobian()**, definite and indefinite integrals by **int()**. Example of symbolic calculus: study of a function. Symbolic Linear Algebra: **rank()**, **det()**, **inv()**, "\", **colspace()**, **null()**, **eig()**, **all()**, **any()**. Solve symbolic differential equations by **dsolve()**. Examples.

SC2_02 – *Linear Spaces and subspaces*

[SC2_02a] Basics of Algebra (abelian groups and fields). Linear (or vector) space definition. Linear combination of two vectors. Examples of Linear Spaces. Subspaces of a linear space and their Characterization Theorem. Examples: Null Space of a matrix. Linear combination of n vectors (span{…}). Examples (with MATLAB code): subspace spanned by a single vector, by two vectors.

[SC2_02b] Vector parametric equation and scalar parametric equations of a line in R^2, R^3, …, R^n. Parametric equation of a line segment in R^2, R^3. MATLAB Lab.: given a line segment in R^2 and another point, establish if it lies on the segment. Cartesian equation of a line in R^2. Vector parametric equation and scalar parametric equations of a plane in R^2, R^3, …, R^n. Cartesian equation of a plane in R^3. Passing from scalar parametric equations to cartesian equation, and viceversa. Draw with MATLAB a plane given by its cartesian equation. Compute with MATLAB (both numerical and symbolic) the four fundamental subspaces of a matrix: the Column Space, the Null Space, the Row Space and the Left Null Space.

[SC2_02c] Linear independence of vectors. Examples. Characterization Theorem of n linear dependent vectors, and consequences (parallel vectors, coplanar vectors of a Linear Space). Matrix rank and row-reduced echelon form of a matrix as tools to find how many and which rows (and columns) are linearly independent. The function `rref()`. Bases of a Linear Space. Examples. Components of a vector w.r.t. a basis of the space. Change of basis. Theorem about the cardinality of all the bases of a Linear Space.

[SC2_02d] Dimension of a Linear Space. Examples. Properties about the dimension of Linear Subspaces. Dimensions and bases of the four fundamental subspaces of a matrix. Compute with MATLAB (both numerical and symbolic) a basis for the four fundamental subspaces of a matrix.

SC2_03 – *Dot products, norms and their applications*

[SC2_03a] Definition of a dot product between real vectors (positive definite symmetric bilinear form). Check by means of MATLAB Symbolic Math Toolbox if a bilinear form is a dot product.

[SC2_03b] Dot product between complex vectors (positive definite symmetric sesquilinear form). An example of dot product in the linear space $C[a,b]$ of complex-valued functions that are continuous on $[a,b]$. Definition of a vector norm. Examples of vector norms in R^n: Euclidean norm (or 2-norm), uniform norm (or ∞-norm), Manhattan norm (or 1-norm). The MATLAB `norm()` function. Different shapes of neighborhoods in metric spaces depending on the particular norm. Induced vector norm. Parallelogram Law.

[SC2_03c] Induced matrix norms. Examples. Geometrical interpretation of matrix norms. MATLAB Lab.: estimate, by means of *Symbolic Math Toolbox*, an induced matrix norm from its definition. Condition number of a generic matrix. Compare it to the condition number for inversion. Orthogonal vectors. Orthogonal vectors are linearly independent. Proof of equivalent formulas for the standard dot product:

$$\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \sum_{k=1}^{n} u_k v_k = \|\boldsymbol{u}\| \|\boldsymbol{v}\| \cos \theta$$

[SC2_03d] Applications of dot products and vector norms: angle between subspaces (angle between two vectors, between lines, angle between a line and a plane, angle between two planes). The function `subspace()` for the angle between subspaces. Geometric interpretation of the cartesian equation of a line in $\mathbf{R}^2$ and of a plane in $\mathbf{R}^3$ (as an orthogonality condition). Examples. MATLAB Labs.: component of a vector along another; compute (by means of *Symbolic Math Toolbox*) the tangent line to a regular curve in $\mathbf{R}^2$ at a point on the curve; compute also the normal line to a regular curve; compute in $\mathbf{R}^3$ the tangent plane to a regular surface at a point on the surface and the related normal line.

SC2_04 – *Sum and direct sum subspace, complementary subspace and orthogonal complement of a subspace*

[SC2_04a] Definition of sum subspace, direct sum subspace of two subspaces, of complement subspace and of orthogonal complement. Examples. For every subspace, there are several complement subspaces in the same space. Grassman's formula and other properties about the direct sum, and about the orthogonal complement. Intersection of two subspaces is a linear subspace, while their union is not.

[SC2_04b] MATLAB Labs.: compute in $\mathbf{R}^3$ the intersection between two planes, first given their bases, and second given their cartesian equations; compute in $\mathbf{R}^3$ the orthogonal complement of a plane, given a basis; compute its cartesian equation. Application of the MATLAB `view()` function to put the viewer in front of the plane.

[SC2_04c] Fundamental Theorem of Linear Algebra about the four fundamental subspaces of a matrix. Examples.

[SC2_04d] Orthogonal and orthonormal vector systems. Gram-Schmidt orthonormalization algorithm (GSO). Examples of GSO. Matrices with orthonormal columns, and its properties. Examples of GSO in MATLAB (also for function subspaces).

[SC2_04e] GSO to check the linear independence of input vectors. GSO and QR factorization. The MATLAB `qr()` function.

SC2_05 – *Affine Spaces*

[SC2_05a] Affine Spaces: definition, properties and examples. Connections between an Affine Space and a Linear Space. Dimension of an Affine Space; affine reference system; affine coordinates of a point. Affine subspaces. Particular affine subspaces of $\mathbf{R}^n$: points, lines and planes. Parametric equation of a line, and of a plane. Parallelism and intersection of affine subspaces. MATLAB Lab.: algorithms to compute the intersection between two affine subspaces (a plane and a line) of $\mathbf{R}^3$. Affinely independent points. Examples of affine reference systems in $\mathbf{R}^2$ and $\mathbf{R}^3$.

SC2_06 – *Linear mappings*

[SC2_06a] Definition of a Linear Map (homomorphism) and examples. Linear map induced by a matrix. Definition of endomorphism, isomorphism, automorphism and examples. Kernel and Range of a linear map as linear subspaces (Theorem with proof). Examples.

[SC2_06b] Characterization Theorem about injective and surjective linear maps; its particularization to a linear map induced by a matrix. Theorem (with proof): the Column Space and the Row Space of a matrix are isomorphic. Lab.: given a matrix, find the isomorphism between its Column Space and its Row Space. Change-of-basis matrix. Advantage in using an orthonormal basis.

[SC2_06c] Examples of elementary 2D linear maps: uniform scaling (or radial homothety), non-uniform scaling, reflections over cartesian axes. Matrix of an orthogonal reflection over any line r=span$\{\boldsymbol{a}\}$, and MATLAB Lab. Properties of 2D reflection matrices. Another matrix form for a generic 2D reflection (with proof by means of *Symbolic Math Toolbox*). Comparison between an orthogonal reflection over span$\{\boldsymbol{a}\}$ and Householder reflection. Matrix form of a plane rotation. Properties of a rotation matrix. Horizontal and vertical 2D shears. MATLAB Lab.: proof, by means of *Symbolic Math Toolbox*, that a rotation is made up of 2 shears (a vertical and a horizontal shear) and a non-uniform scaling.

[SC2_06d] A translation is not a linear map; it is an affine map. Homogeneous coordinates. Matrix of translation in homogeneous coordinates. Combining the matrices to enhance performance: roto-translation in cartesian and homogeneous coordinates. Matrix form of an orthogonal projection onto a line. Application: distance between a point and a line. MATLAB Lab.: factorization of a 2D linear map induced by a matrix (t_A) into elementary linear maps. Permutation matrix and its properties.

[SC2_06e] Examples of elementary 3D linear maps: uniform and non-uniform scaling, reflections across the cartesian planes, and across the cartesian axes; rotation centered at O around a cartesian axis. Properties of a rotation matrix: proper and improper rotation. Rotation matrix, in $\mathbf{R}^3$, around an axis

span{$\boldsymbol{a}$} (normalized vector) and by an angle θ. Compute from the matrix the rotation axis and the rotation angle. Examples. Matrix form of an orthogonal reflection across a line *r*, and across a plane π. Properties of 3D reflection matrices. Examples. Matrix form of an orthogonal projection onto a plane π=span{$\boldsymbol{u,v}$}. Its simplified formula if the basis vectors are orthonormal.

[SC2_06f] Generalized inverses of a matrix (g-inverses). ABCD Theorem and construction of a generalized inverse. Examples. Another kind of generalized inverse: pseudoinverse matrix (Moore-Penrose inverse) and its existence and unicity theorem. Examples. The MATLAB `pinv()` function and computing the pseudoinverse by SVD factorization. Another kind of generalized inverse: left and right (one-sided) inverse, existence theorem, uniqueness theorem. MATLAB examples. General solution of a compatible underdetermined linear system. Least-norm solution of underdetermined linear systems. MATLAB examples.

SC2_07 – *Affine mappings*

[SC2_07a] Definition of an affine map (a compatible map whose induced map is linear), of an affinity and examples. Representation Theorem of an affine map between Affine Spaces of finite dimensions over the same field K. Affine combination of points. Elementary 2D real affine maps (in cartesian and homogeneous coordinates). How to detect an affine map between $\mathbf{R}^n$ and $\mathbf{R}^m$, given a suitable number of points in the domain and their image points in the codomain. When is the problem uniquely solved? Examples. Elementary affinities: isometries and others. Property for the matrix of an isometry.

[SC2_07b] Examples of 2D elementary affinities when applied to the vertices of a rectangle (both w.r.t. cartesian ad homogeneous coordinates): homothety, non-uniform scaling, rotation, horizontal and vertical shear, translation.

[SC2_07c] The MATLAB `patch()` function to draw a parallelepiped. Examples of 3D elementary affine maps: homothety, non-uniform scaling, reflection across XY-plane, rotation by an angle θ around X-axis, around Y-axis, translation.

SC2_08 – *Conformal mappings*

[SC2_08a] Steps in solving a problem by means of a computer. Domain and problem transformations. Examples. Complex limit of a complex function, and complex derivative. Examples. Holomorphic functions. Cauchy-Riemann Equations (w.r.t. *z* and its complex conjugate). Equivalent statements for holomorphic functions. The Laplace PDE (partial differential eq.) and armonic functions.

[SC2_08b] Complex form of 2D mappings. Examples. Conformal and anticonformal maps. Theorem about holomorphic functions and conformal maps. Conformal and anticonformal mappings and Laplace Equation. Examples. Jacobian matrix of a 2D map and its application to the image of the tangent to a curve in $\mathbf{R}^2$. Examples. Theorem on local invertibility of a complex map. Critical points. Example: the quadratic map.

[SC2_08c] Law on local arc transformation, in the complex plane, by means of a holomorphic function: transformation of infinitesimal angles and segments. Consequence: if a complex map is described by a holomorphic function at a point where its derivative does not vanish, then the map is conformal at that point. Geometric interpretation of the derivative of a complex function. Example.

[SC2_08d] Elementary complex maps: rotation, quadratic map, complex conjugate map (reflection across the x-axis), exponential map, inversion map.

[SC2_08e] The extended complex plane. 2D stereographic projection. The Riemann Sphere, the 3D stereographic projection and its property about generalized circles. Examples. *Moebius transformations* T_M between extended complex planes. The matrix *M* of a T_M. Is T_M a conformal map? The inverse map of a Moebius transformation. Properties of a T_M: fixed points (and eigenvectors of *M*), generalized circles and their images. Examples. Find a T_M given 3 points and their images (with MATLAB Lab.).

[SC2_08f] Applications of a Moebius mapping: example 1 about the domain simplification, example 2 about straightening a crooked image. MATLAB Lab.: straighten the curved side of a rectangle or of a rectangular curved grid. Combination of two local maps, such that we pass from one to another with continuity. Recalls about managing RGB images in MATLAB. Joukowski mappings. Symmetric and non-symmetric Joukowski maps. Their application to airfoil. Examples.

SC2_09 – *Eigenvalues and eigenvectors*

[SC2_09a] Definition of eigenvalues and eigenvectors. Find the eigenvalues of a matrix as roots of the characteristic polynomial. Example. The MATLAB `eig()`, `charpoly()` and `roots()` functions. d'Alembert's Theorem on the number and type of eigenvalues of a matrix $n \times n$. The spectrum and the spectral radius of a matrix. Gershgorin's Circle Theorem to estimate the location of eigenvalues in the plane. Examples.

[SC2_09b] Find the eigenvectors related to an eigenvalue and the corresponding eigenspace as a Null Space. Examples. The MATLAB `eig()` function (both numerical and symbolic) to compute eigenvalues and eigenvectors of a matrix. The case of symmetric matrices. Algebraic and geometric multiplicity of an eigenvalue, and related properties. Properties of eigenvalues/eigenvectors of any square matrix. Properties of eigenvalues/eigenvectors of a real symmetric matrix. Conjugate transpose (or hermitian transpose) matrix and unitary (or hermitian) complex matrices. Proof of the property: "All the eigenvalues of a hermitian matrix are real".

[SC2_09c] Examples, with MATLAB, on properties of eigenvalues/eigenvectors. Geometrical interpretation of eigenvalues/eigenvectors and eigenspaces. Examples. A symmetric matrix 2×2 and its ellipse.

[SC2_09d] Eigenvalues/eigenvectors of particular symmetric matrices obtained as product of a rectangular matrix by its transpose ($A^\mathsf{T}A$ and AA^T). Eigenvalues/eigenvectors of some elementary 2D linear maps and geometrical interpretation of eigenspaces: rotation, orthogonal reflection across x-axis, orthogonal reflection across a line. Proof of the properties of a 2D orthogonal reflection across a generic line span$\{\boldsymbol{a}\}$: its eigenvalues are always ± 1, and the eigenspaces are always span$\{\boldsymbol{a}\}$ and span$\{\boldsymbol{a}\}^\perp$. Eigenvalues/eigenvectors of some elementary 3D linear maps and geometrical interpretation of eigenspaces: orthogonal projection on a plane $\mathscr{R}(A)$.

[SC2_09e] Similar matrices. Diagonalizability criterion and example on the diagonalization of a matrix 2×2: the case of a generica matrix and of a symmetric matrix. Spectral Theorem for symmetric real matrices (orthogonally diagonalizable). MATLAB examples. Relation between SVD and diagonalization. MATLAB examples.

[SC2_09f] Consequences and applications of the diagonalization of a matrix: compute A^n efficiently and accurately; compute the team ranking of a tournament; detect the number of connected components of an undirected graph.

SC2_10 – *Principal Component Analysis* (*PCA*)

[SC2_10a] Outline of inferential statistics: data matrix, sample mean vector, centered data matrix and standardized matrix, scatter matrix, covariance matrix and correlation matrix from a statistical point of view and from a Linear Space point of view. MATLAB examples (`mean()`,`std()`,`normalize()`, `cov()`,`corrcoef()`,`pca()` functions). PCA and eigenvalues/eigenvectors of the covariance matrix, PCA and Singular Value Decomposition of the centered matrix. Example on correlation between variables.

[SC2_10b] What is PCA? How does it work? Gradient properties of the standard scalar product (with proofs). Incremental PCA algorithm derivation: how to find the first principal direction.

[SC2_10c] Incremental PCA algorithm (contd.) as minimization of the reconstruction square error, and its connection with maximization of sample variance. The first principal direction is spanned by an eigenvector related to the maximum eigenvalue. For the second principal direction use the same algorithm after the dimensionality reduction using the *complementary projection matrix*. MATLAB illustration of the Incremental PCA algorithm using the *power method*. Geometrical interpretation of PCA. Full PCA algorithm and reduced PCA subspace (for dimensionality

reduction). Apply PCA to centered data matrix or standardized data matrix: what is better (scatter matrix, covariance matrix or correlation matrix)? Compute eigenvalues and eigenvectors by means of SVD factorization of centered data matrix.

[SC2_10d] PCA application: MATLAB Lab. on "*Eigenfaces*" algorithm for dimensionality reduction and fast face recognition.

[SC2_10e] Regression line (of y w.r.t. x, and of x w.r.t. y) in $\mathbf{R}^2$. Regression line or Least Squares line in $\mathbf{R}^2$ and Regression plane or Least Squares plane in $\mathbf{R}^3$ as minimization of the sum of squared residuals. MATLAB examples. Comparison between regression lines w.r.t. cartesian axes in $\mathbf{R}^2$, and among regression planes w.r.t. cartesian planes in $\mathbf{R}^3$. PCA vs Least Squares.

SC2_11 – *Linear Least Squares*

[SC2_11a] Noisy data: interpolation vs LS fitting. Definition of **the best approximation** in a subspace to a vector of the normed Linear Space. Famous norms in the: discrete and finite case (vector $x \in \mathbf{R}^n$), discrete and infinite case (sequence $\{x_k\}_k \in \ell^p$), continuous case (function). Theorem for the existence and uniqueness of the best approximation w.r.t. 2-norm in a finite dimensional subspace, and its geometrical interpretation. The system of Normal Equations and Gram matrix.

[SC2_11b] The best linear approximation in the discrete and finite case: solve incompatible systems by means of the *Least Squares method*. Solve incompatible systems with MATLAB. Definition of a LS solution. Verify, by means of Symbolic Math Toolbox, that a LS solution minimizes the residual norm. LS method applied to solvable systems. Property: "The LS solution of an incompatible system is the same as the solution of Normal Equations" (with proof).

[SC2_11c] Normal Equations in the continuous finite case: examples on computing the best approximation to a function w.r.t. 2.norm.

[SC2_11d] More on Normal Equations. Gram matrix of the subspace basis. Simplified Normal Equations in the case of orthogonal basis vectors, or of orthonormal basis vectors. Examples. Consequences of basis orthonormalization: Normal Equations and QR factorization, conditioning of Normal Equations. Examples. Normal Equations and Singular Value Decomposition (full column rank). Comparison (w.r.t. accuracy and efficiency) of LS solutions, of an incompatibile system, computed in MATLAB with backslash operator (\), by solving Normal Equations, by means of QR factorization, by means of SVD.

[SC2_11e] More on the *Least Squares Method* for overdetermined systems. Properties of the set of LS solutions of a system. The Least Norm solution of underdetermined Normal Equations. Solve incompatible systems in MATLAB: the cases of full (col) rank matrix, and of rank deficient matrix. MATLAB Lab. on the best fit circle to noisy data. Example of Linear Least Squares method applied to a non-linear fitting problem (the wind tunnel experiment).

[SC2_11f] Best linear approximation w.r.t. 2-norm: the case of infinite dimension subspaces. Performance of the residual error as the subspace dimension increases. How to assure the convergence in 2-norm? More assumptions on the linear space need: the linear space must be a Hilbert Space, and it must be equipped with a complete orthonormal system w.r.t. the 2-norm. From Bessel's inequality to Parseval's equality. The Hilbert Space $L^2([-\pi,+\pi])$ of square summable functions over the interval, and the trigonometric (or exponential) function system. Brief notes on convergence in norm (uniform convergence and mean-square convergence). Examples. Sufficient conditions for the existence and convergence of a Fourier Series. MATLAB graphical examples.

SC2_12 – *Discrete Fourier Transform*

[SC2_15a] Definition of Discrete Fourier Transform (DFT) and of Inverse Discrete Fourier Transform (IDFT) as particular matrix-vector products (scalar and matrix forms). First properties of the DFT matrix. DFT (`fft`) and IDFT (`ifft`) in MATLAB. The Fast Fourier Transform algorithm ad its idea as a "Divide and Conquer Algorithm". Main properties of the DFT and MATLAB examples. Applications of the convolution property.

SC2_13 – *Fourier Series*

[SC2_13a] MATLAB examples about summable, non-summable and square summable functions. Examples on partial sums of Fourier Series of summable, square summable and continuous functions. Definition of the Fourier Series of $f(x)$ in $[-\pi, +\pi]$ with period 2π: real and complex form. Fourier coefficients. Angular and circular frequencies. N^{th} degree (N must be even) trigonometric polynomials: real and complex form. Connection between trigonometric and algebraic polynomials. Evaluate a trigonometric polynomial by means of the MATLAB `polyval()` function. Example on the behavior of a Fourier Series of a non-periodic function. Fourier Series in other intervals. Sufficient conditions for the existence, the convergence in quadrati mean and the pointwise (or uniform) convergence of a Fourier Series. Examples of Fourier Series. Gibbs phenomenon of partial sums of a Fourier Series. The Dirac delta function and its Fourier Series (comb function).

[SC2_13b] Properties of Fourier coefficients and examples. Applications of Time Shifting property, of Differentiation property, and of Convolution property. Example of numerical approximation of Fourier coefficients by numerical quadrature (wrong algorithm). Definition of the Discrete Fourier Transform (DFT) and of the Inverse Discrete Fourier Transform (IDFT): matrix form and scalar form.

[SC2_13c] Numerical approximation of Fourier coefficients in $[-T/2, +T/2]$: wrong algorithm by means of quadrature, and right algorithm by means of DFT. Derivation of the right algorithm and its computational steps. Examples of numerical Fourier coefficients and of partial sums of Fourier Series. Fourier Synthesis using Fourier Series: algorithm steps. Examples. A first application to signal filtering. Examples of Windowing and Aliasing errors.

SC2_14 – *Fourier Transform*

[SC2_16a] The idea behind the Fourier Transform. Definition of the Fourier Transform (FT) of $f(x)$. Sufficient condition for the existence of Fourier Transform and Inverse Fourier Transform (IFT) written w.r.t. the angular frequency and the circular frequency. The MATLAB symbolic `fourier()` and `ifourier()` functions. Examples of FT: of a gaussian, of signum, of rect pulse, of triangle pulse, of even decay function, of decay pulse, of trigonometric functions, of Dirac delta, of the constant 1 function, of Heaviside function. The Heaviside (or unit step) function, its connection to Dirac delta function, and its application to represent a signal that switches on at a specified time and stays switched on indefinitely.

[SC2_16b] Fourier Transform of a comb function (or pulse train). In MATLAB Signal Processing Toolbox `pulstran()` function. **Superposition** of functions. Example in a finite case. *Fourier Spectrum* and *Power Spectrum*. Main properties of the Fourier Transform. Application of the convolution property to the rect pulse function and its double and triple convolution (MATLAB Symbolic Math Toolbox Lab.); application of the convolution property to the rect pulse function and its double and triple convolution (numerical approximation algorithm by means of DFT and IDFT) to reduce the noise.

[SC2_16c] Windowing and aliasing effects. Examples. The Nyquist-Shannon Sampling Theorem. The Nyquist frequency. Example of aliasing.

[SC2_16d] Connection between the Fourier Transform and Fourier coefficients for time-limited functions. Algorithm for the numerical approximation of Fourier Transform samples. MATLAB examples of FT and IFT. Applications: low-pass filter, study of the periodicity of equispaced data, dual tone multi-frequency keypad. The Short Time Fourier Transform (STFT) to detect a phone number from a multi-frequency signaling in an audio file.

[SC2_16e] STFT in MATLAB: the `spectrogram()` function (in Signal Processing Toolbox). Application of STFT to sounds: detect musical notes in audio signals.